# 5. Inverter-fed induction machines

*Source: ELIN EBG Motoren GmbH, Austria*

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# 5. Inverter-fed induction machines

## 5.1 Basic performance of variable-speed induction machines

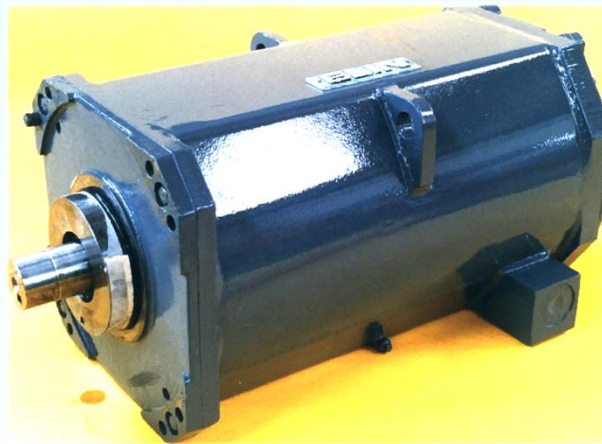## 5.2 Drive characteristics of inverter-fed standard induction motors

## 5.3 Features of special induction motors for inverter operation

## 5.4 Influence of inverter harmonics on motor performance

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# 5. Inverter-fed induction machines

## 5.1 Basic performance of variable-speed induction machines



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# Stator winding air gap flux density distribution

**Voltage-source inverters** with DC voltage link:

400 V grid, $U_d = (3/\pi) \cdot \sqrt{2} \cdot 400 = \underline{\underline{540}}$ V

With neglected stator resistance (*Kloss* function) break down torque is independent of frequency, if voltage fundamental varies linear with frequency:
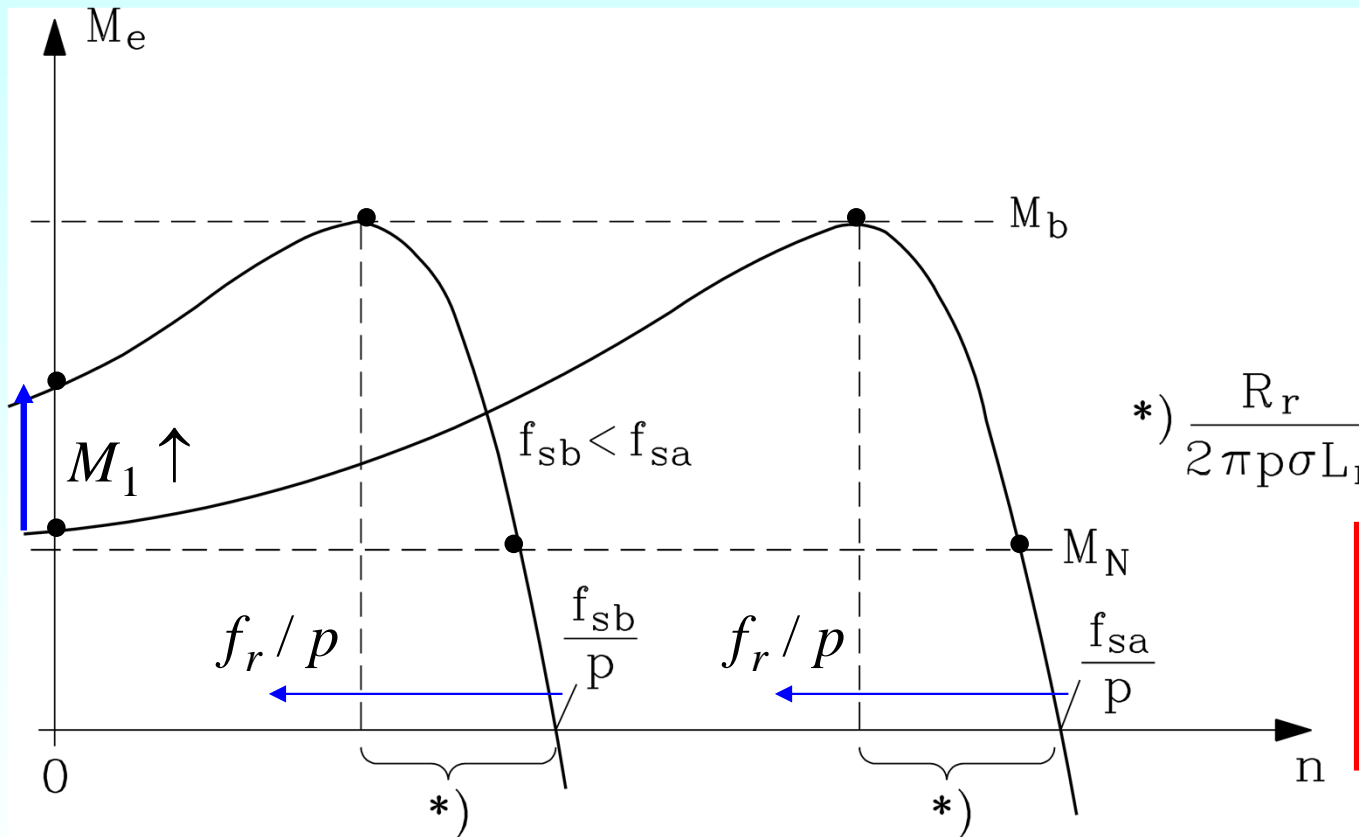


$$*) \quad \frac{R_r}{2\pi p \sigma L_r} = \frac{f_b}{p}$$

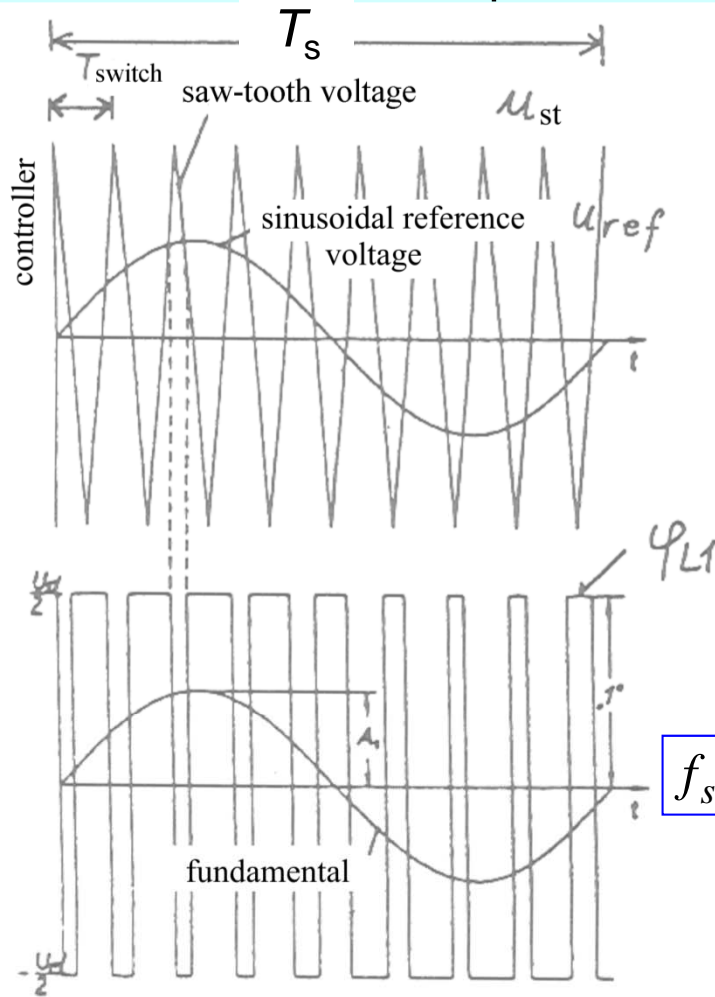$$\boxed{U_s \sim f_s} \quad f_r = s \cdot f_s$$

$$R_s = 0: \quad \frac{M_e}{M_b} = \frac{2}{\dfrac{s_b}{s} + \dfrac{s}{s_b}} = \frac{2}{\dfrac{\omega_b}{\omega_r} + \dfrac{\omega_r}{\omega_b}}$$

$$R_s = 0: \quad M_b = \pm \frac{m_s}{2} \frac{p}{\omega_s} U_s^2 \frac{1-\sigma}{\sigma X_s} = \pm \frac{m_s}{2} \frac{p}{2} \Psi_s^2 \frac{1-\sigma}{\sigma L_s} \qquad \frac{U_s}{\omega_s} = \frac{\Psi_s}{\sqrt{2}} \qquad \frac{s_b}{s} = \frac{s_b f_s}{s f_s} = \frac{\omega_b}{\omega_r} = \frac{f_b}{f_r}$$
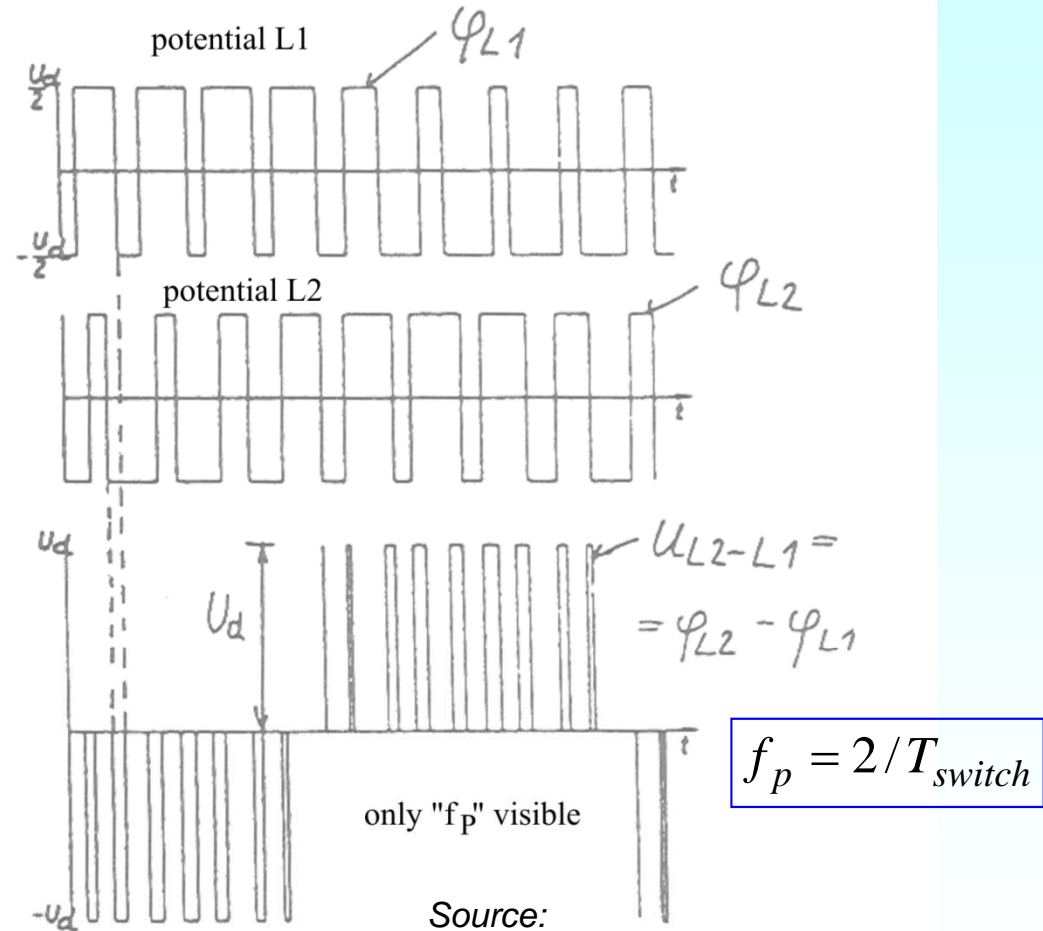
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# Generation of PWM voltage

a) Comparison of saw tooth and reference signal lead to PWM control signal for power switches: Potential $\varphi_{L1}(t)$ at terminal L1 varies with that PWM signal

b) Difference of two terminal potentials delivers line-to-line voltage $u_{L1-L2}(t)$



$$f_s = 1/T_s$$
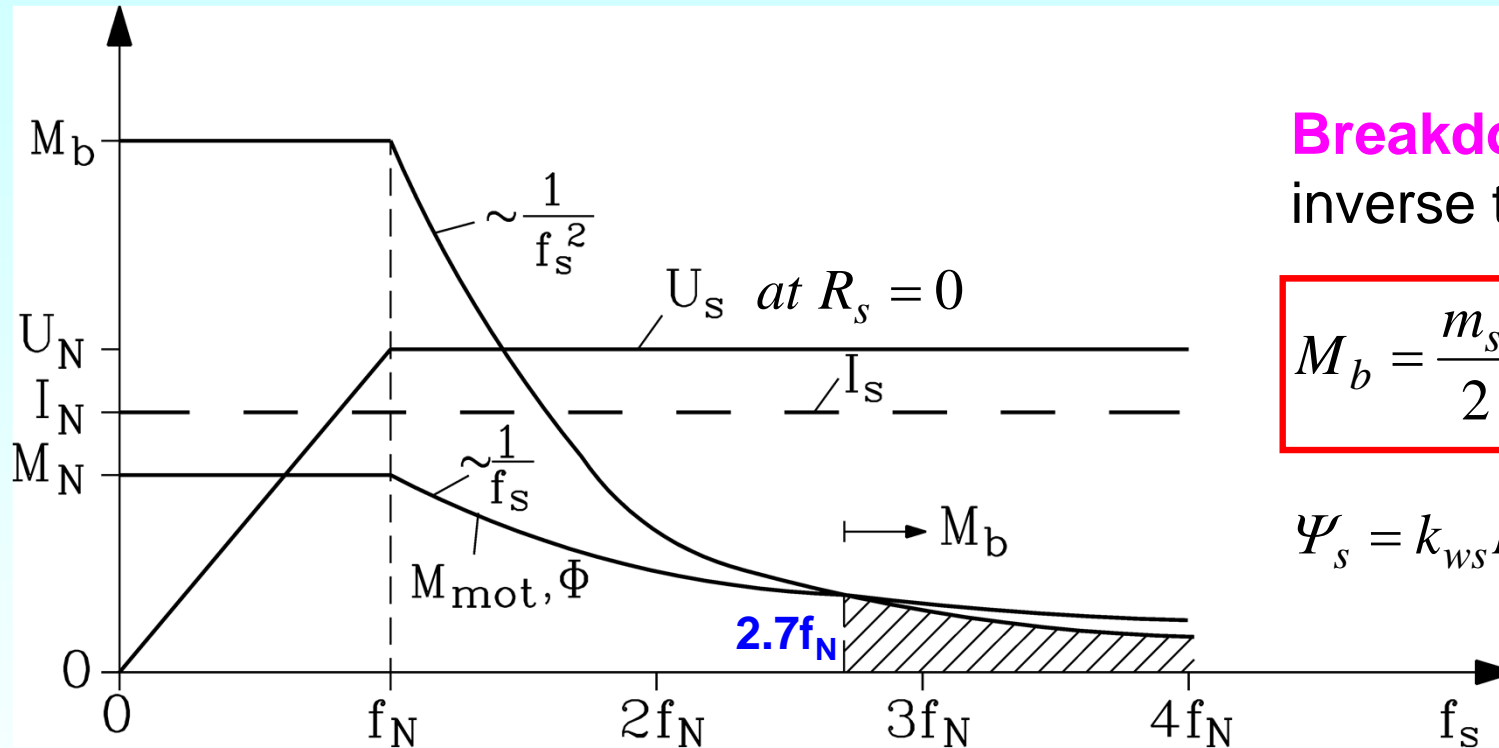
$$f_p = 2/T_{switch}$$

only "$f_p$" visible

*Source:*
*Kleinrath. H.; Springer, 1980*

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# Constant power range

**Voltage stays constant:** Power $\sim U \cdot I$ = const; torque decreases inverse to frequency: $M_{eN} = P_N \cdot p /(\omega_s - \omega_{r,N}) \rightarrow M_e \approx M_{mot} = P_N \cdot p / \omega_s \Leftrightarrow M_e \sim I_s \Psi_s \sim I_s U_s / \omega_s$

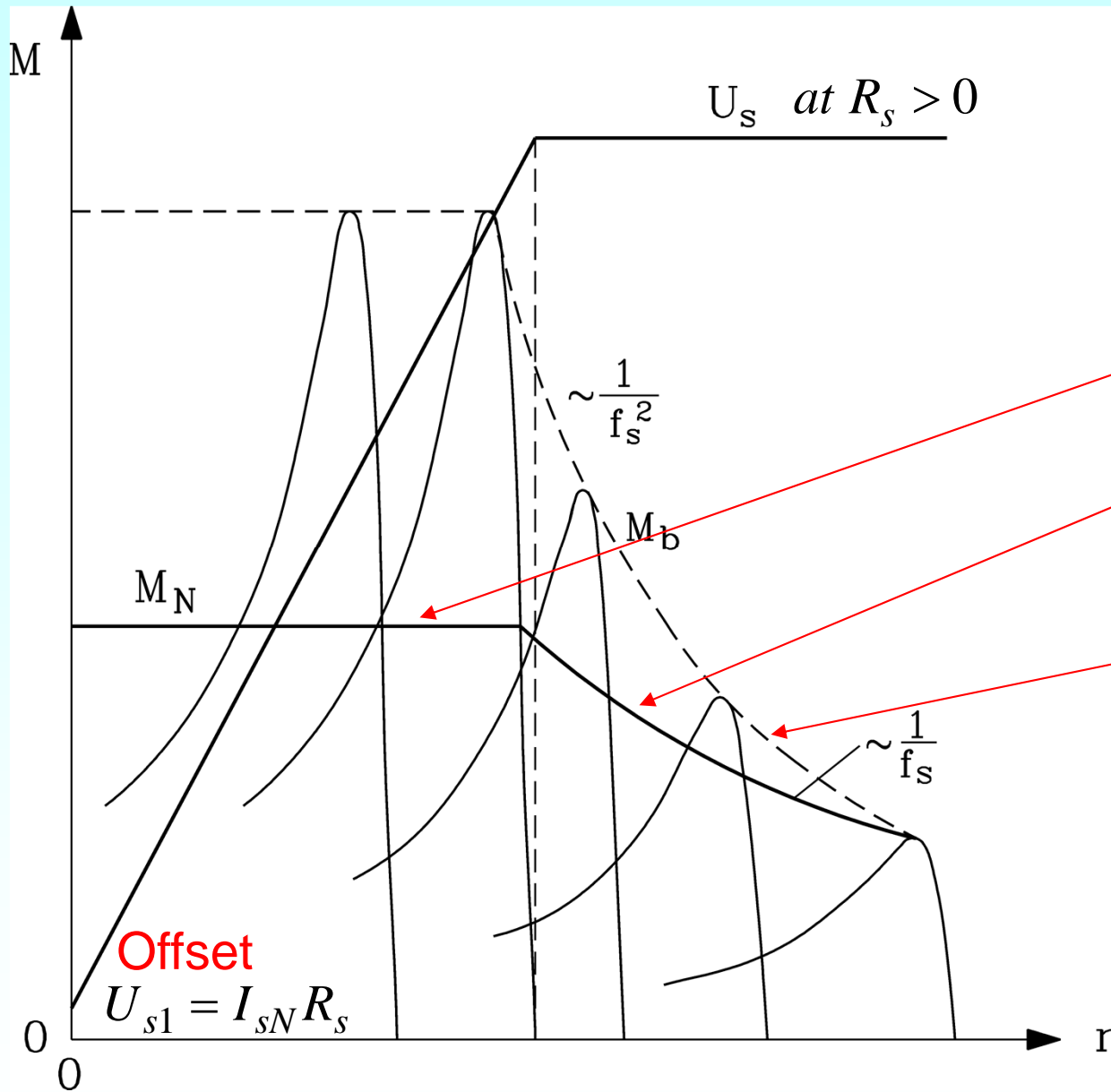**Breakdown torque** decreases inverse to square of frequency:

$$M_b = \frac{m_s}{2} \frac{p}{\omega_s} U_{s,\max}^2 \frac{1-\sigma}{\sigma \omega_s L_s} \sim 1/\omega_s^2$$

$$\Psi_s = k_{ws} N_s \Phi$$

Constant rated power range is limited, where decreasing breakdown torque reaches torque demand for rated power: e.g. 2.7$f_N$.

Usually maximum frequency $f_{s,\max}$ is defined, where $M_b/M_N$ = 1.6 (60% overload margin).

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# Variable speed torque-speed curves with voltage limit

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# Variable speed induction motor at $R_s > 0$
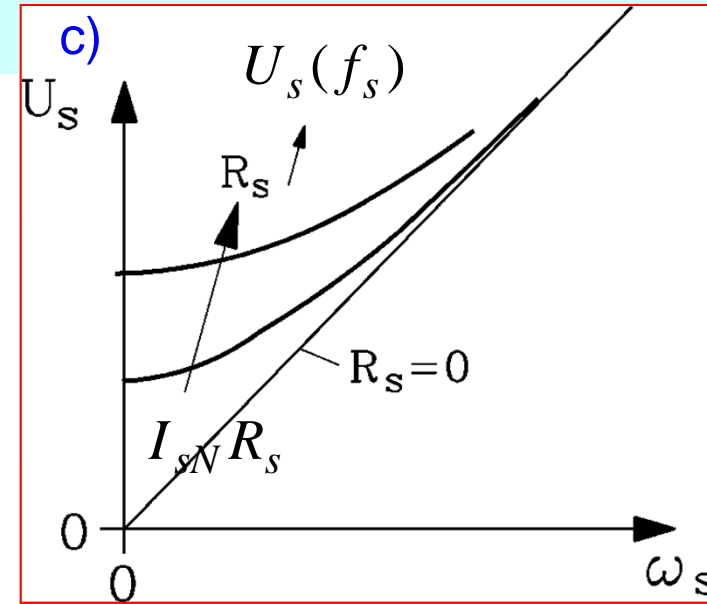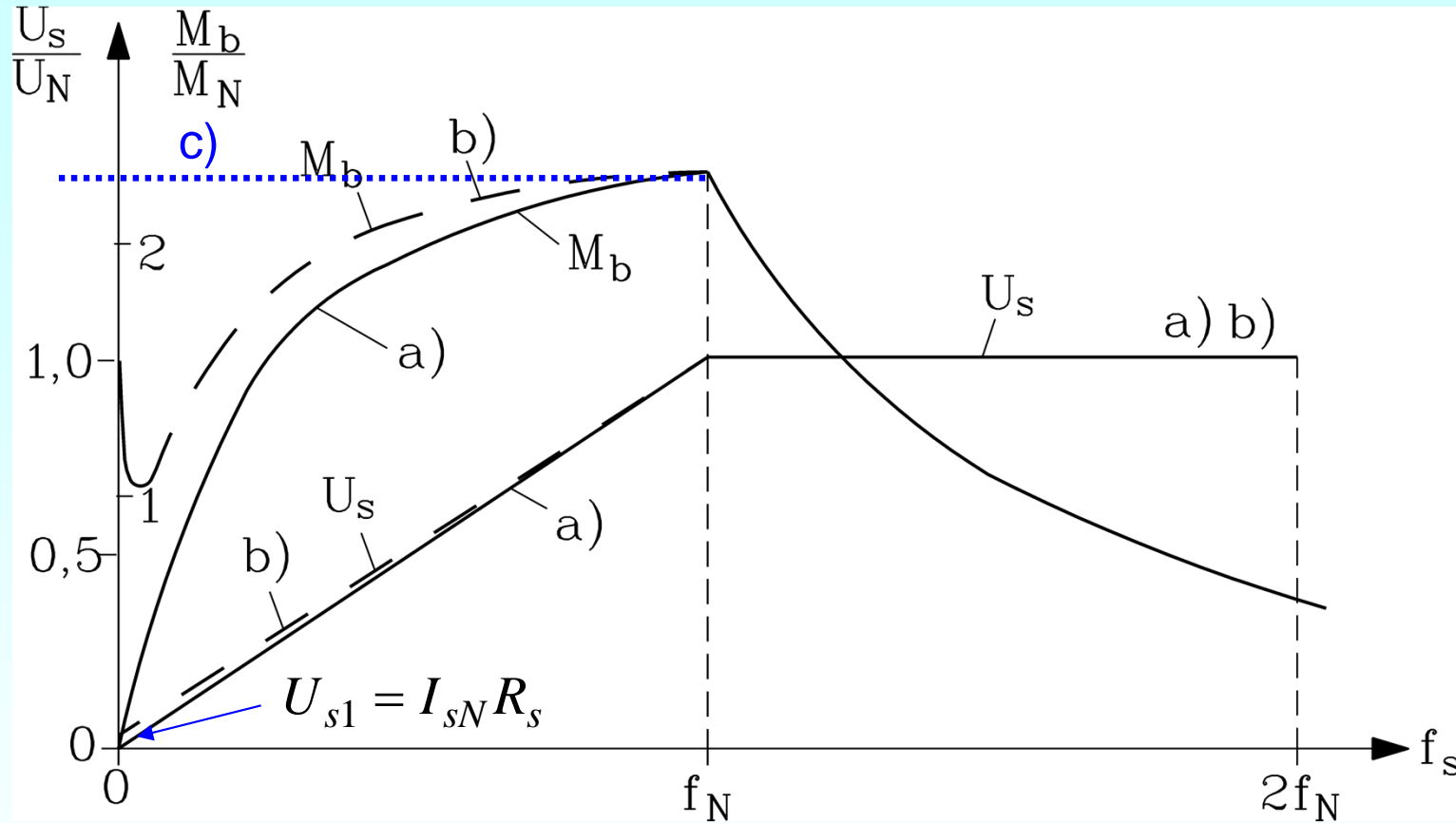


$0 \le f_s \le f_N$ :

$a)\, U_s \sim f_s$

$b)\, U_s = (U_N - U_{s1}) \cdot \dfrac{f_s}{f_N} + U_{s1}$

$c)\, U_s(f_s)$

a) Linear rise of voltage with frequency $U_s \sim f_s$ :
   Breakdown torque decreases with decreasing speed

b) Additional voltage offset $U_{s1} = R_s I_s$ at zero speed to compensate resistive voltage
   drop: Breakdown torque stays above rated torque

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# 5.2 Drive characteristics of inverter-fed standard induction motors



*Source:
Siemens AG*

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# Loss variation in inverter-fed standard induction motors

- **Cooling air flow** of shaft-mounted fan rises linear with speed: $\dot{V} \sim n/n_N \sim f_s/f_N$

- **Iron losses** in stator: rise with square of flux, with frequency exponent $x \cong 1.8$:

$$P_{Fe} \sim (\Psi/\Psi_N)^2 \cdot (f_s/f_N)^x \quad \text{Hysteresis} \sim f, \text{ eddy currents } \sim f^2$$

- **Friction losses** (bearings $\sim n \ldots n^2$), **windage losses** (fan $\sim n^3$): exponent $y = 2{,}5 \ldots 3$:

$$P_{fr+w} \sim (n/n_N)^y \sim (f_s/f_N)^y$$

- **Stray load losses** (caused by space harmonic effects): $z \cong 1.5 \ldots 2$

$$P_{ad1} \sim (n/n_N)^z \cdot (I_s/I_N)^2$$

- tooth flux pulsations
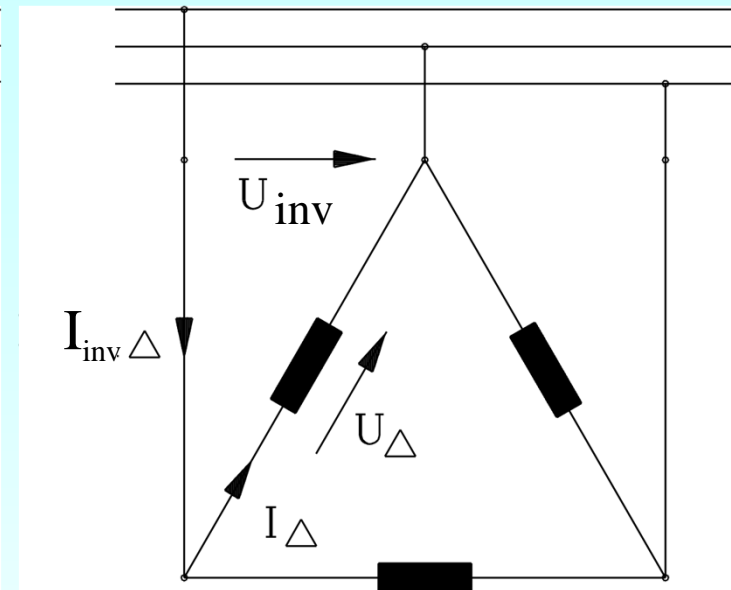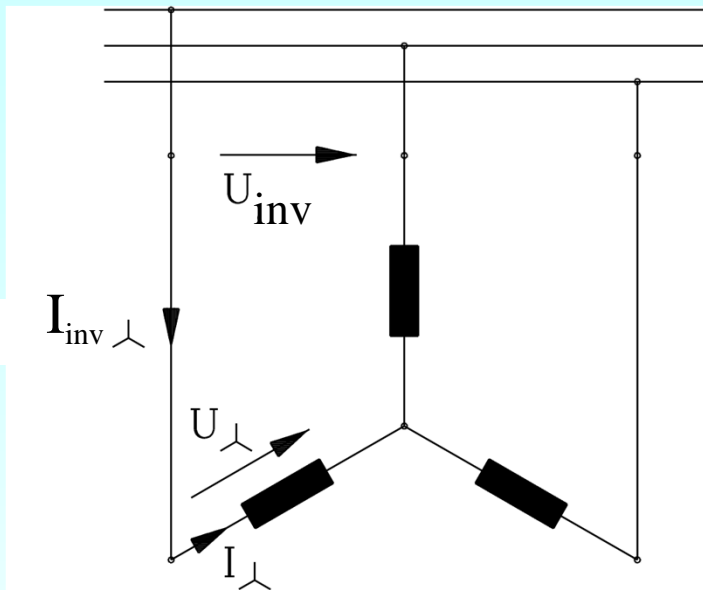- harmonic rotor currents $I_{r\nu}$
- harmonic inter-bar currents $I_{q\nu}$

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# Variable air flow in inverter-fed standard induction motors with shaft-mounted fan

- $f_s > f_N$ : **<u>Flux weakening</u>**: $\Psi \sim 1/f_s$ :

**Iron & copper losses constant;
frequency limit: breakdown/rated power reaches 160%.**

- $f_{th} < f_s < f_N$ : **<u>Constant torque range</u>**:

**Iron losses and air flow decrease: at $f_{th}$, e.g.** $f_{th} \sim 0.5 f_N$ **thermal limit.**

- $f_s < f_{th}$ : **<u>Reduced torque operation:</u>**

**Reduction of current to decrease copper losses due to low air flow**

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# Variable speed induction motor with shaft-mounted fan cooling



Typical example of motor characteristics

Thermal limit at low speed

$M_b/1,3$

$M_b/1,6$

$M_{th}$

$M_b/1,6$

$M_N$

$\sim 0,8 \cdot M_N$

$U_s$

$\sim 0,6 \cdot M_N$

*Source: Siemens AG*

$f_{th} = 0.3 f_N$

$f_s$ /Hz

0  5    15              50  60                    100

- Cooling air flow of shaft-mounted fan rises linear with speed: $\dot{V} \sim n/n_N \sim f_s/f_N$

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# Y-D (star-delta) to increase power by speed



$$U_s \sim f_s$$

- **Star:** Phase voltage $U_Y = U_{inv}/\sqrt{3}$ , phase current $I_Y =$ inverter current $I_{inv,Y}$.
- **Delta:** Phase voltage $U_\Delta =$ Line-to-line voltage $U_{inv}$, Phase current $I_\Delta = I_{inv,\Delta}/\sqrt{3}$

$$U_Y \sqrt{3} = U_\Delta \qquad f_{s\Delta} = \sqrt{3} f_{sY} \quad \text{for same flux} \Rightarrow \quad \text{\textbf{Inverter current:}} \quad I_Y \sqrt{3} = I_\Delta$$

Increase of phase voltage in "delta" by 73% allows increase of speed by 73% from 50 Hz to 87 Hz for constant torque. So power increases by 73%, but also current !
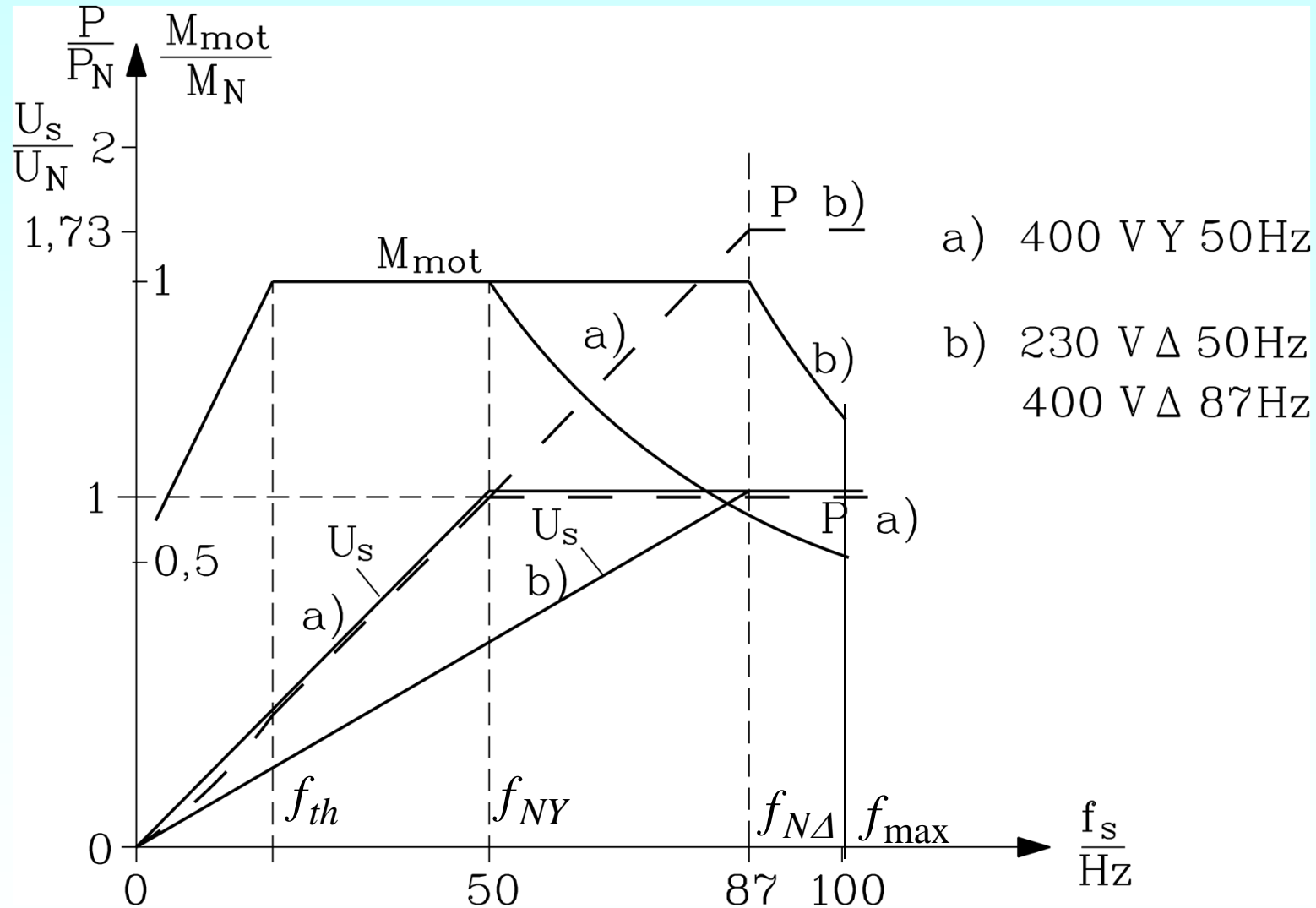
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# Motor power increase with delta connection

*Example:* 4-pole induction motor, power factor 0.85, efficiency 90%

| stator winding connection | *star* | *delta* |
|---|---|---|
| inverter maximum output voltage line-to-line $U_{LL,\max}$ | 400 V | 400 V |
| motor maximum phase voltage $U_{s,\max}$ | 230 V | 400 V |
| motor frequency $f_s = f_N$ at $U_{s,\max}$ | 50 Hz | 87 Hz |
| motor rated phase current $I_{sN}$ | 100 A | 100 A |
| motor rated line current $I_{sN}$ | 100 A | 173 A |
| Motor rated torque $M_N$ | 336 Nm | 336 Nm |
| motor output power $P = 2\pi f_N M_N / p$ | 52.3 kW | 90.6 kW |
| inverter power rating $S = \sqrt{3} U_{LL\max} I_{sN}$ | 69 kVA | 119.4 kVA |

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# Increase of constant torque range & output power

By switching the winding from a) star to b) delta connection !



a) 400 V Y 50 Hz

b) 230 V Δ 50 Hz
   400 V Δ 87 Hz

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# Application: Variable speed drive for pumping

*Example:*
The flow rate $Q$ shall be changed!

Pump / Kreiselpumpe — Throttle valve / Drosselventil — Bypassventil — By-pass valve

**Pump losses**

**Throttle losses**

**Hydraulic flow losses**

Pumpenkennlinie (volle Drehzahl)

Anlagenkennlinie (eingedrosselt)

Anlagenkennlinie (Volllast)

Eindrosseln

Pumpenkennlinie (reduzierte Drehzahl)

Pump head at HIGH speed $n_1$

Drehzahlreduzierung

Pump head at LOW speed $n_2$

Fördermengenbedarf

height difference

Förderhöhe H

head $p$ (H)

flow rate $Q$ Durchfluss Q →

*Source: KSB, Frankenthal*

**Flow rate variation**

**a) by throttling:** Pump operates at fixed speed

**b) by speed variation:** Lower pump speed = lower flow rate. No throttling losses = better overall efficiency. **Up to 60% lower total losses!**

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# Variable speed pumping reduces losses



**Electric energy consumption (%)**

**100%**

Stromverbrauch für eine Pumpe mit einem Motor mit Frequenzumrichter (blau) im Vergleich zu einem Motor mit mechanischer Zuflussregelung (rot)

**Throttling**

Steuerung einer Pumpe mit mechanischen Stellgliedern

**Energy savings 60%**

**Speed variation of pump**

Steuerung mit drehzahlvariablem Antrieb

**100%**

**Flow rate (%)**
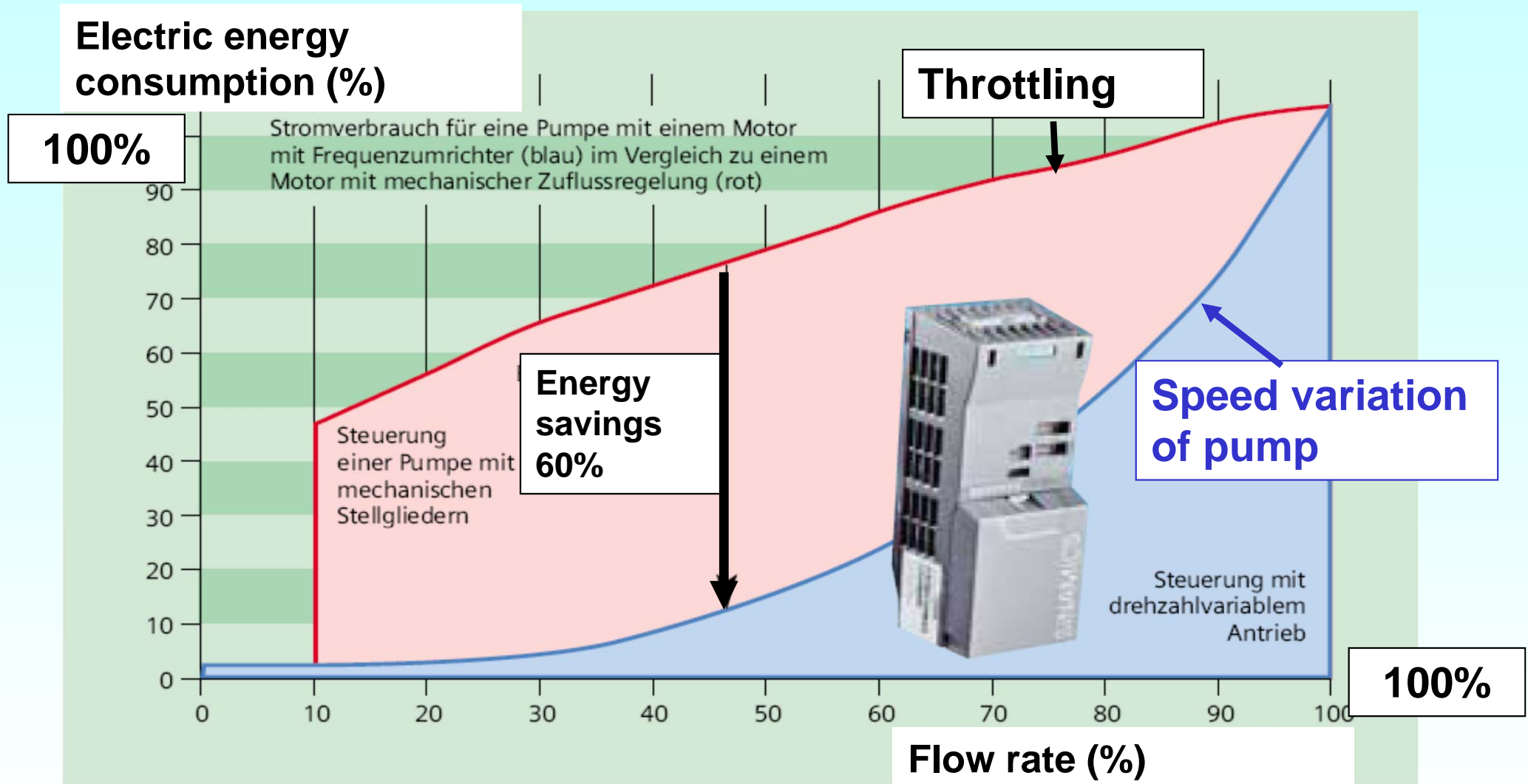
*Source: Siemens AG, Pictures of the future*

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# Variable speed elevators save energy

## *Elevator:*

1 Ton pay-load, 17 m delivery head, 5 stops

a) Old drive:
- Fixed speed drive at the grid: 8.8 kW-induction motor,
- Speed variation by pole-changing („slow-fast")
- Standard gear
- Mechanical braking during stops (Braking energy as heat)

b) New drive:
- 7.5 kW induction motor with raised efficiency
- Speed variation via inverter-feeding
- Low loss gear with synthetic oil
- Energy recovery during braking via the inverter
  (with Active Front End)

**Total energy saving per ride: 81 % at full load (best case)**

**Pay-back time for the more expensive "New drive" 5.5 years at a rate of 400 daily rides!**

*Source: ZVEI, Frankfurt, Germany*

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# 5.3 Features of special induction motors for inverter operation



*Source:*
*ELIN EBG Motoren*
*GmbH, Austria*

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# Induction motor design: Grid vs. inverter-operation

- Equivalent circuit motor parameters for <u>line-</u> vs. <u>inverter</u> operated induction machines

| *Aim  Grid / Inverter* | *air gap* | $L_\sigma$ | $R_r$ |
|---|---|---|---|
| **Big breakdown torque** | - | small | - |
| **Small magnetizing current** | small | - | - |
| **low starting current** | - | big | big |
| **big starting torque** | | small | big |
| **Low additional losses** | big | big | small |

- Low leakage: many slots, no skew, no deep slots, big slot openings
- Small magnetizing current: small air-gap, low iron saturation

**Line-operation:**

- Big starting torque: big current displacement, deep rotor slots, special rotor cage
- Good starting performance: skewing is necessary to minimize harmonic torque.

**Inverter operation:**

$$M_b = \frac{m_s}{2}\frac{p}{\omega_s}U_{s,\max}^2\frac{1-\sigma}{\sigma\omega_s L_s} \sim 1/\omega_s^2$$

- Small current displacement: round or oval rotor slots
- Low additional losses: Avoid skewing to avoid inter-bar currents.
- Sufficient leakage: small slot openings, stray inductance must limit current harmonics.

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# Tooling machine: Variable speed main spindle drive



motor speed sensor

belt

position sensor $\gamma$

spindle

$n_{act}$  actual speed value

speed control

angular position control

set-point speed value

$n_{set}$

inverter

numerical control NC

$\gamma_{act}$  actual angle value

program
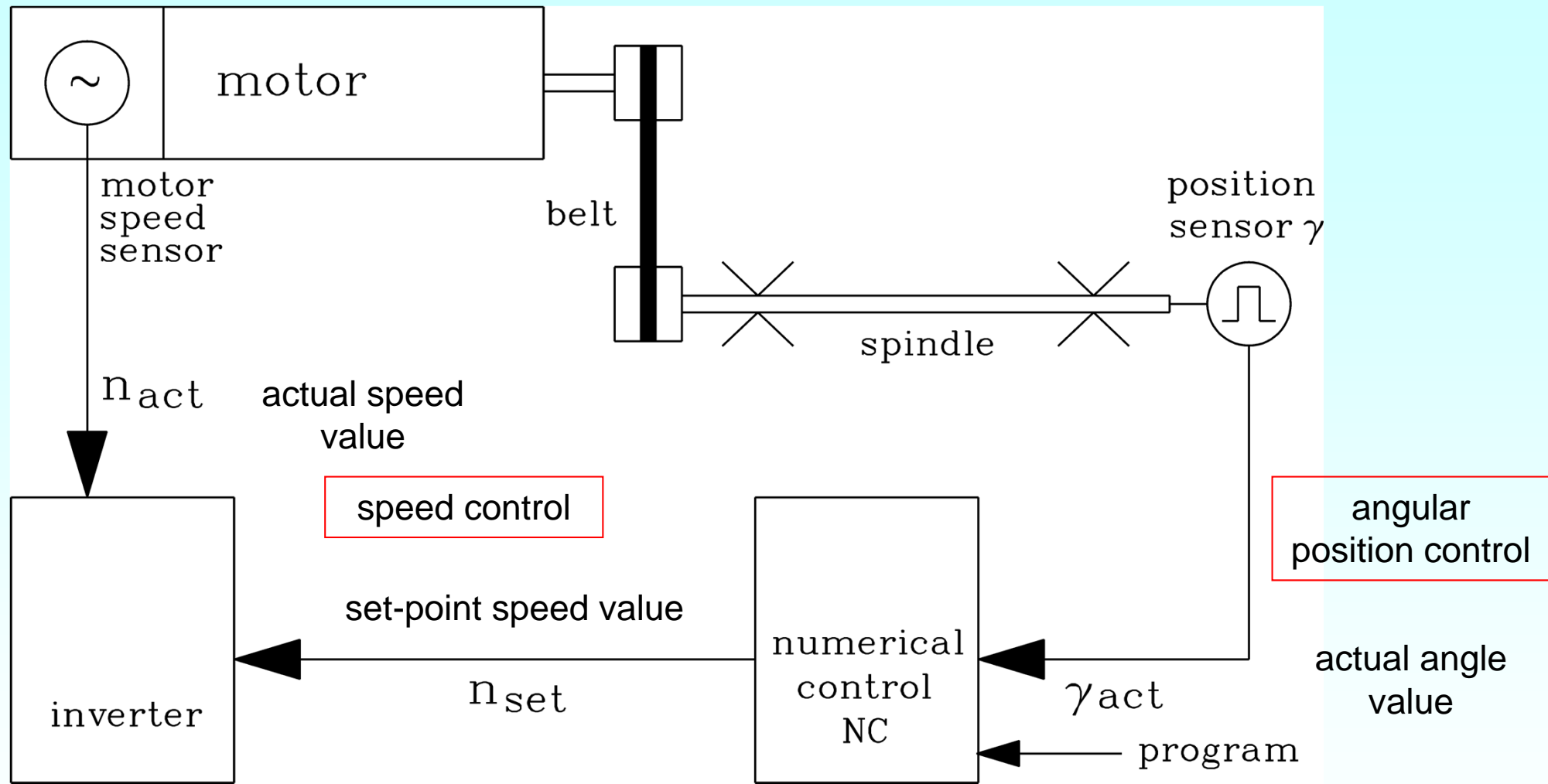
Source: Siemens AG

- **Belt** as mechanical gear for big torque at the spindle at low speed

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# Increase of constant power range by switching a mechanical gear

**_Example:_**

$i = 4$:

$n_N = 500/\text{min}$

$n_{max} = 7500/\text{min}$

$n'_N = 125/\text{min}$

Extension of constant power range from

$500 \ldots 7500 = 1:15$

to $125 \ldots 7500/\text{min} = 1:60$
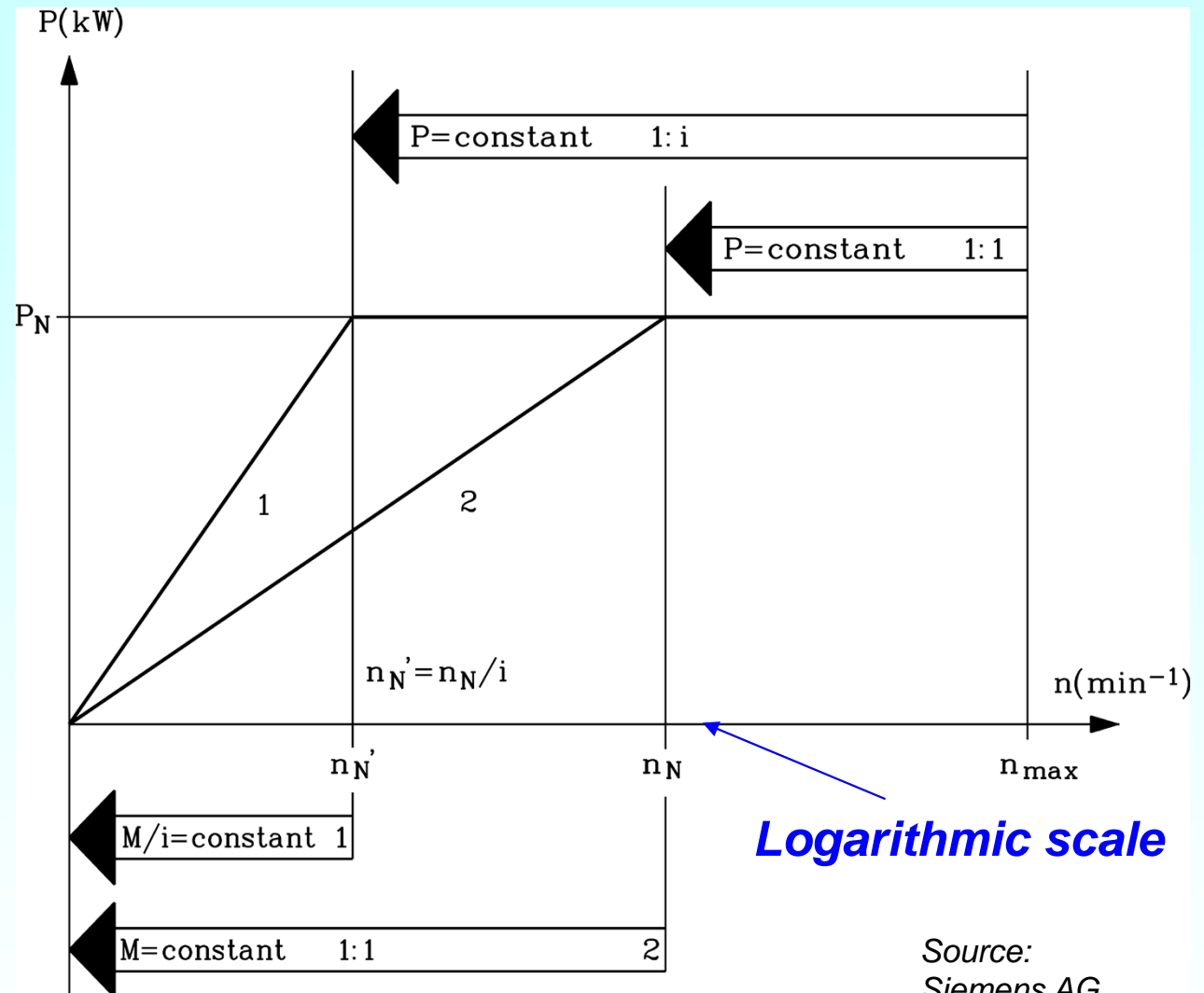


*Source:*
*Siemens AG*

- Gear transfer ratio $i > 1$:  Increase of constant power range at low speed e.g. $i = 4$

# Planetary gear
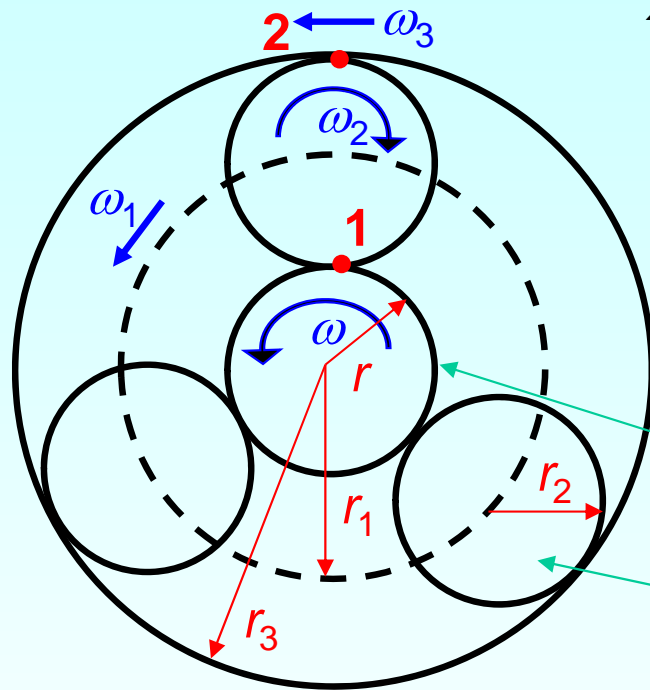
$$1: \quad \omega \cdot r = \omega_1 \cdot r_1 + \omega_2 \cdot r_2$$
$$2: \quad \omega_3 \cdot r_3 = \omega_1 \cdot r_1 - \omega_2 \cdot r_2 \Bigg\} \quad \omega_1 = \frac{\omega \cdot r + \omega_3 \cdot r_3}{2 \cdot r_1} \quad \boxed{i = \frac{\omega_1}{\omega} = \frac{r + r_3 \cdot (\omega_3 / \omega)}{2 \cdot r_1}}$$

$$Case\ I: \quad \omega_3 = 0: \quad i = \frac{r}{2r_1}$$

$$Case\ II: \quad \omega_3 = \omega: \quad i = \frac{r}{2r_1} + \frac{r_3}{2r_1}$$



hollow wheel

sun wheel

planetary wheel
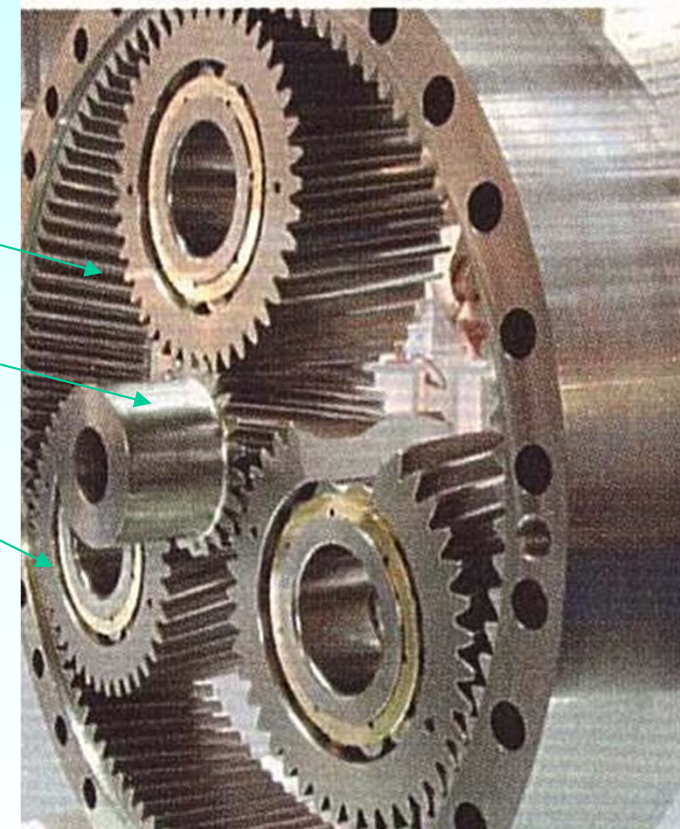
Source: NKE, Germany

Special case:

$$r_2 = r \rightarrow r_1 = 2r, \quad r_3 = 3r$$

$$\boxed{\begin{array}{l} Case\ I: \quad \omega_3 = 0: \quad i = \dfrac{1}{4} \\[2mm] Case\ II: \quad \omega_3 = \omega: \quad i = \dfrac{1}{4} + \dfrac{3}{4} = 1 \end{array}}$$

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# Switching planetary gear with *i* = 1 and *i* = 4

1: gear housing     2: motor

3: sun wheel

4: planetary wheel

5: hollow wheel

6: planetary wheel support

7: switching unit

8: shifting sleeve
position I: gear ratio 1: *i*,
position II: gear ratio 1: 1

9: motor shaft     10: gear shaft

11: housing     12: belt pulley
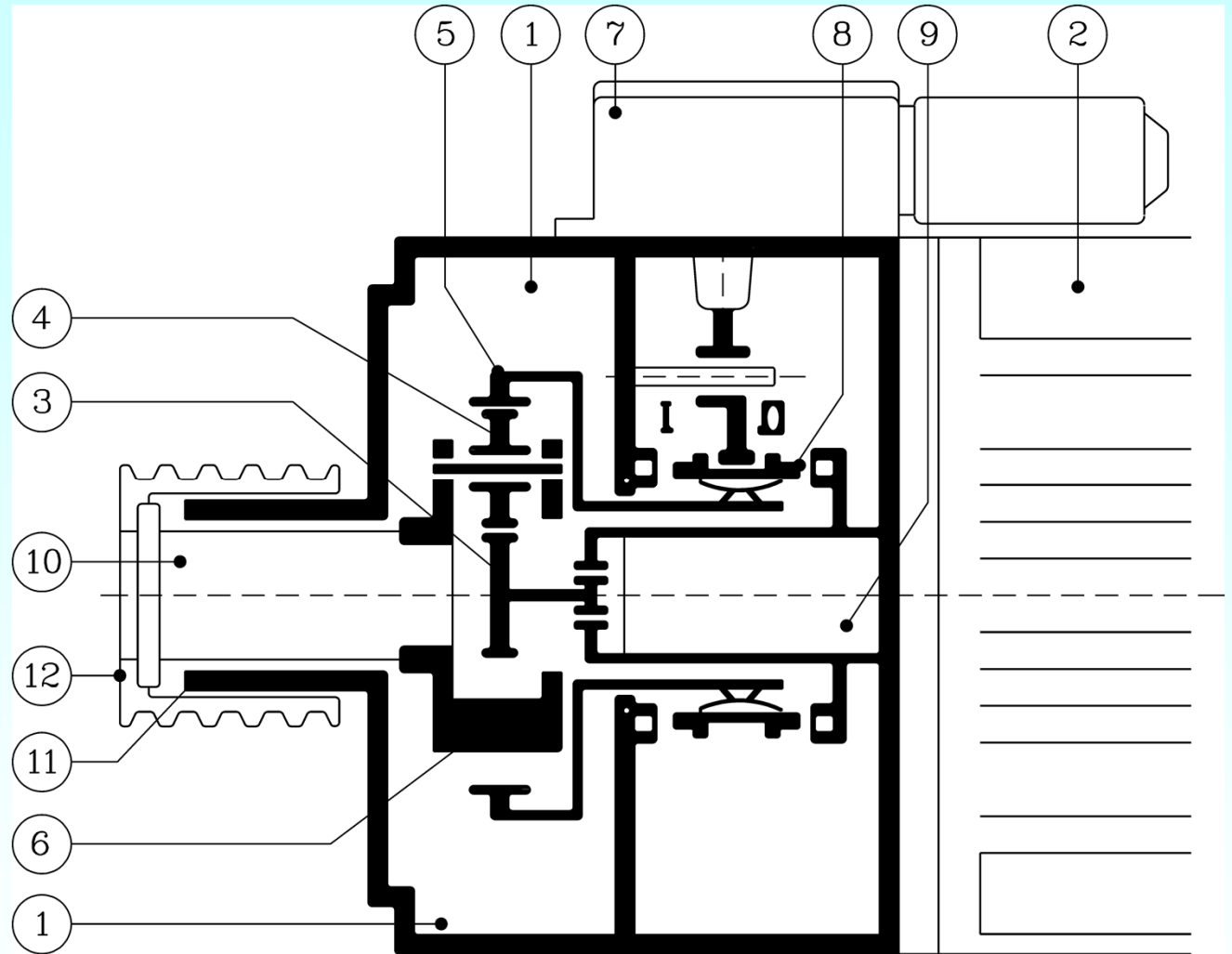


The motor has a holding brake at switched-off power.

*Source:*
*Siemens AG*

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# Induction motors for wide field weakening range

- Over-sizing of motor: $M_b/M_N$ increased

- Increase of inverter current rating $I_s$: $N_s$ reduced,
  $I_s N_s$ = const. for constant rated torque
  = increased conductor cross section $A_c$ at constant current
  density $J_s = I_s/A_c$ (slot fill factor $k_Q = N_c A_c/A_Q$ = const., I
  inductance $L_s \sim (N_s)^2$ decreased, $M_b \sim 1/L_s$ increased

- Star-delta switching of stator winding

- Series-parallel switching of stator winding

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# Star-delta switching of stator winding for same motor volume to increase constant power range by 1:3

a)



here: $I=178\%$

9kW

$\boxed{1:9}$

$N_s$ reduced: $U_{N,mot} < U_{N,inv}$

Oversized inverter rating

$N_s = 66$

$n/\min^{-1}$

1500    13500



(a)   (b)

$I_N$

30A   170%

17A   100%

66   111   $N_s$

c)

b)



P

$N_s = 111$

"$N_s/1.7$"

9kW   2550

Factor "3"

$U_{N,mot} = U_{N,inv}$

$n/\min^{-1}$

1500   3500   5000    13500
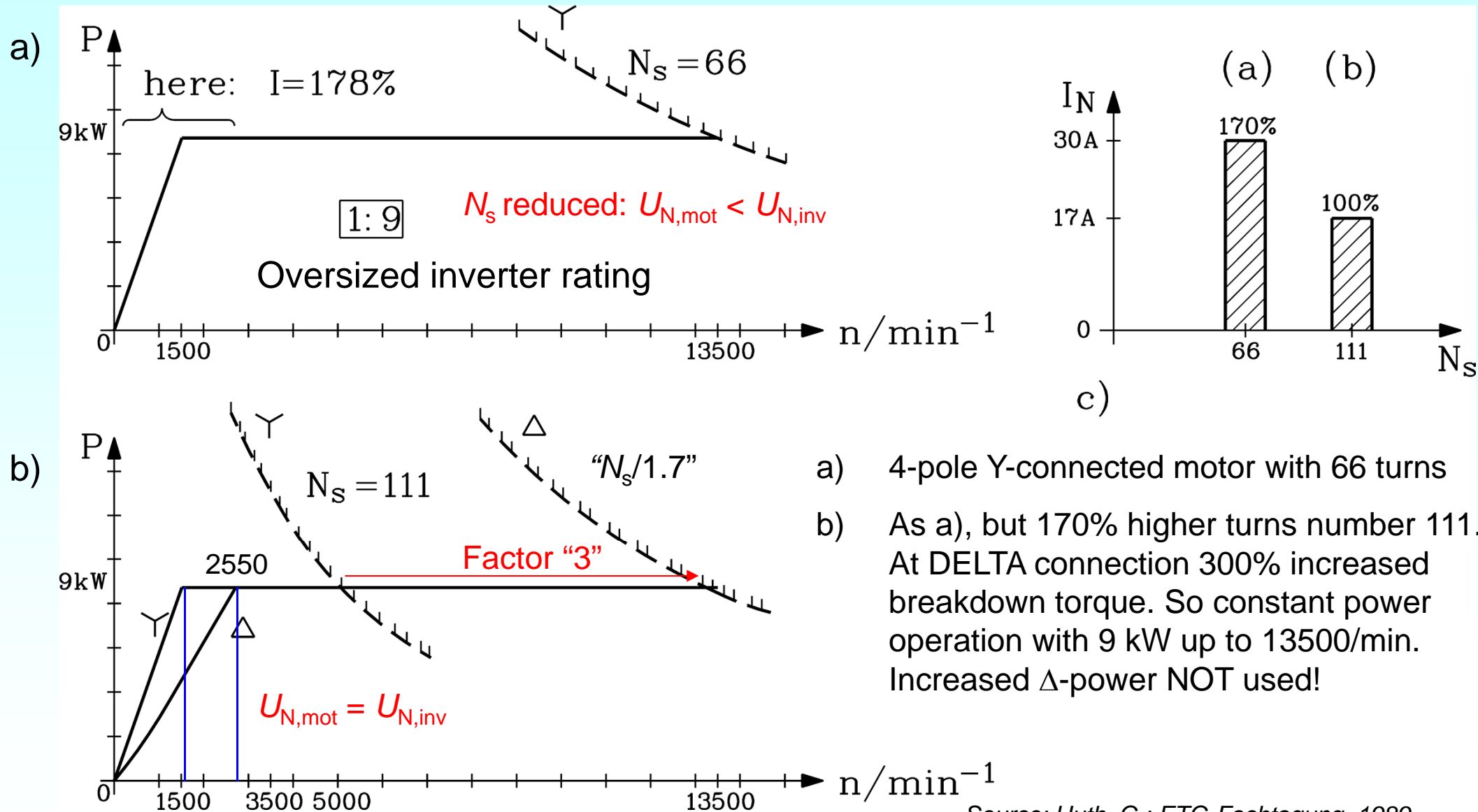
a)   4-pole Y-connected motor with 66 turns

b)   As a), but 170% higher turns number 111. At DELTA connection 300% increased breakdown torque. So constant power operation with 9 kW up to 13500/min. Increased Δ-power NOT used!

*Source: Huth, G.; ETG-Fachtagung, 1989*

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# Series-parallel switching of stator winding

High performance 4-pole induction motor with star-delta switch to enlarge constant power range, rated speed at Y: 500/min, rated frequency 16.7 Hz

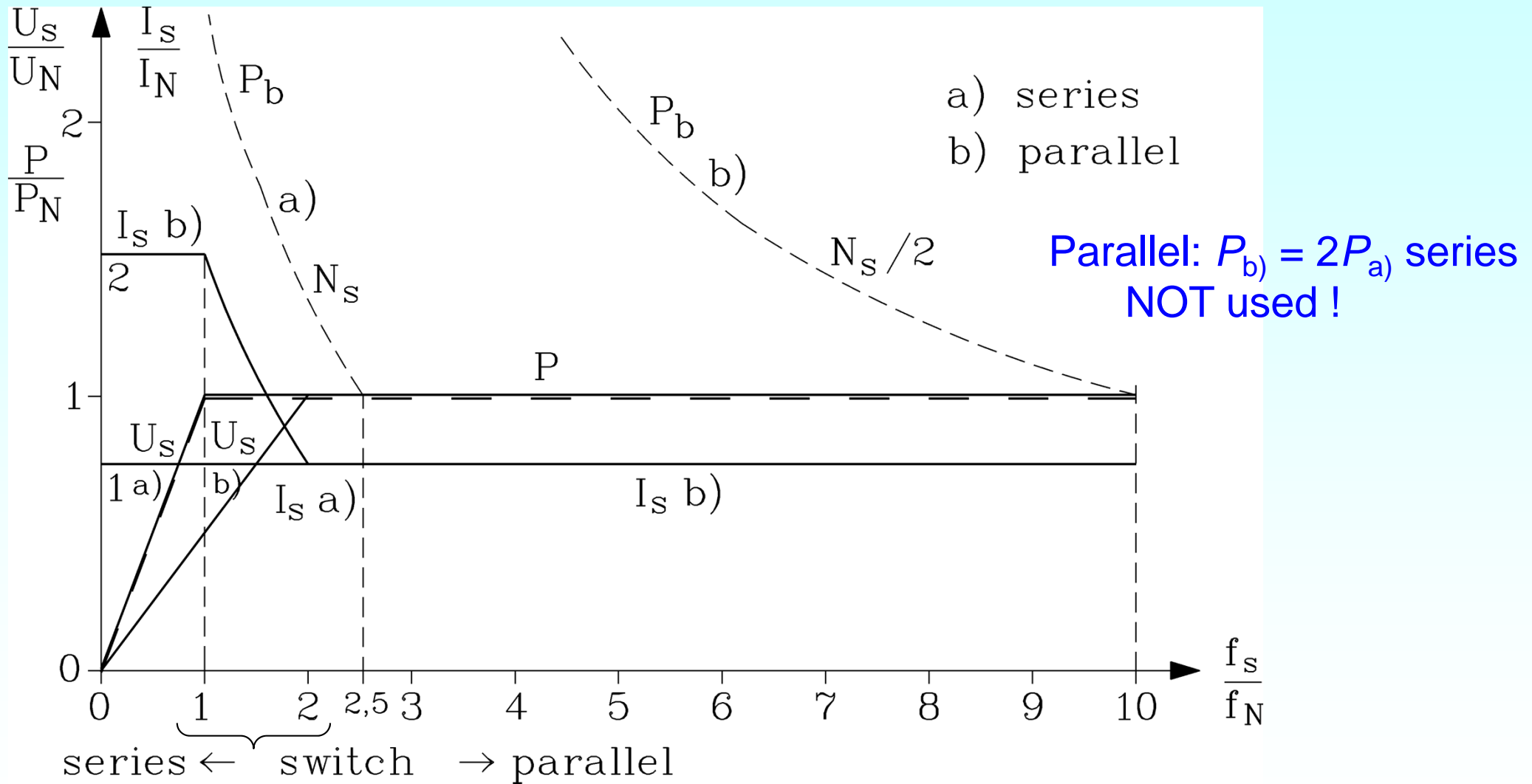Standard induction 4-pole motor, D connected winding, rated speed 1460 /min, rated frequency 50 Hz
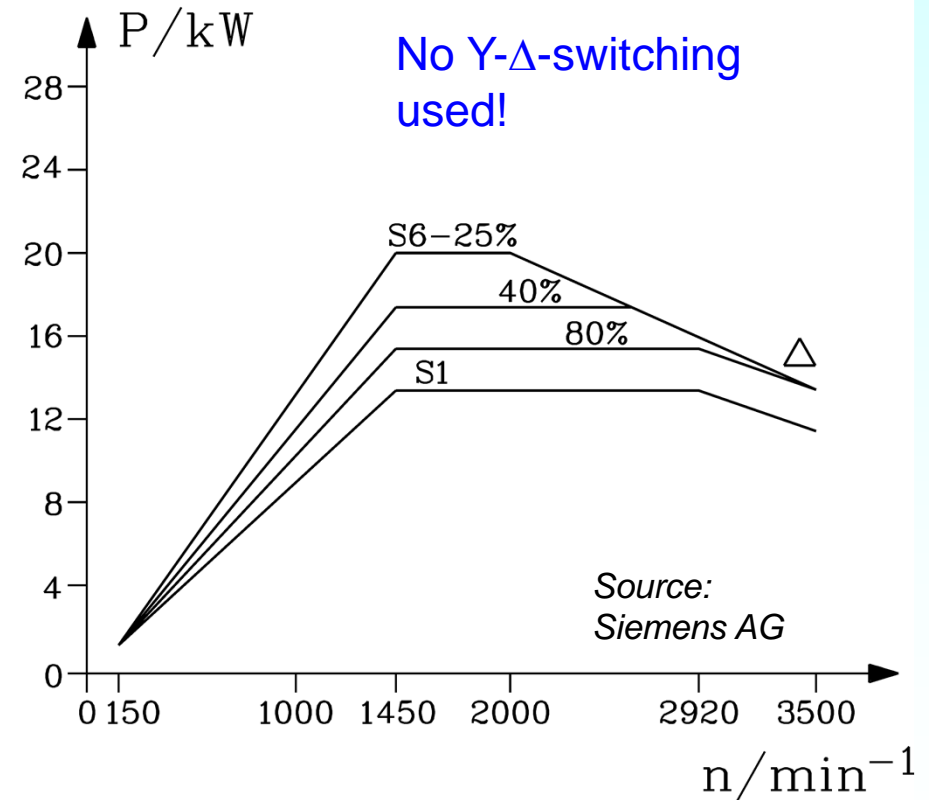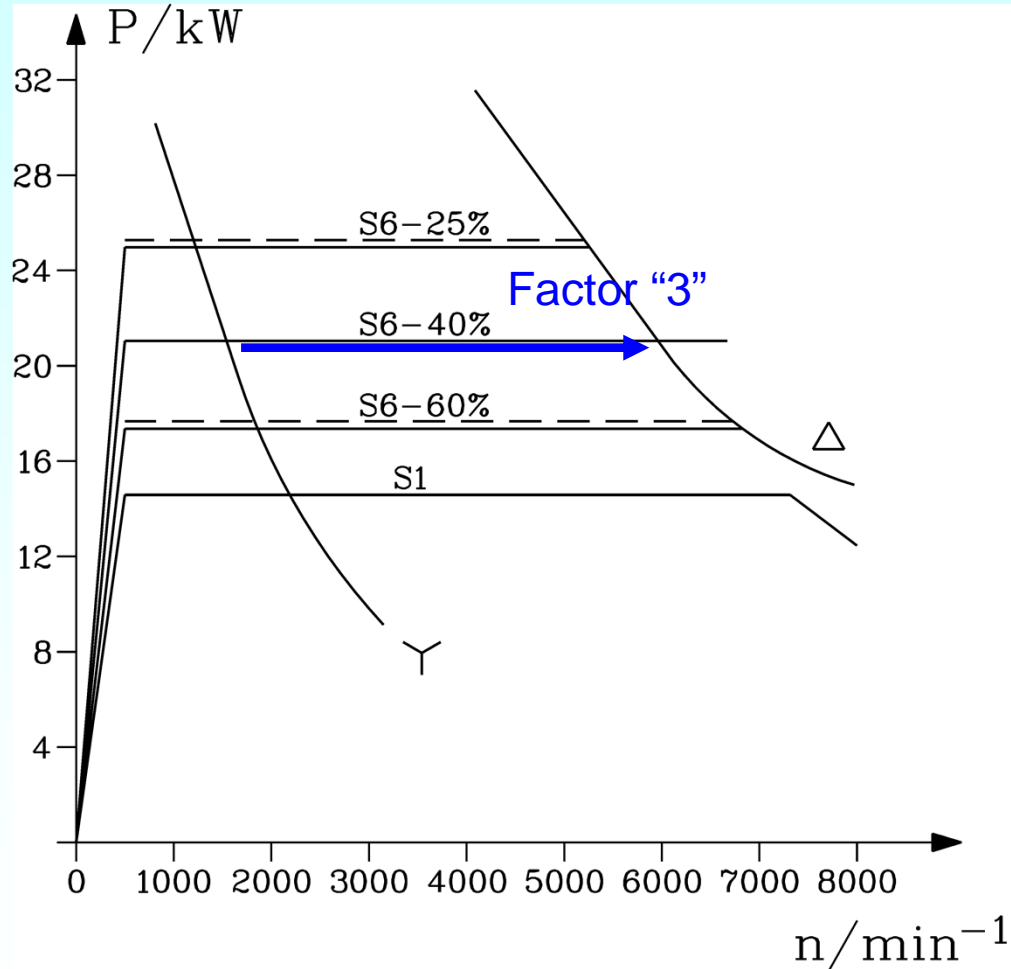
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# Series-parallel switching of stator winding (1)



Parallel: $P_{b)} = 2P_{a)}$ series

P − − −
U ———
I - - - - -

a) series ⎫ connection for
b) parallel ⎬ same torque $M_N$

Series: $N_s$
Parallel: $N_s/2$

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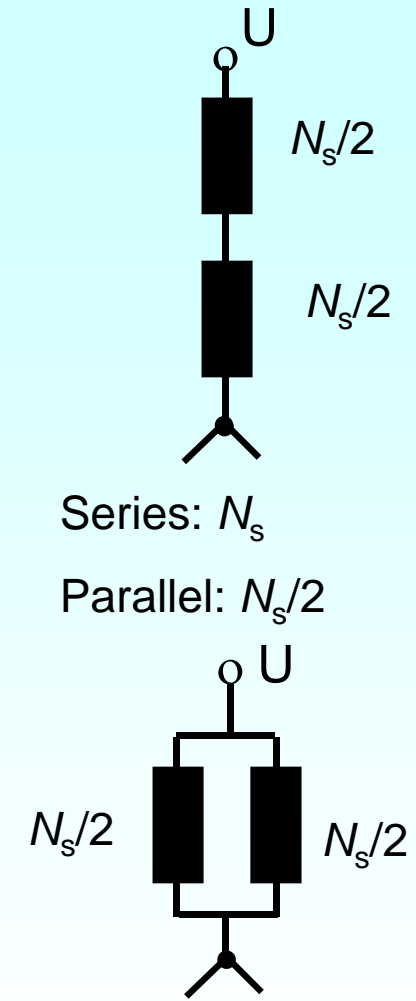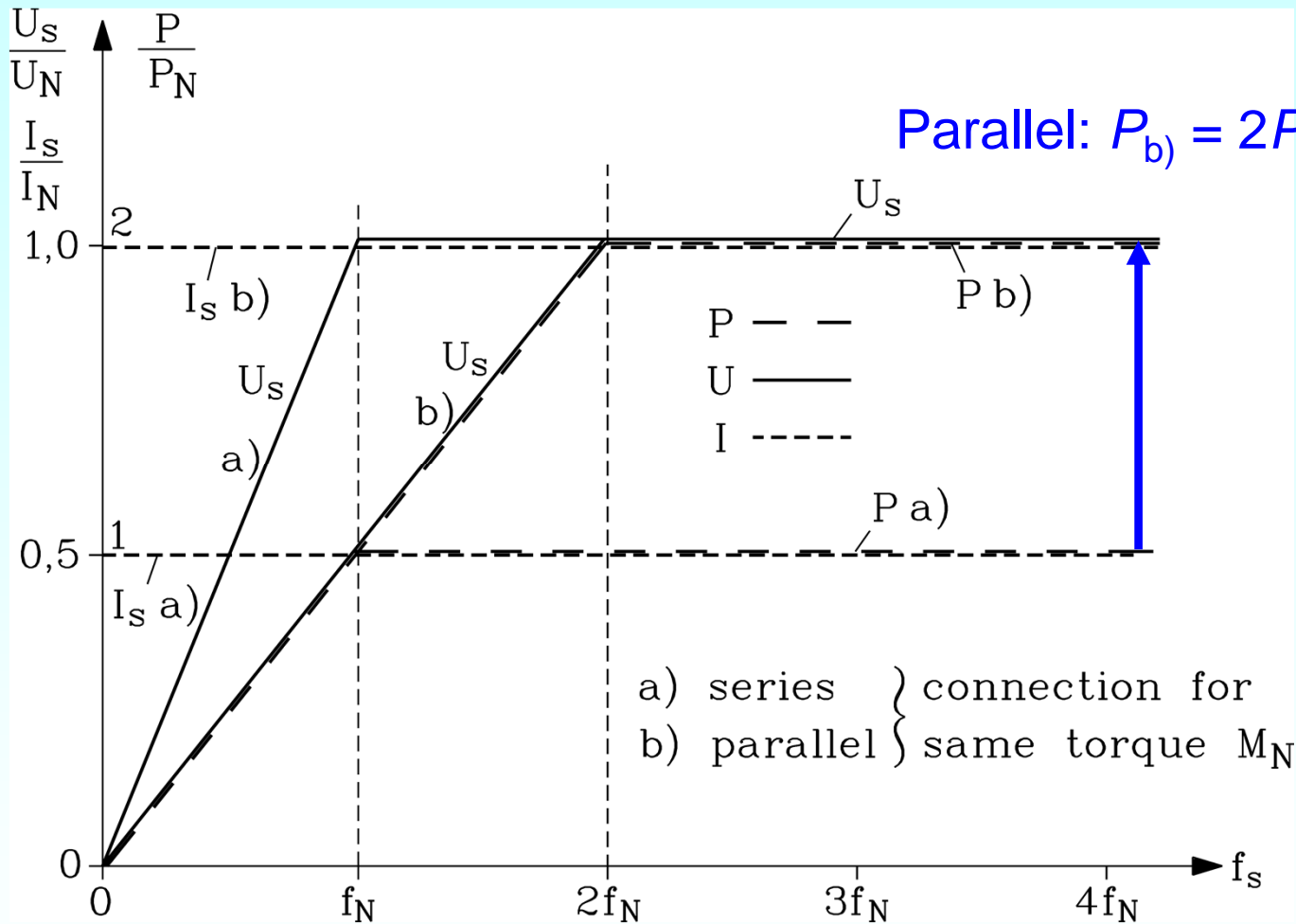
# Series-parallel switching of stator winding (2)

- Increasing constant power range by factor 4
- Increase of field weakening by factor 4 from 1:2.5 to 1:10

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# 5. Inverter-fed induction machines

## 5.4 Influence of inverter harmonics on motor performance



*Source:*
*ELIN EBG Motoren GmbH,*
*Austria*

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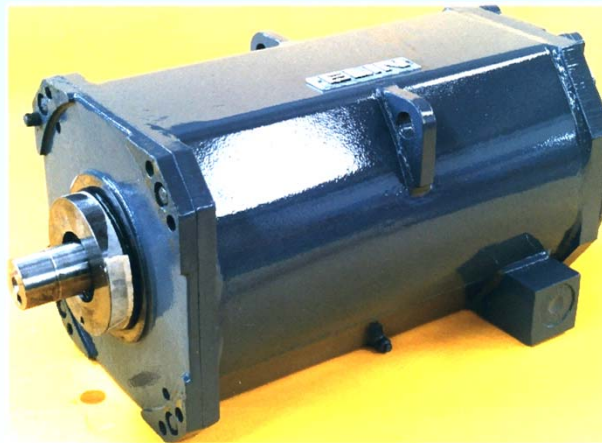
# Inverter with voltage six step operation
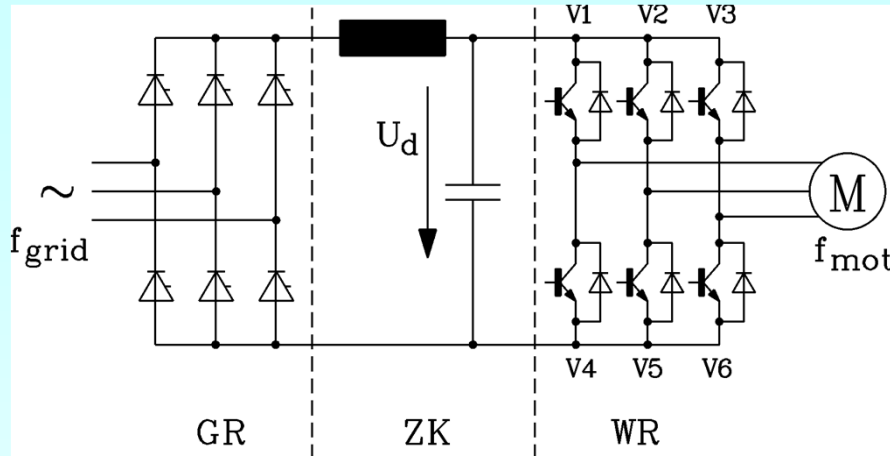


- **Bridge rectifier with thyristors on grid side** GR (firing angle $\alpha$) generates variable DC voltage $U_d$ in DC link ZK; voltage smoothed by capacitor.

- **Inverter WR** generates by six-step switching from $U_d$ a **block shaped** line-to-line output voltage between terminals L1, L2, L3.

*Source: Kleinrath. H.; Springer, 1980*

a)

b)

- **DC link voltage $U_d$** is changed by $\alpha$ **proportional** with output frequency $f_{mot} = 1/T$
- **Grid side energy feed-back only possible with 2$^{nd}$ anti-parallel thyristor bridge:** At $\alpha > 90°$ positive $U_d$ and negative $I_d$ give negative dc link power = power to the grid (gener. braking).

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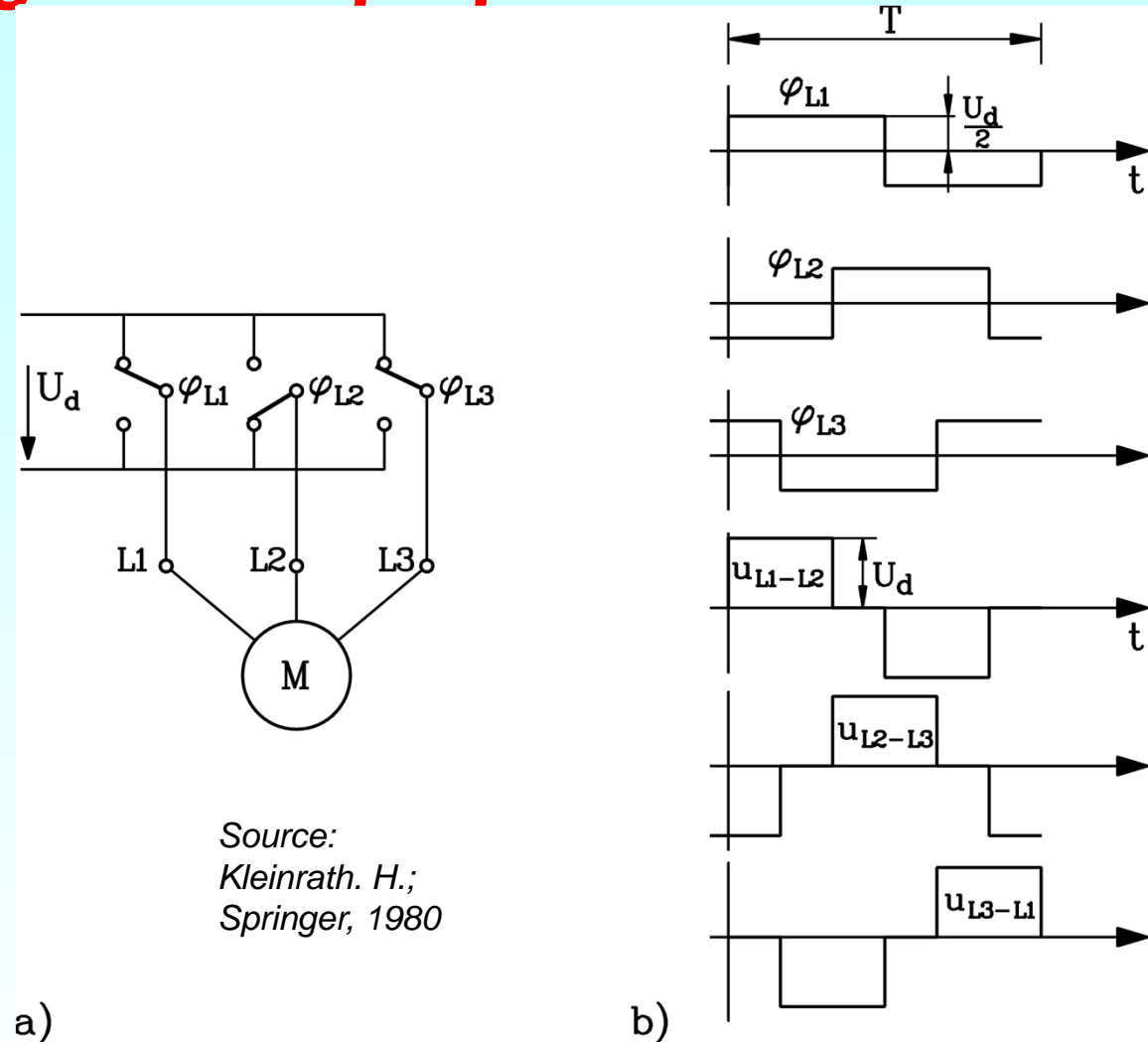
# Voltage harmonics at six-step operation (see Chapter 1)

● **Inverter output phase voltage:** $u_{S1} - u_{S2} = u_{L1-L2};\quad u_{S2} - u_{S3} = u_{L2-L3};\quad u_{S1} + u_{S2} + u_{S3} = 0;$
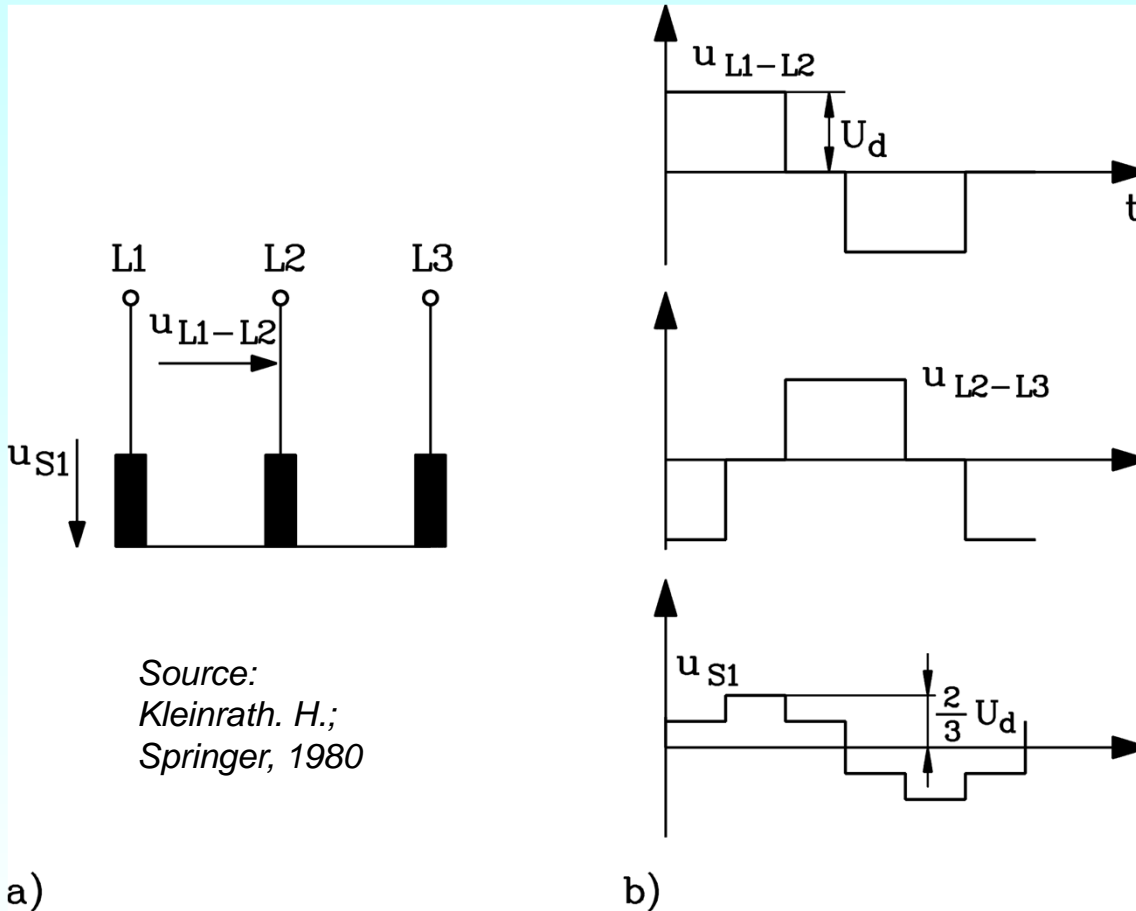
we get:   $u_{S1} = \dfrac{2u_{L1-L2} + u_{L2-L3}}{3}$

● Block shaped line-to-line voltage, expanded as *FOURIER*-series:

$$u_L(t) = \sum_{k=1,-5,7,...}^{\infty} \hat{U}_{L,k} \cdot \cos(k \cdot \omega_s t)$$

$$k = 1 + 6g, \qquad g = 0, \pm 1, \pm 2, ...$$

⇒   **k = 1, -5, 7, -11, 13, ...**

$$\hat{U}_{L,k} = \frac{2}{\pi} \sqrt{3} \frac{U_d}{k}$$

*Source:*
*Kleinrath. H.;*
*Springer, 1980*

a)

b)

*Electrical machine is fed with a mix of harmonic voltages of different amplitude, frequency and phase angle. Only fundamental (ordinal number k = 1) is desired. Voltage harmonics (|k| > 1) cause harmonic currents in electric machine with additional losses, torque pulsation, vibrations and acoustic noise.*

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# Harmonic voltage systems (see Chapter 1)

**k = 1:**

$$u_{U1}(t) = \hat{U}_1 \cdot \cos(\omega t)$$

$$u_{V1}(t) = \hat{U}_1 \cdot \cos(\omega t - 2\pi/3)$$

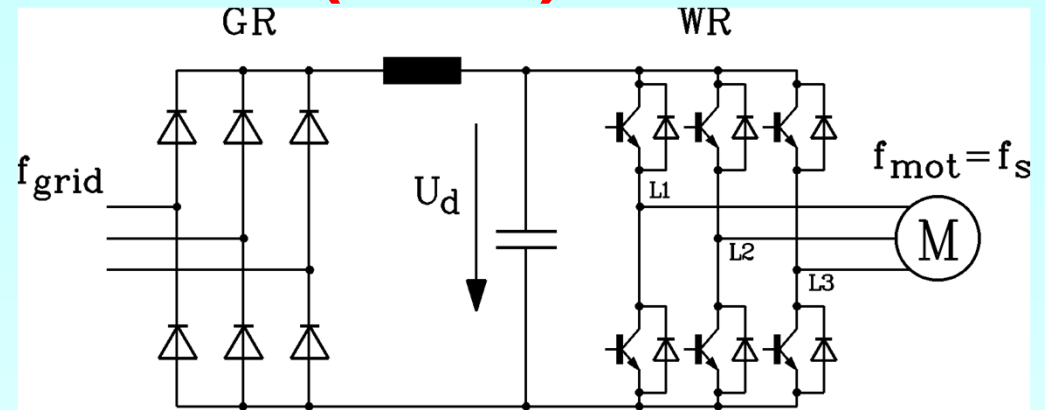$$u_{W1}(t) = \hat{U}_1 \cdot \cos(\omega t - 4\pi/3)$$

U – V – W
Positive sequence system

**k = 5 ⇒ k = -5:**

$$u_{U5}(t) = \hat{U}_5 \cdot \cos(5\omega t)$$

$$u_{V5}(t) = \hat{U}_5 \cdot \cos(5(\omega t - 2\pi/3)) = \hat{U}_5 \cdot \cos(5\omega t + 2\pi/3)$$

$$u_{W5}(t) = \hat{U}_5 \cdot \cos(5(\omega t - 4\pi/3)) = \hat{U}_5 \cdot \cos(5\omega t + 4\pi/3)$$

U – W – V
Negative sequence system

- **General rule:** Positive and negative systems occur alternatively: Ordinal number *k* has a positive or negative sign: *k* =+1, -5, +7, -11, +13, …

$$u_{Uk}(t) = \hat{U}_k \cdot \cos(k\omega t)$$

$$u_{Vk}(t) = \hat{U}_k \cdot \cos(k\omega t - 2\pi/3)$$

$$u_{Wk}(t) = \hat{U}_k \cdot \cos(k\omega t - 4\pi/3)$$

$$k = 1 + 6g \qquad\qquad g = 0, \pm 1, \pm 2, ...$$

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# Pulse width modulation (PWM)



- **At grid side: Diode rectifier** GR
(= firing angle $\alpha = 0$): generates constant
DC link voltage $U_d$ , which is smoothed by
capacitor:

$$U_d \sim U_{grid} = const.$$

- Motor side inverter WR generates from $U_d$ **by pulse width modulation** a line-to-line voltage between L1, L2, L3. Width of pulses is defined by comparison of **saw tooth signal** $u_{SZ}$ (switching frequency $f_{sch}$) with AC reference signal $u_{ref}$ , which pulsates with desired stator frequency $f_s$ . With comparator a **PWM-signal** is generated to control power switches. *Reference signal is most often sine wave.*

- Amplitude A1 of $u_{ref}$ defines amplitude of fundamental of PWM voltage at motor terminal. So it is varied **proportional** to $f_{mot}$ .

- Grid side: $\cos \varphi = 1$ No power flow back into grid possible. (For that a grid-side inverter and a grid-side inductance is necessary !). Therefore generator braking power has to be dissipated in "brake"-resistors, which are connected in parallel with capacitor in DC link.

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# Generation of PWM voltage

a) Comparison of saw tooth and reference signal lead to PWM control signal for power switches: Potential $\varphi_{L1}(t)$ at terminal L1 varies with that PWM signal

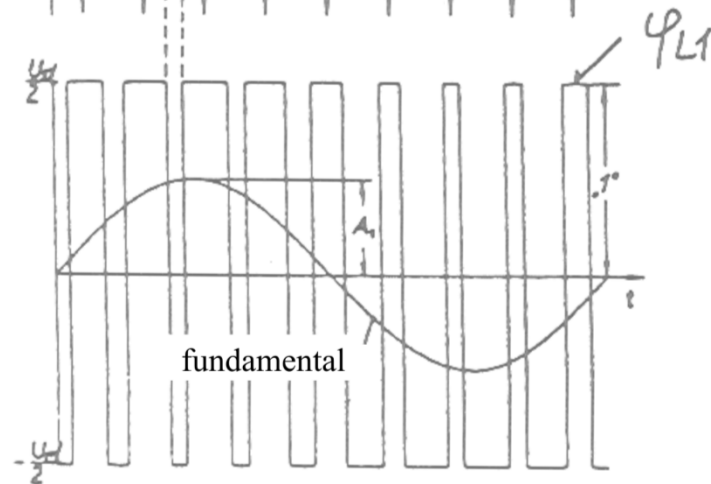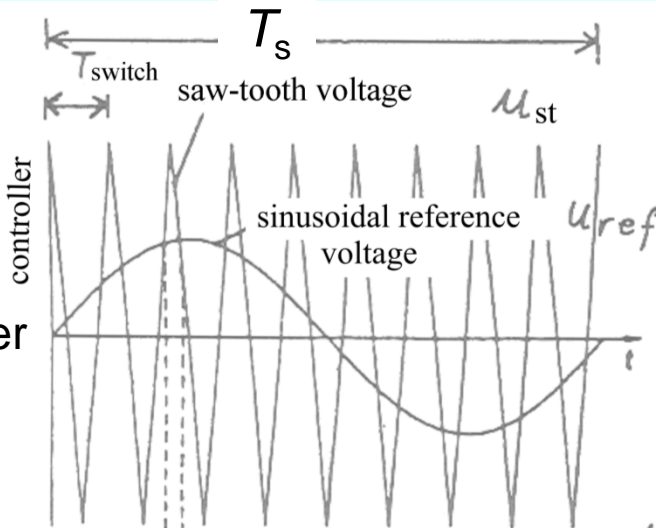b) Difference of two terminal potentials delivers line-to-line voltage $u_{L1-L2}(t)$
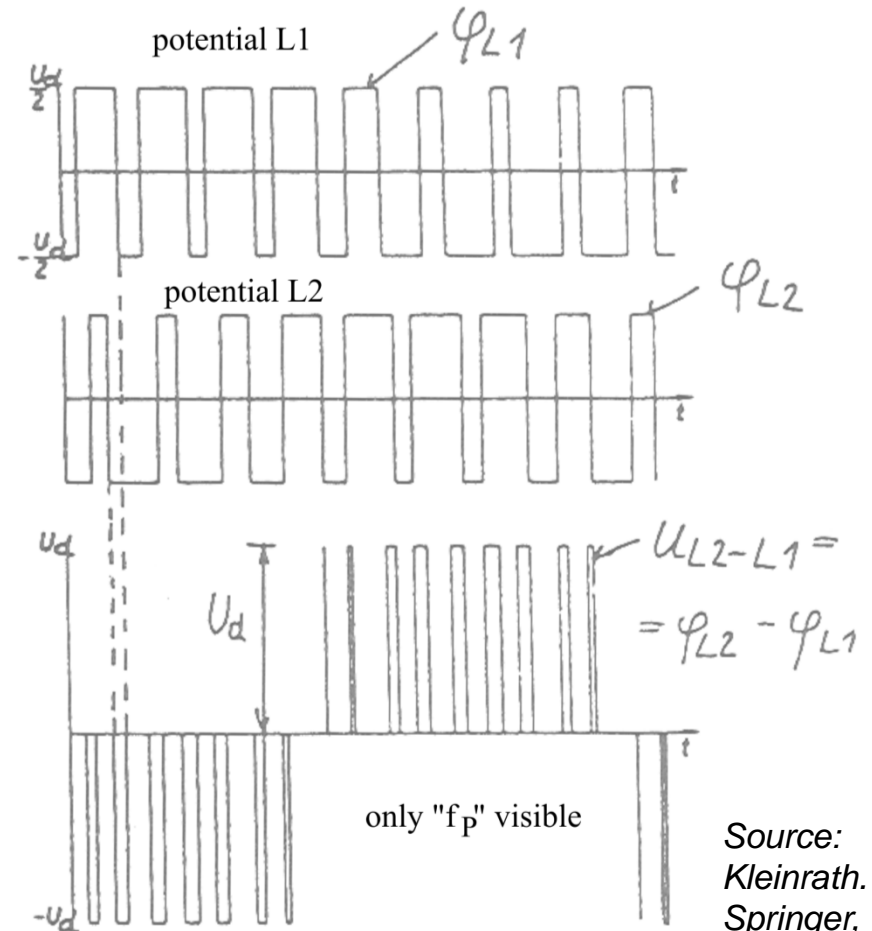
*Example:*

Synchronous switching mode

$T_s/T_{switch}$ = integer

$f_s = f_{mot} = 1/T_s$

$f_T = 1/T_{switch}$



a) $m = A_1 = \hat{U}_{phase}/(U_d/2)$ modulation degree    b)

$f_p = 2/T_{switch}$

*Source:*
*Kleinrath. H.;*
*Springer, 1980*

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# Voltage harmonics: Six-step and PWM

- **Six-step modulation:** FOURIER spectrum of line-to-line inverter output voltage:

| $k$ | 1 | -5 | 7 | -11 | 13 |
|---|---|---|---|---|---|
| $\hat{U}_{Lk} / \hat{U}_{L1}$ | 1 | -0.2 | 0.14 | -0.1 | 0.08 |

- **PWM:** FOURIER spectrum of terminal electric potential $\varphi_{L1}(t)$ and of line-to-line voltage $u_{L1-L2}(t)$ (at modulation degree $m = A_1 = 0.5$ and switching frequency ratio $f_T/f_s = 9$)

| $\lvert k \rvert$ | 1 | 3 | 5 | 7 | 9 | 11 | 13 | 15 | 17 | 19 |
|---|---|---|---|---|---|---|---|---|---|---|
| $\hat{\varphi}_k / (U_d / 2)$ | 0.5 | $<10^{-5}$ | 0.001 | 0.09 | 1.08 | 0.09 | 0.002 | 0.04 | 0.36 | 0.36 |
| $\hat{U}_{L,k} / \hat{U}_{L,k=1}$ | 1 | 0 | 0.002 | 0.18 | 0 | 0.18 | 0.004 | 0 | 0.72 | 0.72 |

*Spectrum of terminal potential $\varphi_L$ shows big amplitude of fundamental, of switching harmonic (k = 9) and at **about twice switching frequency $f_p = 2\, f_T$** (k = 17 and 19).*

$$k = \left\lvert \frac{f_p}{f_s} \pm 1 \right\rvert \quad \Rightarrow \quad k = \lvert 18 \pm 1 \rvert = 17, 19$$

*Voltage harmonics with ordinal numbers, divisible by 3, do **not** occur in line-to-line voltage, because $f_T/f_s$ is divisible by 3 !*
*At high switching frequency $f_T$ the amplitudes of all low voltage harmonics $f_k < f_T$ are small.*
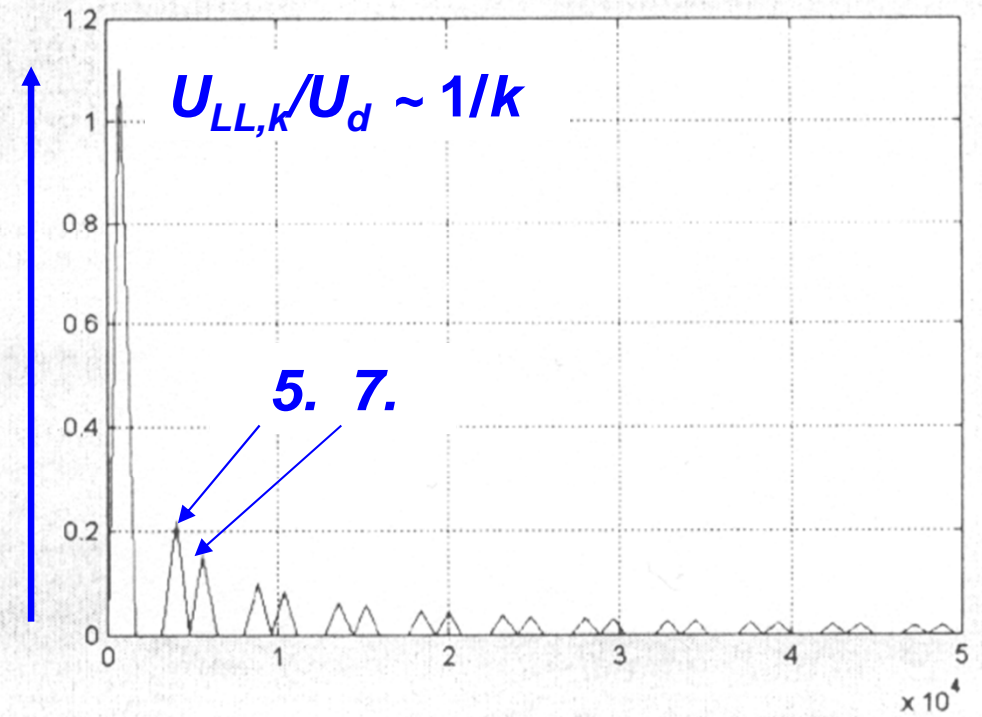
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# FOURIER-Spectrum of voltage harmonics: Six-step operation



$u_{LL}/U_d$
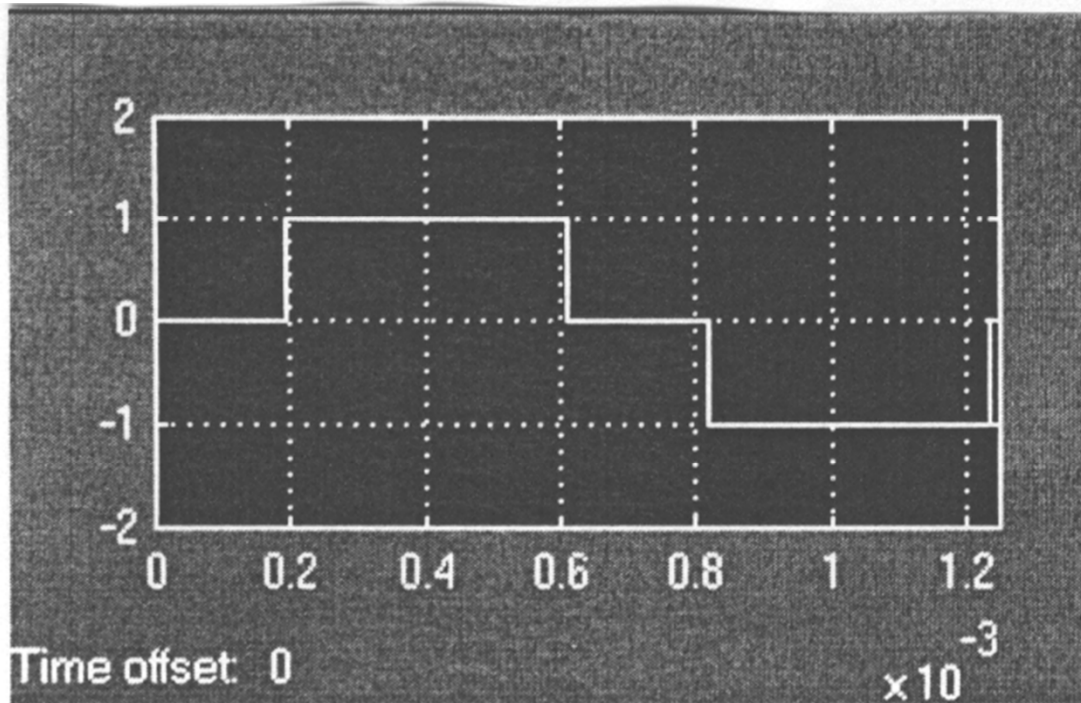
$f_s = 800\ \text{Hz}$

$t\,/\,\text{s}$

$U_{LL,k}/U_d \sim 1/k$

5. 7.

$f\,/\,\text{Hz}$

*Source:*
*T. Lu, PhD Thesis, TU Darmstadt, 2004*

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# FOURIER-Spectrum of voltage harmonics: PWM at 50% modulation degree

$$k = 1 : m = \hat{U}_{phase,1}/(U_d/2) = 0.5 : \hat{U}_{LL,1}/U_d = \sqrt{3} \cdot m/2 = 0.43$$



$u_{LL}/U_d$
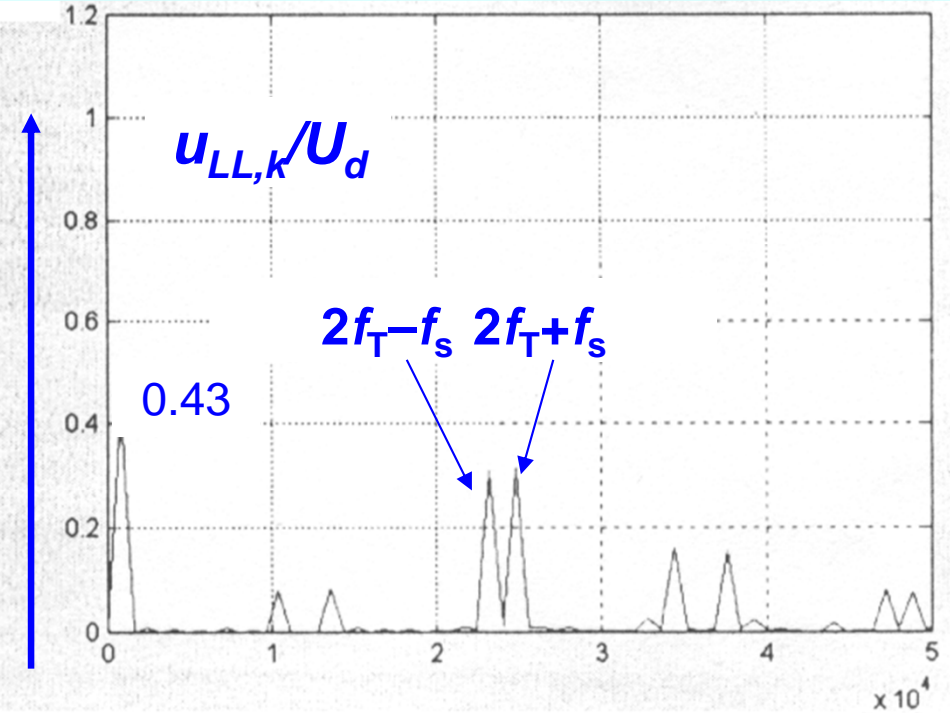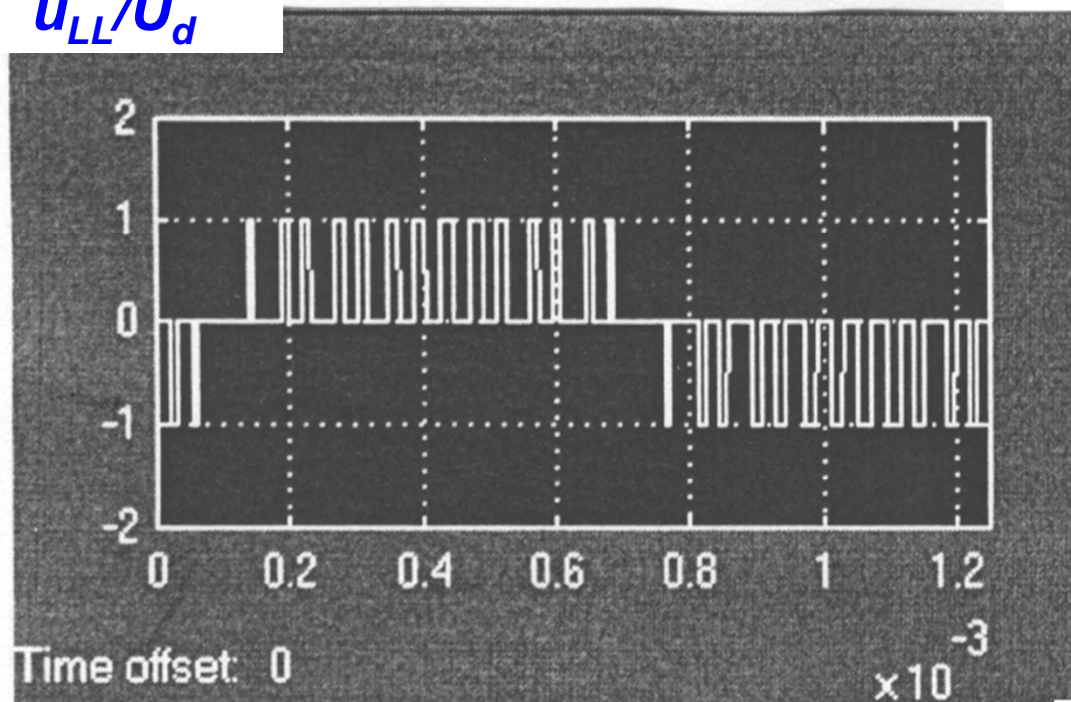
$u_{LL,K}/U_d$

$2f_T - f_s$   $2f_T + f_s$

0.43

$t / s$

$f / Hz$

**Fundamental frequency $f_s$ = 800 Hz**

**Switching frequency $f_T$ = 12000 Hz = 15$f_s$**

*Source:*
*T. Lu, PhD Thesis, TU Darmstadt, 2004*

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# FOURIER-Spectrum of voltage harmonics: PWM at 100% modulation degree

$$k = 1 : m = \hat{U}_{phase,1}/(U_d/2) = 1 : \hat{U}_{LL,1}/U_d = \sqrt{3} \cdot m/2 = 0.866$$



$u_{LL}/U_d$
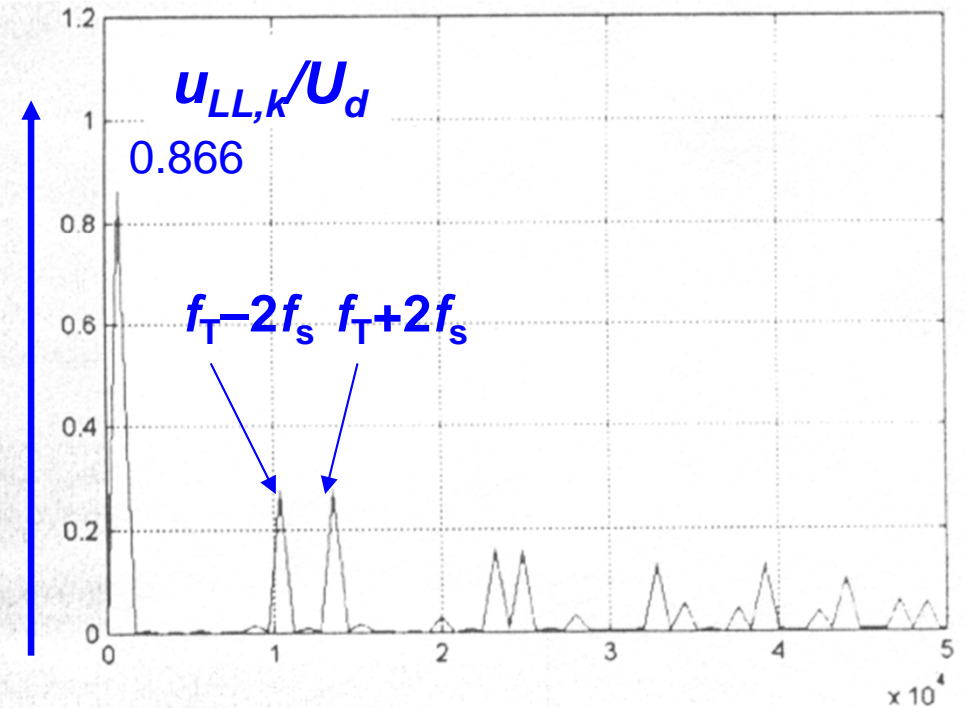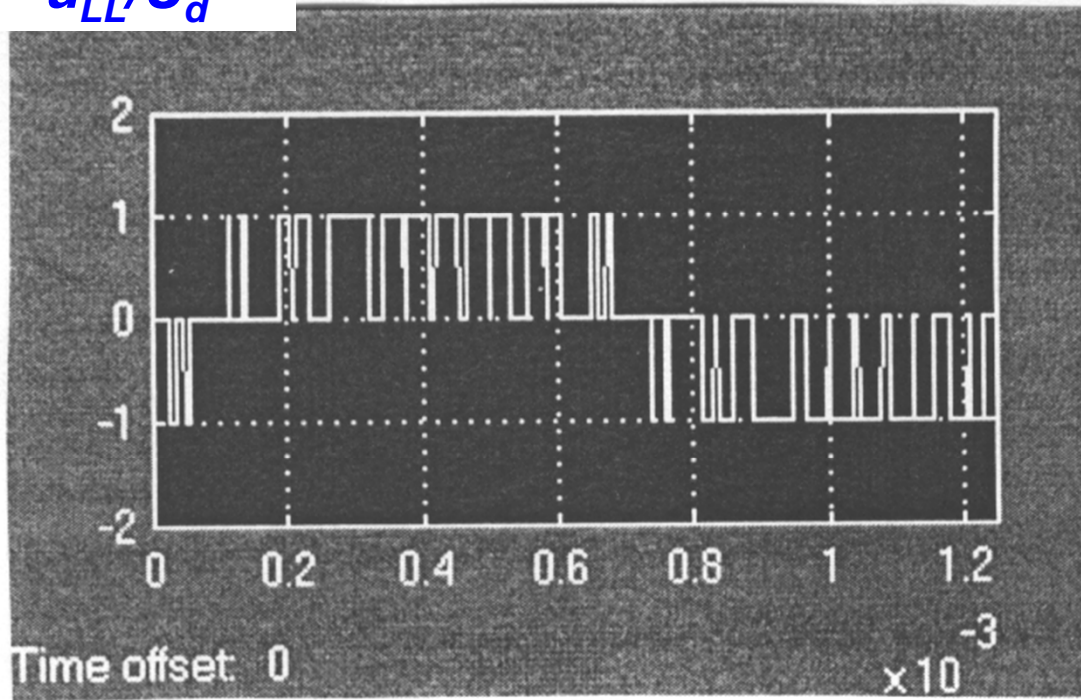
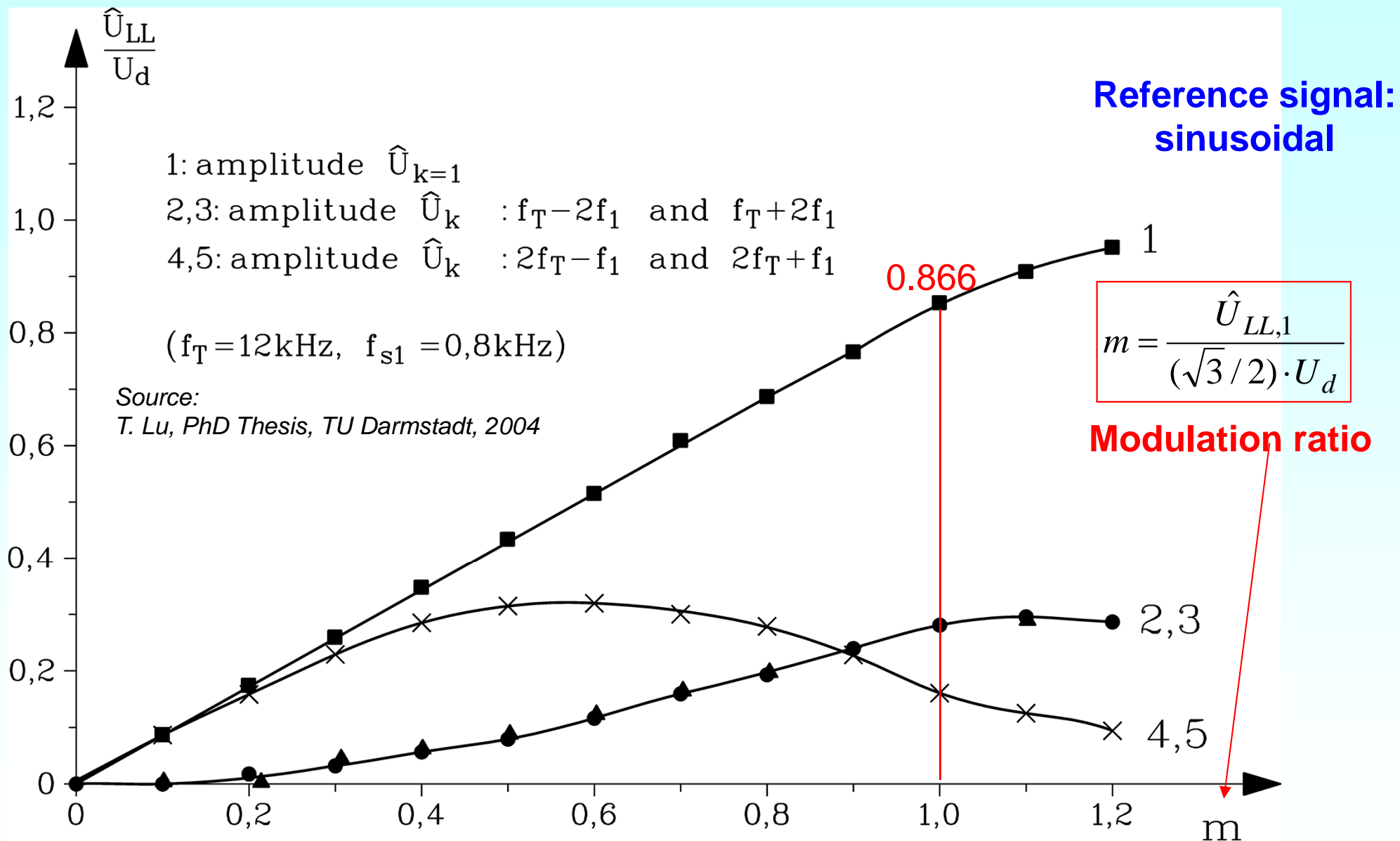$u_{LL,K}/U_d$

0.866

$f_T - 2f_s$  $f_T + 2f_s$

$t / s$

$f / Hz$

**Fundamental frequency $f_s$ = 800 Hz**

**Switching frequency $f_T$ = 12000 Hz = 15$f_s$**

*Source:*
*T. Lu, PhD Thesis, TU Darmstadt, 2004*

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# Example: Voltage harmonics at "synchronous" PWM



$\dfrac{\hat{U}_{LL}}{U_d}$

Reference signal: sinusoidal

1: amplitude $\hat{U}_{k=1}$

2,3: amplitude $\hat{U}_k$ : $f_T - 2f_1$ and $f_T + 2f_1$

4,5: amplitude $\hat{U}_k$ : $2f_T - f_1$ and $2f_T + f_1$

$(f_T = 12\,\text{kHz}, \quad f_{s1} = 0,8\,\text{kHz})$

*Source:*
*T. Lu, PhD Thesis, TU Darmstadt, 2004*

0.866

$$m = \dfrac{\hat{U}_{LL,1}}{(\sqrt{3}\,/\,2)\cdot U_d}$$

Modulation ratio

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# *Example:* Current harmonics at PWM



Reference signal: rectangular

Reference signal: trapezoidal

Current ripple amplitude decreases inverse with increasing switching frequency

Switching ratio:

$f_T/f_s = 6$

Switching ratio:

$f_T/f_s = 9$

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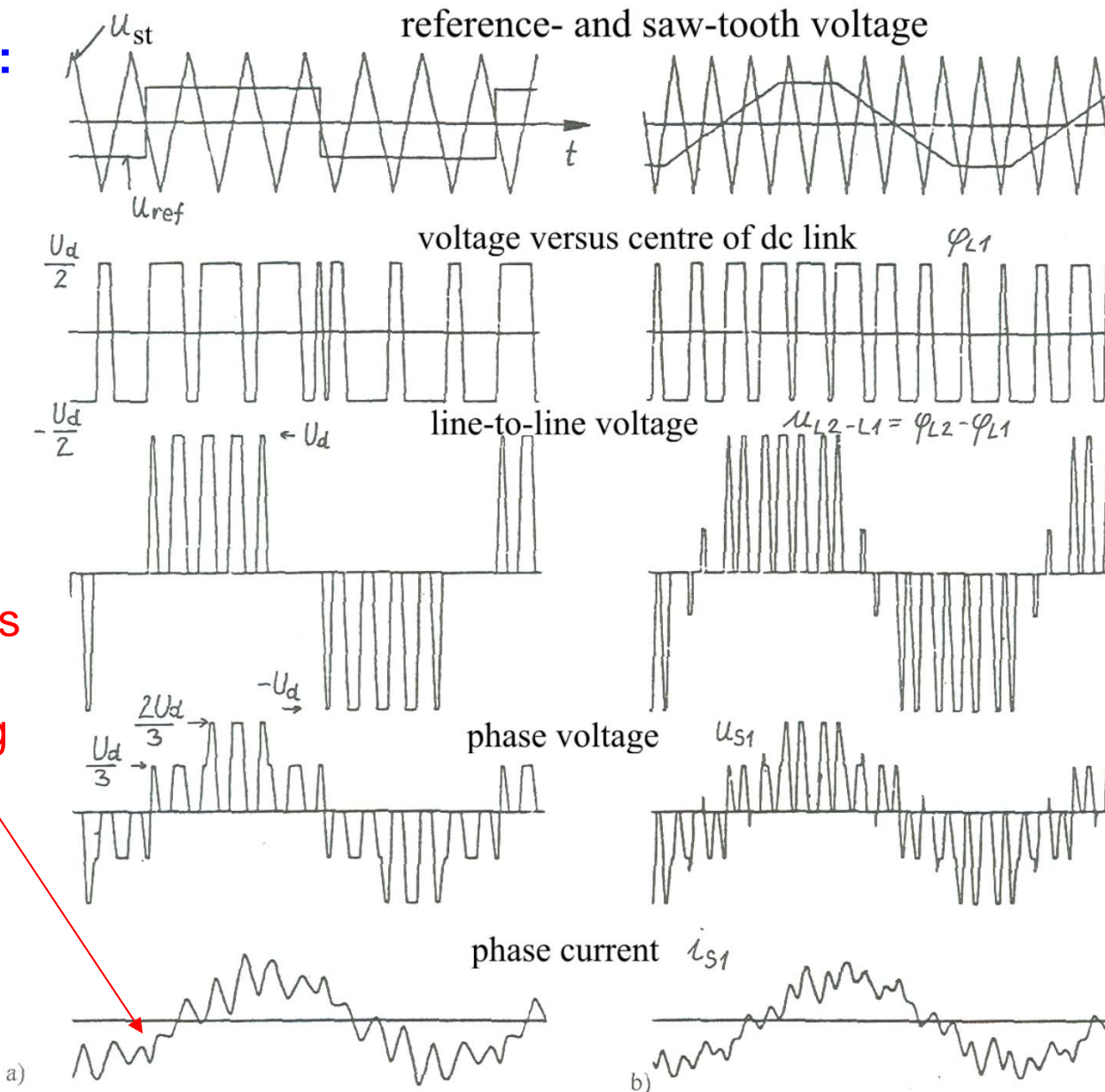
# Voltage harmonics cause current harmonics

- The *voltage harmonics per phase* $U_{s,k}$ (frequency $k$-times fundamental frequency $kf_s$) cause current harmonics per phase $I_{s,k}$ in stator winding.
- These 3-phase harmonic current systems excite in the air gap a "high-speed" magnetic field wave (with pole count $2p$ due to the winding):

  $k^{th}$ **synchronous velocity ("high speed"):** $n_{syn,k} = k \cdot f_s / p$

- **Rotor slip with $k^{th}$ high-speed field** $s_k$ :

$$s_k = \frac{n_{syn,k} - n}{n_{syn,k}} = \frac{k n_{syn} - n}{k n_{syn}} = 1 - \frac{1}{k} \cdot \frac{n}{n_{syn}} = 1 - \frac{1}{k} \cdot (1-s) \approx 1$$

*As harmonic slips $s_k$ are nearly unity, independent of base slip $s$, harmonic current amplitudes $I_{s,k}$ are nearly independent from load. Current harmonics are already present at no-load ($s = 0$) to full extent.*
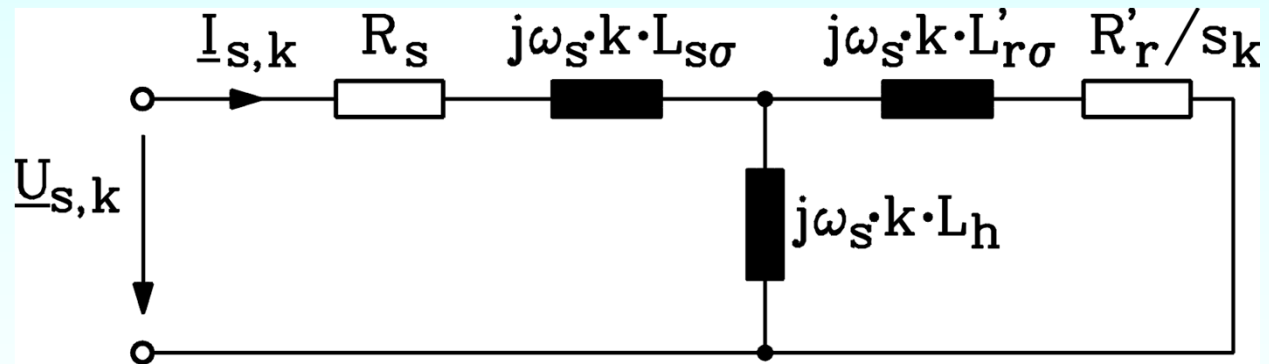


High speed fields $2p$ induce the rotor cage, causing rotor current harmonics with high frequency: $f_{rk} = s_k f_{s,k} \approx f_{s,k}$ ; causing big eddy currents in rotor bars and **big additional rotor losses !**

$$s_k \approx 1 \quad \Rightarrow \quad I_{s,k} \approx \frac{U_{s,k}}{\sqrt{(R_s + R'_r)^2 + (k\omega_s)^2 \cdot (L_{s\sigma} + L'_{r\sigma})^2}} \approx \frac{U_{s,k}}{|k|\omega_s (L_{s\sigma} + L'_{r\sigma})}$$

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# *Example:* Current harmonics at six-step modulation

- Amplitudes of current harmonics at six step operation:

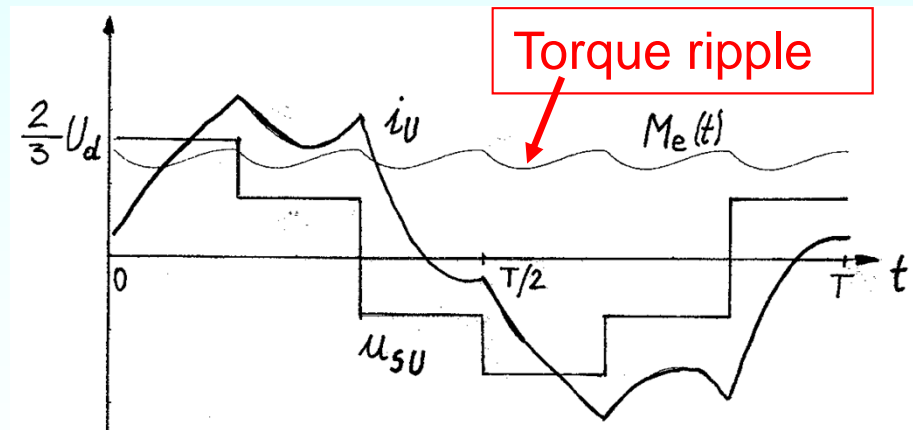$$I_{s,k} \approx \frac{U_{s,k}}{|k|\omega_s(L_{s\sigma}+L'_{r\sigma})} \sim \frac{1}{|k|^2}$$

| $k$ | 1 | -5 | 7 | -11 | 13 |
|---|---|---|---|---|---|
| $\left\| \hat{U}_{Lk}/\hat{U}_{L1} \right\|$ | 1 | 0.2 | 0.14 | 0.1 | 0.08 |
| $s=1, s_k \approx 1:$ $\quad I_{s,k}/I_{s,k=1}$ | 1 | 0.04 | 0.02 | 0.008 | 0.006 |

- *Amplitudes of current harmonics decrease with inverse of square of ordinal number k, because the leakage inductances* **smooth** *the shape of current (= they reduce the current harmonics !)*



Source:
Kleinrath. H.;
Springer, 1980

Torque ripple
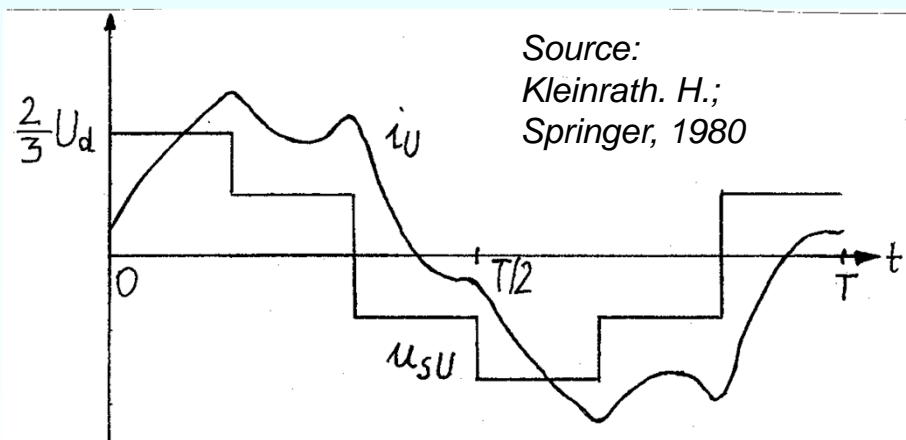
FOURIER sum of 25 current harmonics

Exact solution of dynamic machine equation with space vector theory

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# Additional machine losses at inverter-supply

- Additional cage losses:

Current displacement: AC deep bar resistance increases by ratio $k_{Rk} \cong h_{bar} / d_{E,k}$ with

**penetration depth** (conductivity $\kappa$, permeability $\mu$ of rotor bar conductor)

$$d_{E,k} = 1/\sqrt{\mu \cdot \kappa \cdot \pi \cdot f_{rk}} \approx 1/\sqrt{\mu \cdot \kappa \cdot \pi \cdot |k| \cdot f_s} \qquad .$$

Losses in bar conductors for $k^{th}$ harmonic of voltage/current:

$$P_{ad,r,inv,k} = Q_r \cdot R_{r\sim} \cdot I_{r,k}^2 = Q_r \cdot k_{Rk} \cdot R_{r=} \cdot I_{r,k}^2 \cong Q_r \cdot k_{Rk} \cdot R_{r=} \cdot \frac{U_{s,k}^2}{(|k|\omega_s(L_{s\sigma} + L'_{r\sigma}))^2}$$

$$P_{ad,r,inv,k} \sim \sqrt{|k|} \cdot U_{s,k}^2 / |k|^2$$

- Additional "iron losses": Rotor surface eddy current losses; losses in the end shields, ...

- Additional inter-bar current losses: rather small, as only field fundamental waves $\nu = 1$ are of considerable amplitude for $k \neq 1$. But field waves $\nu = 1$ do not produce much inter-bar currents in skewed cages.

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# Influence of rotor slot shape on additional cage losses AT LOW SWITCHING FREQUENCIES $f_T$ = HIGH POWER MOTORS

_Example:_ $f_{sN}$ = 50 Hz, Model motor 15 kW for investigating losses in high-power motors, synchronous switching, $f_T$ = 750 Hz ($f_T/f_s$ = 15):
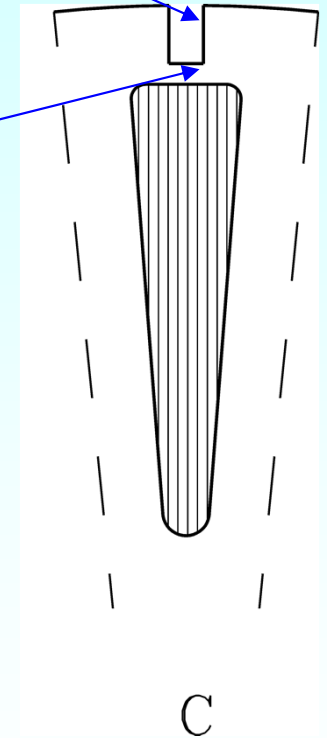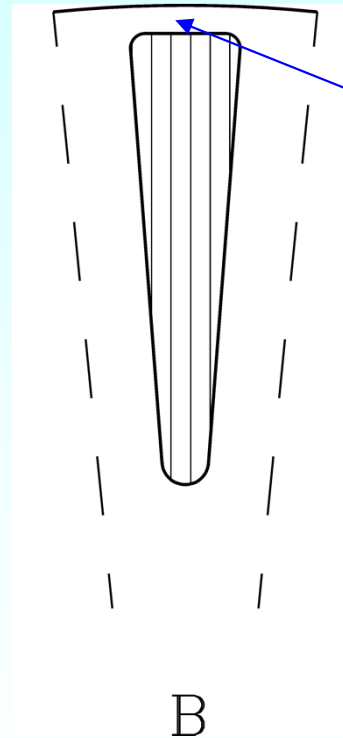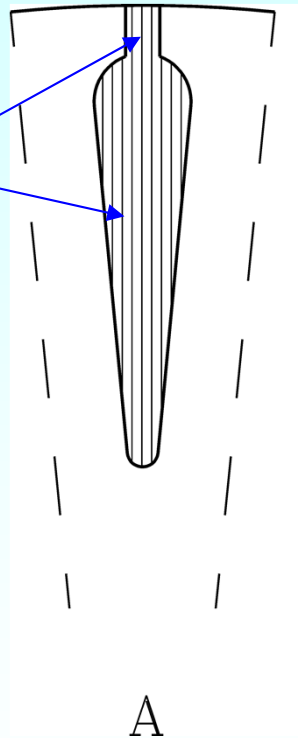Additional cage losses: numerically determined:
Slot shape A: <u>190 W</u>, slot shape C: <u>60 W</u>.

- Slit gives independence of leakage inductance of bridge saturation

- surface eddy currents
- current displacement

- Closed bridge increases leakage inductance = reduction of harmonic currents

_Source:_
_Arkkio, A.; ICEM, 1992_



A                      B                      C

„not so good"          „good"                 „better"

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# Increase of switching frequency to decrease add. losses

_Example_:

- 2-pole motor, 3 kW, 380 V Y; voltage source inverter 8.3 kVA, asynchronous switching ($f_T$ = const.) with rating 8.3 kVA, 400 V.
- Motor is operated at $f_s$ = 50 Hz, slip $s$ = 4.5% at 10 Nm, 3 kW.
- Motor and inverter efficiency were measured by directly measuring input and output power both of motor and inverter for different inverter switching frequencies.

| | grid operation | inverter operation | | |
|---|---|---|---|---|
| $f_s$ / Hz | 50 | 50 | 50 | 50 |
| $f_T$ / Hz | - | 2 400 | 4 800 | 9 600 |
| $f_p = 2f_T$ / kHz | | 4.8 | 9.6 | 19.2 |
| Efficiency motor | 81.9 % | 81.3 % | 81.4 % | 81.4 % |
| Efficiency inverter | - | 96.9 % | 96.8 % | 95.9 % |
| Overall efficiency | **81.9 %** | **78.8 %** | **78.8 %** | **78.1 %** |

_Facit:_

_At $f_p$ = 9.6 kHz the motor efficiency is higher (motor heating is lower), and the overall efficiency is the same as at 4.8 kHz. At 19.2 kHz the current ripple reduction in the motor is already negligible, but inverter switching losses increase considerably._

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# Measured PWM voltage

Measured *Fourier* spectrum of line-to-line voltage, $f_T$ = 2.4 kHz, $f_p = 2f_T$ = 4.8 kHz, 2-pole induction motor, 3 kW, load operation, $U_{LL,rms}$ = 403 V
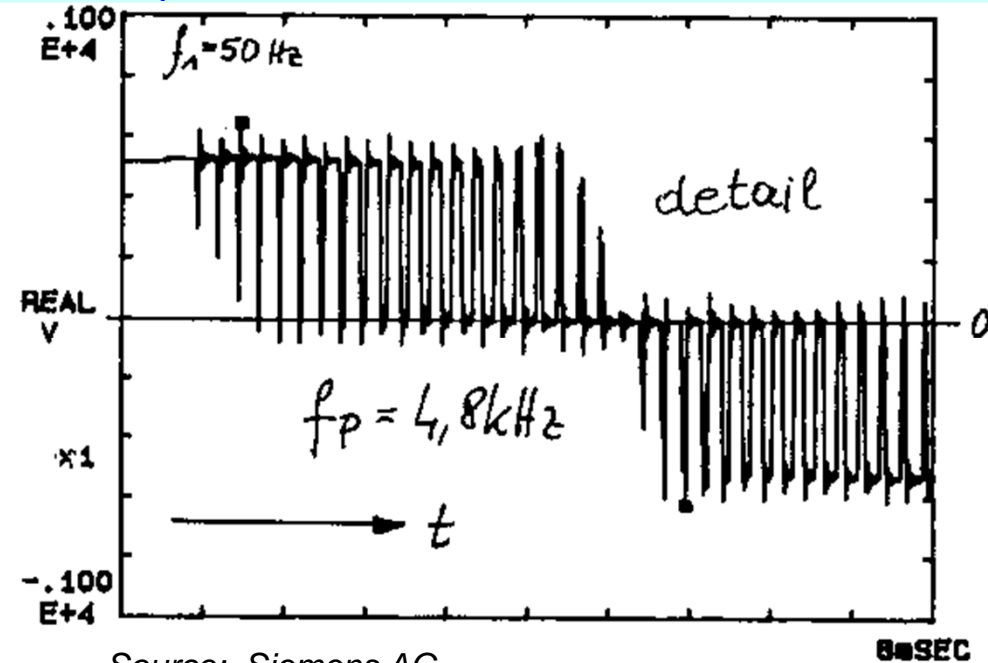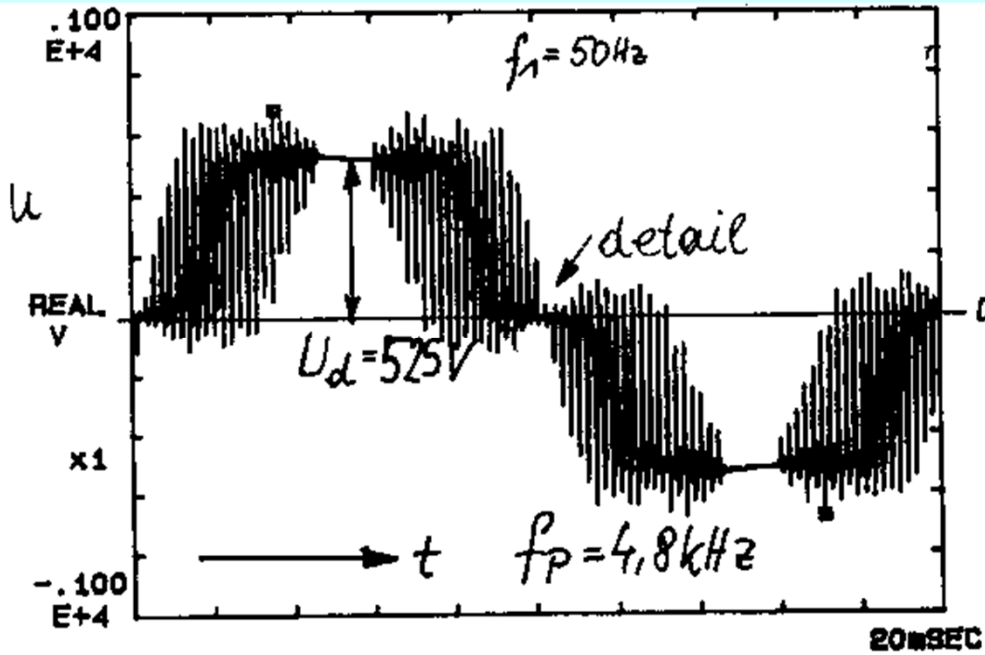


*Source:  Siemens AG*
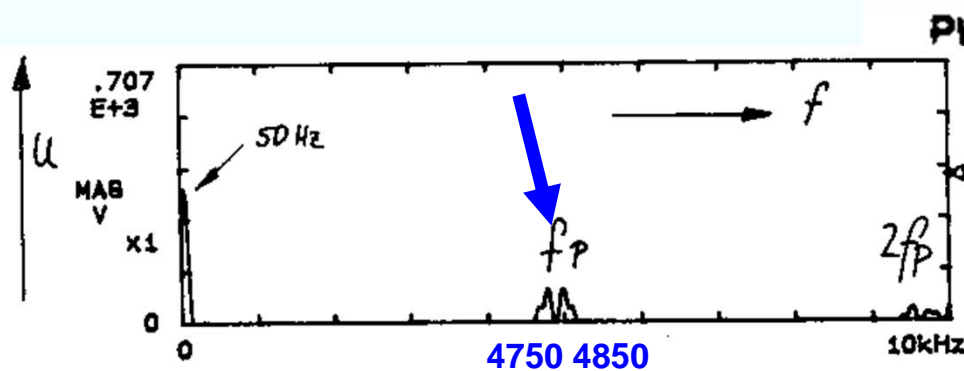


| | | |
|---|---|---|
| 1 | 50Hz | .372E+3  V |
| 2 | 4975 | .913E+2 ←4850 Hz |
| 3 | 4775 | .695E+2 ←4750 Hz |
| 4 | 5075 | .431E+2 |
| 5 | 9525 | .386E+2 |
| 6 | 9725 | .248E+2 |
| 7 | 9825 | .213E+2 |
| 8 | 275 | .293E+1 |

PWR SPECTRUM

4750 4850

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# Measured motor current at PWM feeding

Measured *Fourier* spectrum of phase motor current, $f_T = 2.4$ kHz, $f_p = 2f_T = 4.8$ kHz, 2-pole induction motor, 3 kW, load operation, r.m.s. current 6.25 A



Source: Siemens AG
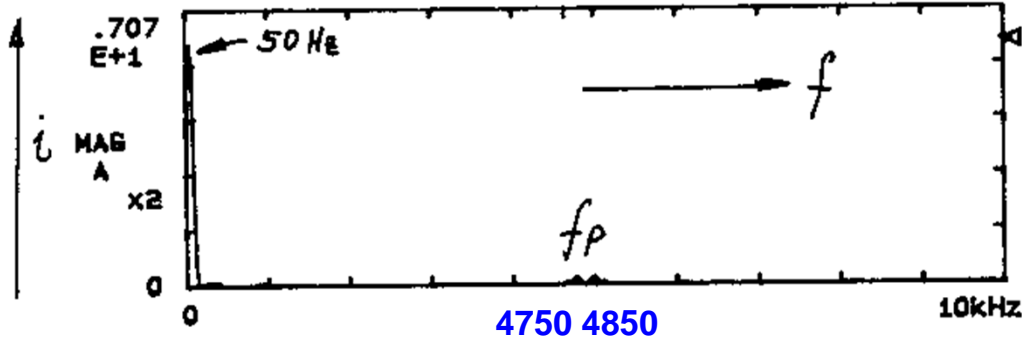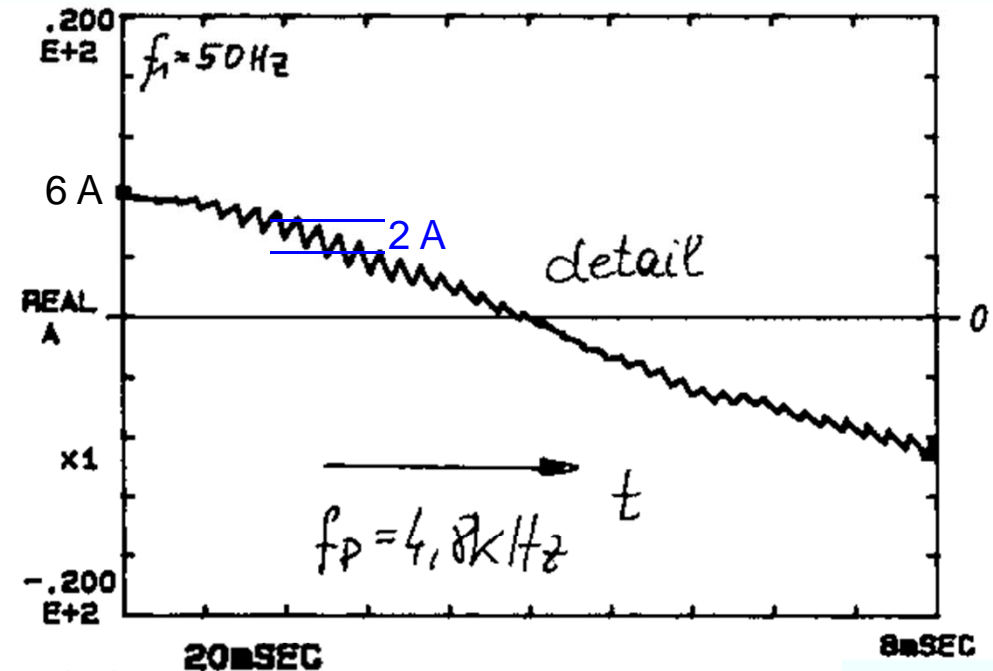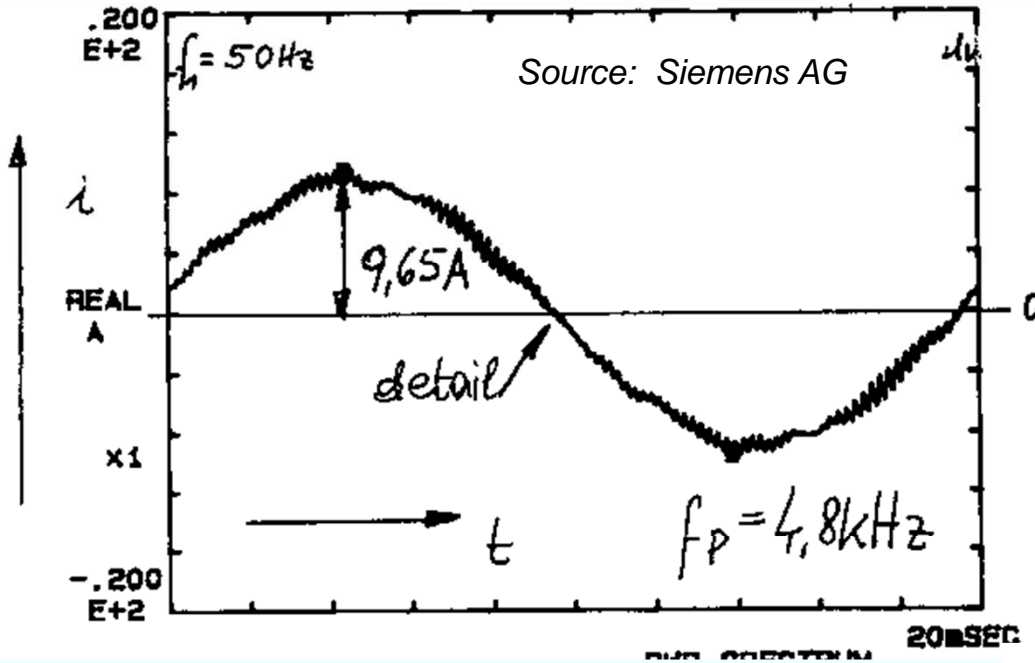
| PWR SPECTRUM | | |
|---|---|---|
| 1 | 50Hz | .620E+1 A |
| 2 | 4775 | .182E+0 ← 4750 Hz |
| 3 | 4975 | .183E+0 ← 4850 Hz |
| 4 | 350 | .113E+0 |
| 5 | 250 | .913E-1 |
| 6 | 925 | .849E-1 |
| 7 | 1125 | .515E-1 |
| 8 | 650 | .281E-1 |
| 9 | 9550 | .352E-1 ← 36 mA |
| 10 | 9750 | .267E-1 ← |

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# Determination of harmonic stator/rotor currents

*Example:*

Measured *Fourier* spectrum of line-to-line voltage & phase motor current, $f_T = 2.4\,kHz$

| | | $U_{LL,k}$ | $I_{s,k}$ | $I_{s,k} / I_{s,k=1}$ |
|---|---|---|---|---|
| $f_s$ | 50 Hz | 372 V | 6.2 A | 100 % |
| $2f_T - f_s$ | 4750 Hz | 89.5 V | 0.18 A | 2.9 % |
| $2f_T + f_s$ | 4850 Hz | 91.3 V | 0.18 A | 2.9 % |
| $4f_T - f_s$ | 9550 Hz | 38.6 V | 0.035 A | 0.6 % |
| $4f_T + f_s$ | 9650 Hz | 24.8 V | 0.027 A | 0.4 % |

-Time harmonic current:

e.g. at 9550 Hz: $I_{sk} = \dfrac{U_{LL,k} / \sqrt{3}}{\omega_{sk} \cdot (L_{s\sigma} + L'_{r\sigma})} = \dfrac{38.6 / \sqrt{3}}{2\pi \cdot 9550 \cdot 0.00961} = \underline{\underline{38.6}}\,mA$ (measured: 35 mA)

- Peak-to-peak current ripple: Two phases in series: $2L_\sigma$ active. $\quad L_\sigma = L_{s\sigma} + L'_{r\sigma}$

$u = L \cdot di / dt \quad \Rightarrow \quad U_d = 2L_\sigma \cdot \Delta i / \Delta t, \quad \Delta t \cong T_p / 2 = 1/(2 f_p):$

$\Delta i = \dfrac{U_d}{4 L_\sigma f_p} = \dfrac{525}{4 \cdot 0.00961 \cdot 4800} = \underline{\underline{2.8 A}}$

$2.8 A /(2 \cdot \sqrt{2}) \approx 1 A \approx \sum_k \left| \underline{I}_k \right|$

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# Measured PWM voltage at increased switching frequency

Measured *Fourier* spectrum of line-to-line voltage, $f_T$ = 4.8 kHz, $f_p = 2f_T$ = 9.6 kHz, 2-pole induction motor, 3 kW, load operation, $U_{LL,rms}$ = 401 V
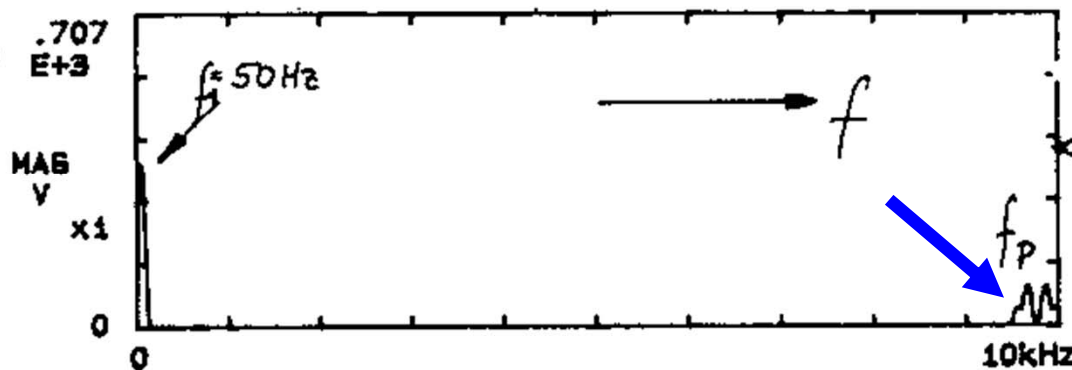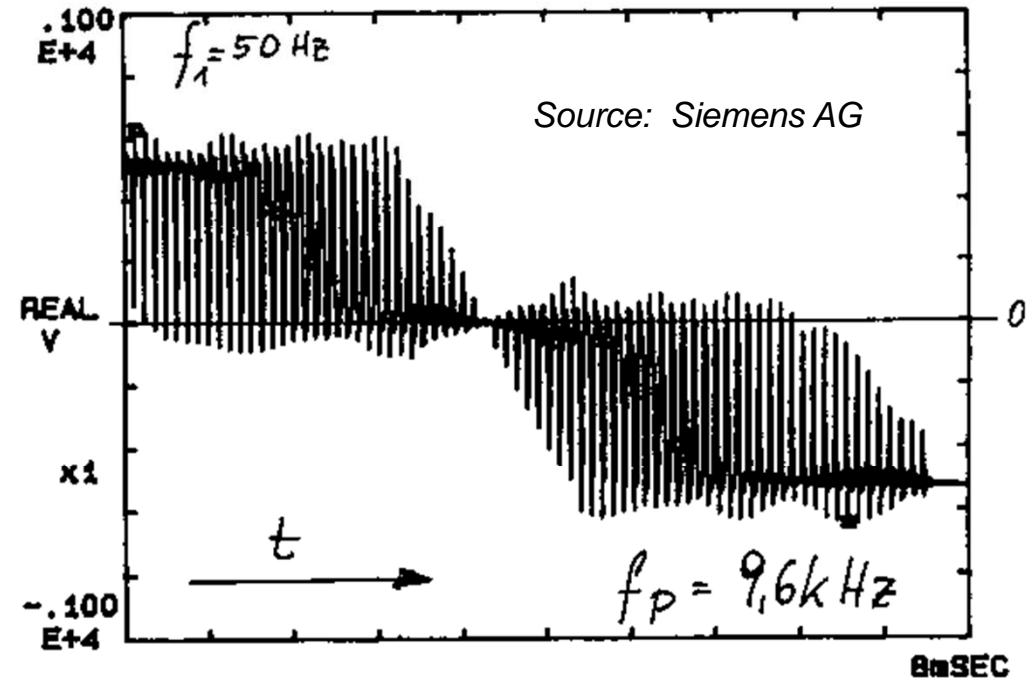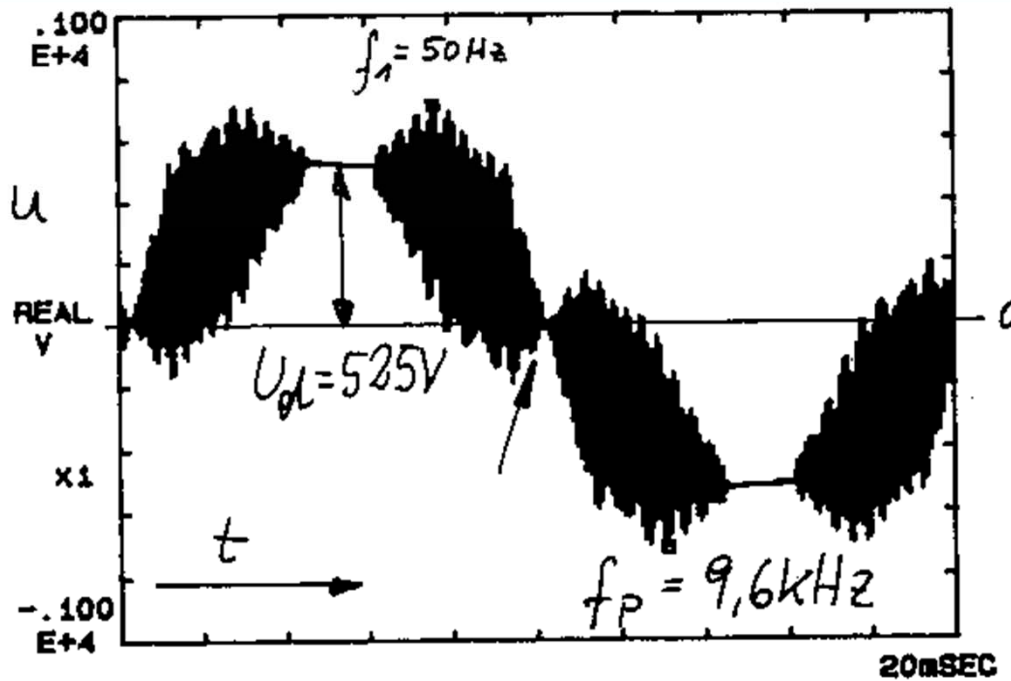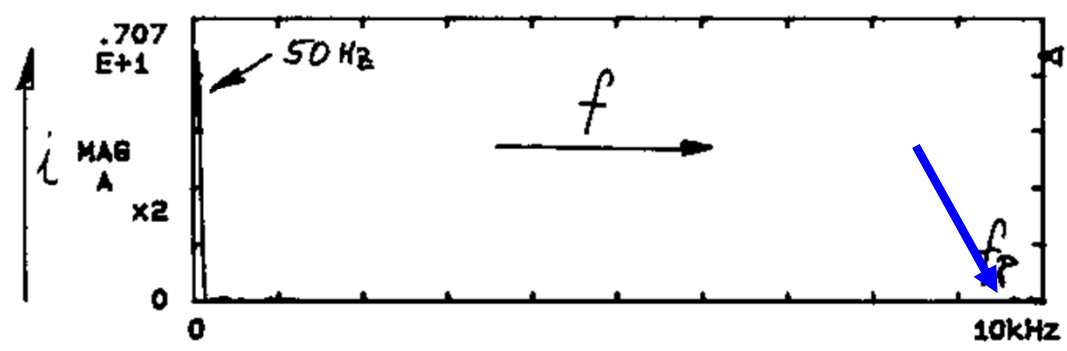


| SPECTRUM | | |
|---|---|---|
| 1 | 50Hz | .372E+3 V |
| 2 | 9675 | .893E+2 |
| 3 | 9675 | .899E+2 |
| 4 | 9975 | .421E+2 |
| 5 | 9575 | .419E+2 |
| 6 | 275 | .258E+1 |
| 7 | 325 | .227E+1 |
| 8 | 675 | .222E+1 |
| 9 | 9275 | .243E+1 |

9650 Hz
9550 Hz

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# Measured motor current at increased switching frequency

Measured *Fourier* spectrum of phase motor current, $f_T = 4.8$ kHz, **$f_p = 2f_T = 9.6$ kHz,**
2-pole induction motor, 3 kW, load operation, **r.m.s. current 6.27 A**



*Source: Siemens AG*

$f_p \rightarrow 2f_p: \Delta i \rightarrow \Delta i/2$
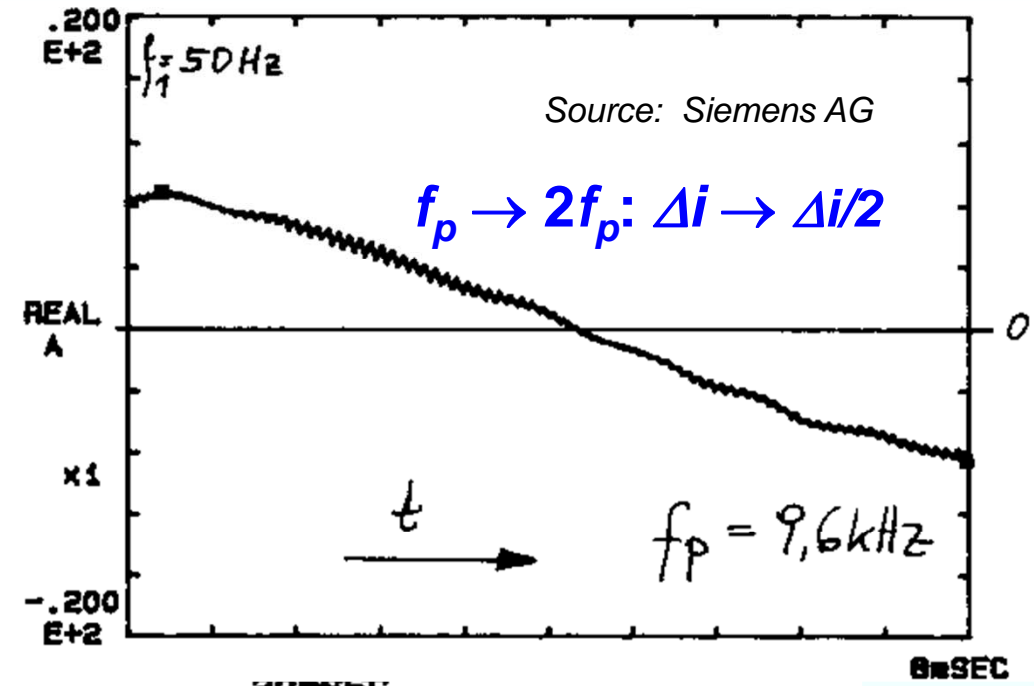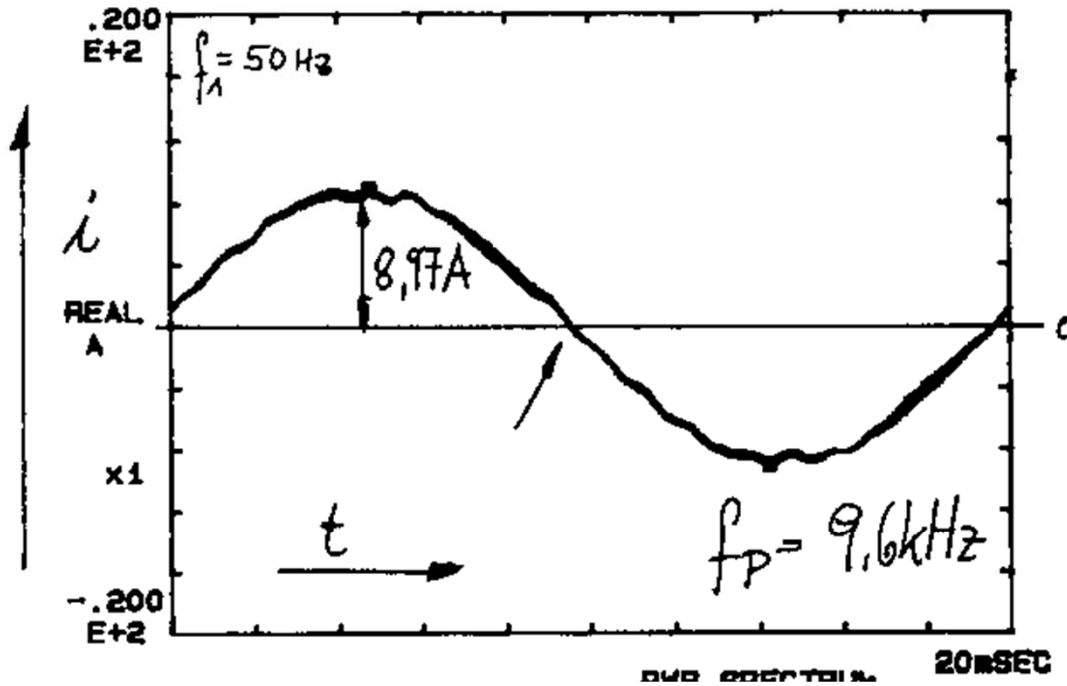
**9650 Hz**

| SPECTRUM | | |
|---|---|---|
| 1 | 50Hz | .625E+1 A |
| 2 | 375 | .915E-1 |
| 3 | 900 | .912E-1 |
| 4 | 9675 | .101E+0 ← |
| 5 | 9675 | .102E+0 ← |
| 6 | 1125 | .495E-1 |
| 7 | 9975 | .481E-1 ← |
| 8 | 675 | .292E-1 |

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# Measured motor current at a) PWM and b) grid operation

Measured *Fourier* spectrum of phase motor current, $f_T = 4.8$ kHz, $f_p = 2f_T = 9.6$ kHz,
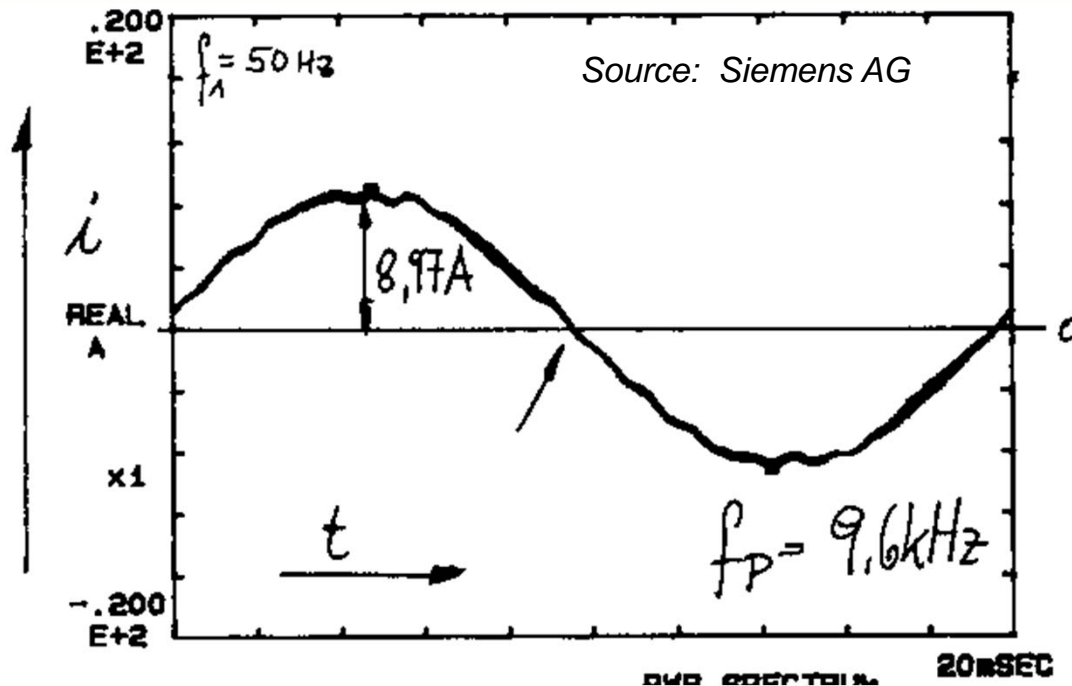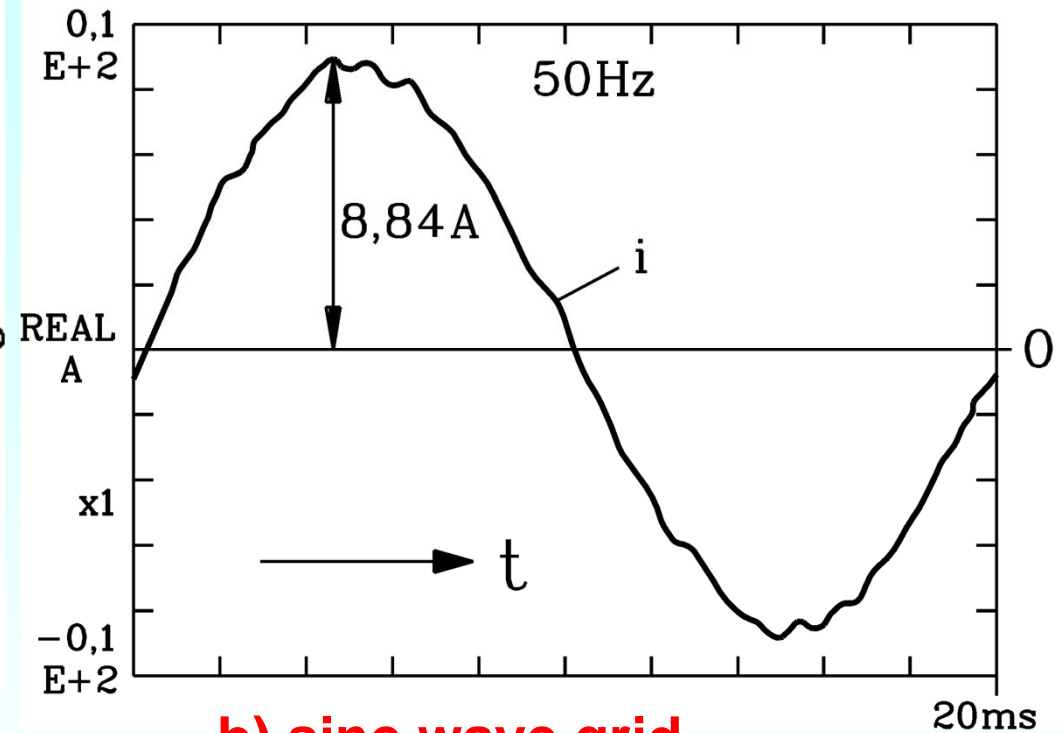2-pole induction motor, 3 kW, load operation, r.m.s. current 6.27 A



**a) PWM**                    **b) sine wave grid**

- Nearly no difference between current wave form at PWM with high switching frequency and sine wave voltage operation.
- Harmonic content in the stator current is caused mainly by rotor slot harmonics of the air-gap field, excited by the fundamental rotor current.
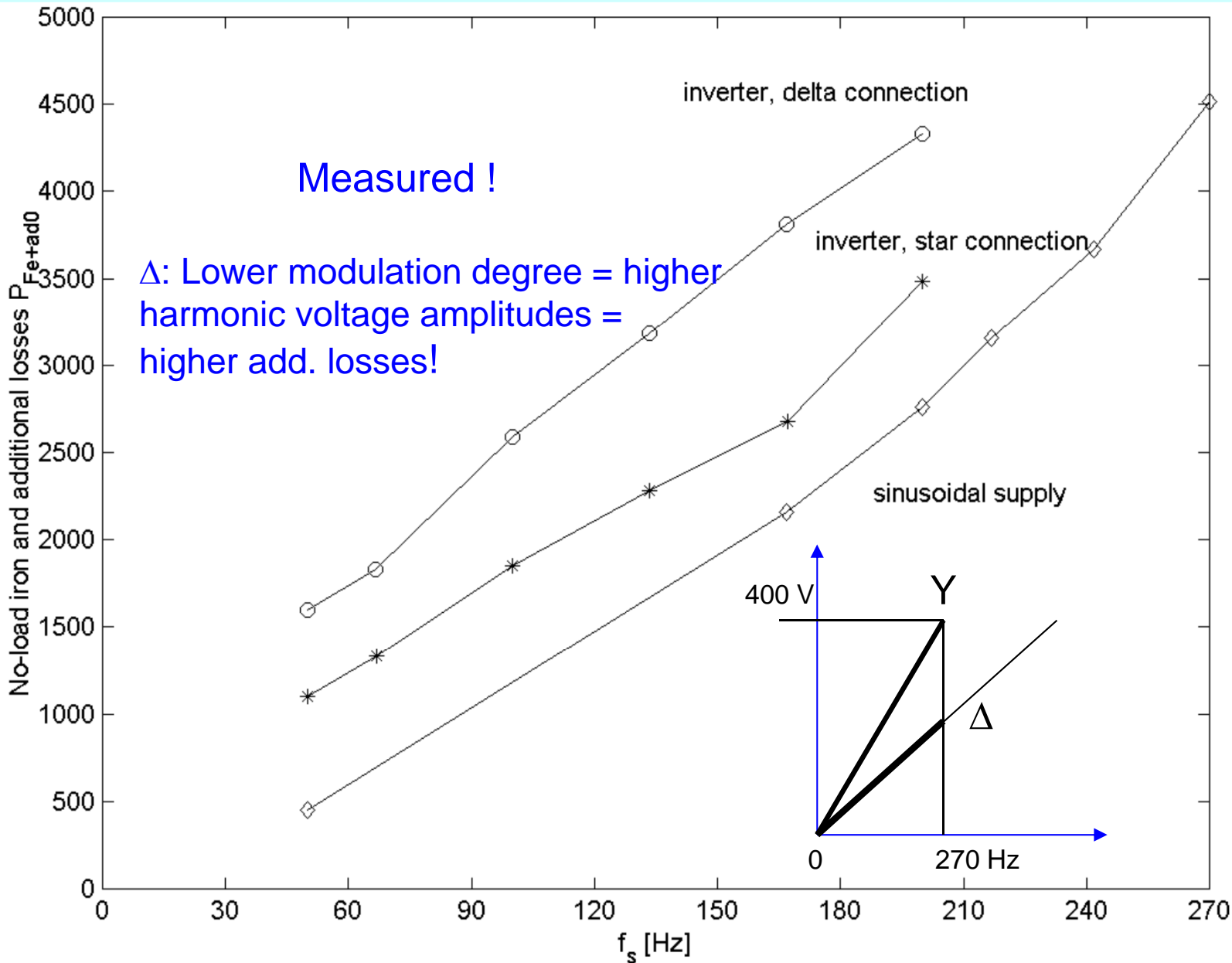
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# No-load iron and additional losses at sinus and inverter supply



Measured !

$\Delta$: Lower modulation degree = higher harmonic voltage amplitudes = higher add. losses!

inverter, delta connection

inverter, star connection

sinusoidal supply

400 V    Y

$\Delta$

0    270 Hz

No-load iron and additional losses $P_{Fe+ad0}$

$f_s$ [Hz]

270 kW, 400 V, 2-poles, high-speed induction machine,
max. 16000/min

Comparison of no-load iron and additional losses
a) Sinus supply
b) IGBT-PWM-Inverter-supply, 4 kHz, star connection Y,
c) delta connection D.

Dependence of stator fundamental frequency $f_s$. During measurements air gap flux density had been maintained constant.

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# Inverter-induced torque ripple

- Fundamental air gap field amplitudes $B_{\delta sk,\nu=1}$ of stator harmonic currents $I_{sk}$ produce with rotor harmonic currents $I_{rk}$ tangential *Lorentz*-forces, and therefore additional torque components.

- Stator field $B_{\delta sk=1,\nu=1}$, excited by fundamental current $I_{s,k=1}$, generates with rotor harmonic current $I_{rk}$ of DIFFERENT ordinal number $k$ a **pulsating** torque $M_{e,1k}$, which is proportional to product $M_{e,1k} \sim B_{\delta sk=1,\nu=1} \cdot I_{rk}$ or due to $B_{\delta sk=1,\nu=1} \sim I_{sk=1}$ it is $M_{e,1k} \sim I_{s1} \cdot I_{rk}$ . Pulsating frequency: $\boxed{f_{1k} = \left| f_s - f_{sk} \right|}$

- As $f_{1k} = \left|1-k\right|f_s = 6\left|g\right|f_s$, the pulsating torque occurs with multiples of six time stator fundamental frequency.

- Each two harmonics contribute with their torque amplitudes to one resulting pulsating torque with the sum of both torque amplitudes:
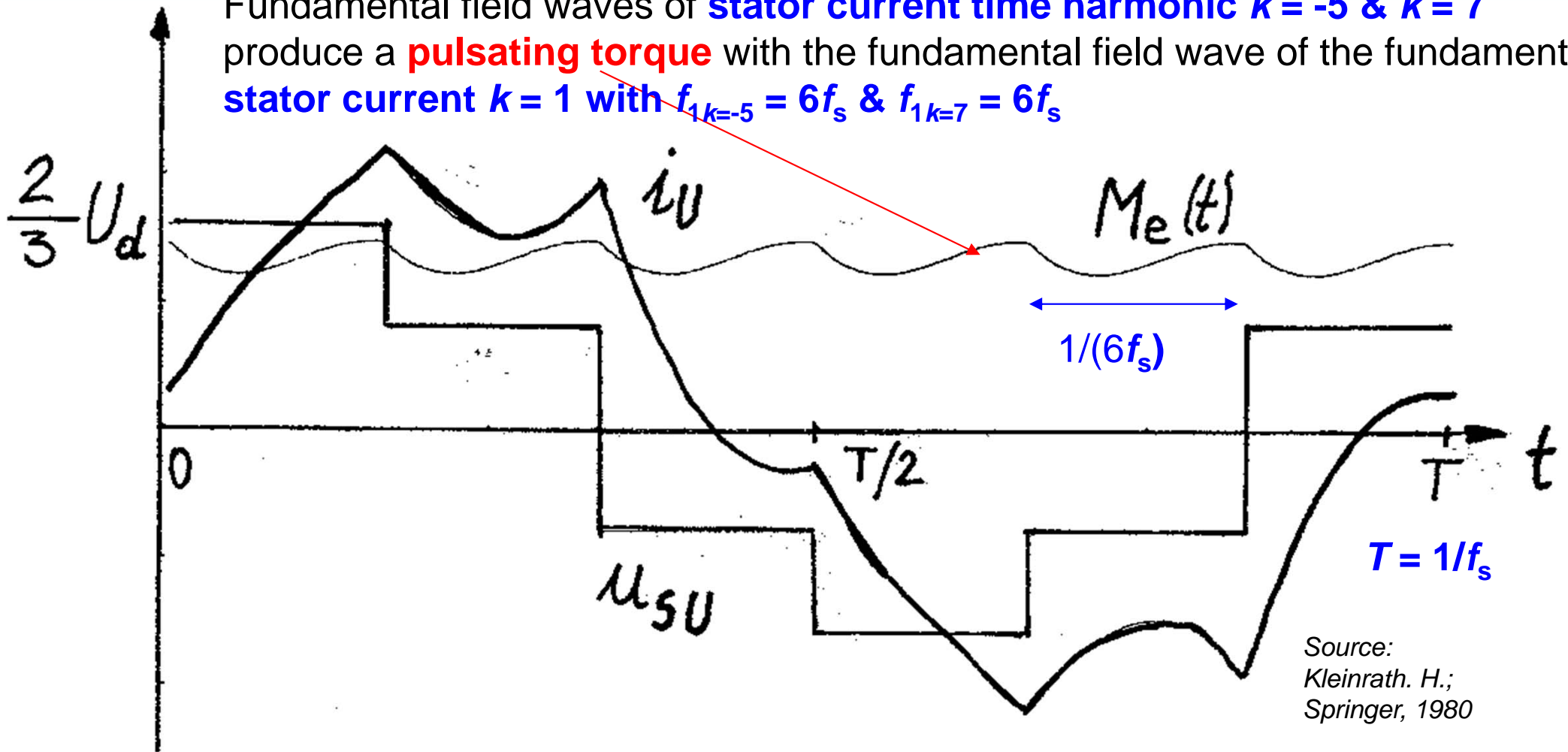
*Example:*

$k = -5$ and $k = 7$: $\hat{M}_{e,1,-5}$, $\hat{M}_{e,1,7}$, frequency $6f_s$

resulting amplitude $\hat{M}_{e,6f_s} = \hat{M}_{e,1,-5} + \hat{M}_{e,1,7}$ .

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# Torque ripple amplitude at six-step operation

Fundamental field waves of **stator current time harmonic $k = -5$ & $k = 7$** produce a **pulsating torque** with the fundamental field wave of the fundamental **stator current $k = 1$ with** $f_{1k=-5} = 6f_s$ & $f_{1k=7} = 6f_s$



$$\frac{2}{3} U_d$$

$i_U$

$M_e(t)$

$1/(6f_s)$

$u_{SU}$

$T/2$

$T = 1/f_s$

*Source:*
*Kleinrath. H.;*
*Springer, 1980*

**Ripple frequency = 6-times fundamental frequency and multiples**

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# Air-gap torque ripple amplitude

$$M_{e,1k} = \frac{p\,\tau_p \cdot Q_r \cdot \hat{I}_{rk} \cdot B_{\delta s} \cdot l_{Fe}}{2\pi} \cdot \sin((1-k)\omega_s t)$$

$$B_{\delta s} = B_{\delta s,k=1,\nu=1}$$

*Example:*

8-pole induction motor, 440 V Y, 60 Hz, 2.6 kW, 5.6 A, 28.4 Nm, 860/min, inverter supply at $f_s = 50$ Hz, $U_d = 525$ V, PWM, switching frequency $f_T = 2.4$ kHz

Motor data: $m_s = 3$, $Q_s/Q_r = 48/44$, $l_{Fe} = 80$ mm, $L_\sigma = 19.6$ mH, $k_{ws} = 0.933$, $N_s = 344$,

stator field fundamental: $B_{\delta s} = 1$ T

$$f_{1k} = \left| f_s - f_{sk} \right| = \left| 1 - k \right| f_s = 96 f_s = 4.8 kHz$$

$$\hat{I}_{rk} = -\frac{m_s N_s k_{ws}}{Q_r/2} \cdot \frac{\hat{U}_{s,k}}{|k|\omega_s L_\sigma}$$

$2f_T/f_s \pm 1 = 95, 97$: $k = -95, 97$

Air gap torque ripple: 1.68/28.4 = 5.9 %
Shaft torque ripple: 0.0015 Nm ( = 0.005 %)
due to rotor inertia and elastic coupling

| $|k|$ | $f_k$ | $\hat{U}_{s,k}$ | $\hat{I}_{rk}$ | $\hat{M}_{e,1k}$ | $\hat{M}_{e,6|g|f_s}$ | $f_{1k}$ |
|-------|-------|-----------------|----------------|------------------|-----------------------|----------|
| - | Hz | V | A | Nm | Nm | Hz |
| 95 | 4750 | 73.1 | 5.5 | 0.85 | | |
| 97 | 4850 | 74.2 | 5.4 | 0.83 | | |
| | | | | | 1.68 | 4800 |

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# Shaft-torque much smoother than air-gap torque



$$\gamma = \gamma_M - \gamma_L \qquad \omega = 2\pi f_{1k}$$

$$\omega_0 = \sqrt{c \cdot \frac{J_M + J_L}{J_M \cdot J_L}}$$
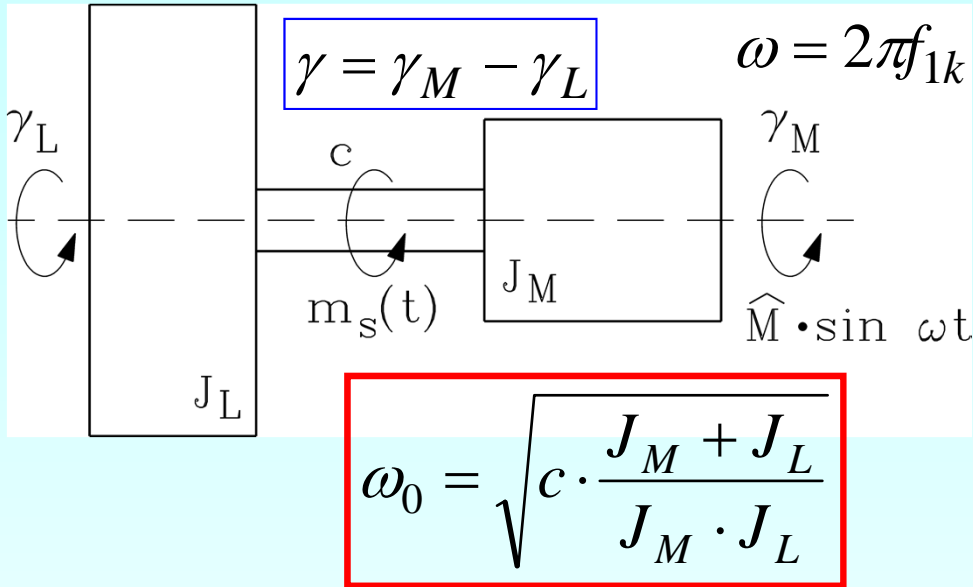
Rotor of motor coupled to rotating load via an elastic coupling

- coupling stiffness $c$

- inertia of motor and load $J_M$, $J_L$
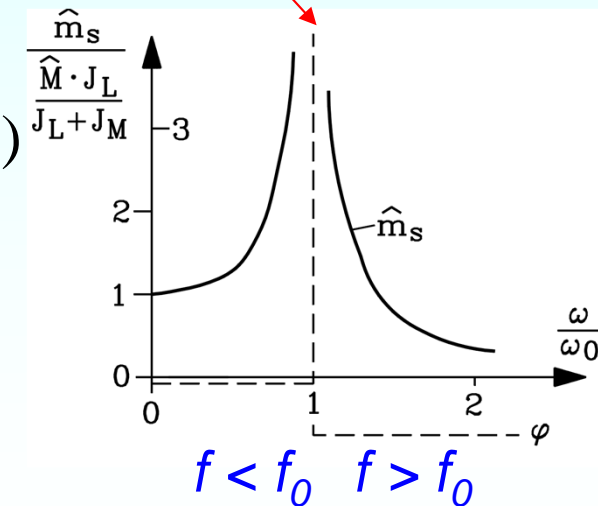
**Resonance frequency $f_0$:**
big … small motors: 20 Hz … 200 Hz

**Pulsating shaft torque:** $\quad m_s(t) = c \cdot \gamma(t) = \dfrac{\hat{M}}{J_M} \cdot \dfrac{c}{\omega_o^2 - \omega^2} \cdot \sin(\omega t)$

**Low exciting frequency $f < f_0$:** $\quad m_s(t) \cong \hat{M} \cdot \dfrac{J_L}{J_M + J_L} \cdot \sin(\omega t)$

**High exciting frequency $f > f_0$:** $\quad m_s(t) \cong -\dfrac{\hat{M}}{J_M} \cdot \dfrac{c}{\omega^2} \cdot \sin(\omega t)$



$f < f_0 \quad f > f_0$

*Facit: Switching frequency torque ripple smoothed strongly by rotor inertia and elastic coupling above $f_0$ !*

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# Air-gap torque ripple amplitude

Shaft torque ripple is below torsion resonance $f_0$ determined by ratio of load versus motor inertia, above resonance by the inverse square of pulsation frequency

$$f_{1k} < f_0: \ \hat{M}_s \cong \frac{\hat{M}_{e,6|g|f_s} \cdot J_L}{J_L + J_M} \qquad\qquad f_{1k} > f_0: \ \hat{M}_s \cong \frac{\hat{M}_{e,6|g|f_s} \cdot c}{J_M \cdot (2\pi f_{1k})^2}$$

*Example:*

$f_0 = 200$ Hz, $f_s = 10$ Hz, $J_L = J_M$

a) $f_{1k} = 6f_s = 60$ Hz $< 200$ Hz      b) $f_{1k} = 2f_T = 4.8$ kHz $> 200$ Hz.

Torque ripple:

- in air gap:      3.3%                          5.9%

- at the shaft:    1.7%                          0.005%

*Although $6f_s$-torque ripple in the air gap is lower, the shaft torque ripple with $6f_s$ is dominating over high harmonic torque components.*

*Due to speed and frequency variation the torsion resonance might be hit.*

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# Measured shaft torque ripple: Low *n*, high $J_L$

*Example:*  **Low speed 20/min**

Induction motor: 2 pole, 750 W, 2850/min, 2.51 Nm, 400 V D, 50 Hz, 1.6 A
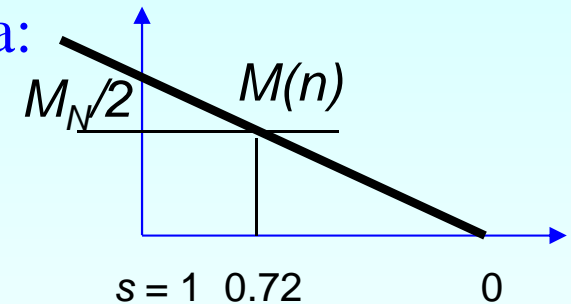
Voltage-source IGBT Inverter: 1500 VA, 2.2 A, 400 V

Motor operated at $f_s = 1.2$ Hz, $n = 20$/min, $s = 0.72$, $M = 1.24$ Nm = 50% rated torque

a) PWM, asynchronous switching: $k = 1 + 6g = 1, -5, 7, -11, 13, ...$   $g = 0, \pm1, \pm2, ...$

b) PWM voltage output pattern is not symmetrical to abscissa:

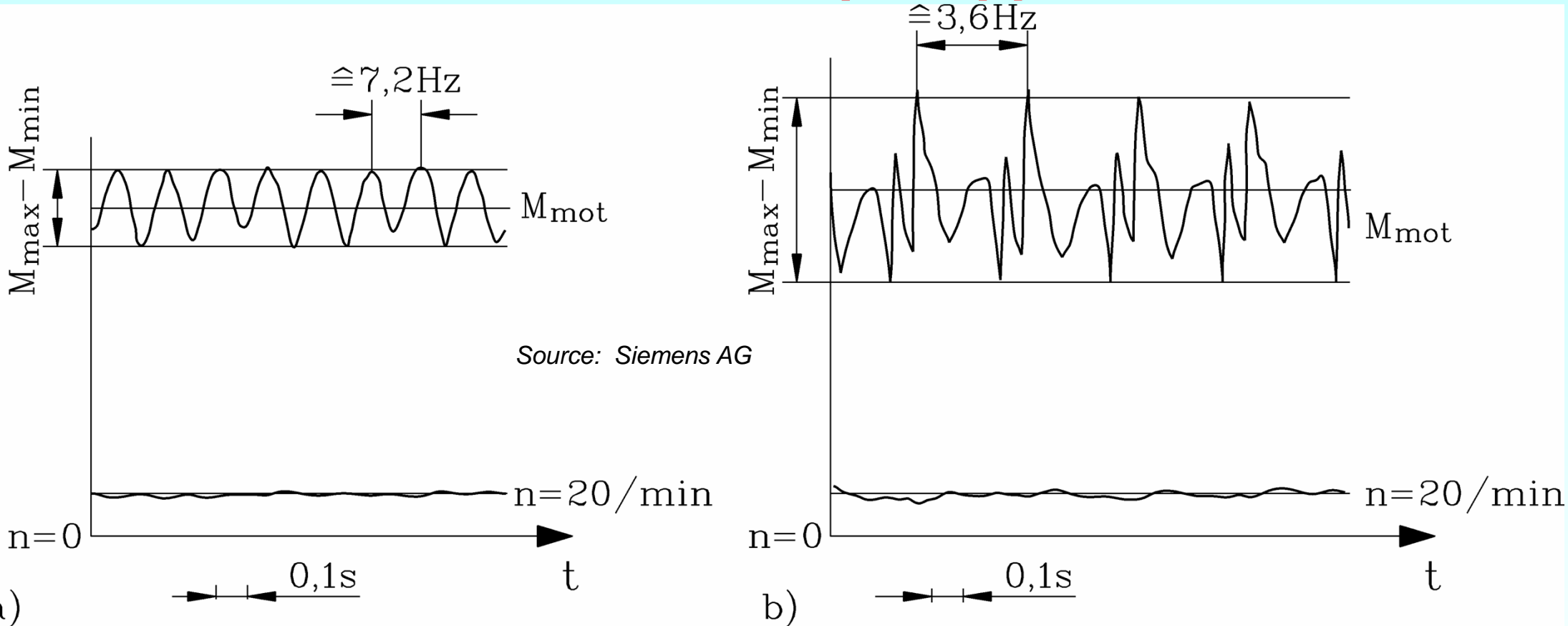Voltage harmonics with even ordinal numbers occur:

$k = 1 + 3g = 1, -2, 4, -5, 7, -8, 10, -11, 13, ...$   $g = 0, \pm1, \pm2, ...$

$M_N/2$   $M(n)$

$s = 1\ 0.72$   $0$

| | a) symmetrical PWM | b) asymmetrical PWM |
|---|---|---|
| Lowest torque ripple frequency | $f_{1k} = 6\|g\|f_s = 6f_s =$ $= 6 \cdot 1.2 = 7.2 Hz$ | $f_{1k} = 3\|g\|f_s = 3f_s =$ $= 3 \cdot 1.2 = 3.6 Hz$ |
| Measured torque ripple $\hat{w}_M$ | 14.4% | 35.0% |
| Torque ripple amplitude | **0.18 Nm** | **0.43 Nm** |

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# Measured torque ripple



Source: Siemens AG

a)

b)

| | a) symmetrical PWM | b) asymmetrical PWM |
|---|---|---|
| Lowest torque ripple frequency | $f_{1k} = 6\|g\|f_s = 6f_s =$ $= 6 \cdot 1.2 = 7.2 Hz$ | $f_{1k} = 3\|g\|f_s = 3f_s =$ $= 3 \cdot 1.2 = 3.6 Hz$ |
| Torque ripple amplitude | **0.18 Nm** | **0.43 Nm** |

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# Inverter-induced acoustic noise

- In addition to the magnetically excited acoustic noise at sinus voltage supply, additional air gap waves $\nu = 1$ of the current harmonics $I_{sk}$ will add to acoustic noise with new tonal frequencies, mainly with inverter pulse frequency $f_p = 2f_T$.

**Stator fundamental field waves ($\nu = 1$):**

a) excited by magnetizing current $\underline{I}_m = \underline{I}_s + \underline{I}'_r$ with stator frequency $f_s$ :

$$B_{\delta sk=1,\nu=1}(x_s,t) = B_{\delta s} \cdot \cos\left(\frac{\pi x_s}{\tau_p} - 2\pi f_s t\right) = B_{\delta s} \cdot \cos\left(\frac{2p\pi x_s}{2p\tau_p} - 2\pi f_s t\right) = B_{\delta s} \cdot \cos\alpha$$

b) excited by magnetizing current $\underline{I}_{mk} = \underline{I}_{sk} + \underline{I}'_{rk}$ with stator harmonic frequency $f_{sk}$ :

$$B_{\delta sk,\nu=1}(x_s,t) = B_{\delta sk} \cdot \cos\left(\frac{2p\pi x_s}{2p\tau_p} - 2\pi \cdot k \cdot f_s t\right) = B_{\delta sk} \cdot \cos\beta$$

Magnetic pull: $f_n(x_s,t) = \dfrac{B^2(x_s,t)}{2\mu_0} \sim \left(\sum\limits_{k=1}^{\infty} B_{\delta sk}\right)^2 \quad \Rightarrow \quad f_{n,1k} \sim \sum\limits_{k,k'=1}^{\infty} B_{\delta sk}^2 + 2B_{\delta sk}B_{\delta sk'} \qquad k \neq k'$

**Radial force density waves,** causing oscillating pull on iron stack: $f_{n,1k} \sim \cos(\alpha \pm \beta)$

$$f_{n,1k}(x_s,t) = \frac{B_{\delta s}B_{\delta sk}}{2\mu_0} \cdot \cos(2r \cdot \frac{\pi x_s}{2p\tau_p} - 2\pi f_{Ton,k}t)$$

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# Vibration mode of inverter-induced acoustic noise

$2p = 4$, $2r = 0$ or $8$.



**Tonal frequency** $f_{Ton,k}$ :

$$f_{Ton,k} = f_s \cdot |1 + k| \qquad \text{for} \qquad 2r = 4p$$

$$f_{Ton,k} = f_s \cdot |1 - k| \qquad \text{for} \qquad 2r = 0$$
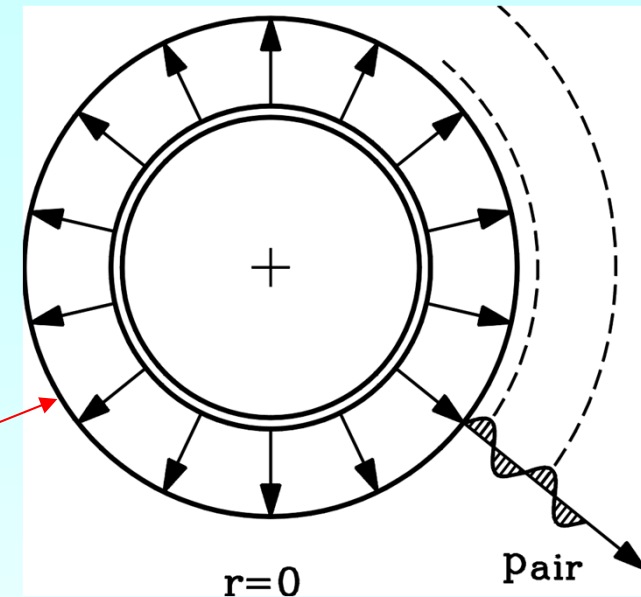
Dominating current harmonic amplitudes at frequencies $f_{sk} = 2f_T \pm f_s$ .

Dominating acoustic noise at

$$f_{Ton,k} = f_s \cdot |1 - k| = f_{sk} - f_s = 2f_T \quad \text{or} \quad f_{Ton,k} = 2f_T - 2f_s .$$

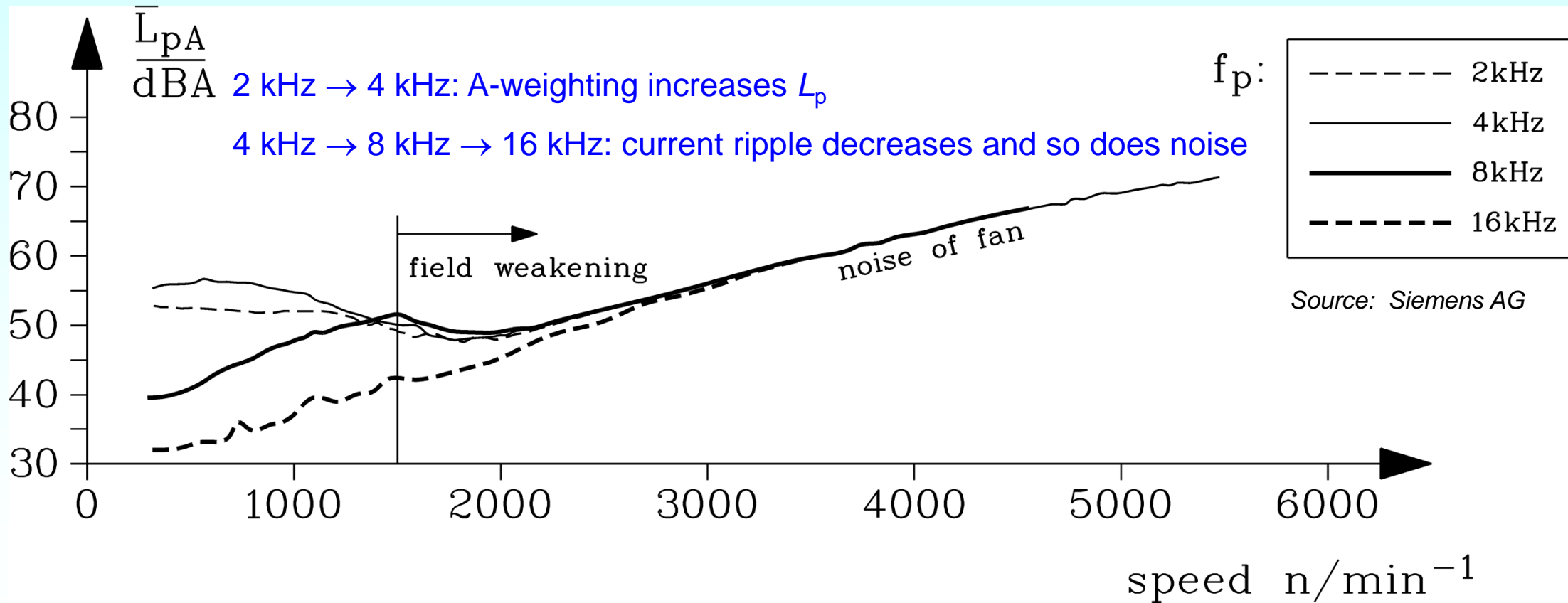*Inverter caused magnetic noise is usually excited with twice switching frequency ("pulse frequency"). Modal vibration is r = 0, so the sound is far reaching and well audible.*

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# Influence of switching frequency on acoustic noise at inverter-operation

Measured A-weighted sound pressure level at 1 m distance:

250 W, PWM inverter-fed induction motor,

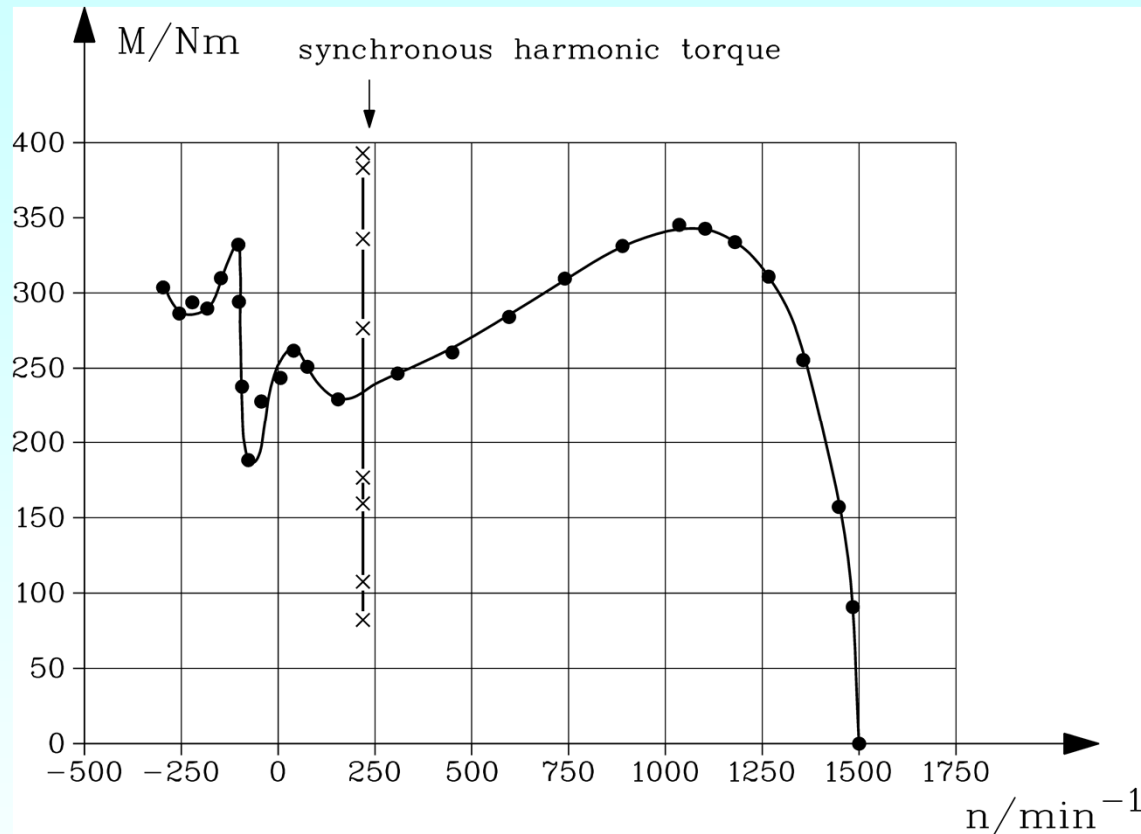Varying switching frequency $f_T$ and pulse frequency $f_p = 2f_T$ .



Source: Siemens AG

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# Influence of space harmonics at inverter-operation

- Asynchronous and synchronous harmonic torque occur at big slip: $s = 0.8 ... 1.2$

- At low speed, full-load slip increases up to unity, so space harmonic torques are in the range of motor operation.

- **Motor current: ~ rated current (not – as during line-start – ~ 500% of $I_N$). So parasitic torque ~ $I^2$ components are negligible.**

*Example:* Unskewed 4-pole cage induction motor, 15 kW, 380 V, D, 50 Hz, 30 A, $Q_s/Q_r = 36 / 28$. Synchronous harmonic torque at slip 0.86: harmonics $\nu = -\mu = 13$, speed 215/min.

|  | *a) 50 Hz line operation* | *b) 1.5 Hz inverter operation* |
|---|---|---|
| Synchronous speed | 1500/min | 45/min |
| speed at rated torque 98 Nm | 1460/min | 5/min |
| slip at rated torque 98 Nm | 2.6 % | 88.9 % |
| rotor frequency at 98 Nm | 1.3 Hz | 1.3 Hz |
| current at slip 0.86 (synchronous harmonic torque) | 550 % rated current | 100% rated current |
| speed at synchronous torque | (1-0.86)1500=215/min | (1-0.86)45=6.3 /min |
| Synchronous torque amplitude | **150 Nm** | **150·(1/5.5)² = 5 Nm** |

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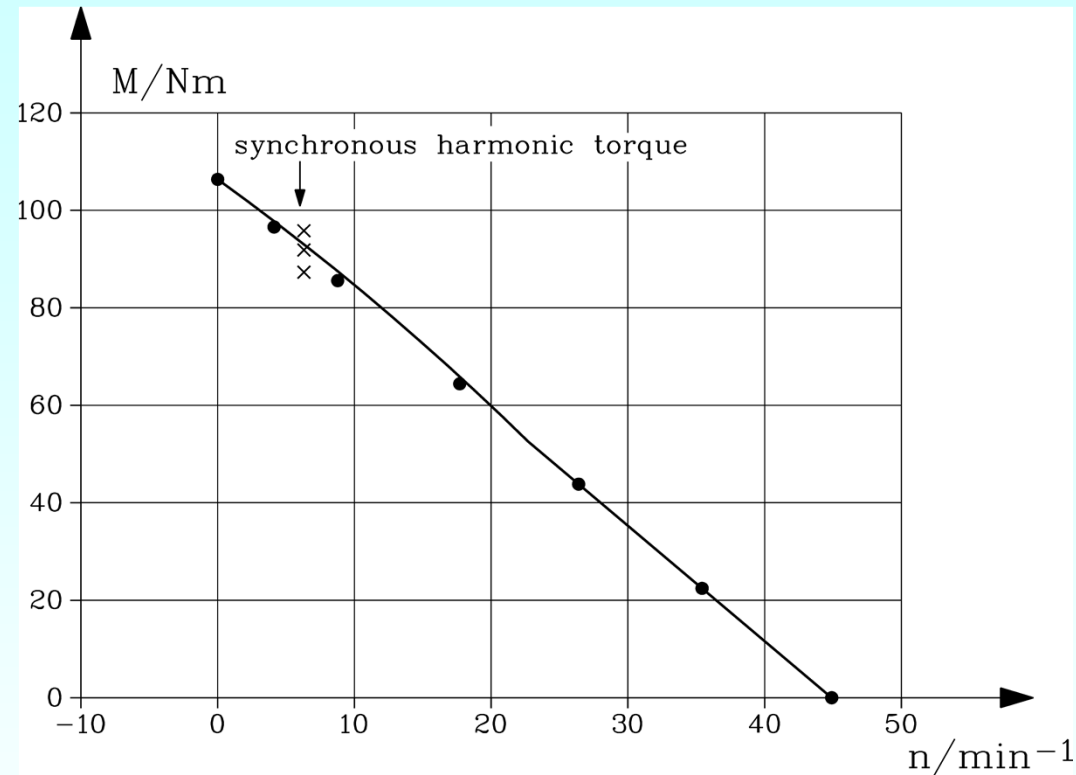Institut für Elektrische Energiewandlung • FB 18

# SMALL harmonic torque at inverter operation with rated current



**50 Hz**

**550 % rated current**

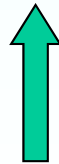**Big** synchronous harmonic torque

**1.5 Hz**

100 % rated current

**Small** synchronous harmonic torque

*Source:*
*Arkkio, A.; ICEM, 1992*

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# Unskewed induction motors for inverter operation

- <u>Inverter-fed motors</u> start via frequency control with rated current, so the harmonic torques are small. **Therefore no skewing is necessary!**

- **Advantage** of unskewed induction motors:

  - Minimization of inter-bar currents = **reduction of additional losses !**

  - Maximum magnetic coupling between stator and rotor winding = Increase of main inductance, no skew leakage reactance = **total leakage flux is reduced !**

- As in unskewed induction motors there are nearly no inter-bar currents, the rotor slot number may be bigger than the stator slot number: $Q_r > Q_s$!

  - **Advantage:** At $Q_r > Q_s$ the ratio $Q_r/p$ increases. The rotor air-gap field gets more sinusoidally distributed = the rotor harmonic leakage flux is reduced = **total leakage flux is reduced !**

- Reduction of total leakage flux = reduction of *BLONDEL*´s coefficient $\sigma$ = **increase of breakdown torque $M_b$** according to *KLOSS*´s function!

$$M_b \approx \frac{m_s}{2} \frac{p}{\omega_s} U_s^2 \frac{1-\sigma}{\sigma X_s}$$

*Example:* Six-pole unskewed cage induction machine:
$Q_s = 36, \ Q_r = 42 > 36$

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