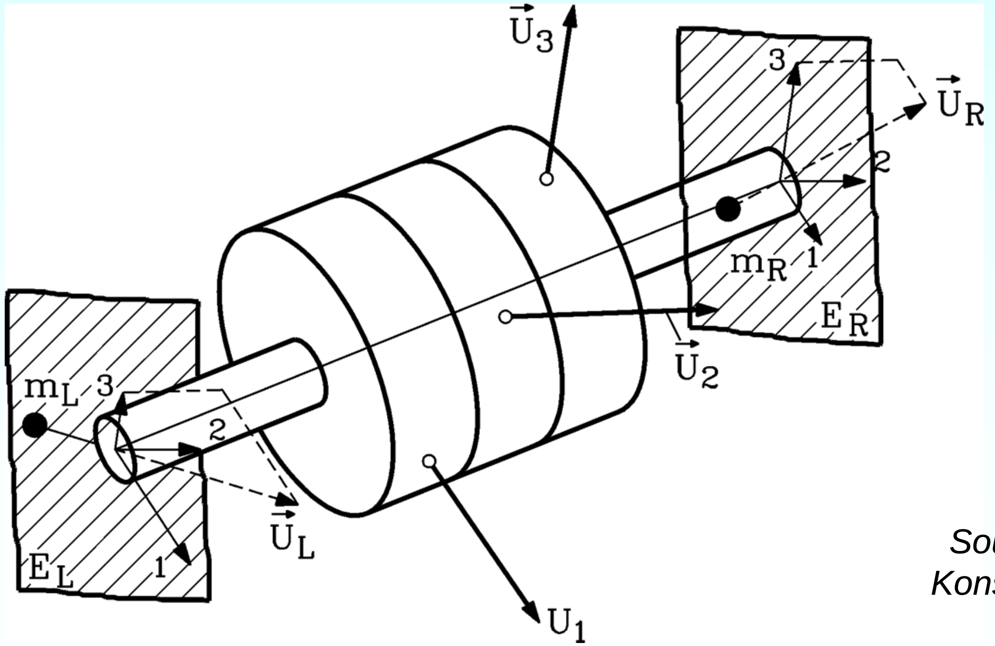
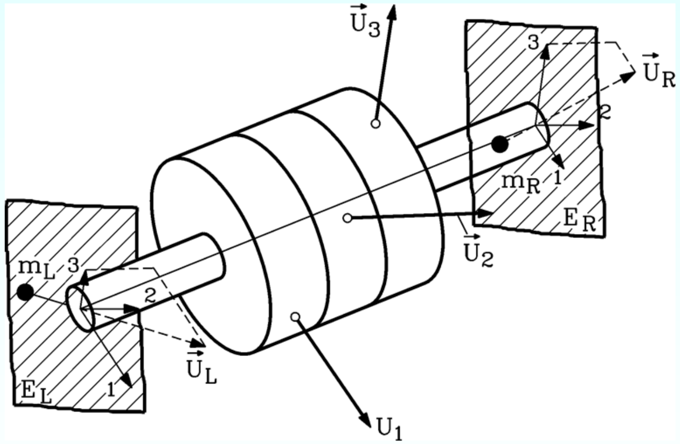
7. Mechanical motor design



Source: Wiedemann-Kellenberger, Konstruktion elektrischer Maschinen, Springer, 1968

7. Mechanical motor design

7.1 Rotor balancing



Source: Wiedemann-Kellenberger, Konstruktion elektrischer Maschinen, Springer, 1968

Rotor imbalance

- Rotor mass centre of gravity NOT located on the rotational axis
- Dislocated by a displacement e_S.
- **Centrifugal force** *F* of rotor mass m_r: $F = m_r \cdot e_S \cdot \Omega_m^2$
- Direction of force *F* rotates with rotational speed.
- It may excite mechanical vibrations with frequency $f = n$, which – when hitting natural vibration frequency of motor system – causes resonance.
- **Imbalance** may lead to

- increased bearing stress,
- additional rotor loading and
- increased machine vibrations.

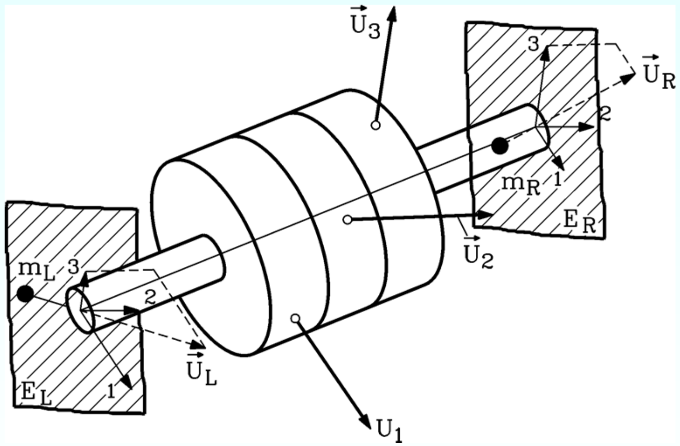
Mathematical models for rotor balancing

rotor model	***rotor***	***bearings***
rigid rotor model	**rotor body shows no deformation under force load (= geometry does not change shape under force load)**	**rigid bearings show no deformation under force load**
elastic rotor model	**rotor body is deformed under force load (= *Young*´s modulus of elasticity *E* of rotor material is not infinite)**	**rigid bearings**
elastic bearing model	**elastic rotor body**	**bearing geometry is deformed under force load (deformation is ruled by *Young*´s modulus of elasticity *E* of bearing material and end shields)**

- Different mathematical models for rotor system (= rotor body and bearings)
- If **maximum speed n_{max} below** natural bending frequency f_{b1}, elastic rotor bending is negligible: rigid rotor model applies !

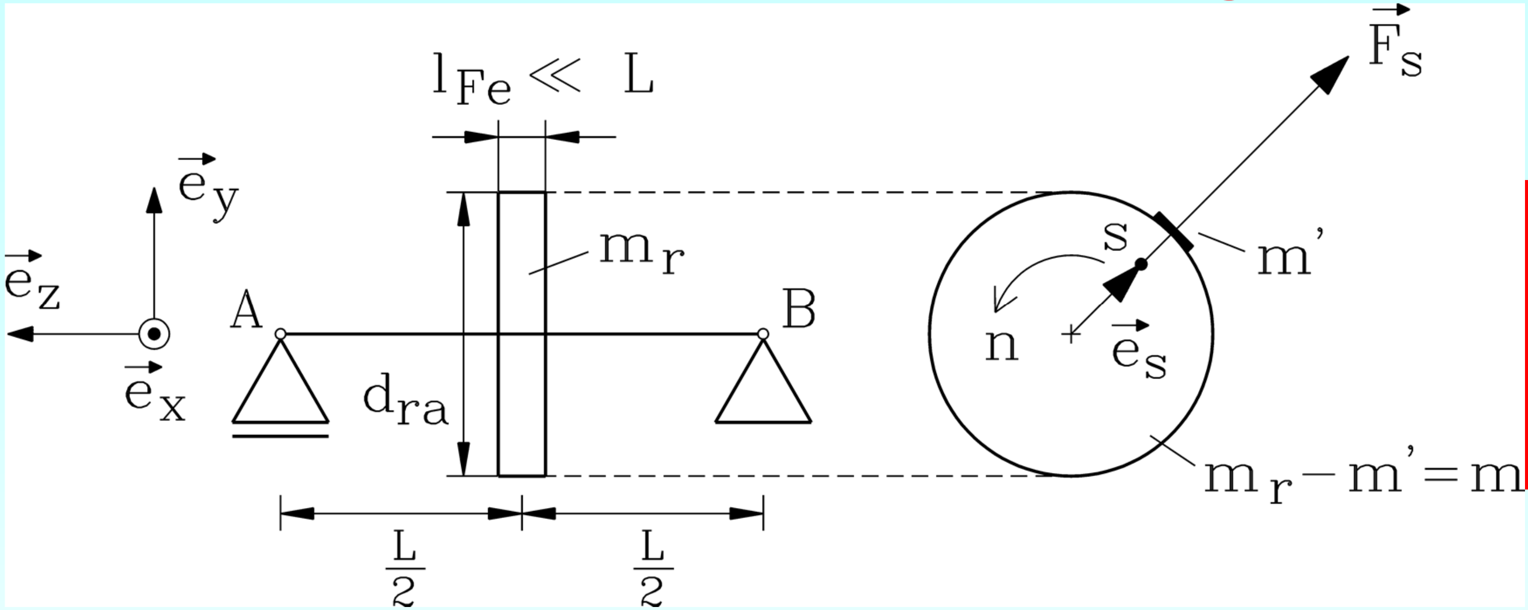
7. Mechanical motor design

7.1.1 Rigid rotor balancing



Source: Wiedemann-Kellenberger, Konstruktion elektrischer Maschinen, Springer, 1968

Static imbalance of rigid rotor bodies



$$\vec{F}_S = m_r \cdot \vec{e}_S \cdot \Omega_m^2 \quad \Rightarrow$$

$$\vec{U}_S = \frac{\vec{F}_S}{\Omega_m^2} = m_r \cdot \vec{e}_S$$

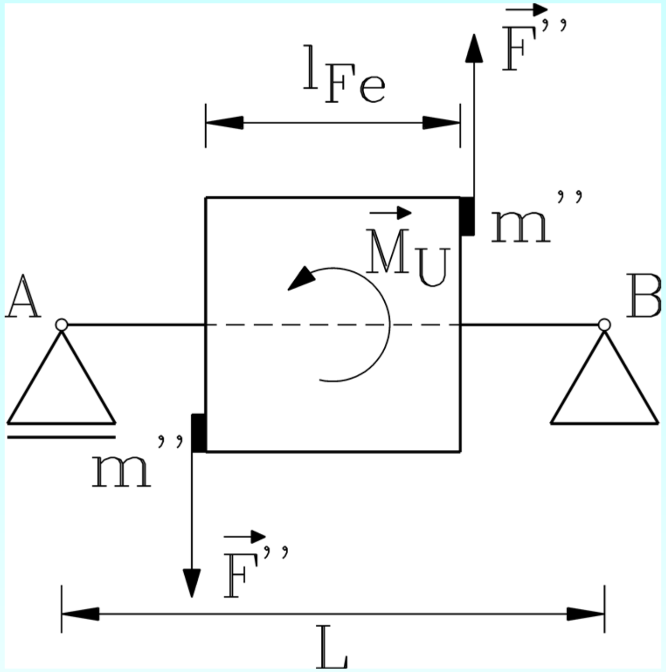
Rigid body disc rotor on stiff shaft with centre of gravity *S* dislocated from rotational axis by distance e_s (left: loose bearing A, right: fixed bearing B).

Static imbalance: The maximum torque exerted by gravity on the disc at stand still occurs at disc position where additional mass is in horizontal plane:

$$M = m' \cdot g \cdot d_{ra} / 2 = m_r \cdot g \cdot e_S \qquad \Rightarrow \qquad e_S = (m' / m_r) \cdot (d_{ra} / 2)$$

Imbalance can be detected at stand still ("static").

Dynamic imbalance M_U



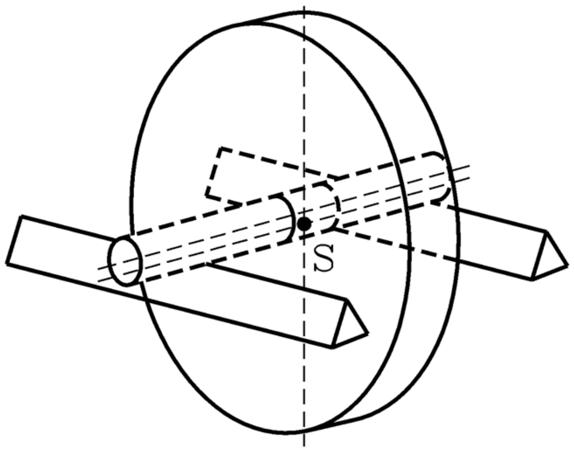
$$M'' = m'' \cdot (d_{ra}/2) \cdot \Omega_m^2 \cdot l_{Fe} \quad \Rightarrow$$
$$M_U = \frac{M''}{\Omega_m^2} = \frac{m'' \cdot d_{ra} \cdot l_{Fe}}{2}$$

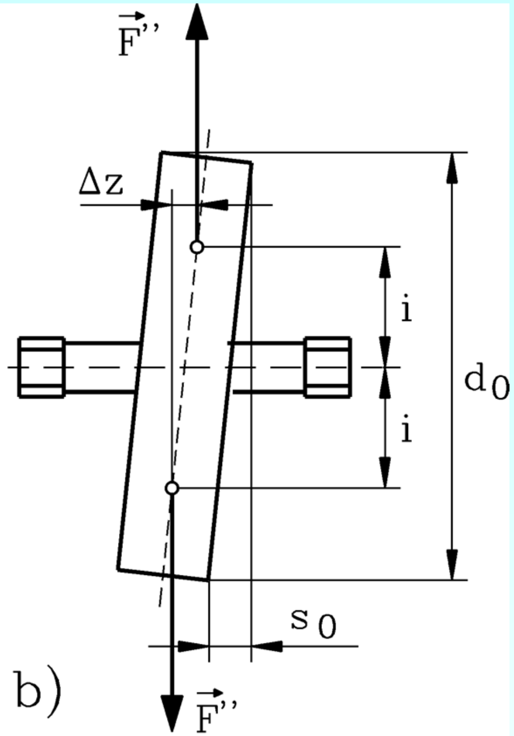
Rigid body cylindrical rotor with centre of gravity *S* located on rotational axis, but uneven distributed mass along rotor axis, represented here by two masses *m*'' , which lead to imbalance torque *M*'', when rotor is rotating.

*Due to **static imbalance** additional bearing force is **IN PHASE** in both bearings (common mode force), oscillating with rotational frequency.*
***Dynamic imbalance** M_U leads to additional oscillating bearing forces with opposite sign in both bearings (= 180° PHASE SHIFT) = differential mode.*

Examples for static and dynamic imbalance

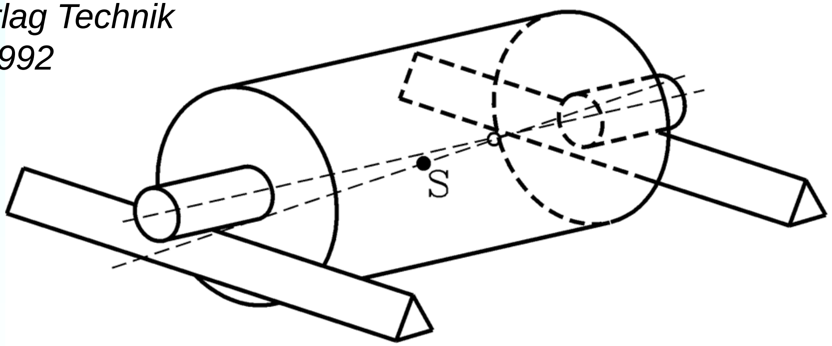
Static imbalance





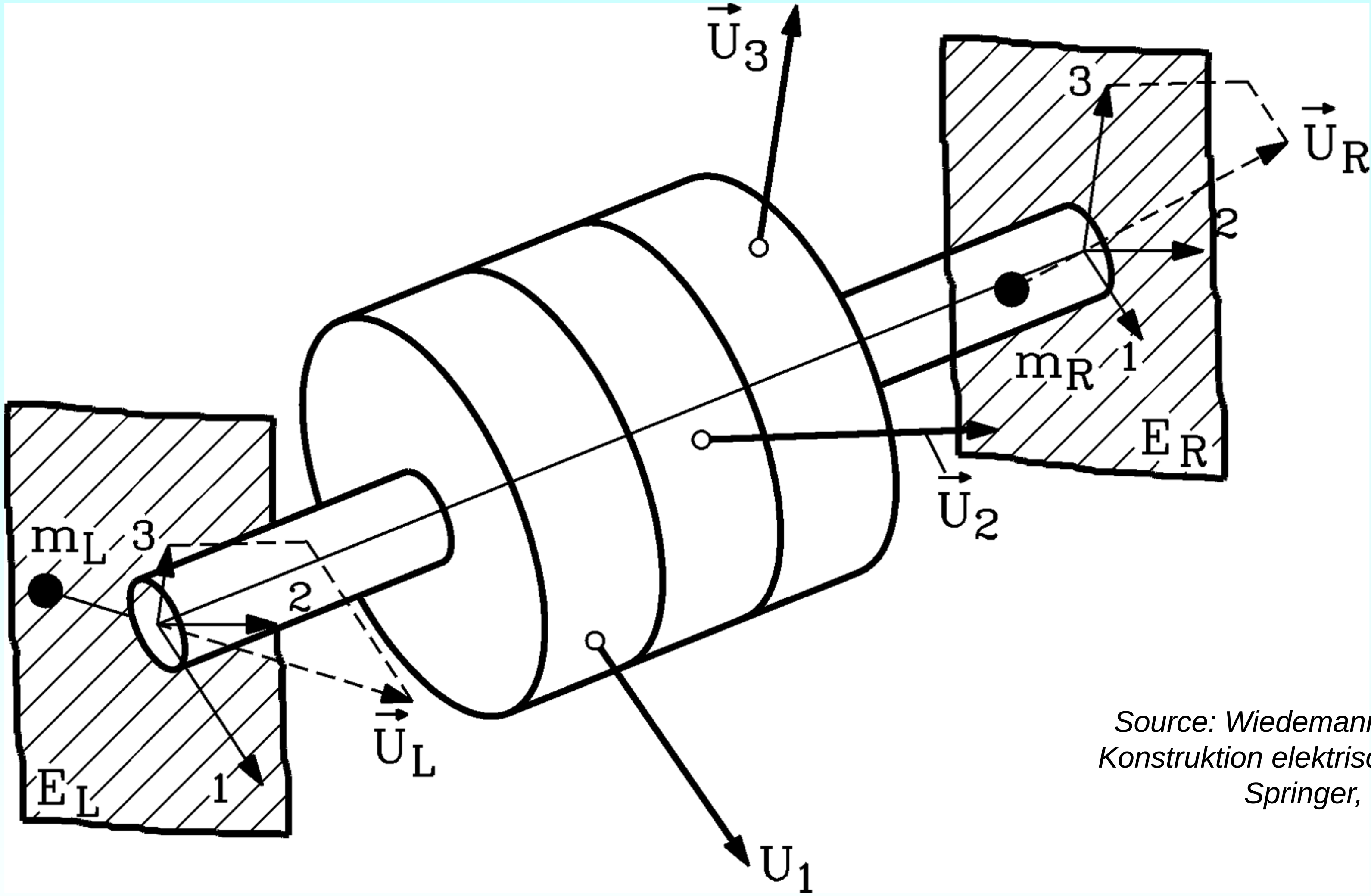
Dynamic imbalance

Source: Lingener, Auswuchten - Theorie und Praxis, Verlag Technik GmbH Berlin, 1992



Mixed static and dynamic imbalance

“Real” imbalance distribution inside a rotor



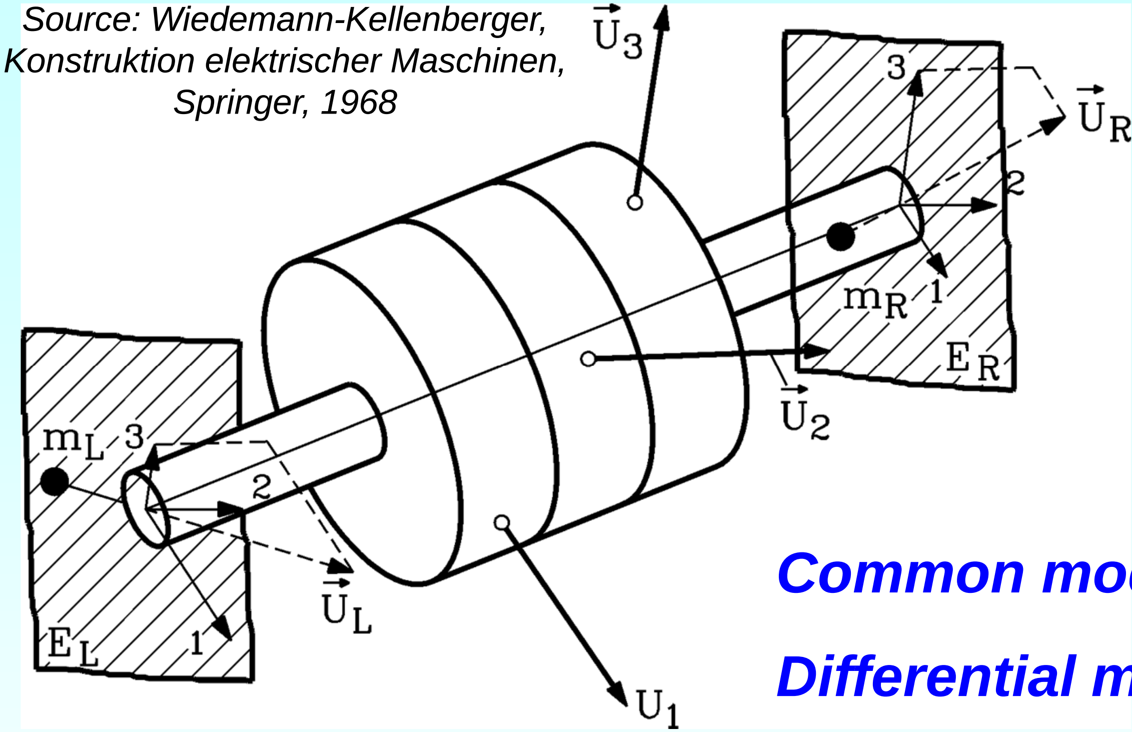
Source: Wiedemann-Kellenberger, Konstruktion elektrischer Maschinen, Springer, 1968

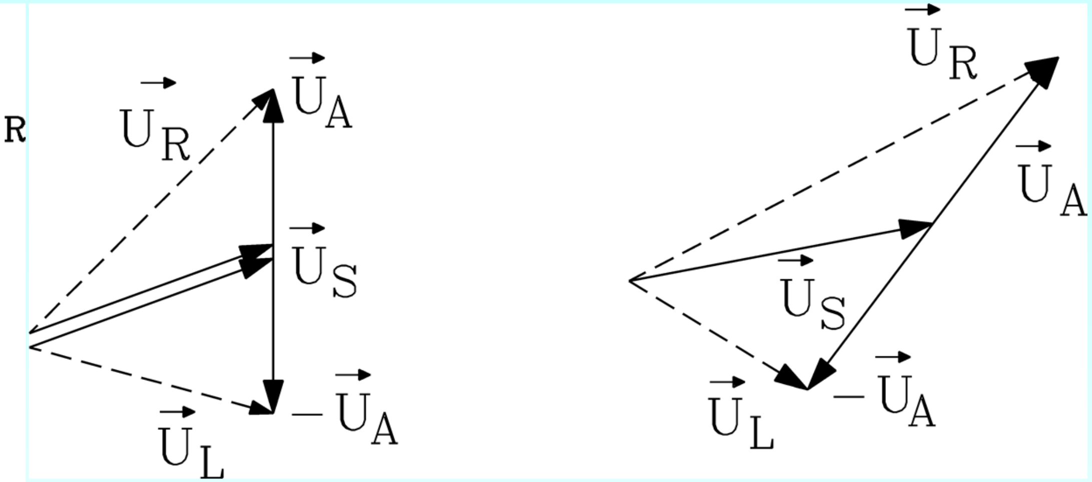
- Rotor is considered as sequence of narrow discs (here: 3 discs)

- Each disc has a static imbalance *U* (no dynamic imbalance, as discs are narrow)

Imbalance forces in two parallel planes

Source: Wiedemann-Kellenberger, Konstruktion elektrischer Maschinen, Springer, 1968





Common mode = symmetric component: F_S.

Differential mode = anti-symmetric component: F_A.

- ***Imbalance bearing force in left and right bearing are not directed in the same direction, but in arbitrary one.***

- ***But each bearing force may always be decomposed into a sum of common and a differential mode component: F_S, F_A.***

- ***So rigid rotor imbalance is a superposition of a static and dynamic imbalance.***

Need for rotor balancing

Static imbalance U_S is the proportional coefficient between square of angular mechanical frequency and centrifugal force. It is independent of speed.

Example:

Rotor mass m_r = 60 kg, rotor outer diameter d_{ra} = 200 mm, m' = 20 g,

$$e_S = (m'/m_r)\cdot(d_{ra}/2) = (20/60000)\cdot 0.1 = \underline{\underline{33.3}}\mu\text{m}$$

$$U_S = m_r \cdot e_S = 60\cdot 33.3\cdot 10^{-6} = \underline{\underline{2000}}\ \text{g·mm}$$

Centrifugal force at n = 2000/min:

$$F_S = m_r \cdot e_S \cdot \Omega_m^2 = 60\cdot 33.3\cdot 10^{-6}\cdot(2\pi\cdot 2000/60)^2 = \underline{\underline{88}}\ \text{N}$$

Compare:

Gravity force of rotor is $m_r \cdot g = 60\cdot 9.81 = 589$ N.

At 4000/min centrifugal force is already 60% of gravity force.

A balancing process is always necessary.

Balancing equation for rigid rotor bodies

Measured imbalance bearing forces: $F_L = U_L \Omega_m^2,\ F_R = U_R \Omega_m^2$

(here: For simplification: Imbalance forces are assumed to be oriented in same plane)

Balancing:

Fixing two balancing masses m_1, m_2 at the radii r_1, r_2 in two balancing planes EL and ER ($\vec{U}_1 = m_1 \vec{r}_1, \vec{U}_2 = m_2 \vec{r}_2$)

Their additional centrifugal forces $\vec{F}_1 = \vec{U}_1 \Omega_m^2,\ \vec{F}_2 = \vec{U}_2 \Omega_m^2$ must compensate in the bearings the imbalance bearing forces $\vec{F}_L, \vec{F}_R$.

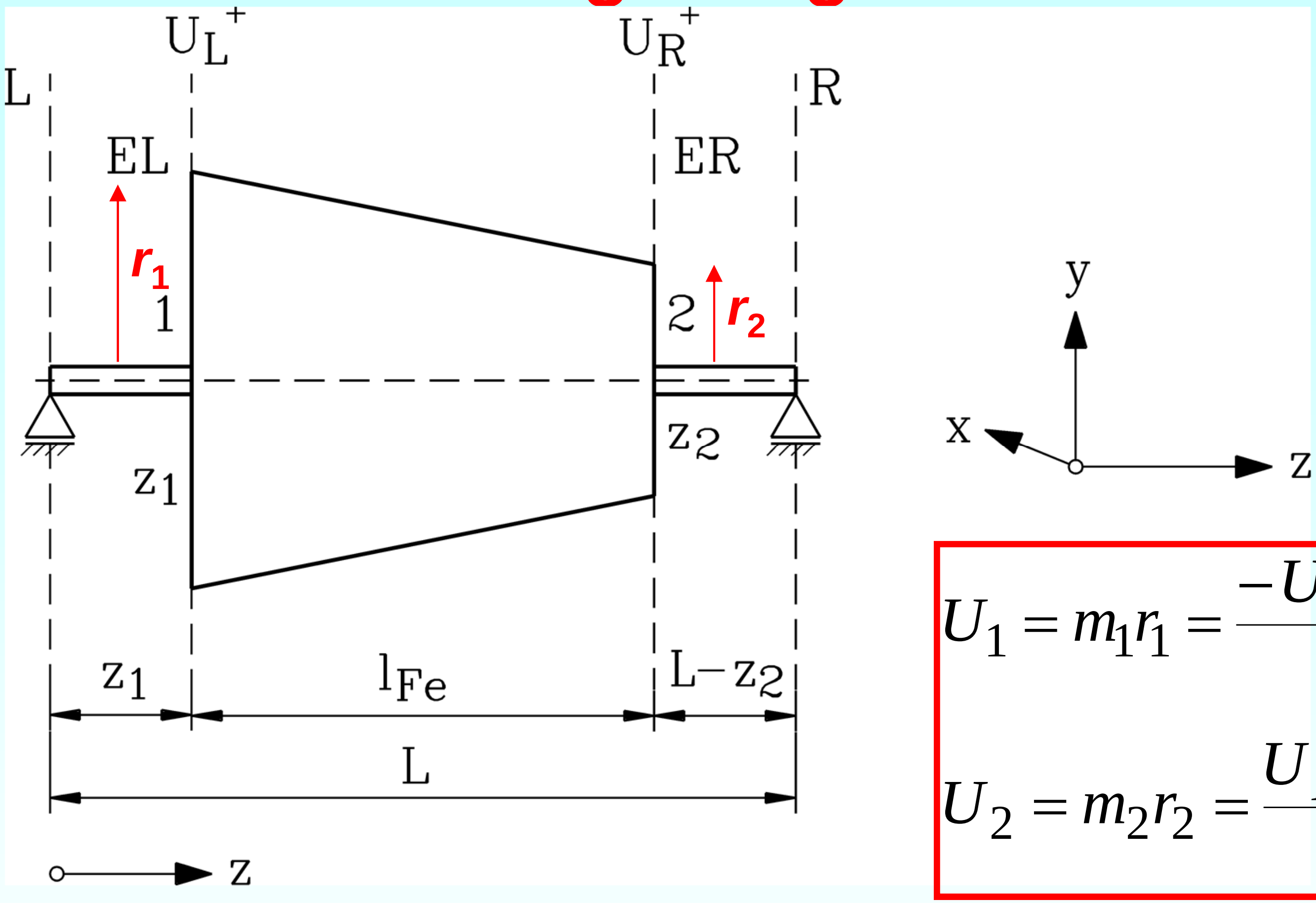
Axial co-ordinates of balancing planes z_1, z_2 & bearing distance L:

Determination of balancing masses:

$$U_1 = m_1 r_1 = \frac{-U_L + U_R \cdot (L / z_2 - 1)}{1 - z_1 / z_2}$$

$$U_2 = m_2 r_2 = \frac{U_L \cdot (z_1 / z_2) - U_R \cdot (L / z_2 - z_1 / z_2)}{1 - z_1 / z_2}$$

Balancing of rigid rotor bodies in two planes



Source: Lingener, Auswuchten - Theorie und Praxis, Verlag Technik GmbH Berlin, 1992

$$U_1 = m_1 r_1 = \frac{-U_L + U_R \cdot (L/z_2 - 1)}{1 - z_1/z_2}$$

$$U_2 = m_2 r_2 = \frac{U_L \cdot (z_1/z_2) - U_R \cdot (L/z_2 - z_1/z_2)}{1 - z_1/z_2}$$

-Balancing planes EL, ER at Axial co-ordinates z_1, z_2

- Determination of balancing masses m_1, m_2 from force and torque equilibrium !

Rotor balancing masses

Example:

Motor 75 kW, 1500/min, rotor mass m_r = 60 kg, rotor outer diameter 200 mm

balancing in two planes at $L/z_2 = 3/2,\ z_1/z_2 = 1/2$

balancing radii: $r_1 = r_2 = d_{ra}/2 = 100\,\text{mm}.$

Measured imbalance bearing forces at 500 /min: $F_L = 1.6\,\text{N},\ F_R = 1.0\,\text{N}.$

(We assume force direction in both bearings to be the same).

Needed balancing masses m_1, m_2 to compensate completely rotor imbalance:

$$U_L = F_L/\Omega_m^2 = 1.6/(2\pi(500/60))^2 = 583.6\,\text{g·mm}$$

$$U_R = F_R/\Omega_m^2 = 1.0/(2\pi(500/60))^2 = 364.8\,\text{g·mm}$$

$$m_1 = \frac{-U_L + U_R\cdot(L/z_2 - 1)}{1 - z_1/z_2}\cdot\frac{1}{r_1} = \frac{-583.6 + 364.8\cdot(3/2-1)}{1-1/2}\cdot\frac{1}{100} = \underline{\underline{-8g}}$$

$$m_2 = \frac{U_L\cdot(z_1/z_2) - U_R\cdot(L/z_2 - z_1/z_2)}{1 - z_1/z_2}\cdot\frac{1}{r_2} = \frac{583.6\cdot(1/2) - 364.8\cdot(3/2-1/2)}{1-1/2}\cdot\frac{1}{100} = \underline{\underline{-1.46g}}$$

Balancing masses are 8 g and 1.5 g, which – due to negative sign – must be fixed opposite to direction of measured bearing forces, or this amount of mass must be removed from the rotor.

Limits for residual imbalance (ISO 1940)

	$\Omega_m \cdot e_S$	***Examples***
	mm/s	
G 4000	4000	Slow turning big *Diesel* engines for ships
G 1600	1600	Big two stroke combustion engines
G 630	630	Big four stroke combustion engines
G 250	250	Fast turning four stroke piston engines
G 100	100	Combustion engines for cars and locomotives
G 40	40	Wheel sets for cars
G 16	16	Cardan transmission shafts
G 6.3	6.3	Fans, pump rotors, standard electric motor rotors
G 2.5	2.5	Rotors of steam and gas turbines, big electric generators, high speed electric motors, turbo prop for air craft
G 1	1	Ultra high speed small motors, grinding spindle drives
G 0.4	0.4	Gyroscopic rotors, special high speed grinding spindle drives

Residual imbalance

- Residual imbalance must stay below limits, defined – depending on the purpose of the rotor – by standard ISO 1940.

- Residual imbalance is given as **circumference speed of centre of gravity**

$$G = \Omega_m \cdot e_S$$

In ideal case: centre of gravity is on rotational axis, so $G = \Omega_m \cdot e_S = \Omega_m \cdot 0 = 0$!

Example:

Limit of residual imbalance (ISO 1940) for:
Electric motor: 2000/min, 100 kW, rotor mass 100 kg.
Table 7.1.3-1 gives:

$$G = 2.5 mm/s \quad \Rightarrow \quad e = G/\Omega_m = 0.0025/(2\pi \cdot 2000/60) = 11.9 \mu\text{m}$$

Residual imbalance:

$$U = 11.9 \cdot 10^{-6} \cdot 100 = \underline{\underline{1194}} g \cdot mm$$

Methods for balancing

• **Special fast-hardening cement as balancing masses:** Wound rotors of small to medium sized DC motors, universal motors, wound rotor induction machines.

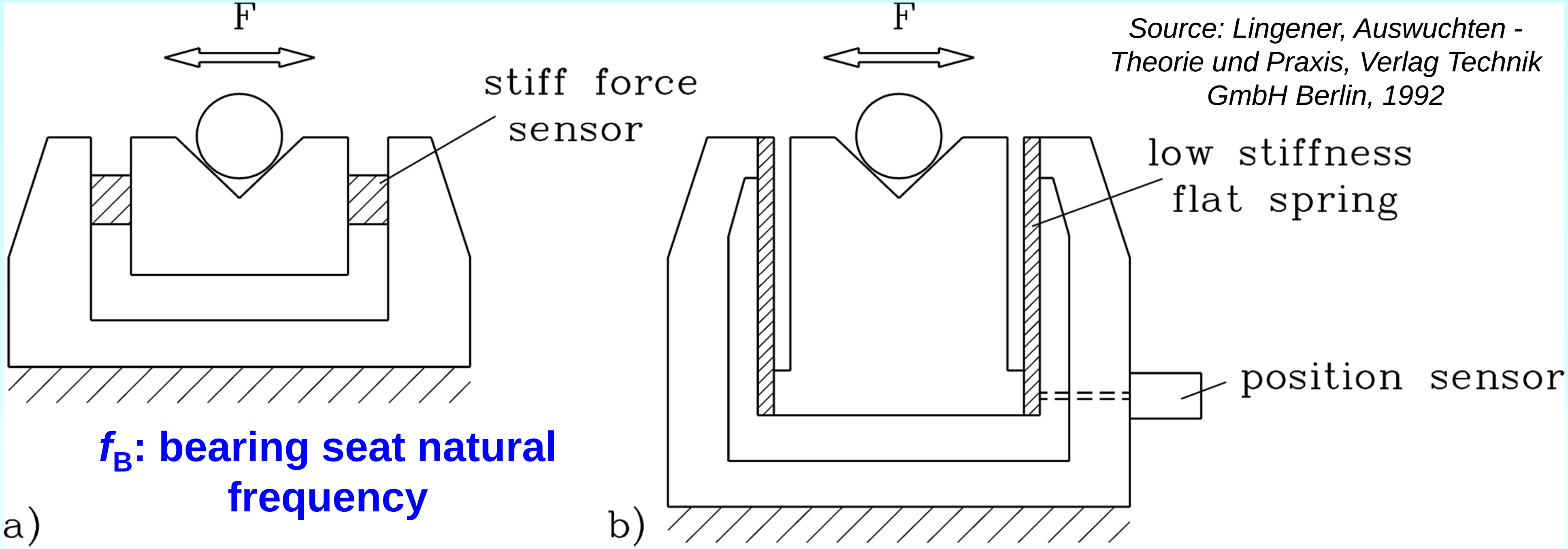
• **Cylindrical noses integrated into the aluminium end rings of the cage:** For fixing rings as balancing masses: Cage induction rotors

• **Negative mass balancing:** Some cage mass is cut (e.g. in the rings). For high speed machines especially, as noses might cause additional air friction.

• **Two discs on the rotor, where the balancing masses are fixed:** Bigger machines.

Source: Breuer-Motoren, Germany

Measuring bearing imbalance forces



bearing force measurement

bearing position measurement

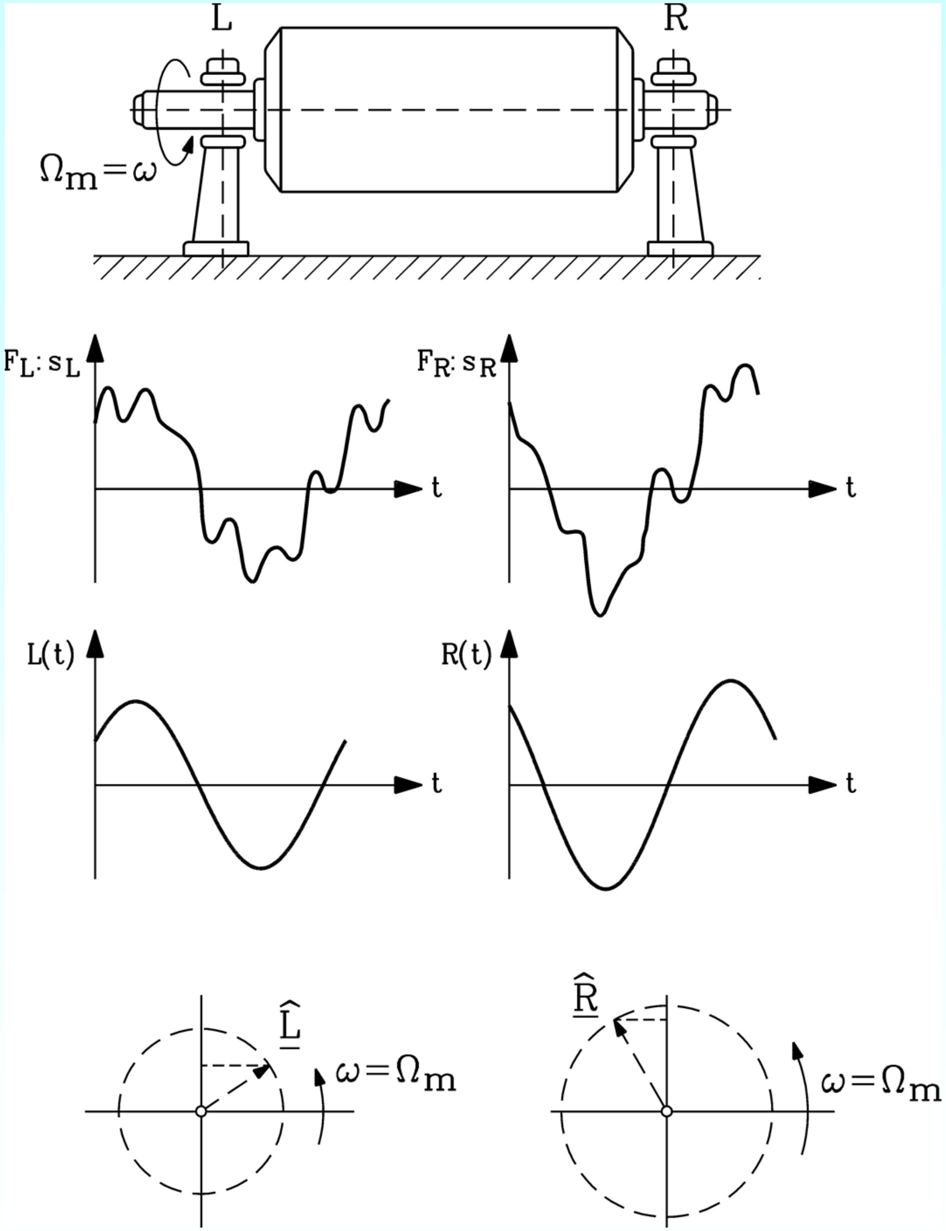
$$\underline{n << f_B:} \quad \hat{X} \approx \frac{\hat{F}}{c_B} \quad \Rightarrow \quad \hat{F} = \hat{X} \cdot c_B = U \cdot \Omega_m^2$$

Stiff bearing seat: **direct force measurement *F* leads to imbalance *U***

$$\underline{n >> f_B:} \quad \hat{X} \approx \frac{\hat{F}}{-\Omega_m^2 \cdot m_r} = -\frac{U}{m_r}$$

Soft bearing seat: **direct position measurement *X* leads to imbalance *U***

Measuring imbalance component of bearing forces



Left and right bearing force pick-up system for measurement of bearing force and imbalance signal

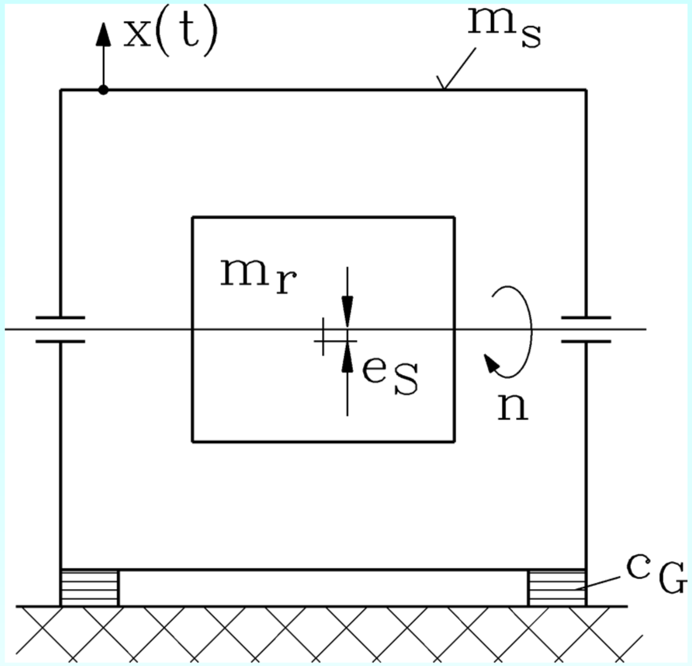
Source: Lingener, Auswuchten - Theorie und Praxis, Verlag Technik GmbH Berlin, 1992

Force and imbalance signal contains also noise or harmonics

As unbalance force direction varies with rotational speed, a *FOURIER* analysis is used to filter the unbalance component

Phase angle and amplitude of unbalance force are used for calculating unbalance phasor

Balancing of complete motor system



Imbalance (e.g. static imbalance):

$$F_S = m_r \cdot e_S \cdot \Omega_m^2 \quad \Rightarrow \quad F_x(t) = F_S \cdot \cos(\Omega_m t)$$

Exciting vibration of whole motor mass in *x*-direction:

$$(m_s + m_r) \cdot \ddot{x} + c_G \cdot x = F_S \cdot \cos(\Omega_m t)$$

Resonance of elastic pads: frequency f_G is much lower than rated speed.

$$x(t) = \hat{X} \cdot \cos(\Omega_m t) \quad , \quad \hat{X} = \frac{F_S}{c_G - \Omega_m^2 \cdot m_{mot}} \quad , \quad f_G = \frac{1}{2\pi} \cdot \sqrt{\frac{c_G}{m_{mot}}}$$

$$\underline{n >> f_G:} \quad \hat{X} \approx \frac{F_S}{-\Omega_m^2 \cdot m_{mot}} = -\frac{U_S}{m_{mot}}$$

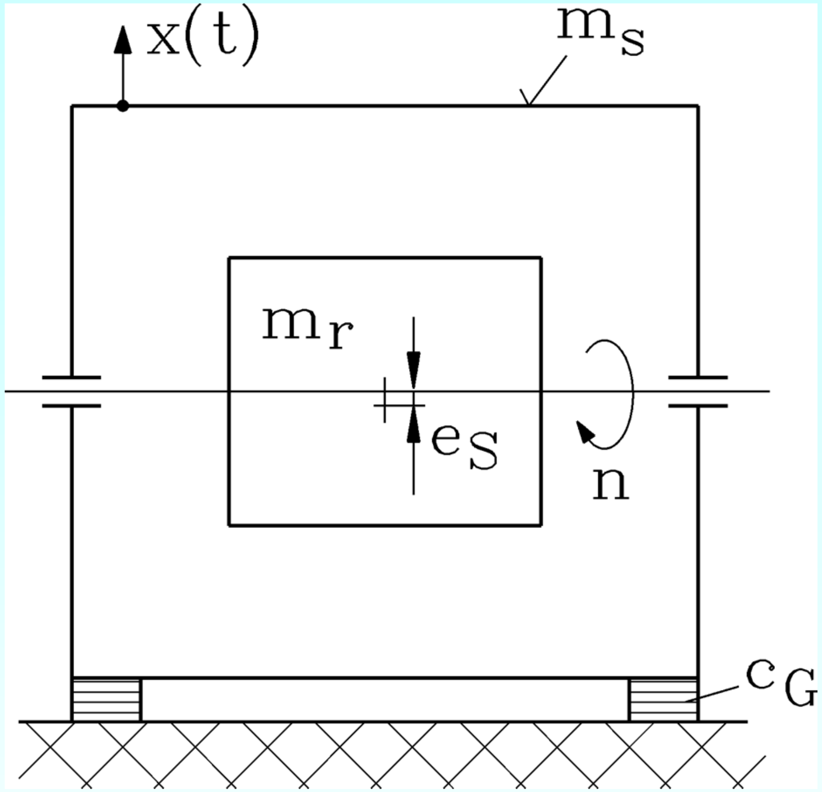
Vibration velocity rises linear with speed, its increase is directly proportional to imbalance.

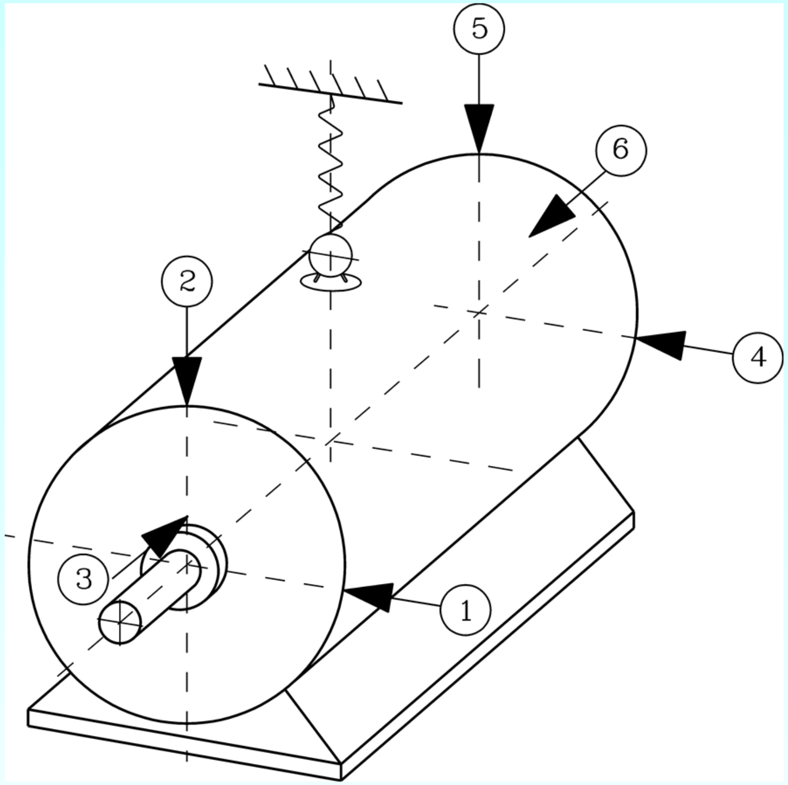
Vibration velocity:

$$v(t) = \dot{x}(t) = \frac{U_S}{m_{mot}} \cdot \Omega_m \cdot \sin(\Omega_m t) \quad \Rightarrow \quad \hat{v} = \frac{m_r}{m_s + m_r} \cdot e_S \cdot \Omega_m$$

Thus elastic mounting of motor allows access of status of imbalance of complete motor.

Alternative for measuring vibration of complete motor





- Motor on elastic pads with low spring constant

- Motor is hanging in springs with low spring constant

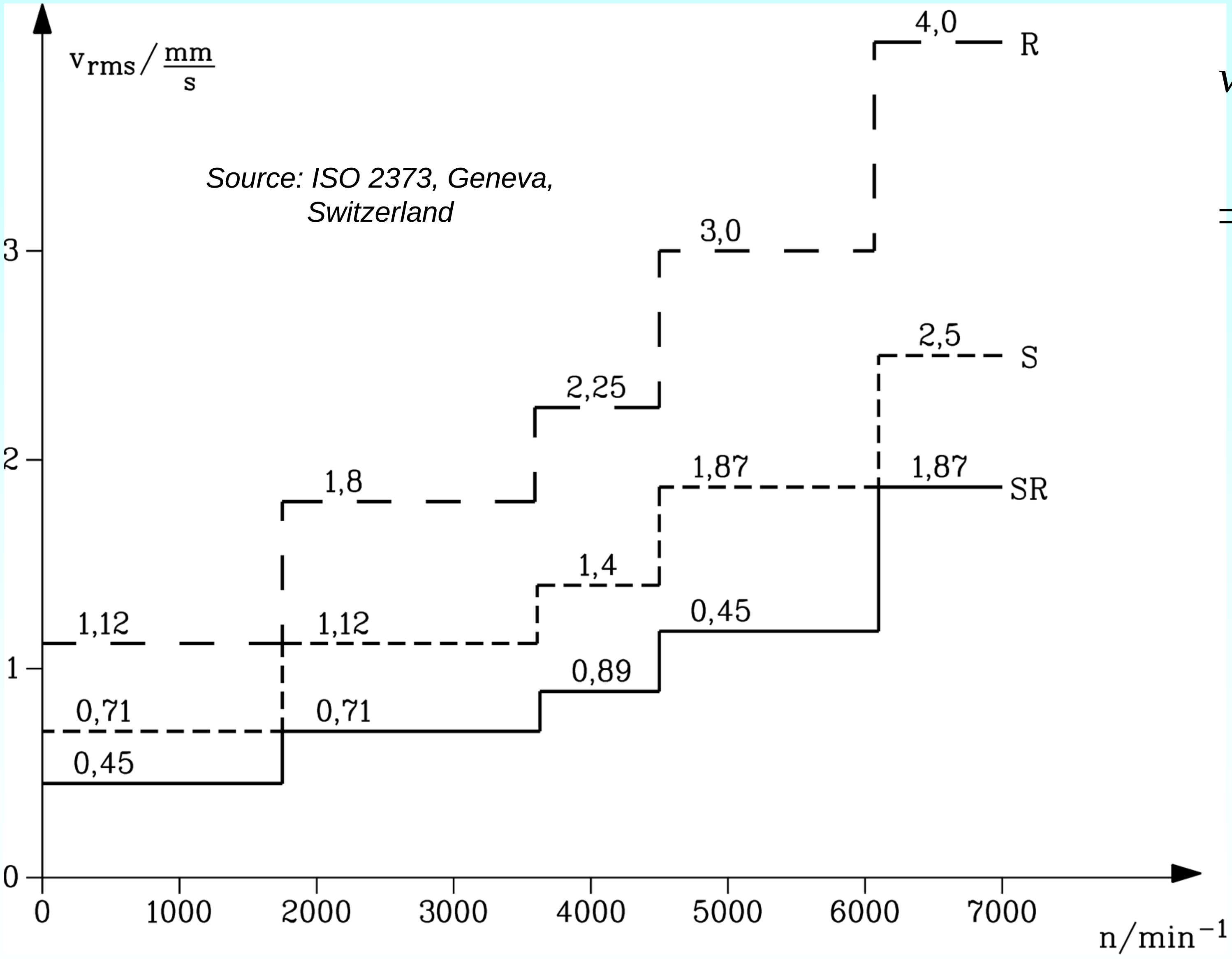
-Used for bigger motors

- Used for smaller motors

Source: Lingener, Auswuchten - Theorie und Praxis, Verlag Technik GmbH Berlin, 1992

- Vibrations of motor are decoupled from basement
- Vibrations lead directly to unbalance

Limiting curves for vibration velocity



$$v(t) = \dot{x}(t) = \frac{U_S}{m_{mot}} \cdot \Omega_m \cdot \sin(\Omega_m t)$$

$$\Rightarrow \quad \hat{v} = \frac{m_r}{m_s + m_r} \cdot e_S \cdot \Omega_m$$

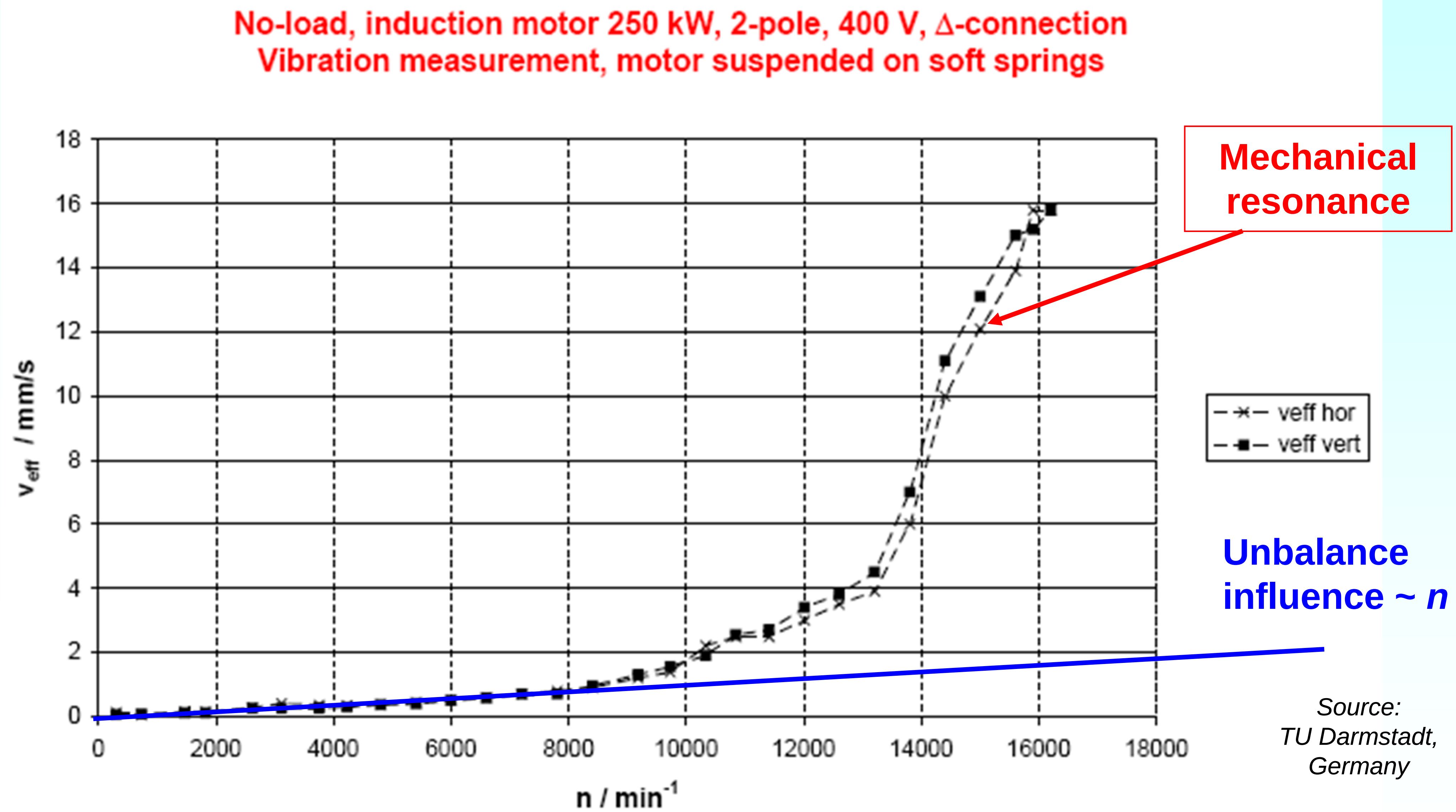
-Vibration velocity is directly proportional to unbalance

- Vibration velocity therefore increases with rotational speed

- For high quality drives low vibration levels (S or SR) are demanded

ISO2373, for motor frame size 160 mm to 180 mm

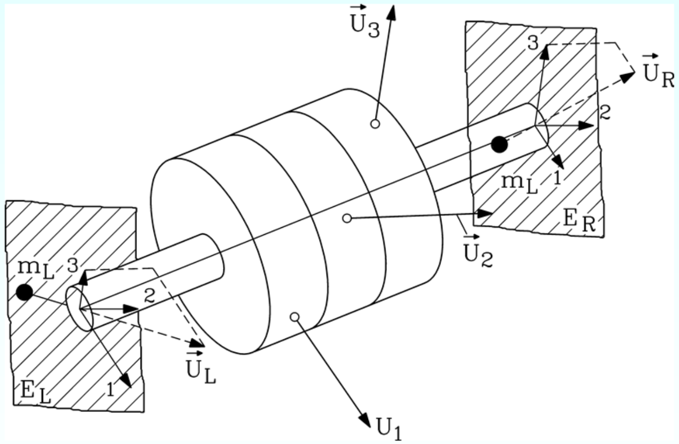
Vibration measurement of soft suspended high-speed induction motor



Source: TU Darmstadt, Germany

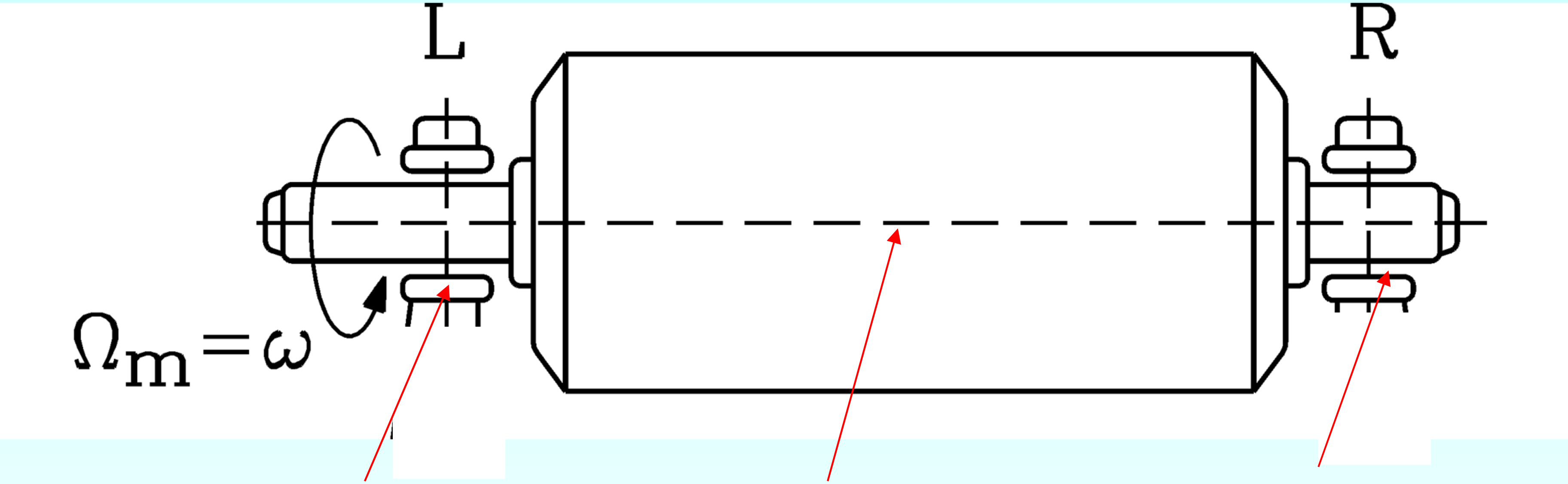
7. Mechanical motor design

7.1.1 Elastic rotor balancing



Source: Wiedemann-Kellenberger, Konstruktion elektrischer Maschinen, Springer, 1968

Elastic rotor properties



Elastic bearing seat ***elastic shaft*** ***elastic bearing seat***

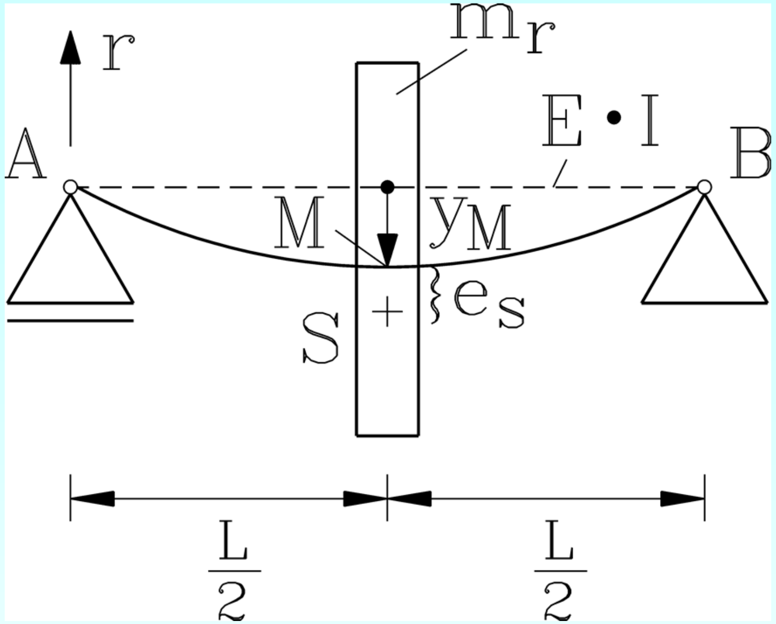
- Rotor iron stack may add to rotor bending stiffness, but main stiffness is determined by rotor shaft

Source: Lingener, Auswuchten - Theorie und Praxis, Verlag Technik GmbH Berlin, 1992

-Natural frequencies:

a) Rotor bending frequencies b) Bearing elastic frequencies

Rotor bending: Elastic rotor lumped mass model



Natural bending differential equation is

$$m_r \cdot \ddot{y} + c_{sh} \cdot y = 0$$

with **natural bending frequency**

$$f_b = \frac{1}{2\pi} \cdot \sqrt{\frac{c_{sh}}{m_r}}$$

$$f_b = \frac{1}{2\pi} \cdot \sqrt{\frac{c_{sh}}{m_r}} = \frac{1}{2\pi} \cdot \sqrt{\frac{59631953}{91.7}} = \underline{\underline{128.3}}\ \text{Hz}$$

Example:

Electric motor 75 kW at 1500/min:

Shaft length / diameter L = 0.7 m, d_{sh} = 80 mm, stack length l_{Fe} = 350 mm, outer diameter d_{ra} = 190 mm, iron mass density ρ = 7850 kg/m^3:

$$m_{sh} = \rho \cdot L \cdot d_{sh}^2 \pi / 4 = 27.6\,\text{kg},\ m_{stack} = \rho \cdot l_{Fe} \cdot (d_{ra}^2 - d_{sh}^2)\pi / 4 = 64.1\,\text{kg},$$

$$m_r = 27.6 + 64.1 = 91.7\,\text{kg},\ I = \pi \cdot d_{sh}^4 / 64 = 2.01 \cdot 10^{-6}\,\text{m}^4,\ c_{sh} = \frac{48 \cdot E \cdot I}{L^3} = 59.63 \cdot 10^6\,\text{N/m}$$

Static rotor bending due to gravity: $y_M = m_r \cdot g / c_{sh} = 15\,\mu\text{m}$

Elastic rotor shaft: Distributed mass model

- **Shaft** has to be considered as cylindrical beam of diameter d_{sh} and length L with DISTRIBUTED mass along the beam. Several natural modes of vibration exist.

$$f_{b,i} = \frac{1}{2\pi} \cdot \left(\frac{i \cdot \pi}{L}\right)^2 \cdot \sqrt{\frac{E \cdot I}{\rho \cdot A}} \quad , \quad i = 1, 2, 3, \ldots$$ Number of mode

- **Iron stack mass** increases the total mass: $f_{b,i,corr} = f_{b,i} \cdot \frac{1}{\sqrt{1 + \frac{m_{stack}}{m_{sh}}}} \quad , \quad i = 1, 2, 3, \ldots$

Example:

Electric motor 75 kW, 1500/min:

$L = 0.7\ \mathrm{m},\ d_{\mathrm{sh}} = 80\ \mathrm{mm},\ \rho = 7850\ \mathrm{kg/m^3},\ m_{sh} = 27.6\ \mathrm{kg},\ m_{stack} = 64.1\ \mathrm{kg},$

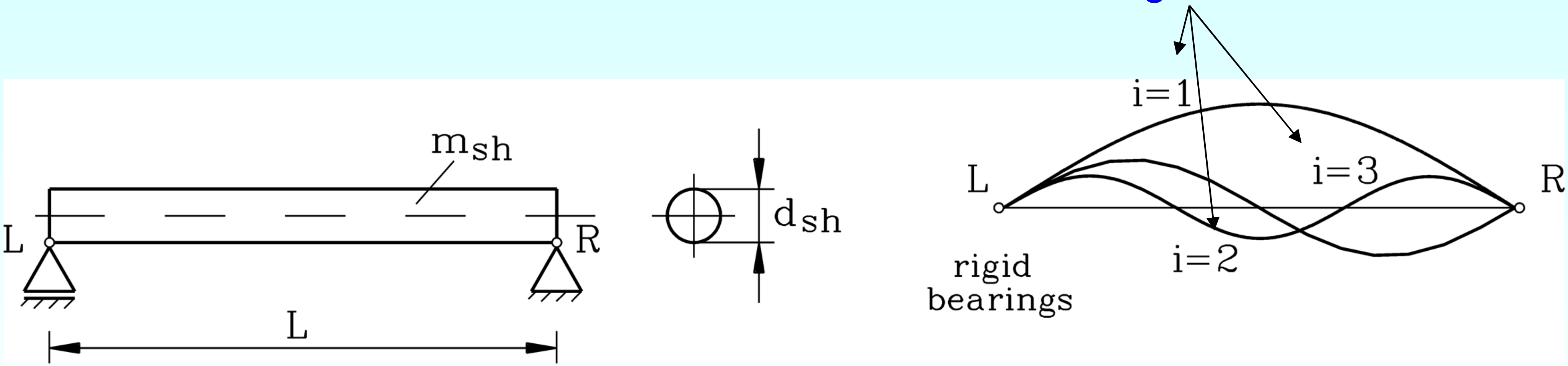
$1 + m_{stack} / m_{sh} = 1 + 64.1 / 27.6 = 3.32,\ A = \pi \cdot d_{sh}^2 / 4 = 5.03 \cdot 10^{-3}\ \mathrm{m^2}$

i	1	2	3
$f_{\mathrm{b,i,corr}}$ / Hz	182	731	1645

Elastic rotor shaft: Distributed mass model

- **Shaft** has to be considered as cylindrical beam of diameter d_{sh} and length L with DISTRIBUTED mass along the beam. Several natural modes of vibration exist.

Number of bending modes *i*



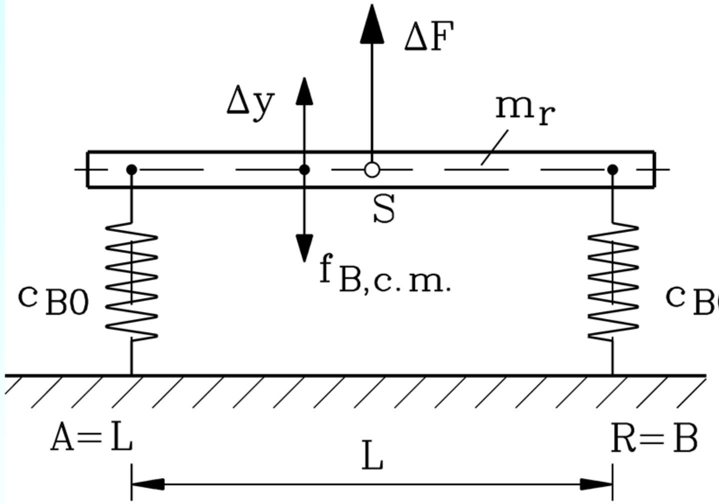
Example:

i	1	2	3
$f_{b,i,corr}$ / Hz	182	731	1645

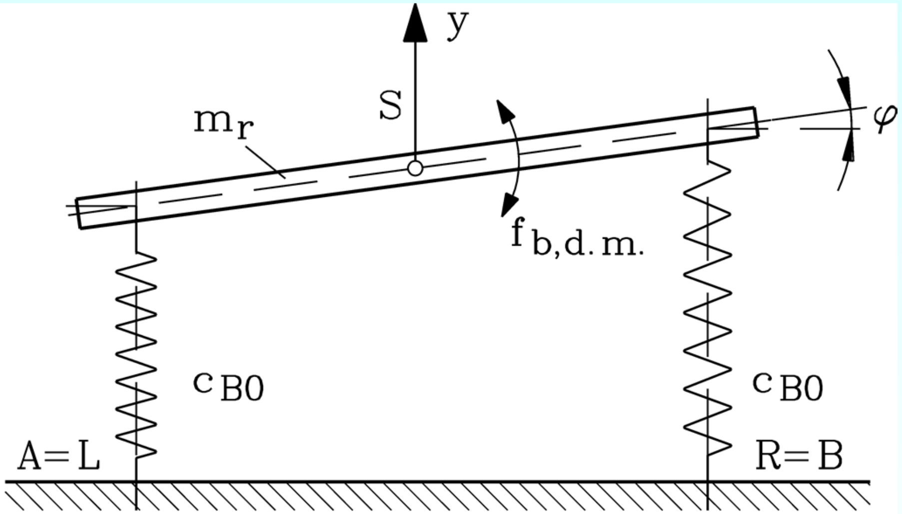
Rigid rotor vibration in elastic bearings

- **For roller bearings** bearing elastic natural frequency is much higher than rotor elastic bending frequency.

- **Magnetic bearings** have lower dynamic stiffness due to control delay. So bearing elastic frequency is lower than rotor bending frequency. So rotor is considered RIGID in elastic bearings ! Rigid body oscillations occur !



a) Common mode vibration



b) Differential mode vibration

Source: Parkus Mechanik fester Körper, Springer, Vienna

$$f_{B,c.m.} = \frac{1}{2\pi} \cdot \sqrt{\frac{c_B}{m_r}}$$

$$f_{B,d.m.} = \frac{1}{2\pi} \cdot \frac{L}{2} \cdot \sqrt{\frac{c_B}{J_x}} > f_{B,c.m.}$$

"Unbalanced magnetic pull" decreases natural bending frequency

- Unbalanced pull directed towards smallest air gap, tends to decrease air gap further.

For $2p \geq 4$: $F_M = \frac{p \cdot \tau_p \cdot l_{Fe}}{2\mu_0} \cdot B_\delta^2 \cdot \frac{e}{\delta}$ Two-pole machines: only 50%.

- Unbalanced pull may be regarded as "negative" spring constant: $F_M = -c_M \cdot y$

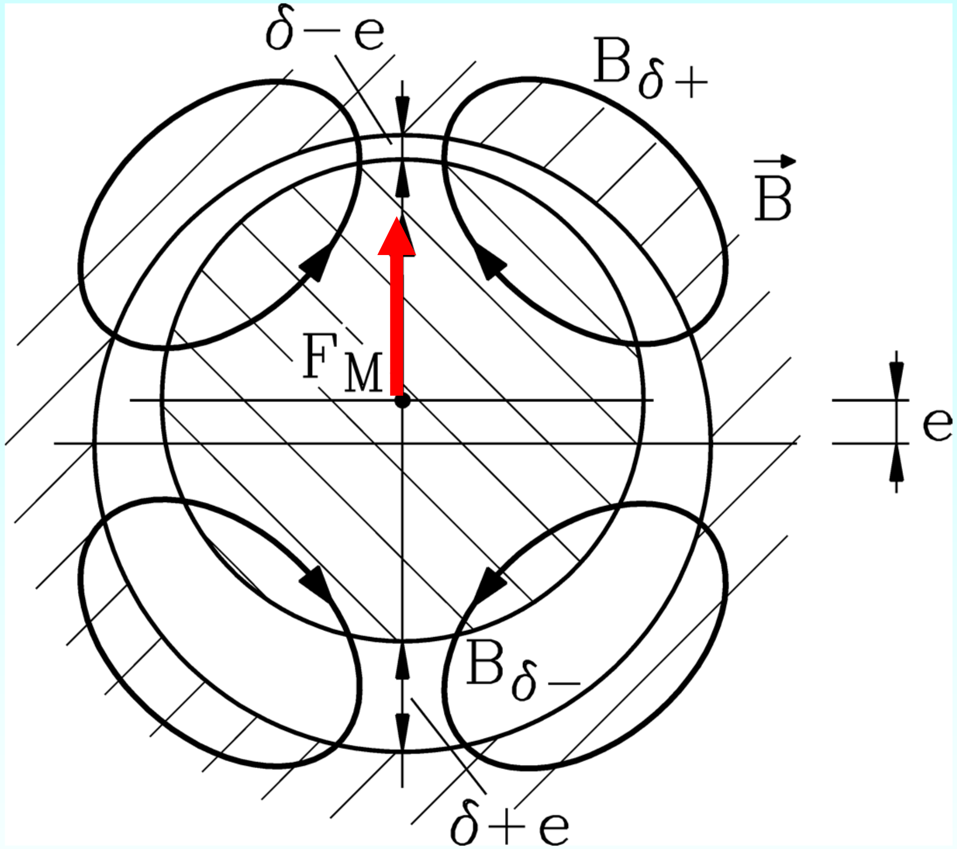
$$c_M = \frac{p \cdot \tau_p \cdot l_{Fe}}{2\mu_0} \cdot B_\delta^2 \cdot \frac{1}{\delta}$$

- Leads to decrease of natural bending frequency:

$$f_{b,M} = \frac{1}{2\pi} \cdot \sqrt{\frac{c_{sh} - c_M}{m_r}} = f_b \cdot \sqrt{1 - \frac{c_M}{c_{sh}}}$$

Unbalanced magnetic pull leads to a considerable decrease of natural bending frequency by about 10% ... 20%, depending on utilization of magnetic circuit.

Unbalanced magnetic pull



- Elastic shaft tends to decrease eccentricity *e*.

- Magnetic pull tends to increase by attracting force eccentricity *e*.

-*Facit:* Magnetic pull acts as a NEGATIVE spring constant.

$$f_{b,M} = \frac{1}{2\pi} \cdot \sqrt{\frac{c_{sh} - c_M}{m_r}} = f_b \cdot \sqrt{1 - \frac{c_M}{c_{sh}}}$$

Example:

Four-pole machine with dynamic eccentricity e due to rotor bending.

Eccentricity rotates with the rotor unbalance.

Example: Decrease of bending eigen-frequency

4-pole electric motor with 75 kW at 1500/min:

L = 0.7 m, d_{sh} = 80 mm, l_{Fe} = 350 mm, d_{ra} = 190 mm, $m_r = 91.7\,\mathrm{kg}$,

air gap flux density amplitude B_δ = 0.9 T, air gap δ = 1.0 mm, pole pitch: $\tau_p = 149\,\mathrm{mm}$

Rotor gravity force: $m_r \cdot g = 900\,\mathrm{N}$

Unbalanced magnetic pull at 10% eccentricity: $e/\delta = 0.1$ **:**

$$F_M = \frac{p \cdot \tau_p \cdot l_{Fe}}{2\mu_0} \cdot B_\delta^2 \cdot \frac{e}{\delta} = \frac{2 \cdot 0.149 \cdot 0.35}{2 \cdot 4\pi \cdot 10^{-7}} \cdot 0.9^2 \cdot 0.1 = \underline{\underline{3360}}\,\mathrm{N}$$

Calculated first natural bending frequency

a) without influence of magnetic pull: $\underline{f_{b1} = 183\,\mathrm{Hz}}$

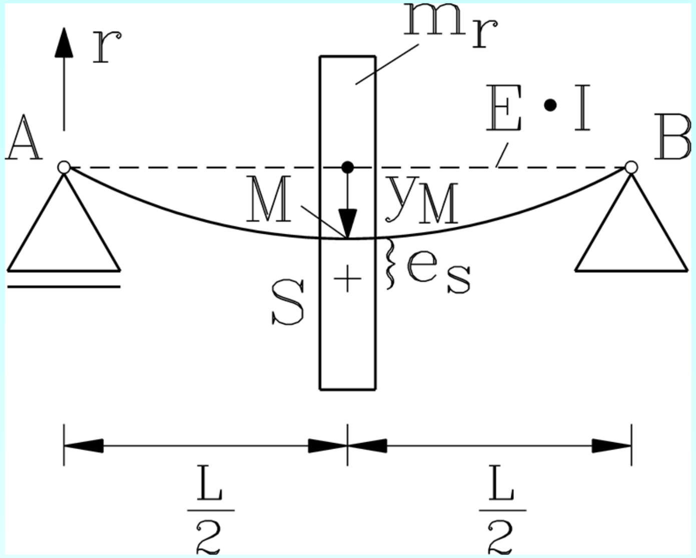
b) with influence of unbalanced magnetic pull:

equivalent shaft stiffness: $c_{sh} = (2\pi f_{b1})^2 \cdot m_r = 121.4 \cdot 10^6\,\mathrm{N/m}$,

$$\text{magnetic stiffness: } c_M = \frac{p \cdot \tau_p \cdot l_{Fe}}{2\mu_0} \cdot B_\delta^2 \cdot \frac{1}{\delta} = 33.6 \cdot 10^6\,\mathrm{N/m}$$

$$f_{b1,M} = f_{b1} \cdot \sqrt{1 - c_M / c_{sh}} = 183 \cdot \sqrt{1 - 33.6/121.4} = \underline{\underline{155.6}}\,\mathrm{Hz}$$

Elastic rotor balancing



Example of disc rotor on elastic shaft (*Laval* rotor)

The shaft bends until equilibrium between centrifugal and elastic force is reached:

$$F_S = F_{c_{sh}} \quad \Rightarrow \quad m_r \cdot (e_S + r_M) \cdot \Omega_m^2 = c_{sh} \cdot r_M$$

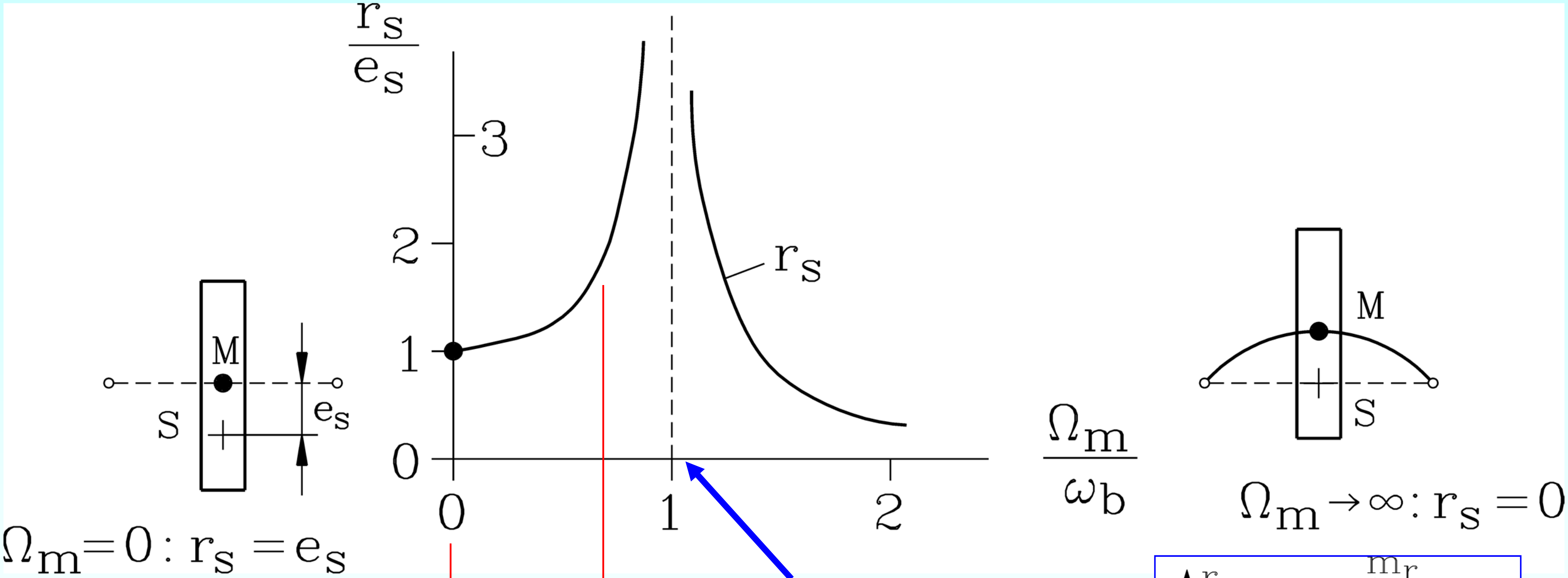
Displacement r_M of centre of rotation *M* from geometrical axis depends on speed

$$r_M = e_S \cdot \frac{\Omega_m^2}{\omega_b^2 - \Omega_m^2} \quad , \qquad \omega_b = 2\pi \cdot f_b$$

Displacement r_S of centre of gravity *S* from geometrical axis depends on speed

$$r_S = e_S \cdot \frac{\omega_b^2}{\omega_b^2 - \Omega_m^2}$$

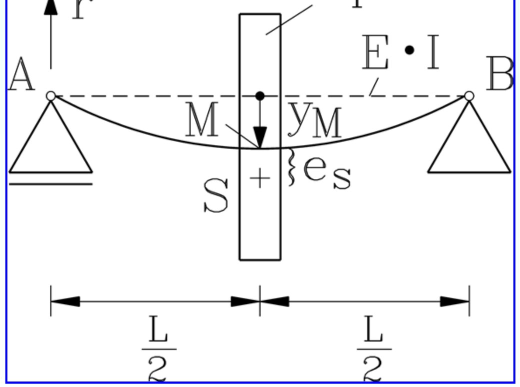
Rotor bending depends on speed



(First) „critical“ speed

Rigid body balancing

Elastic body balancing



Elastic rotor balancing

In a third plane additional balancing masses are fixed. This third plane should be located at the rotor near the location of maximum rotor bending. The centrifugal force of this added imbalance shall act opposite to the centrifugal force of the bent shaft.

Example:

a) 2-pole standard induction motor, 50 Hz, 500 kW:
1st natural bending frequency at $f_{b1} = 35$ Hz.
Elastic balancing with 3 balancing planes is necessary.
b) 2-pole large synchronous turbo generator, 50 Hz, 1000 MW (power plant *Lippendorf*, Germany): 3 natural bending frequencies lie in the frequency range 5 ... 40 Hz:
Elastic balancing with 5 balancing planes is necessary.