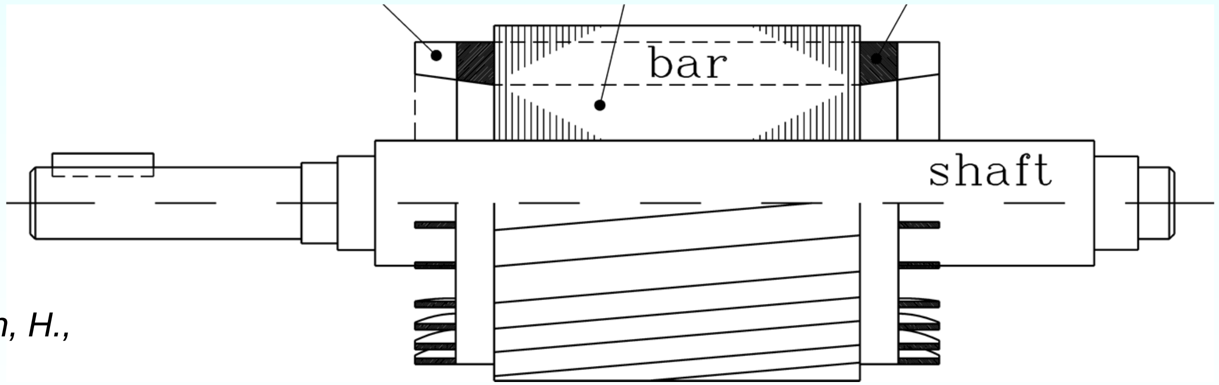
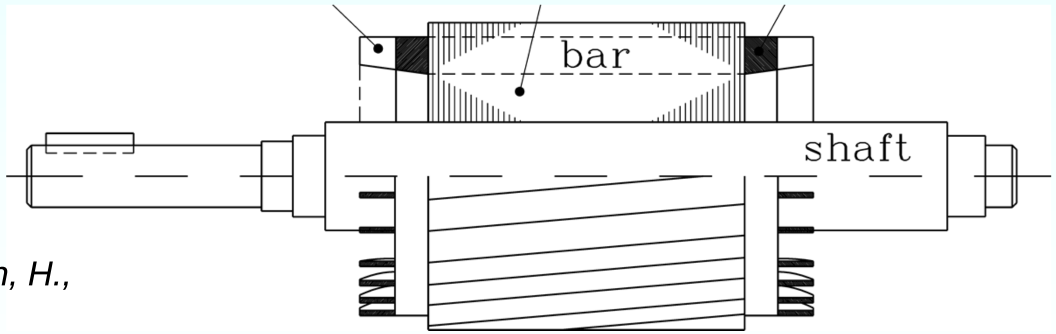
4. Cage induction machines



Source: Kleinrath, H., Studientext

4. Cage induction machines

4.1 Significance and features of induction machines



Source: Kleinrath, H., Studientext

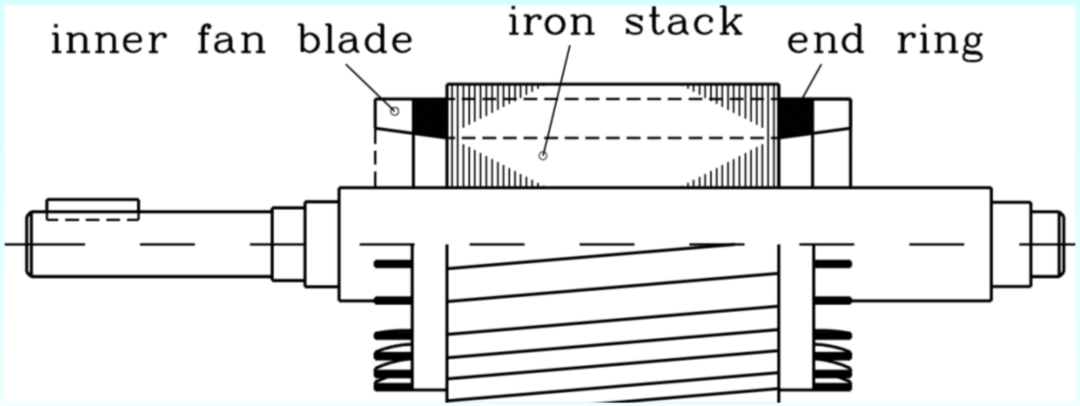
Features of cage induction machines

Fundamental wave model of line-operated induction machine:

Stator: 3-phase winding, frequency f_s: Stator fundamental field rotates with n_{syn}

Rotor: Short circuited rotor bars = rotor cage, rotates with mechanical speed n

Slip: $$s = \frac{n_{syn} - n}{n_{syn}}$$



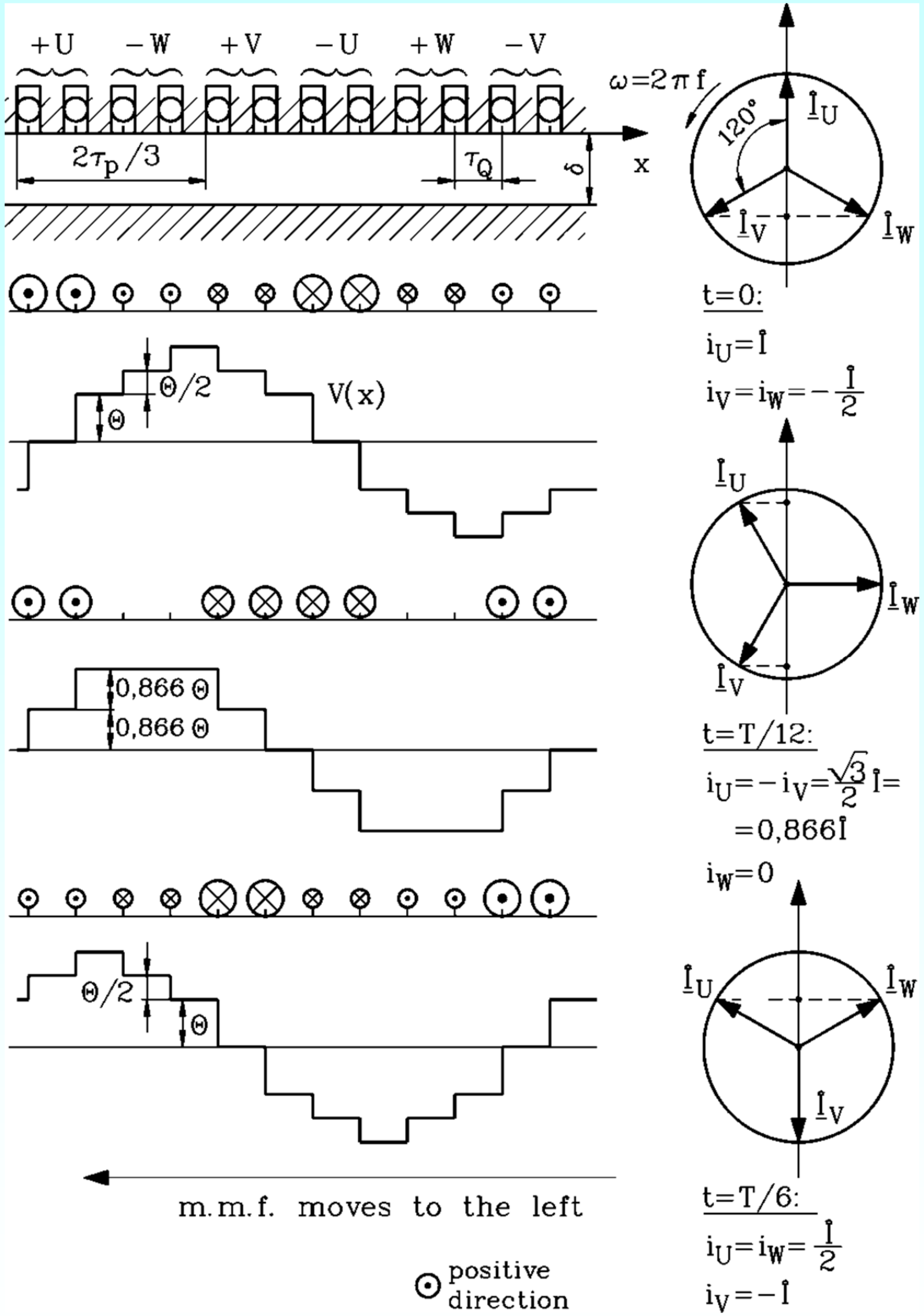
Source: Kleinrath, H., Studientext

Stator fundamental field induces rotor cage loops with rotor frequency: $f_r = s \cdot f_s$

Rotor bar currents produce with stator field tangential LORENTZ forces, yielding electromagnetic torque M_e.

At slip s = 0: No-Load: No torque, at slip s = 1: speed zero = starting

Motor operation: Between s = 1 and s = 0, means: between n = 0 and n_{syn}.



Stator rotating field

- Field curve moves with increasing time t to the left !
- After time T the field curve has passed the distance $2\tau_p$
- Velocity of linear movement is called

$$v_{syn} = \frac{2\tau_p}{T} = 2 f \tau_p$$

synchronous velocity !

Synchronous rotational speed n_{syn} in case of rotating field arrangement:

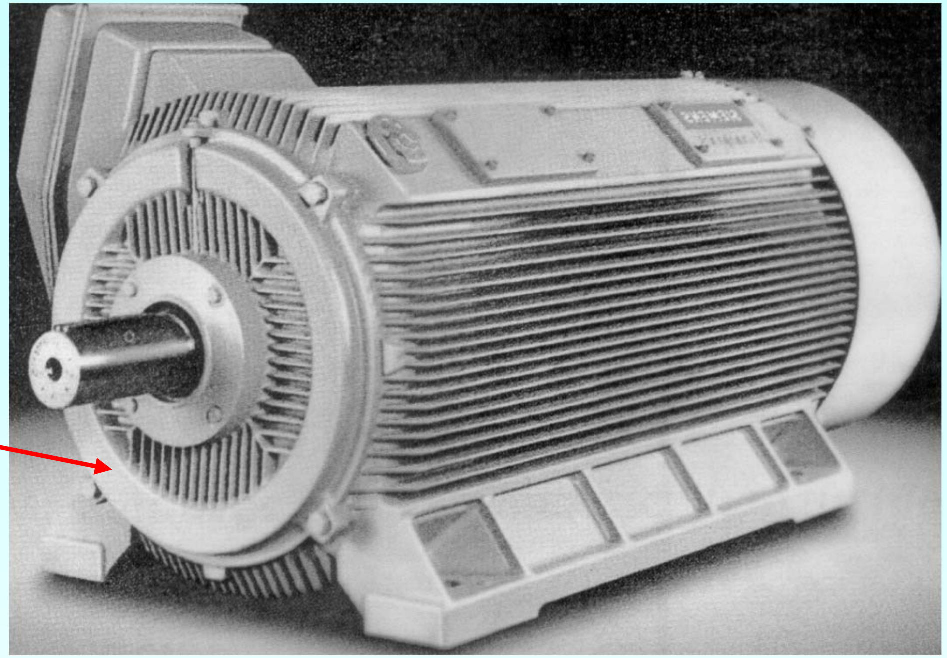
$$\omega_{syn} = 2\pi n_{syn} = \frac{v_{syn}}{d_{si}/2} = \frac{v_{syn}}{p\tau_p/\pi} = \frac{2\pi f}{p}$$

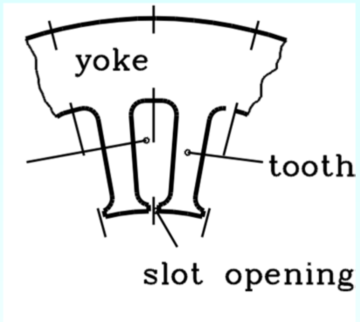
$$n_{syn} = \frac{f}{p}$$

Features of standard motors

Standard motors for pumping

Trans-standard generator 1 MW for wind mill



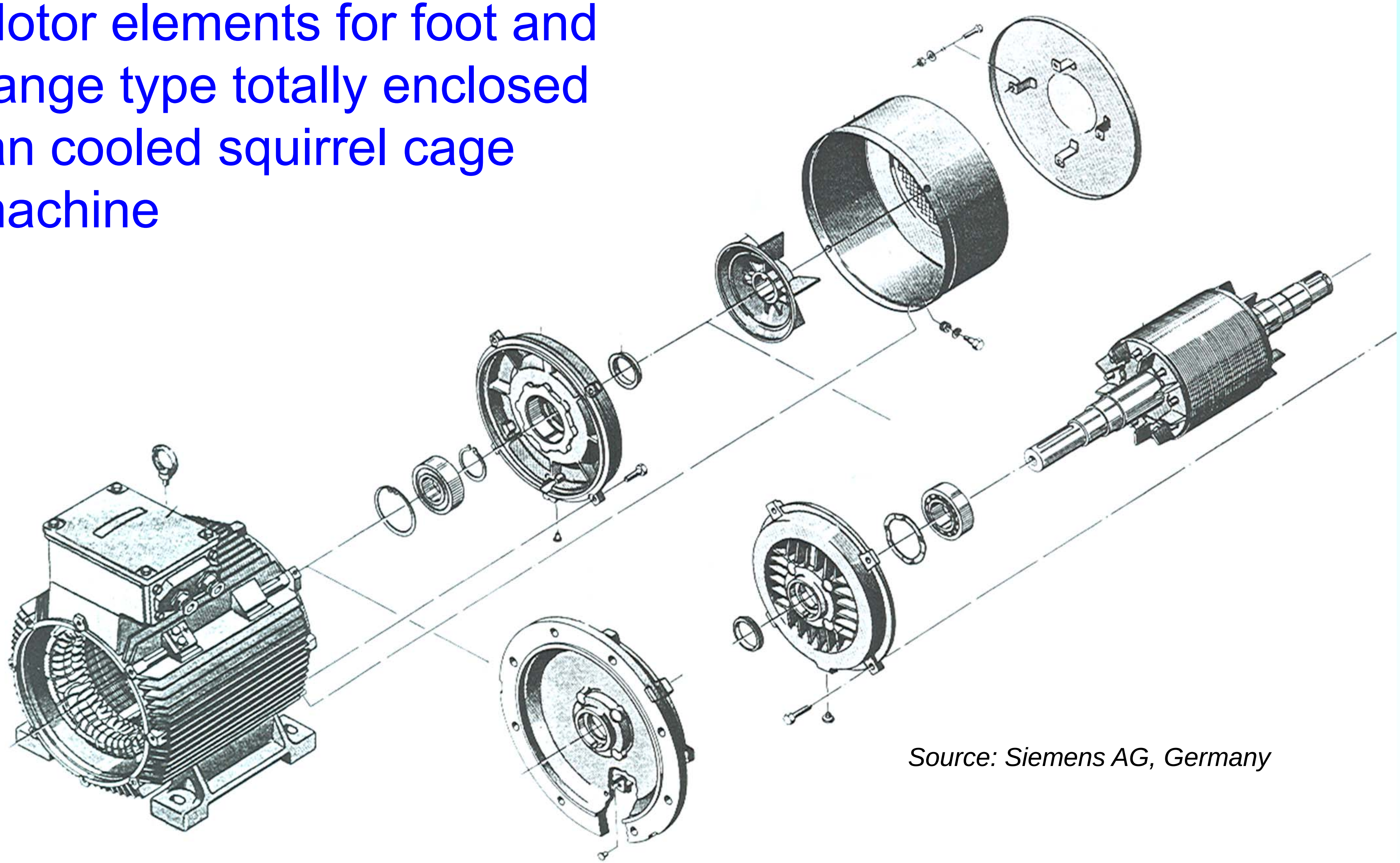


Source: Siemens AG, Germany

- **Standardized** shaft height (= frame size), motor flange dimensions, rated power, shaft end dimensions.
- Standard IEC 72 valid for pole count 2, 4, 6, 8
- **Standardized** between shaft height (frame size) 56 mm and 315 mm for low voltage: < 1000 V: 230 V, 400 V, 690 V.
- **Cooling system: TEFC:** Totally enclosed, shaft mounted fan

Components of standard induction machines

Motor elements for foot and flange type totally enclosed fan cooled squirrel cage machine

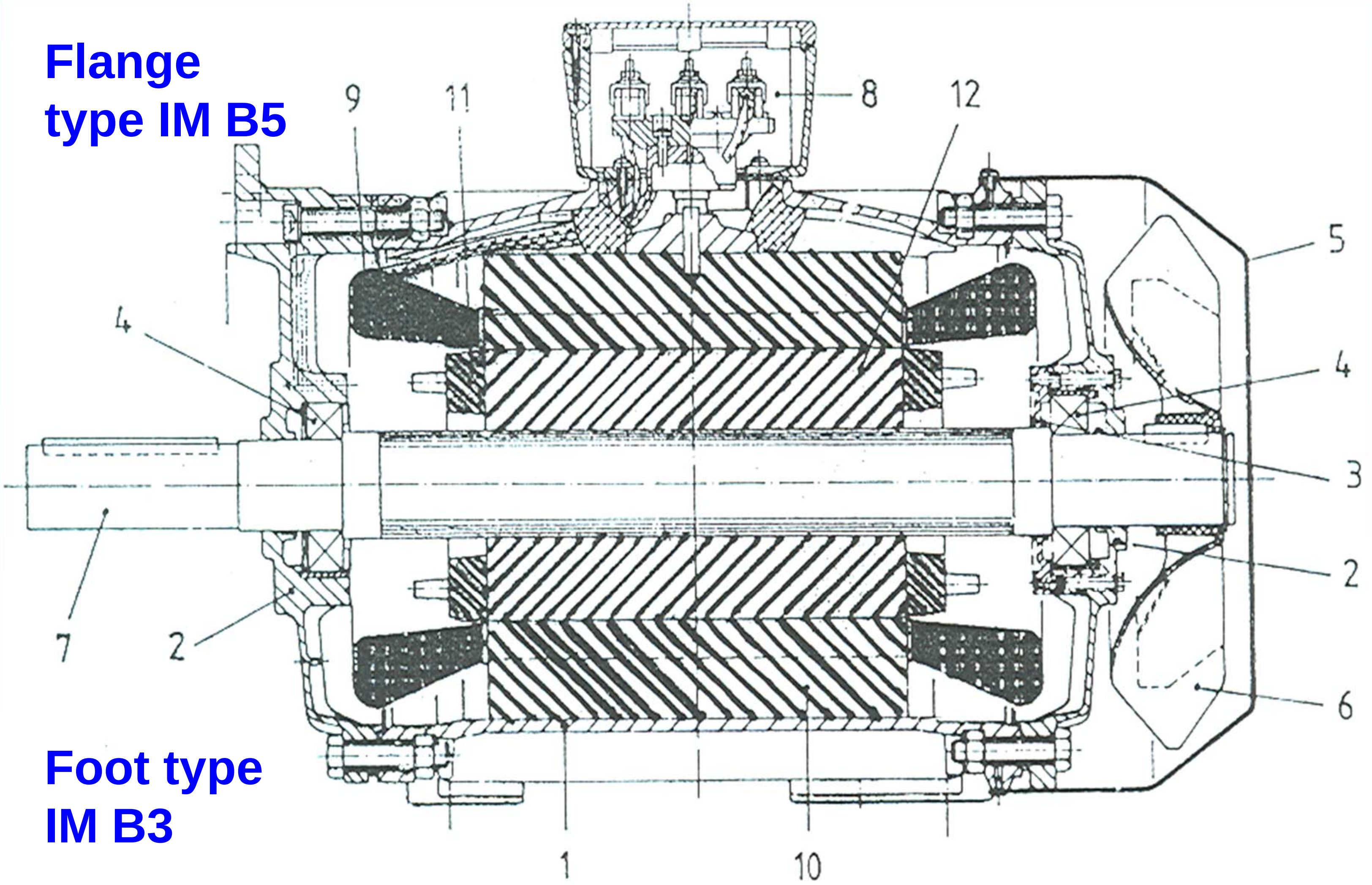


Source: Siemens AG, Germany

Axial cross section of standard induction machine

- Totally enclosed fan cooled squirrel cage machine

Flange type IM B5



Foot type IM B3

Source: Siemens AG, Germany

1: housing

2: end-shield

3: bearing lubrication

4: bearing

5: fan hood

6: shaft-mounted fan

7: shaft

8: terminal box

9: stator winding overhang

10: stator iron stack

11: rotor cage

12: rotor iron stack

Standard induction machines

Standardized shaft heights (mm):

56	63	71	80	90	100	112	132	160	180	200	225	250	280	315

Standardized motor power (= mechanical shaft power = output power) (kW):

...	11	15	18.5	22	30	37	45	55	75	90	110	132	146	...

Example:

Rated data: 7.5 kW, 230 / 400 V, D/Y, 26.5 / 15.2 A, 50 Hz, 1455/min, $\cos\varphi = 0.82$

Shaft height 132 mm, four poles $2p = 4$,

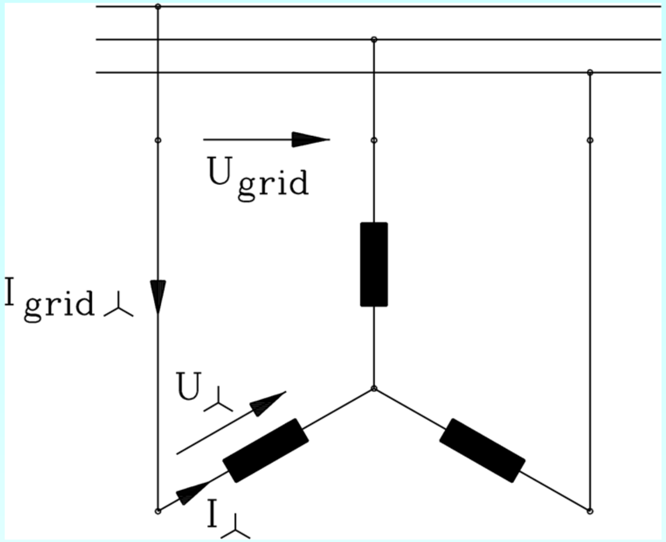
Synchronous speed: $n_{syn} = f / p = 50/2 = 25/s = 1500/\min$

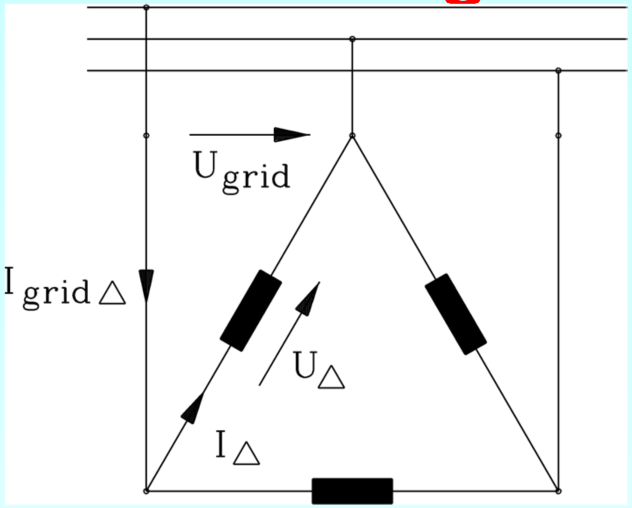
Rated slip: $s_N = (n_{syn} - n_N)/n_{syn} = (1500 - 1455)/1500 = 3\%$

Above motor size 315 mm "trans-standard" machines: Still main dimensions of motor housings are standardized in IEC72 with shaft height 355 mm, 400 mm, 450 mm, but corresponding power ratings vary with different manufacturers, lying in the range of 355 kW ... 1000 kW for 4-pole machines.

For NAFTA market: 460 V/ 60 Hz, different standardized frame sizes in inches, different power ratings in US hp (American horse power: 1 h.p. ≈ 0.7 kW).

Stator winding features





Example:

Rated data: 7.5 kW, 230 / 400 V, D/Y, 26.5 / 15.2 A, 50 Hz, 1455/min, $\cos\varphi$ = 0.82

Efficiency:

$$\eta_N = \frac{P_{out}}{P_{in}} = \frac{P_m}{P_e} = \frac{7500}{8656} = 86.6\%$$

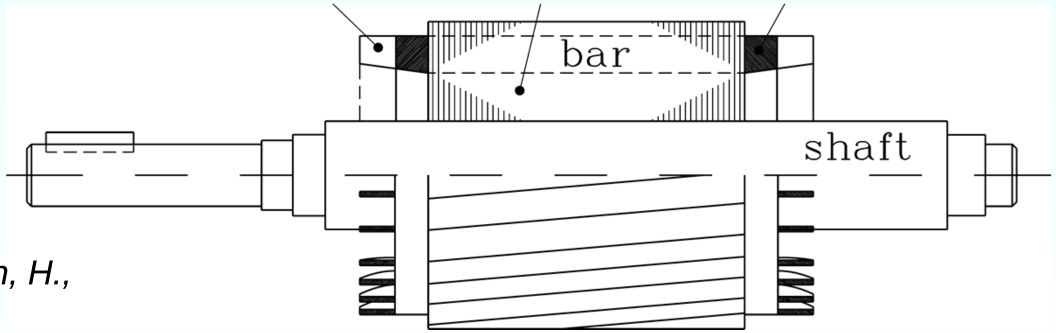
Winding connection	*delta*	*star*
Phase voltage	230 V	230 V
line-to-line voltage	230 V	400 V
Phase current	15.2 A	15.2 A
Line current	26.5 A	15.2 A

Delta connection (D): $P_e = \sqrt{3} U_N I_N \cos\varphi = \sqrt{3} \cdot 230 \cdot 26.5 \cdot 0.82 = 8656$ W

Star connection (Y): $P_e = \sqrt{3} U_N I_N \cos\varphi = \sqrt{3} \cdot 400 \cdot 15.2 \cdot 0.82 = 8656$ W

D and Y: Phase values: $P_e = 3 U_s I_s \cos\varphi = 3 \cdot 231 \cdot 15.2 \cdot 0.82 = 8656$ W.

4.2 Fundamental wave model of line-operated induction machines

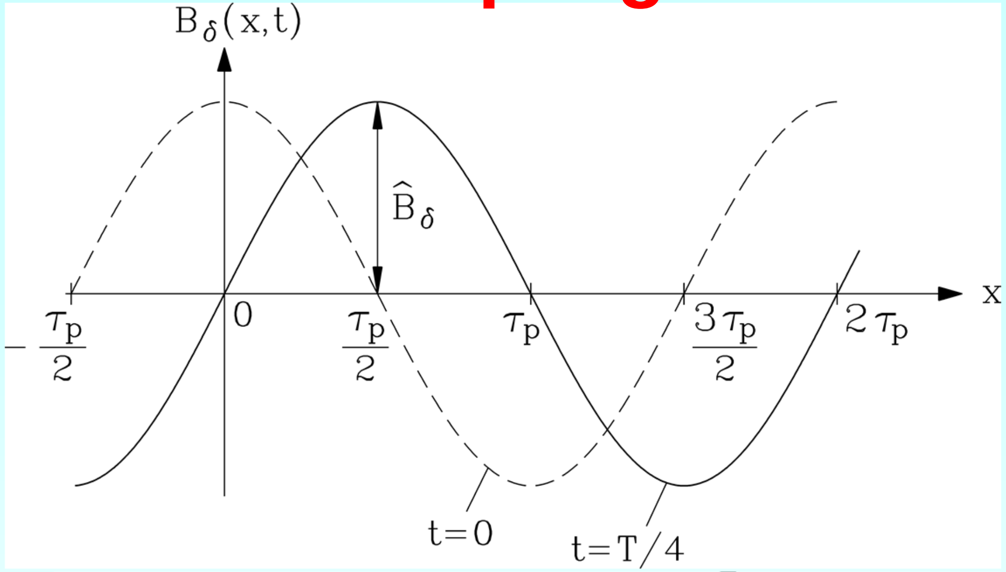


Source: Kleinrath, H., Studientext

Fundamentals of magnetic field - Torque generation

Stator fundamental field:

$$B_s(x_s,t) = \hat{B}_s \cdot \cos\left(\frac{x_s \pi}{\tau_p} - \omega_s t\right)$$

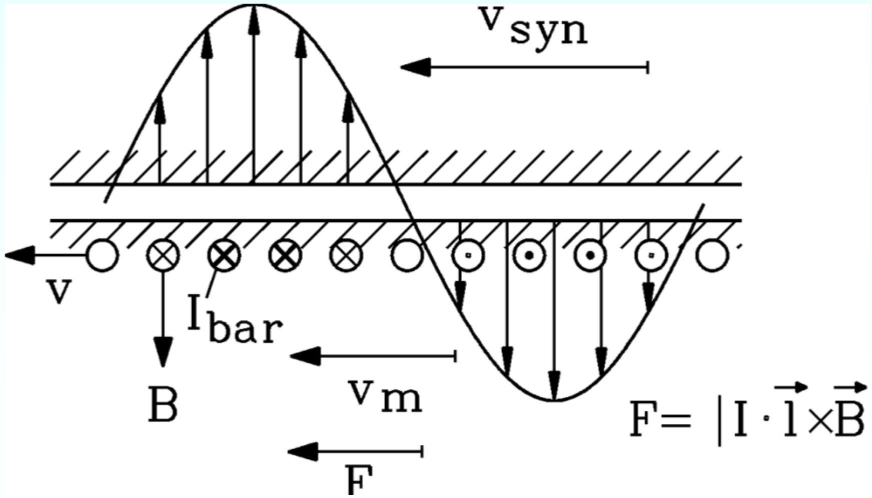


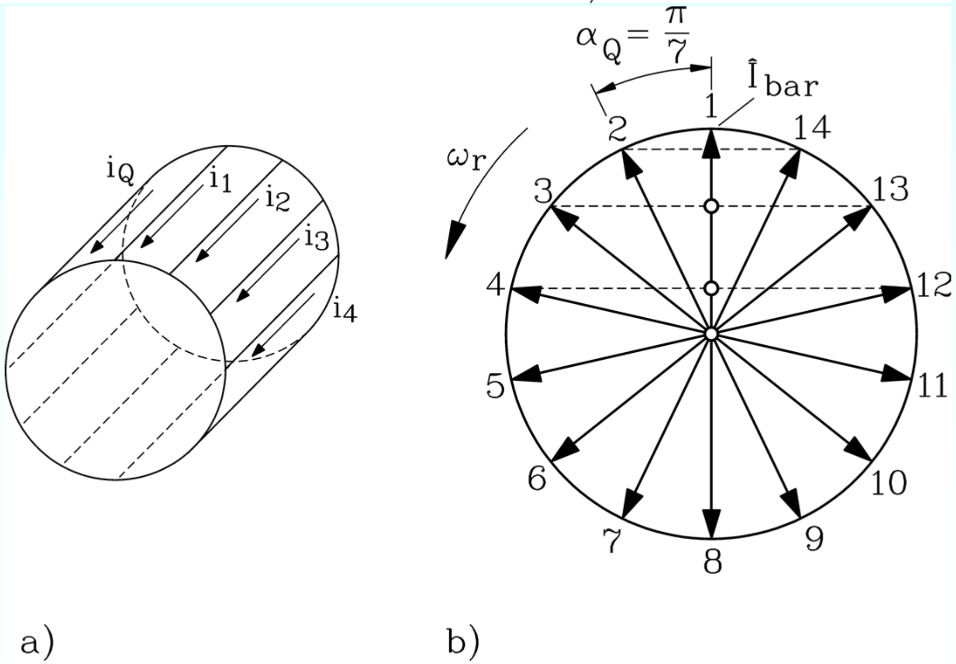
$$\hat{B}_s = \frac{\mu_0}{\delta} \cdot \frac{m_s}{\pi \cdot p} \cdot N_s k_{ws} \hat{I}_s$$

Unsaturated amplitude

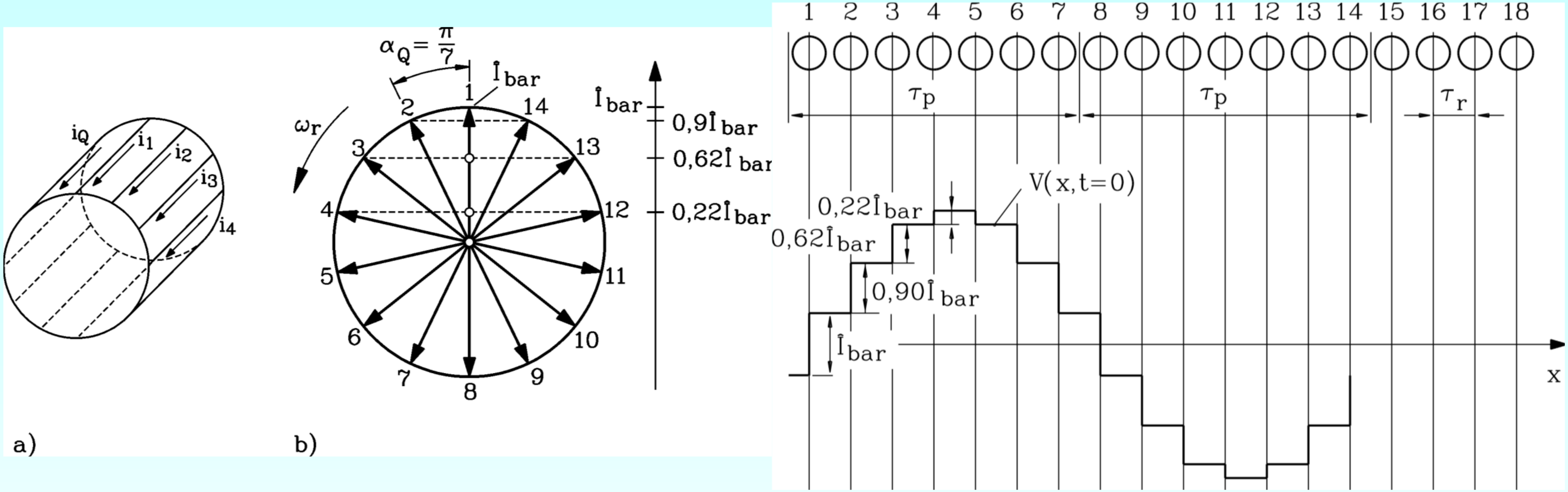
Example:

Rotor cage with Q_r = 28 rotor bars, induced be 4 four-pole stator wave. Phase shift between adjacent rotor bar currents: $2\pi/(Q_r/p) = 2\pi/(28/2) = \pi/7$





Rotor current distribution and rotor field



Squirrel-cage winding with Q_r = 28 bars, $2p$ = 4: Rotor magnetic field calculated form m.m.f.: Determination unsaturated only by air gap: $B_\delta = \mu_0 V / \delta$

- Velocity of rotor field: $v = v_m + v_{r,syn} = 2pn\tau_p + 2f_r\tau_p = 2p \cdot n_{syn}(1-s) \cdot \tau_p + 2 \cdot sf_s \cdot \tau_p =$

$$= 2p \cdot \frac{f_s}{p} \cdot (1-s) \cdot \tau_p + 2 \cdot sf_s \cdot \tau_p = 2f_s\tau_p = v_{syn}$$

Fundamental rotates synchronously with stator field = electromagnetic torque is constant !

Transformer principle of fundamental wave of induction machine

Air gap field amplitude | Induced voltage | Mutual inductance

From stator to rotor:

$$B_s = \frac{\mu_0}{\delta} \cdot \frac{m_s}{\pi \cdot p} \cdot N_s k_{ws} \hat{I}_s \quad \underline{U}_{i,rs} = js\omega_s M_{rs} \cdot \underline{I}_s \quad \Rightarrow \quad M_{rs} = \mu_0 \cdot N_s k_{ws} N_r k_{wr} \cdot \frac{2m_s}{\pi^2 p} \cdot \frac{\tau_p l_{Fe}}{\delta}$$

From rotor to stator:

$$B_r = \frac{\mu_0}{\delta} \cdot \frac{m_r}{\pi \cdot p} \cdot N_r k_{wr} \hat{I}_r \quad \underline{U}_{i,sr} = j\omega_s M_{sr} \cdot \underline{I}_r \quad \Rightarrow \quad M_{sr} = \mu_0 \cdot N_r k_{wr} N_s k_{ws} \cdot \frac{2m_r}{\pi^2 p} \cdot \frac{\tau_p l_{Fe}}{\delta}$$

Along with self induction:

stator air gap field in stator winding, rotor air gap field in rotor winding,

stator stray field in stator winding, rotor stray field in rotor winding,

resistive voltage drop in stator and rotor field we get

a) Voltage equation for one stator phase: $\underline{U}_s = R_s \underline{I}_s + j\omega_s L_{s\sigma} \underline{I}_s + j\omega_s L_{sh} \underline{I}_s + j\omega_s M_{sr} \underline{I}_r$

b) Voltage equation for one rotor bar: $0 = R_r \underline{I}_r + j\omega_r L_{r\sigma} \underline{I}_r + j\omega_r L_{rh} \underline{I}_r + j\omega_r M_{rs} \underline{I}_s$

Transfer ratios of cage induction machine

Definition of **voltage and current transfer ratio** to simplify equations:

$$\ddot{u}_U = \frac{N_s k_{ws}}{N_r k_{wr}}, \quad \ddot{u}_I = \frac{m_s N_s k_{ws}}{m_r N_r k_{wr}}$$

$$\underline{U}_s = R_s \underline{I}_s + j\omega_s L_{s\sigma} \underline{I}_s + j\omega_s L_{sh} \underline{I}_s + j\omega_s (\ddot{u}_I M_{sr})(\underline{I}_r / \ddot{u}_I)$$

$$0 = (\ddot{u}_U \ddot{u}_I R_r)\frac{\underline{I}_r}{\ddot{u}_I} + j\omega_r (\ddot{u}_U \ddot{u}_I L_{r\sigma})\frac{\underline{I}_r}{\ddot{u}_I} + j\omega_r (\ddot{u}_U \ddot{u}_I L_{rh})\frac{\underline{I}_r}{\ddot{u}_I} + j\omega_r (\ddot{u}_U M_{rs})\underline{I}_s$$

Advantage: SIMPLIFICATION: Only ONE magnetizing fundamental inductance L_h:

$$L_{sh} = M_{sr}\ddot{u}_I = \ddot{u}_U M_{rs} = \ddot{u}_U \ddot{u}_I L_{rh} = L_h$$

$$\underline{U}_s = R_s \underline{I}_s + j\omega_s L_{s\sigma} \underline{I}_s + j\omega_s L_h (\underline{I}_s + \underline{I}'_r)$$

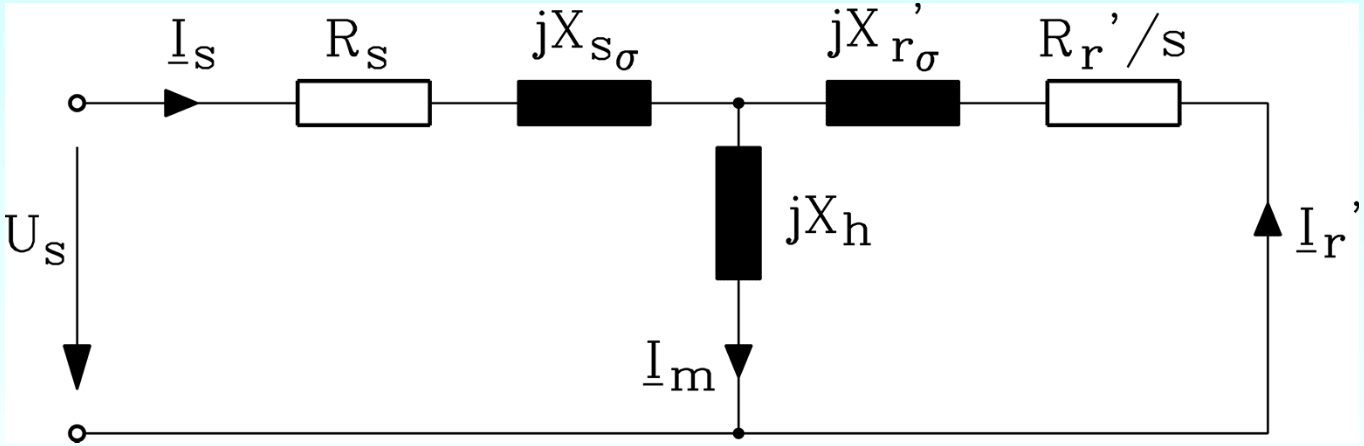
$$0 = R'_r \underline{I}'_r + j\omega_r L'_{r\sigma} \underline{I}'_r + j\omega_r L_h (\underline{I}_s + \underline{I}'_r)$$

Equivalent circuit of fundamental wave induction machine

Use only of stator frequency also for rotor side in equivalent circuit:

$$\underline{U}_s = R_s \underline{I}_s + j\omega_s L_{s\sigma} \underline{I}_s + j\omega_s L_h (\underline{I}_s + \underline{I}'_r)$$

$$0 = (R'_r / s)\underline{I}'_r + j\omega_s L'_{r\sigma} \underline{I}'_r + j\omega_s L_h (\underline{I}_s + \underline{I}'_r)$$



Leakage coefficient:

$$\sigma = 1 - \frac{X_h^2}{X_s X'_r}$$

Stator and rotor current for given voltage and rotor slip:

$$\underline{I}_s = \underline{U}_s \frac{R'_r + jsX'_r}{(R_s R'_r - s\cdot\sigma\cdot X_s X'_r) + j(s\cdot R_s X'_r + X_s R'_r)} \qquad \underline{I}'_r = -\underline{I}_s \frac{jX_h}{\frac{R'_r}{s} + jX'_r}$$

Power balance of equivalent circuit

Electrical input power: $P_e = 3 \cdot \mathrm{Re}\left(\underline{U}_s \cdot \underline{I}_s^*\right)$ $\underline{I}_s^*$: conjugate complex

Stator copper losses $P_{Cu,s}$

Air gap power P_δ, transferred to rotor = rotor copper losses $P_{Cu,r}$ + mechanical output power P_m.

$$P_\delta = P_e - P_{Cu,s} = P_e - m_s R_s I_s^2 = P_{Cu,r} + P_m = m_r R_r I_r^2 + P_m = m_s R_r' I_r'^2 + P_m$$

Equivalent circuit shows $P_\delta = m_s \cdot (R_r' / s) \cdot I_r'^2 = P_{Cu,r} / s \rightarrow P_m = (1-s) \cdot P_\delta$

Electromagnetic torque M_e: $P_m = \Omega_m M_e = (1-s)\Omega_{syn} M_e \rightarrow P_\delta = \Omega_{syn} M_e$

$$M_e = \frac{P_\delta}{\Omega_{syn}} = \frac{P_{Cu,r}}{s \cdot \Omega_{syn}} = \frac{m_s R_r' I_r'^2}{s \cdot \Omega_{syn}}$$

Asynchronous torque M_e: depends on square of stator voltage and on slip s:

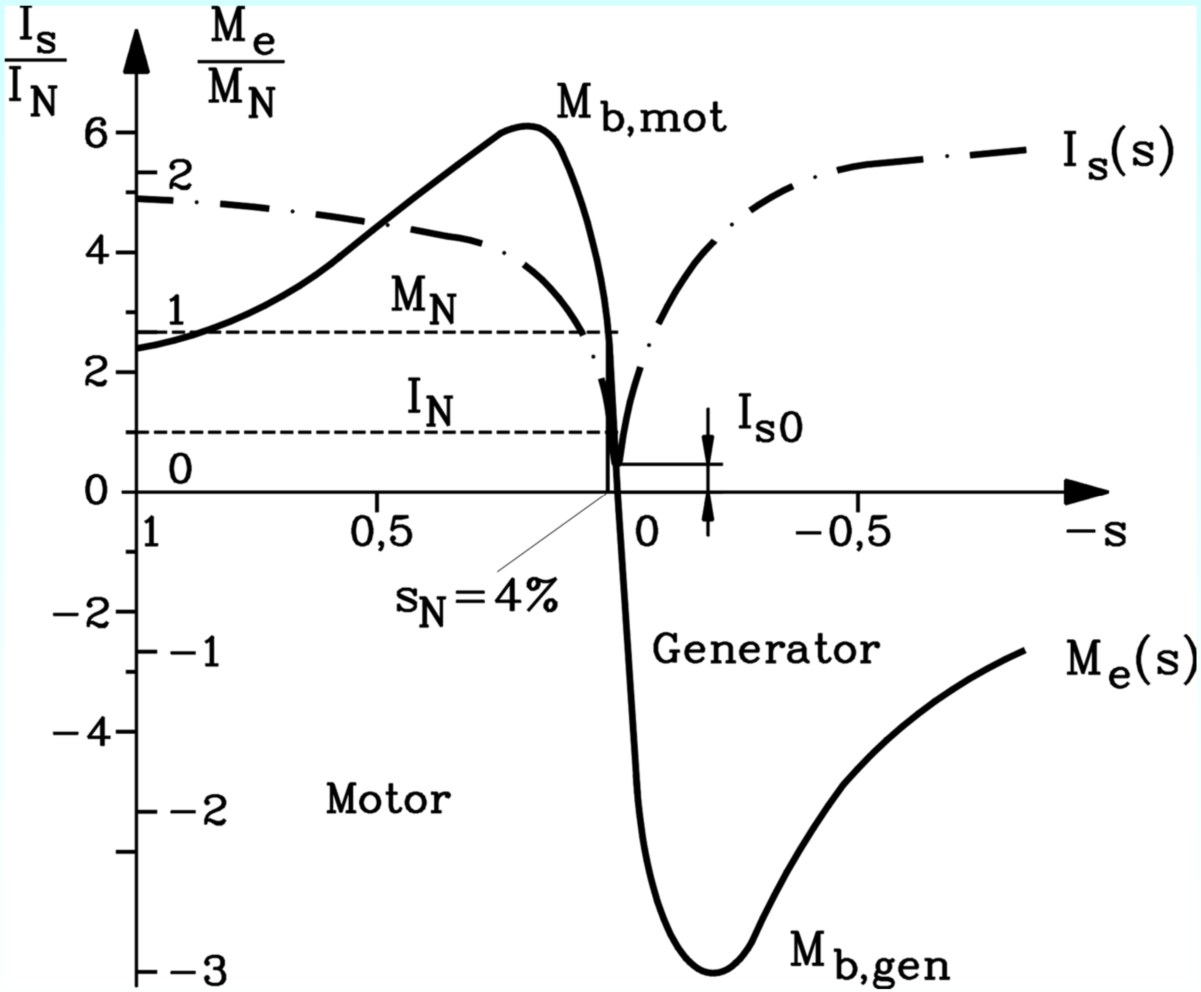
$$M_e = m_s \frac{p}{\omega_s} U_s^2 \frac{s(1-\sigma) X_s X_r' R_r'}{(R_s R_r' - s\sigma X_s X_r')^2 + (sR_s X_r' + X_s R_r')^2}$$

Torque and current depending on speed

Example:

Data: $R_s/X_s = 1/100$, $R_r/X_r = 1.3/100$, $\sigma = 0.067$, $X_s = X'_r = 3Z_N$, $Z_N = U_N/I_N$

Torque and current may be depicted either in dependence of slip or in dependence of rotor speed *n* : $n = (1-s)\cdot f_s / p$



Breakdown torque:

$$M_e(s = s_b) = M_b$$

$$|M_{b,mot}| < |M_{b,gen}|$$

$$|s_{b,mot}| = |s_{b,gen}|$$

Torque properties - *KLOSS* function

- Asynchronous torque depends on the square of stator voltage.
- At no-load ($s = 0$) torque is zero.
- At infinite positive and negative slip torque is also zero.
- At break down slip $\pm s_b$: **motor / generator break down torque $M_{b,mot}$ / $M_{b,gen}$.**

Simplified: $R_s = 0$:

$$M_e = m_s \frac{p}{\omega_s} U_s^2 \frac{(1-\sigma)}{X_s} \frac{sR'_r X'_r}{(s\sigma X'_r)^2 + R'^2_r}$$

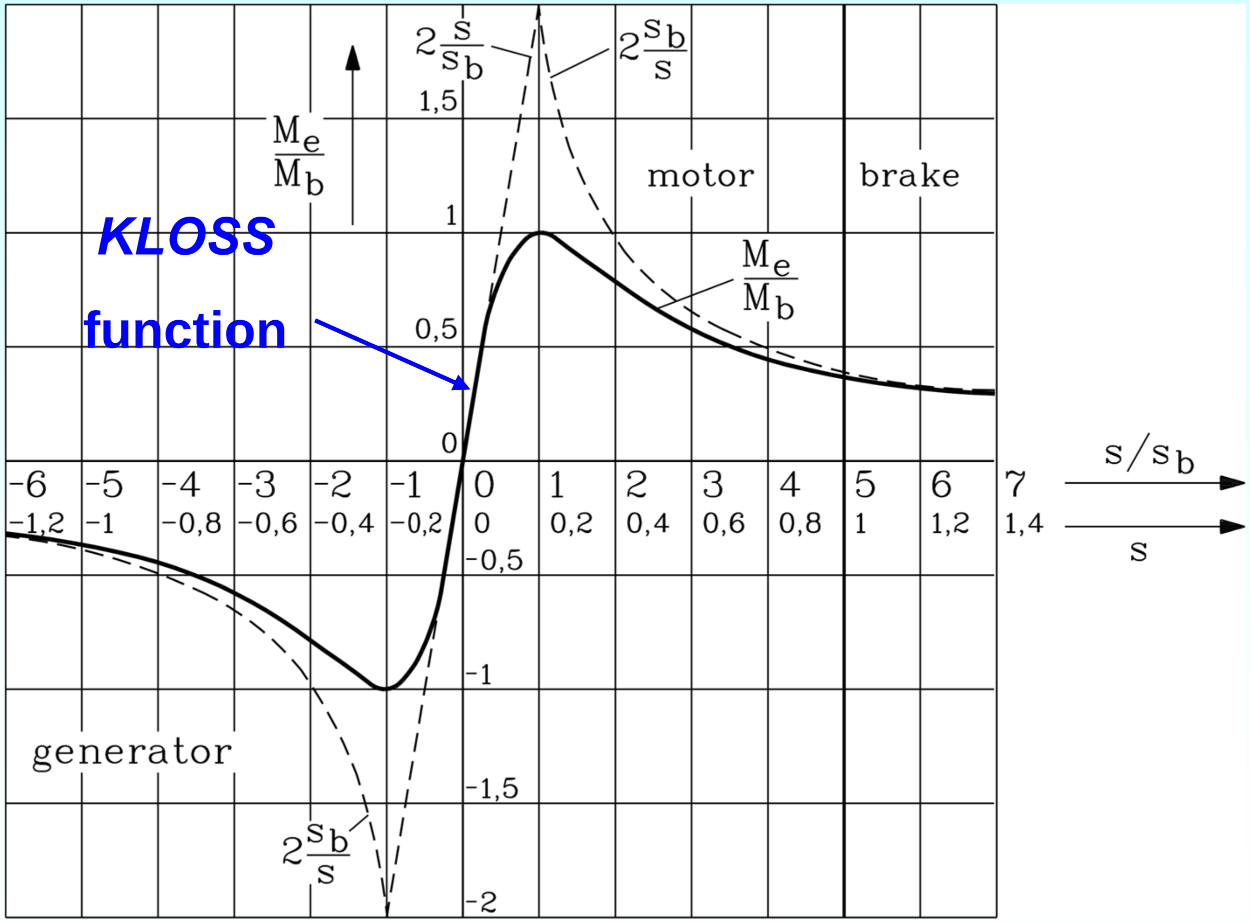
(+: motor, -: generator)

$$R_s = 0: \quad M_b = \pm \frac{m_s}{2} \frac{p}{\omega_s} U_s^2 \frac{1-\sigma}{\sigma X_s}$$

$$s_b = \pm \frac{R'_r}{\sigma X'_r}$$

$$R_s = 0: \quad \frac{M_e}{M_b} = \frac{2}{\frac{s_b}{s} + \frac{s}{s_b}}$$

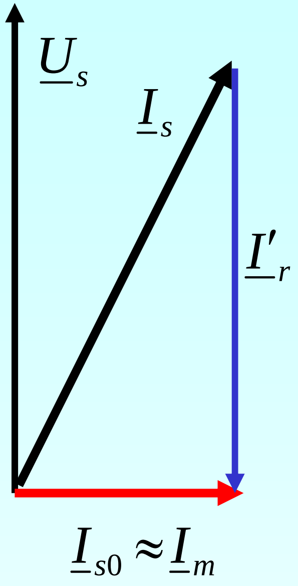
***KLOSS* function**



Simplified torque equation for small slip *s* and $R_s = 0$

Operation slip range: $-2s_N \le s \le 2s_N$: Torque: $M_e \approx M_b \cdot 2 \cdot s / s_b$

$$M_e \approx m_s \frac{p}{\omega_s} U_s^2 \frac{1-\sigma}{X_s} \frac{sX'_r}{R'_r} \sim s \qquad \underline{I}_s\Big|_{R_s=0} = \frac{\underline{U}_s}{j \cdot X_s} \cdot \frac{R'_r + jsX'_r}{R'_r + js \cdot \sigma \cdot X'_r}$$



Stator current for small slip:

$$\underline{I}_s\Big|_{R_s=0} = \frac{\underline{U}_s}{j \cdot X_s} \cdot \frac{1 + jsX'_r / R'_r}{1 + js \cdot \sigma \cdot X'_r / R'_r} = \underline{I}_{s0} \cdot \frac{1 + js \cdot a}{1 + js \cdot \sigma \cdot a} \approx \underline{I}_{s0} \cdot \left(1 + js \cdot \frac{X'_r}{R'_r} \cdot (1-\sigma)\right)$$

$$X'_r / R'_r = a \qquad \frac{1 + js \cdot a}{1 + js \cdot \sigma \cdot a}\Bigg|_{s \cdot \sigma \cdot a \ll 1} \approx (1 + js \cdot a) \cdot (1 - js \cdot \sigma \cdot a) \approx 1 + js \cdot a \cdot (1-\sigma)\Bigg|_{s^2 \ll 1}$$

Rotor current for small slip:

$$\underline{I}_s = \underline{I}_m - \underline{I}'_r \approx \underline{I}_{s0} - \underline{I}'_r \Rightarrow \underline{I}'_r = -\frac{\underline{U}_s}{X_s} \cdot \frac{s(1-\sigma)X'_r}{R'_r}$$

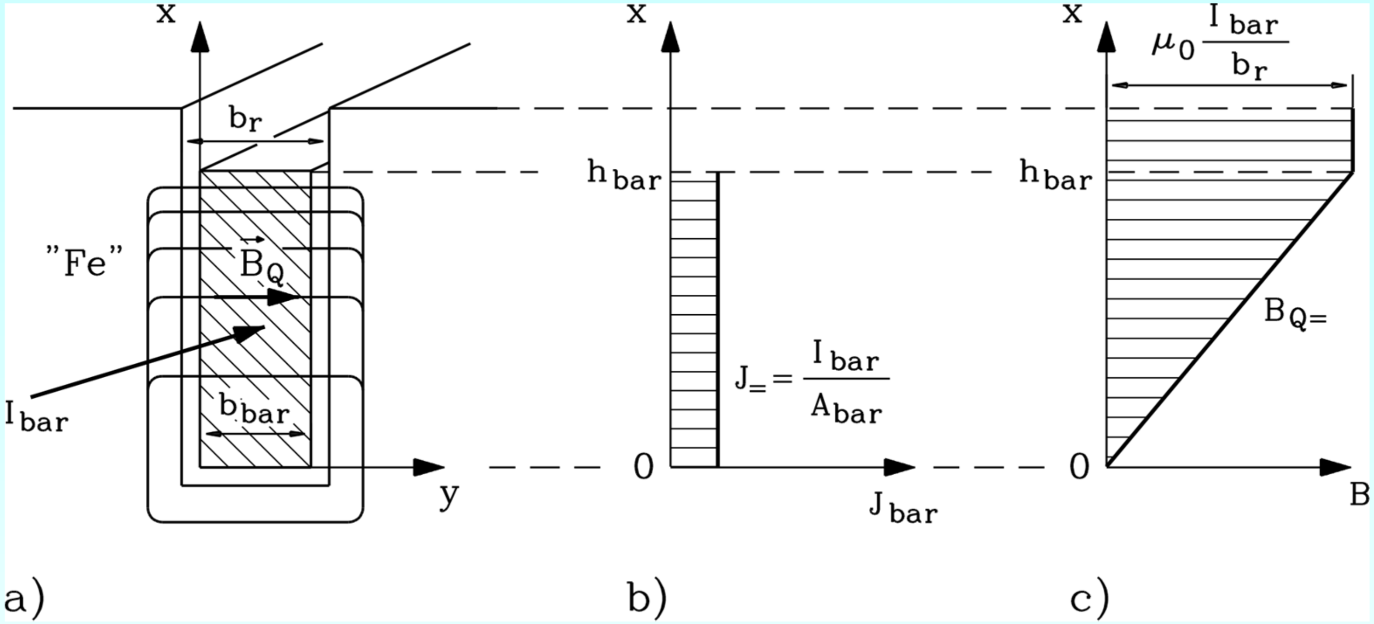
Main flux linkage: $\dfrac{U_s}{\omega_s} = \dfrac{\Psi_s}{\sqrt{2}} \approx \dfrac{\Psi_h}{\sqrt{2}}$

$$M_e \approx m_s \frac{p}{\omega_s} U_s \cdot I'_r = m_s p \cdot \frac{\Psi_h}{\sqrt{2}} \cdot I'_r$$

Torque ~ Main flux linkage per pole x rotor current

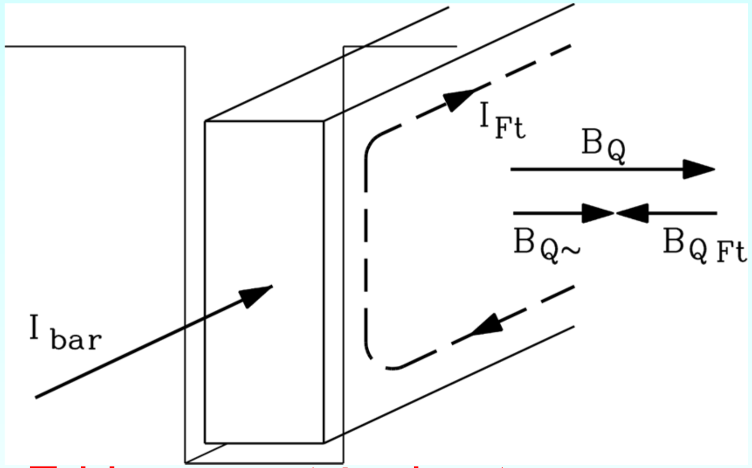
$$M_e \sim \Psi_h \cdot I'_r$$

Rotor bar current displacement effect



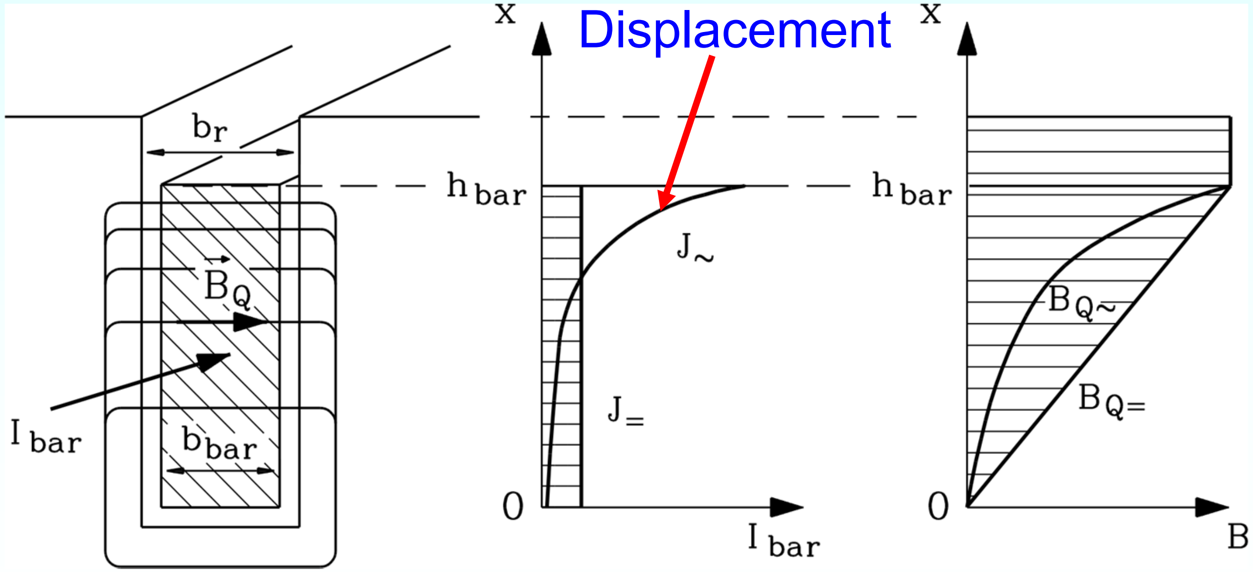
Rotor slot leakage flux Φ_Q due to I_{bar} at small rotor frequency (= small slip):

NO current displacement



Eddy current I_{Ft} due to

$u_i = - d\Phi_Q/dt \sim -dB_Q/dt$



Rotor slot leakage flux at big rotor frequency (= big slip ~ 1):

BIG current displacement in deep rotor bars = increased losses !

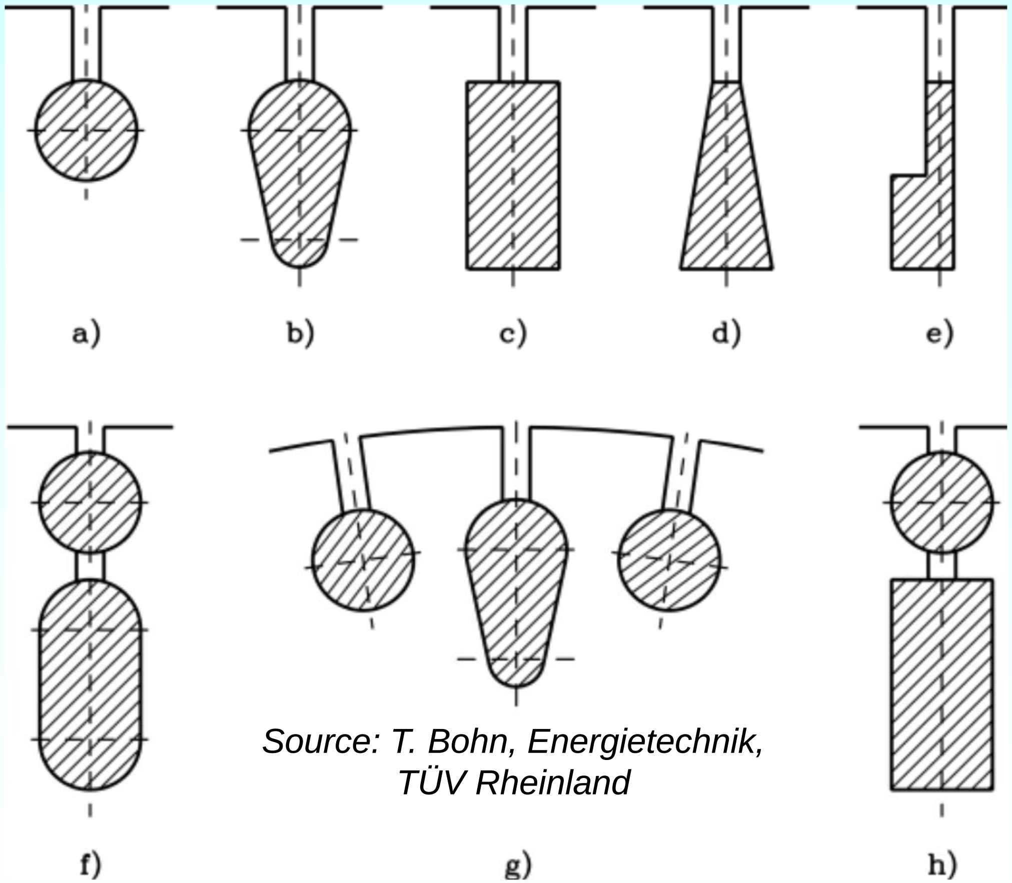
Rotor bar current displacement increases starting torque

Starting torque increases with rotor losses:

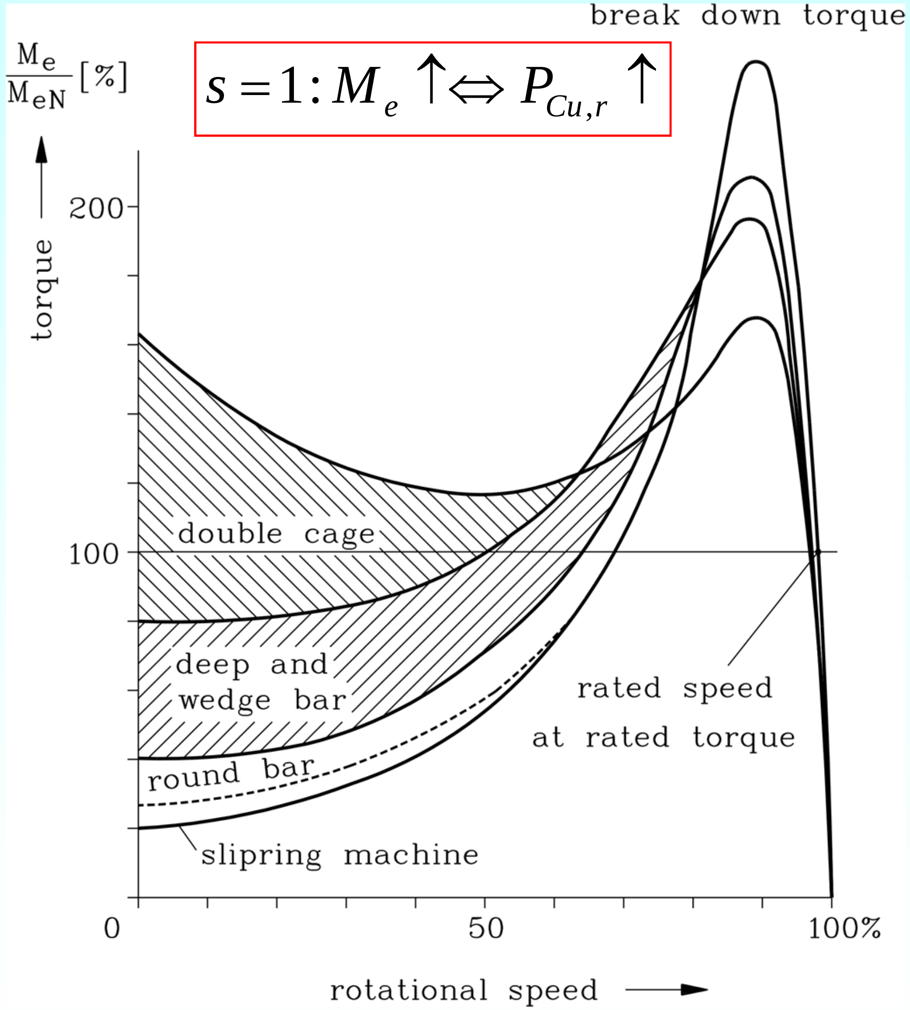
Rotor losses increase due to rotor bar eddy currents =

= due to current displacement !

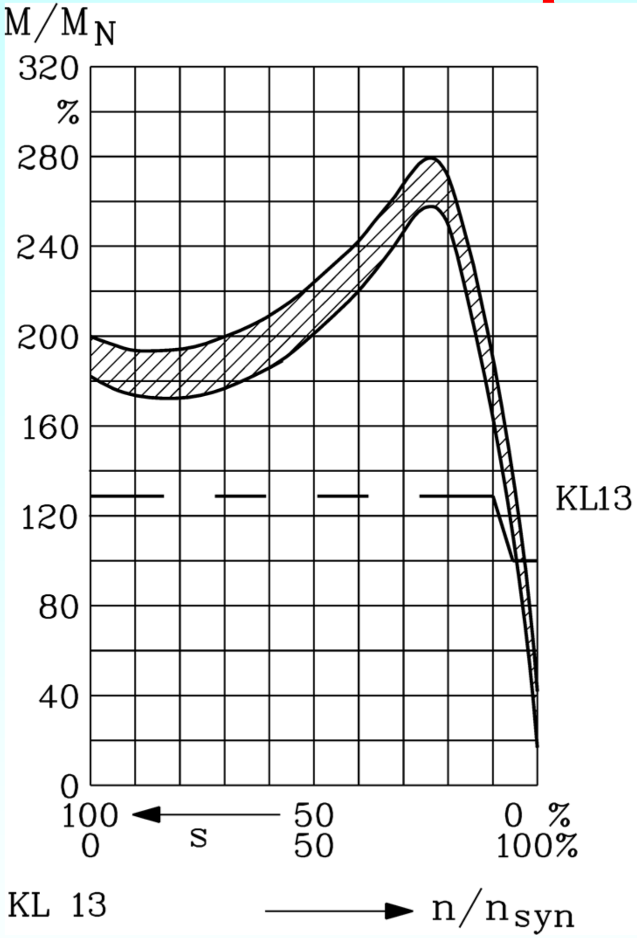
$$M_e = \frac{P_m}{(1-s)\Omega_{syn}} = \frac{m_s R_r' I_r'^2}{s \cdot \Omega_{syn}} = \frac{P_{Cu,r}}{s \cdot \Omega_{syn}}$$



Source: T. Bohn, Energietechnik, TÜV Rheinland



Typical torque-speed curves with current displacement – “Torque classes”



- Double cage die-cast aluminum rotors
- Typical variation of torque curve due to manufacturing tolerances of die casting & inter-bar currents !

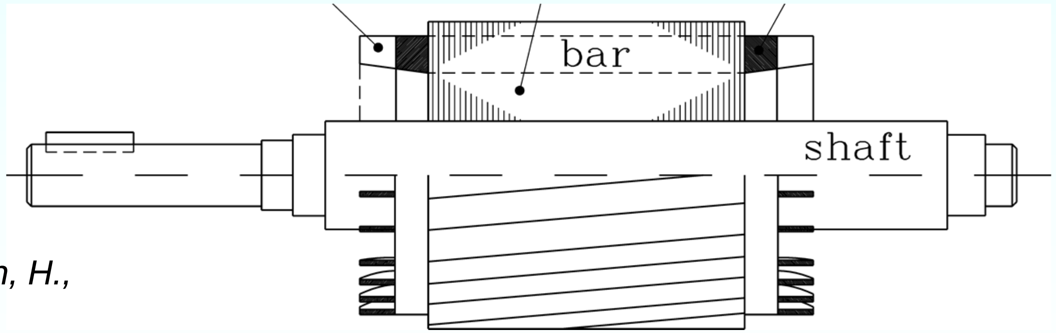
Standard induction motors *M*(*n*)-curves !

Source: Siemens AG, Germany

- Torque class KL160 means: Motor can start at 160% rated torque !
- Typical torque classes: 100% 130%, 160%, 200%

4. Cage induction machines

4.3 Premium efficiency machines



Source: Kleinrath, H., Studientext

Energy savings with industrial drives

Motivation for improvement of efficiency of electric motors, which have already high efficiency, when compared e.g. with combustion machines:

German power consumption in 2011:

a) Electrical energy consumption: 16 % of total energy consumption= 608 TWh

b) Industrial percentage of a): 44 % of a) = 267 TWh

c) Motor percentage of b): 68 % of b) = 182 TWh

We assume:

- Average efficiency increase of 4% by premium efficiency motors
- Realized for 50% of installed drive power

Result:

Energy saving of $0.04 \cdot 0.5 \cdot 182$ TWh = 3.6 TWh per year (= 8760 h)

= Power delivery of a power plant: 3.6 TWh / 8760 h = 415 MW.

Average efficiency of old/new plants at full/partial load: 35 %.

Result:

Saving of 415/0.35 =*1187 MW* thermal input power.

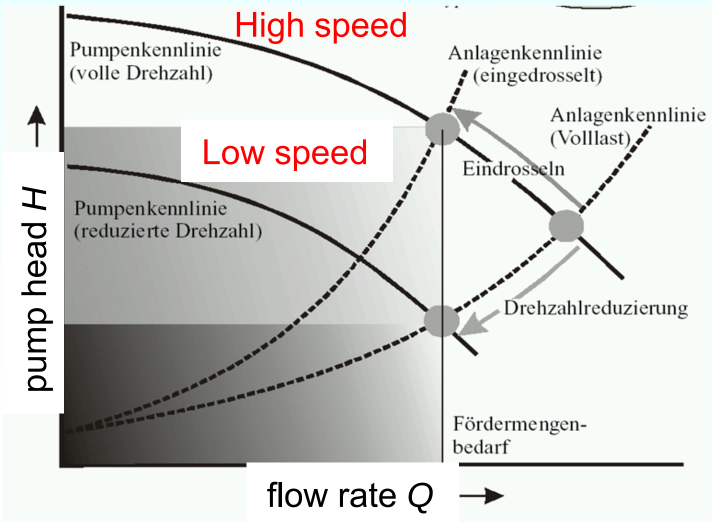
Energy saving potentials in drive technology

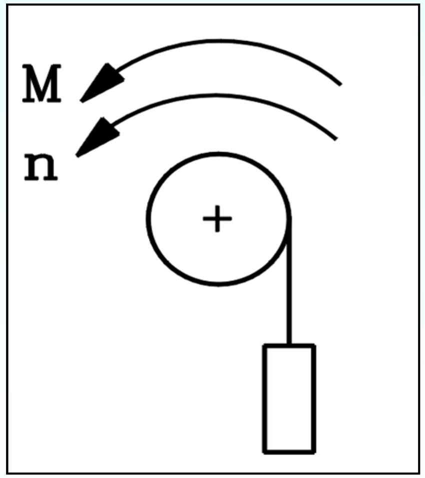
Industrial drive systems:

Estimated energy savings:

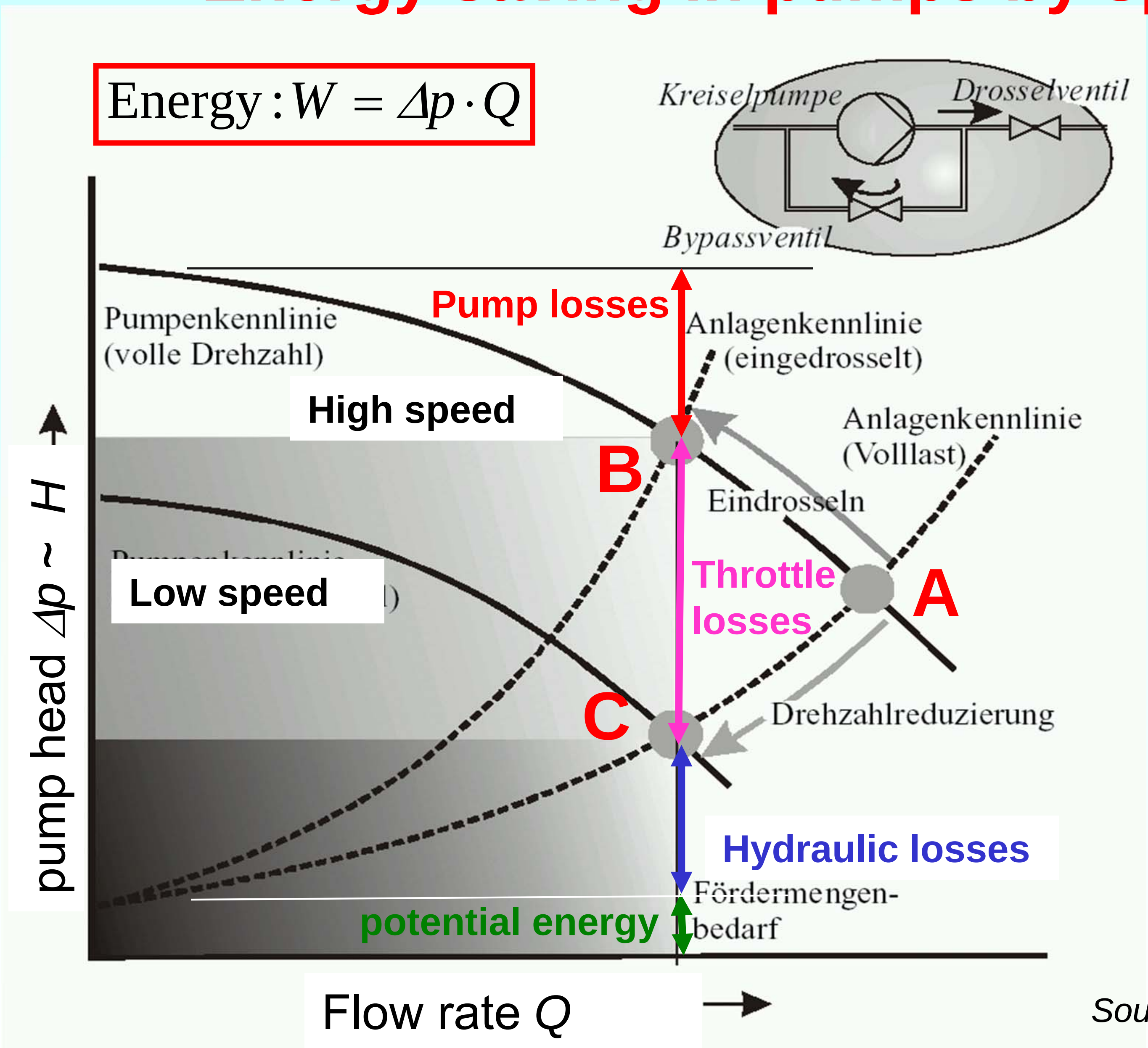
Motor efficiency	Speed variation	System optimization
1.4% … 3%	**8 % … 10%**	**15 … 20 %**
	e.g. Pump application	e.g. elevator system

Sources: Leroy Somer; France; KSB, Germany





Energy saving in pumps by speed variation



Example: Pump:
Volume flow shall be reduced!

a) Volume flow reduced by throttling valve, while pump operates at constant speed:

A → B

b) Volume flow reduced by reducing of pump speed:

A → C

This yields lower total losses by up to 60%!, proportional to light-grey shaded area!

Source: KSB, Frankenthal, Germany

System optimization – ***Example:*** Elevator

Elevator data: (b) invest more expensive than a)

1 Ton of pay-load, 17 m hoisting distance, 5 stops

a) Old drive: - Grid-fed induction motor 8.8 kW for fixed speed,
- two windings for pole changing „slow-fast“
- conventional oil-lubricated gear
- mechanical brake at the stops

b) New drive: - Inverter-fed induction motor 7.5 kW, one winding system
- Speed variation via frequency control
- Low loss gear with synthetic oil lubrication
- Energy feed back during braking via the inverter

Energy saving per tour: 81 % at full load (best case)

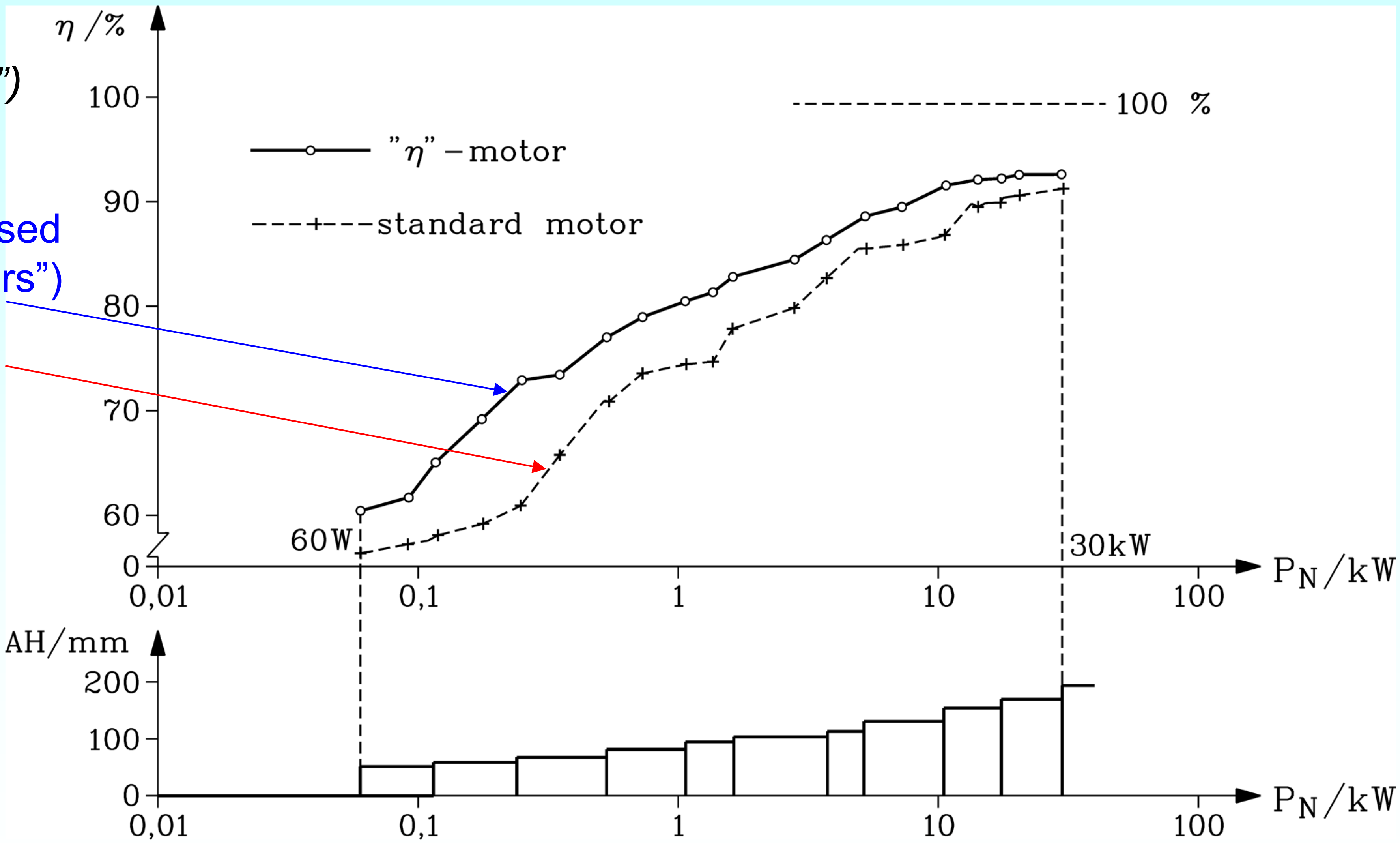
Return on investment at 400 daily tours due to lower losses: after 5.5 years! *Source: ZVEI, Frankfurt/Main, Germany*

History: Catalogue efficiency of 4-pole standard cage induction motors before 1990

- Efficiency η (*"eta"*)
- Shaft height AH

Motors with increased efficiency ("η-motors")

„normal“ efficiency



Source:

Siemens AG, Germany

History on "Increased efficiency induction motors"

- United States: **Energy Policy Act (EPACT)** established in 1997:

At US American market 2-pole and 4-pole squirrel-cage induction motors with increased efficiency values up to a rating of 90 h.p. must be offered by manufacturers.

- Europe: 2000: Community of European Motor Manufacturers (CEMEP) agreed on **a voluntary agreement with the Commission of European Community** (EC) to offer for the European market 2- and 4-pole motors 1 ... 100 kW in **three efficiency classes**.

Cheap standard motors with usual efficiency values:	Efficiency class eff3
Standard motors with increased efficiency:	Efficiency class eff2
Premium efficiency motors at increased motor price:	Efficiency class eff1

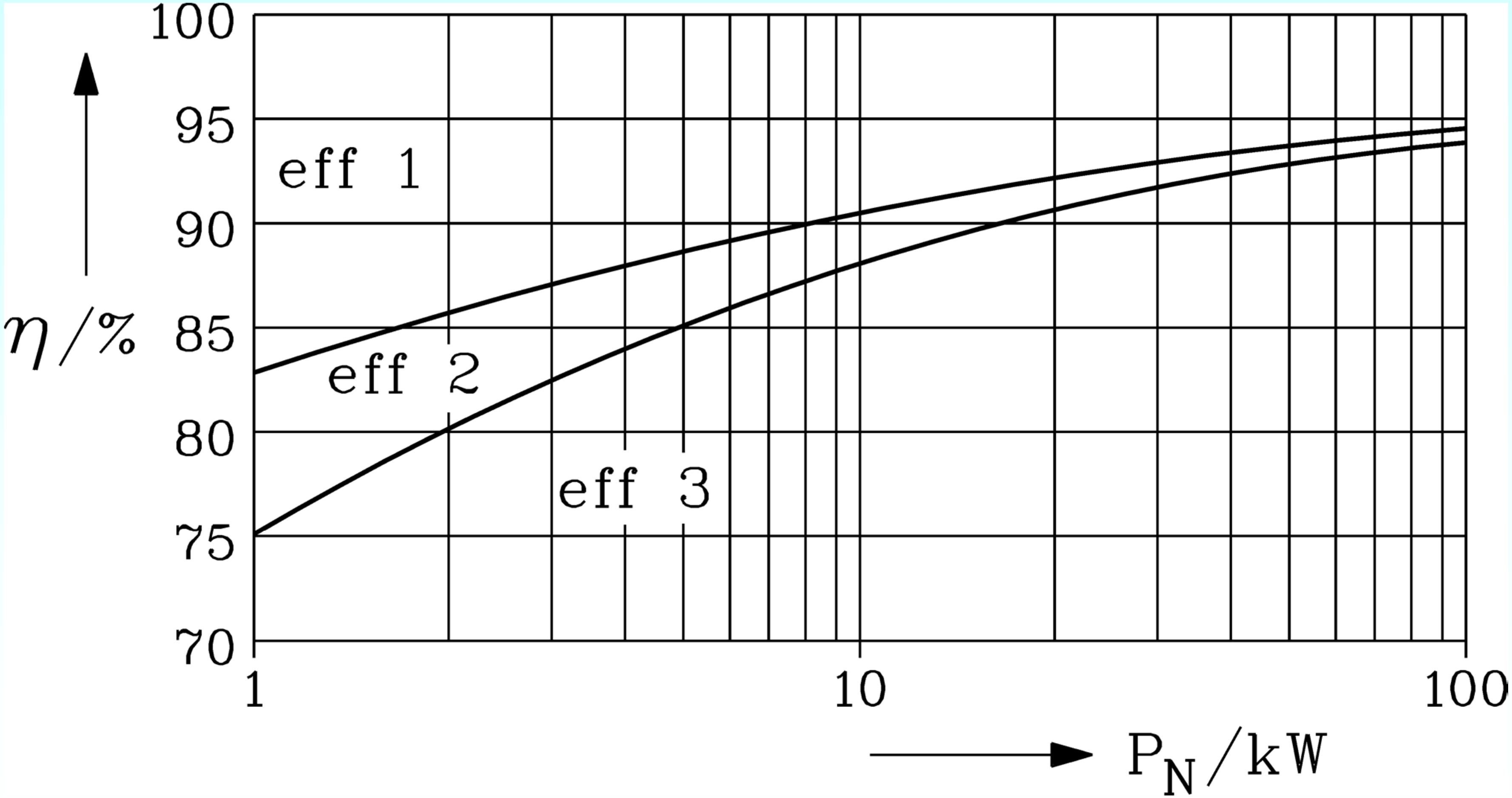
2004:

Sold motors (% of total numbers, round about): eff3: 15 %, eff 2: 80 %, eff 1: 5 %

eff1-motors are TOO expensive !

History: Efficiency classes for induction machines before 2009

Definition of efficiency classes eff1, eff2, eff3 for four pole standard induction motors in power range 1 ... 100 kW according to voluntary agreement between CEMEP and commission of EC

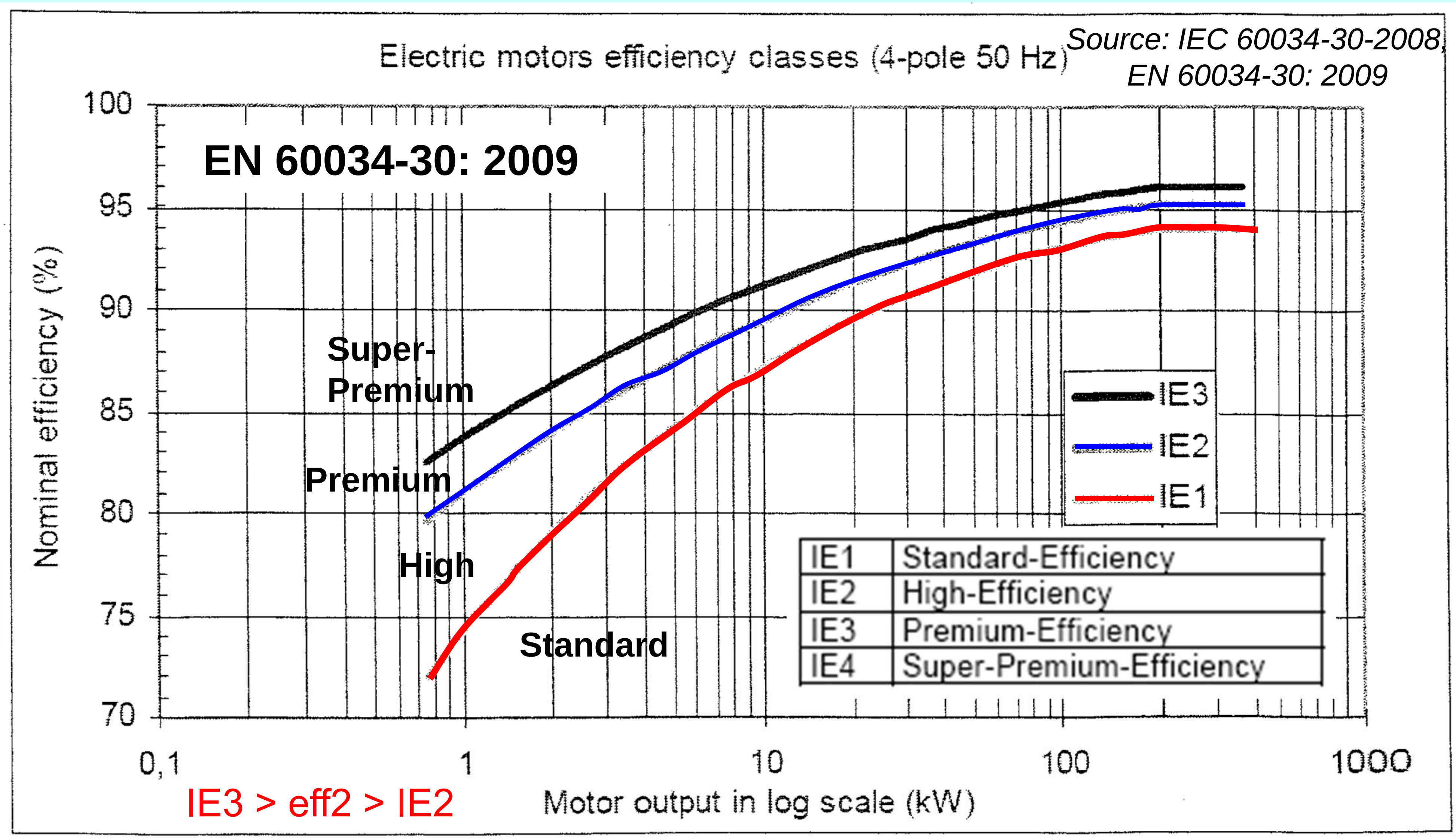


Example:

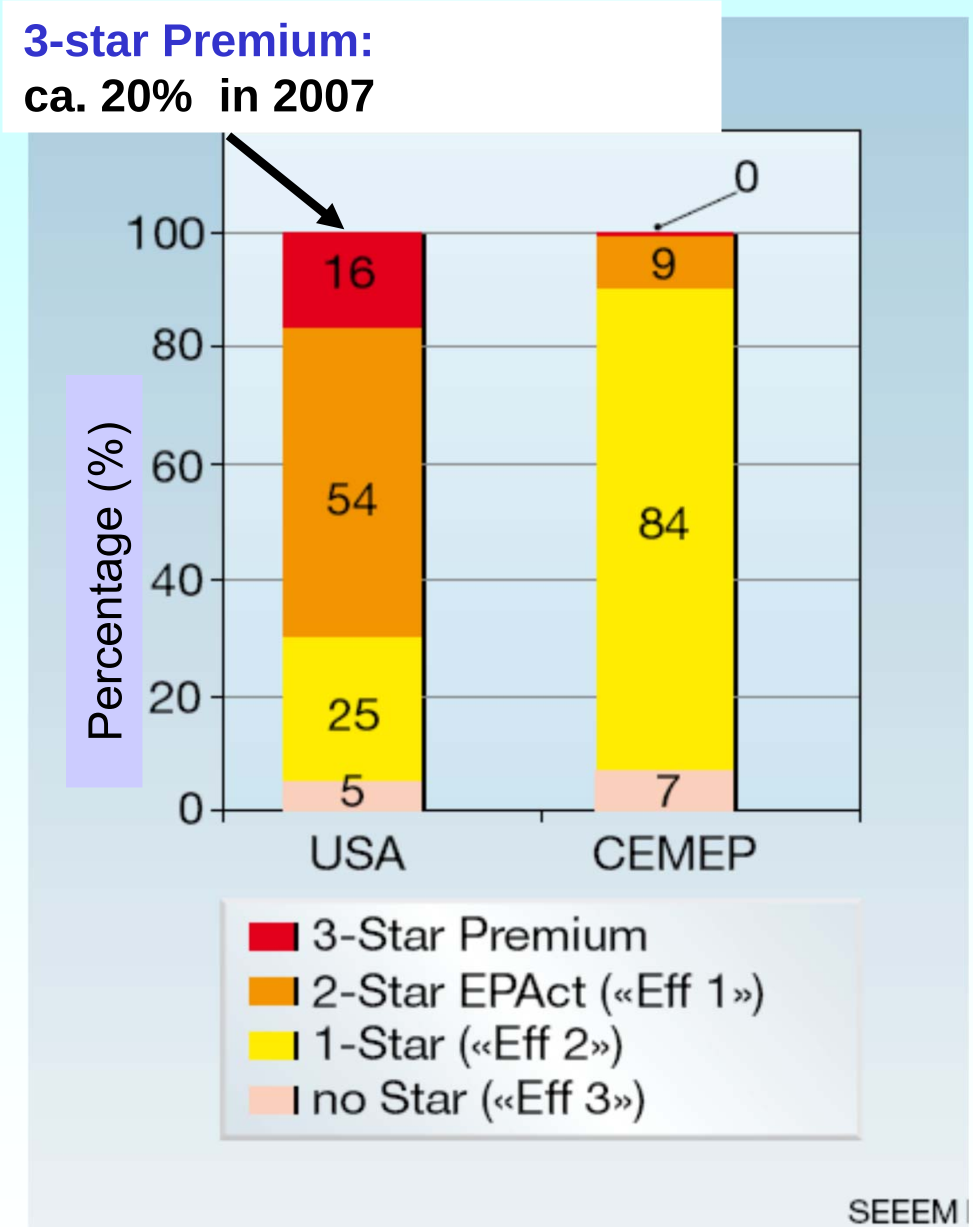
Four-pole cage induction motors

TEFC

Internationally standardized electric motors efficiency classes at 1500/min, 50 Hz (IEC 60034-30-2008)



History: Efficiency classes of sold induction motors to industry

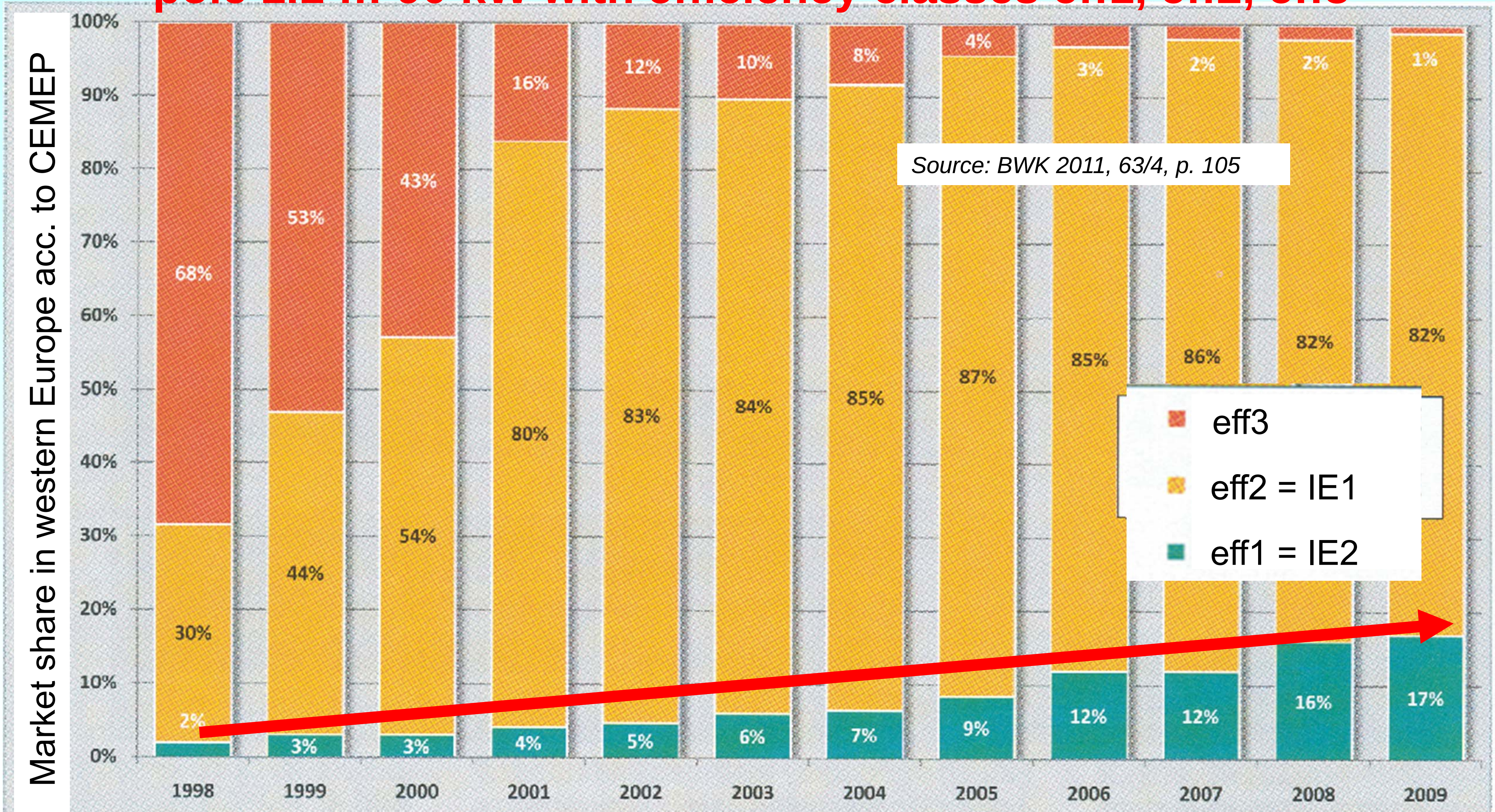


Europe 2005: (CEMEP)
2- & 4-pole standard cage induction machines TEFC, 1 ... 100 kW

USA 2004:
2- & 4-pole standard cage induction motors TEFC, 0.7 ... 200 kW

Source: SEV Bulletin, 2007

Market share of new bought standard induction motors 2 & 4-pole 1.1 … 90 kW with efficiency classes eff1, eff2, eff3



EC regulations on selling of new motors

Minimum Efficiency Performance Standard MEPS acc. to IE1 … IE3-classes!

From 16.6.2011:

New cage induction standard motors (0.75 … 375 kW, 2, 4, 6 poles, S1-operation) must be **IE2** or better!

From 1.1.2015:

New grid-fed cage induction standard motors (**7.5** … 375 kW, 2, 4, 6 poles, S1-operation) must be **IE3** or better! Alternatively inverter-fed IE2-motors may be sold.

From 1.1.2017:

New cage induction standard motors (**0.75** … 375 kW, 2, 4, 6 poles, S1-operation) must be **IE3** or better! Alternatively inverter-fed IE2-motors may be sold.

Loss balance of induction motors

Example:
Small 8-pole motor 2.5 kW

Iron losses 4%

Stator-side losses

Stator I^2R losses 12%

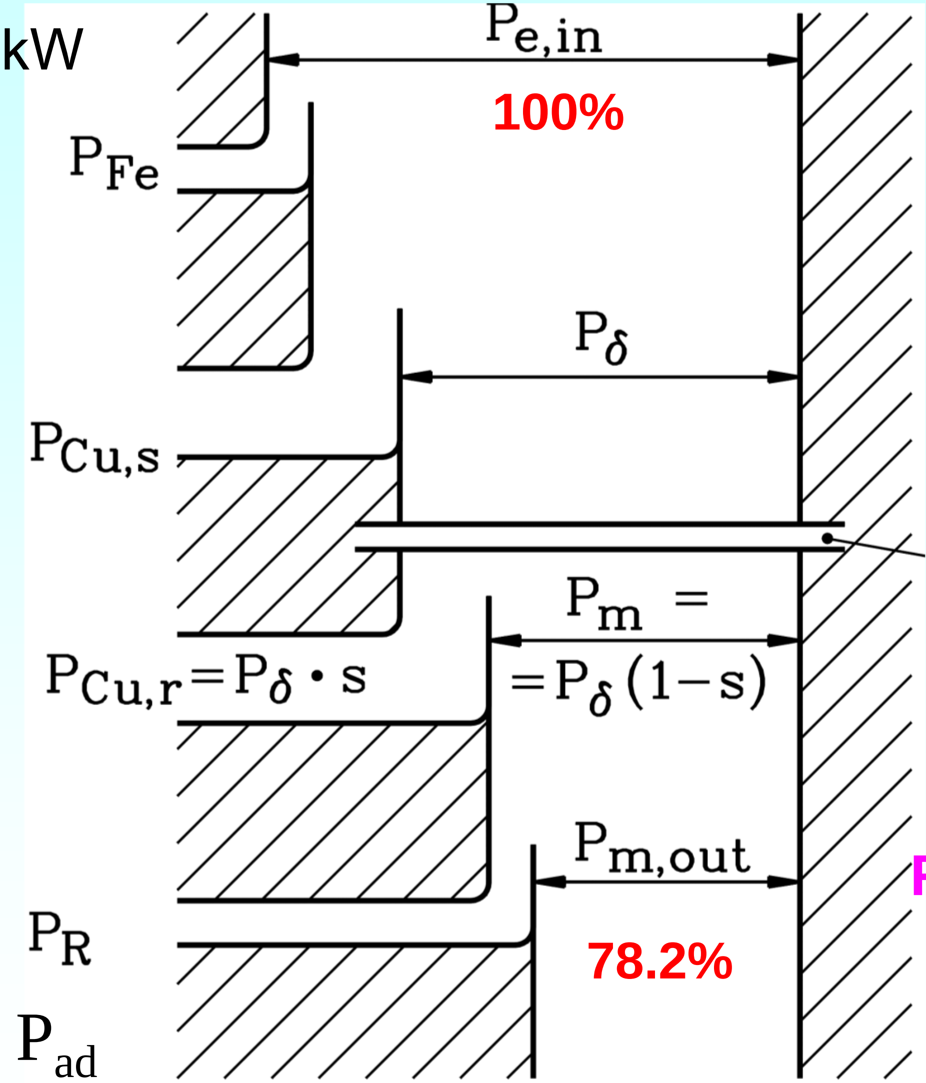
Air gap

Rotor I^2R losses 4%

Friction & windage losses 0.4%

Rotor-side losses

Stray load losses 1.4%



Induction machine losses

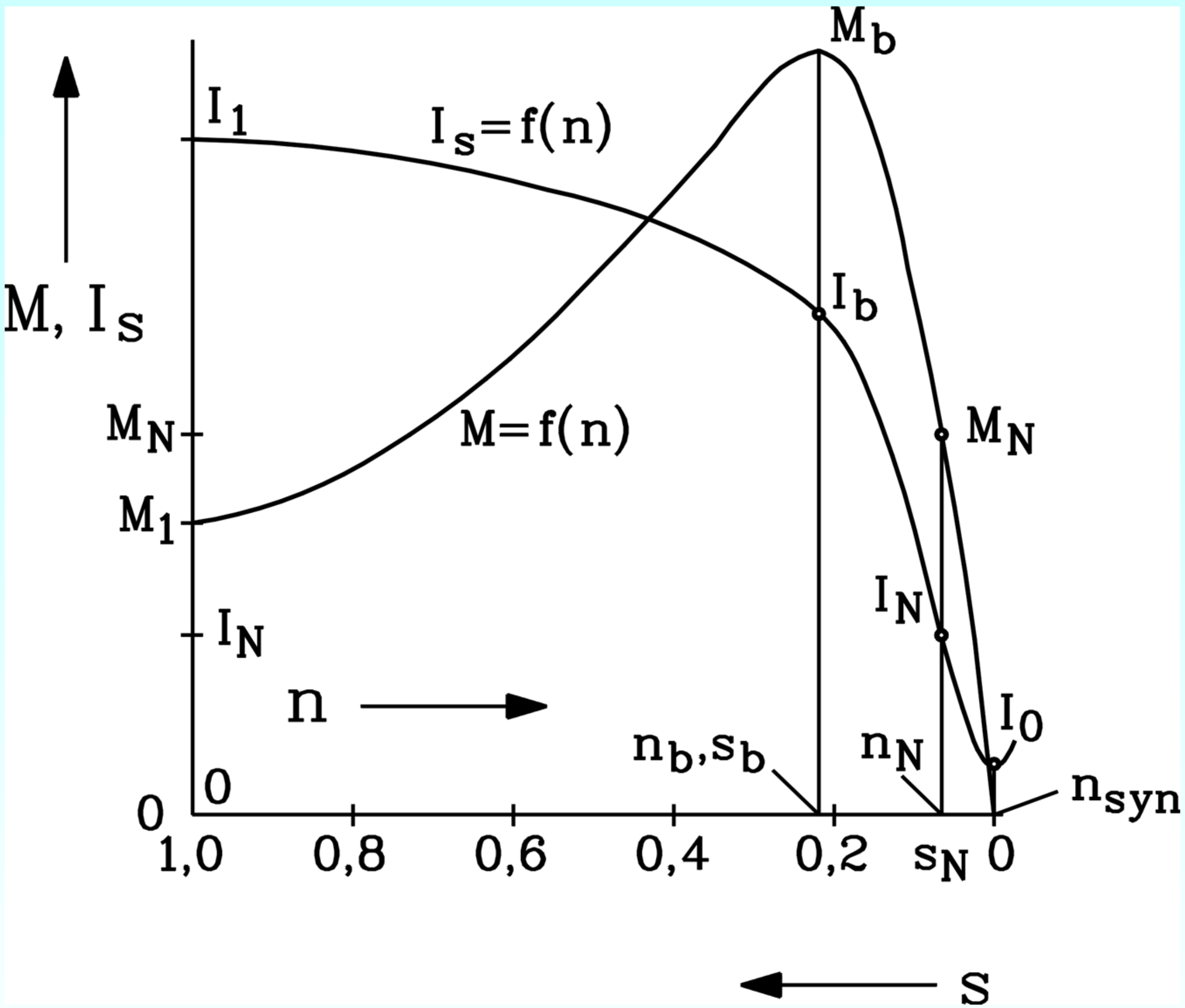
	No-load losses	*Load losses*
Stator losses	Copper losses in winding $P_{Cu,0}$ Iron losses in iron stack P_{Fe} Additional no-load losses	Copper losses in winding $P_{Cu,s}$ Additional load losses
Rotor losses	Friction and windage losses P_{fr+w} Additional no-load losses	Cage losses due to rotor current P_r Additional load losses

Slip at rated power 2.55 kW	s_N = 4.44 %
Speed / torque n / M_s	860 /min / 28.4 Nm
Measured electrical input power P_{in}	3254 W
Stator copper losses $P_{Cu,s}$	385 W (55 %)
Stator iron losses P_{Fe}	133 W (19 %)
Rotor cage losses $P_r = s \cdot P_\delta$	121 W (17 %)
Additional load losses $P_{ad,1}$	47 W (7 %)
Friction and windage losses P_{fr+w}	14 W (2 %)
Total losses P_d	700 W (100 %)
Output power P_{out}	2554 W
Efficiency	**78.49 %**

Example:

- Measured loss balance and efficiency for a:
- Thermal Class B 8-pole induction motor: 60 Hz, 440 V Y
- "Direct" method at rated load (IEC 60034-2)
- ambient temperature 20°C

Typical current and torque performance of a cage induction motor



- Between no-load and double nominal slip the speed variation is less than 10%. Hence speed is nearly constant!
- The torque and current increase proportional with slip!
- Power varies proportional with current!

$$n \approx const. \quad M \sim I_s \sim s$$

$$P = 2\pi \cdot n \cdot M \approx const. \cdot M \sim I_s$$

	Slip	Stator current	Torque
No-load	$s = 0$	I_0 = ca. $0.3 I_N$	$M = 0$
Nominal	$s = s_N$	I_N	M_N
Breakdown	$s = s_b$	I_b = ca. $2.5 I_N$	M_b = ca. $2 M_N$
Starting	$s = 1$	I_1 = ca. $4 I_N$	M_1 = ca. $0.8 M_N$

At which load for a given motor efficiency is maximum ?

No-load losses $P_{d0} = k_0 \cdot P_N$, Load losses $P_{d1} = k_1 \cdot P_N \cdot (M/M_N)^2$, Rated load losses $P_{d1N} = k_1 \cdot P_N$

We assume: $P_{d0} = const.$, $P_{d1} \sim I_s^2 \sim M^2$

Efficiency is: $\eta = \dfrac{P_{out}}{P_{out} + P_{d0} + P_{d1}}$ $\quad P_{out} = P_N \cdot (M/M_N)$ $\quad \boxed{d\eta / dM = 0}$

At load point $M/M_N\big|_{opt} = \sqrt{P_{d0}/P_{d1N}}$ ***maximum efficiency*** is achieved !

At that point no-load and load losses are equal: $P_{d0} = P_{d1}$

- Maximum efficiency:

$$\boxed{\eta_{max} = \frac{\sqrt{P_{d0}/P_{d1N}}}{\sqrt{P_{d0}/P_{d1N}} + 2 \cdot (P_{d0}/P_N)}}$$

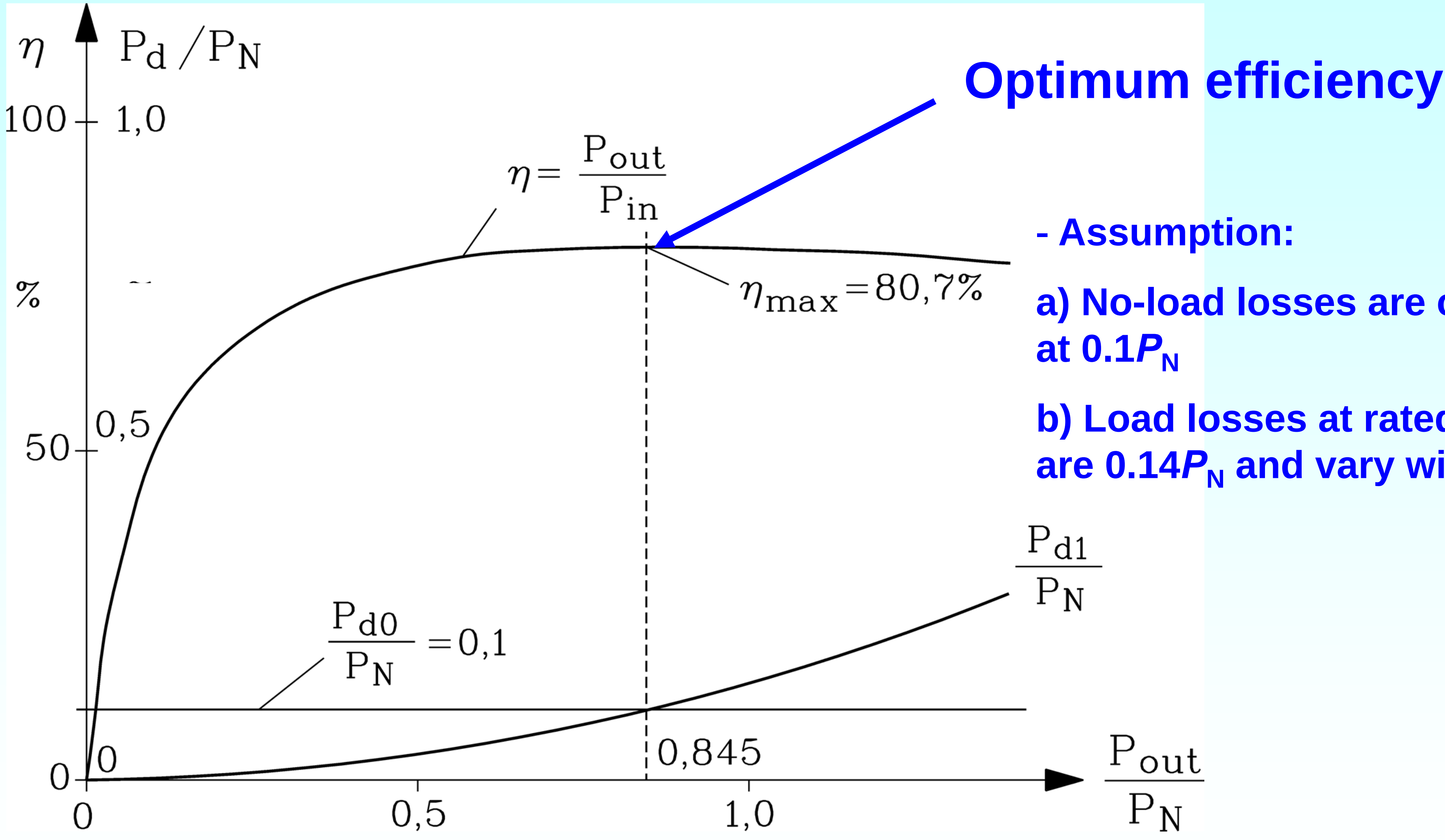
Example: $P_{d0} = 0.06 \cdot P_N$, $P_{d1} = 0.2 \cdot P_N \cdot (M/M_N)^2$

Load M/M_N	0	0.25	0.5	0.75	1.0
Efficiency η	0	77.52 %	81.96 %	81.30 %	79.36 %

At $M/M_N\big|_{opt} = \sqrt{0.06/0.2} = 0.55$ we get $\eta_{max} = \dfrac{\sqrt{0.06/0.2}}{\sqrt{0.06/0.2} + 2 \cdot 0.06} = 0.8203$

Variation of losses P_d and efficiency η

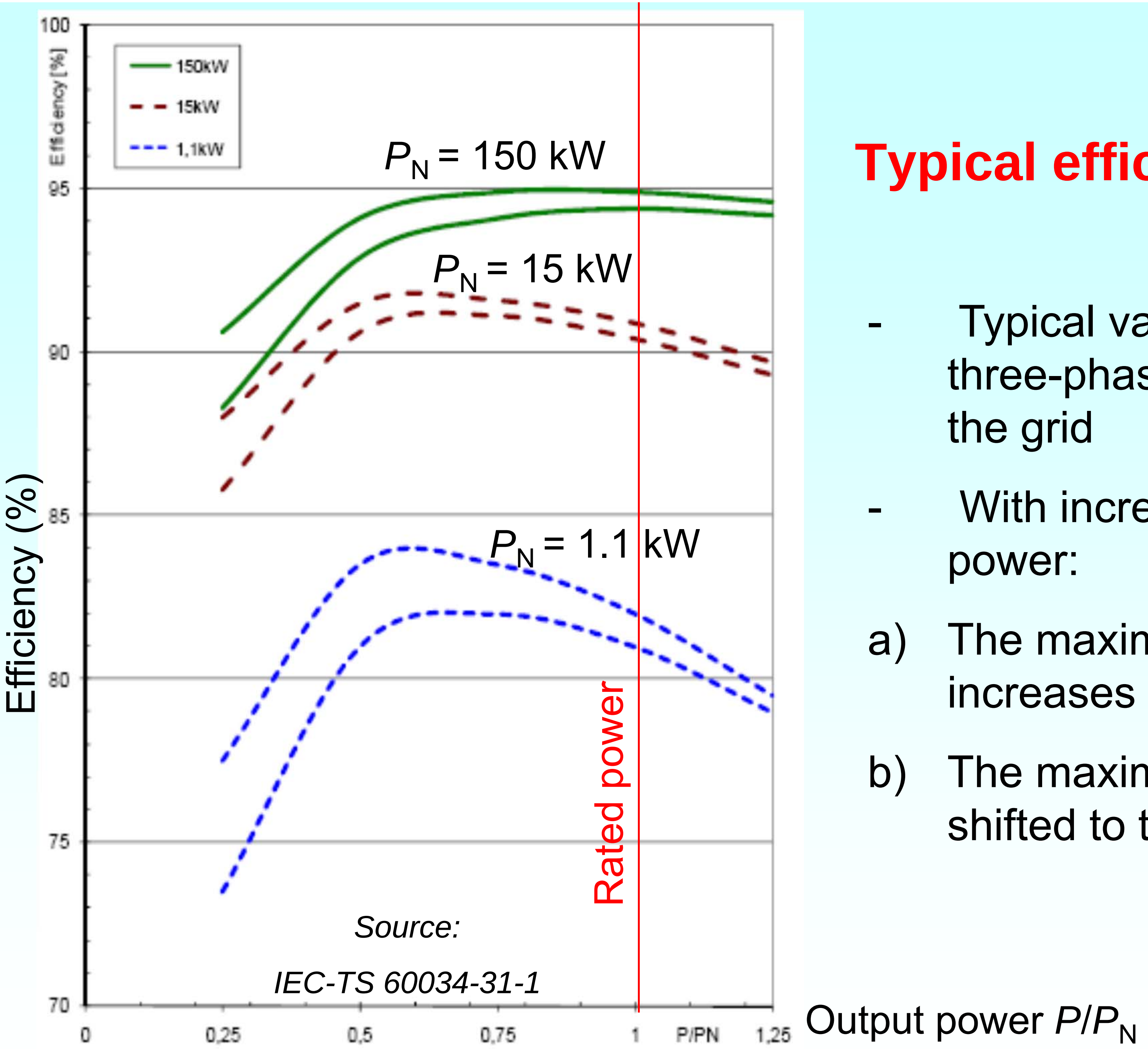
-Variation of losses P_d and efficiency η with output power P_{out}.



- **Assumption:**

a) No-load losses are constant at $0.1P_N$

b) Load losses at rated power are $0.14P_N$ and vary with P^2



Typical efficiency bands vs. load

- Typical values for 2- and 4-pole three-phase cage induction motors at the grid
- With increasing motor size and rated power:

a) The maximum motor efficiency increases

b) The maximum motor efficiency is shifted to the nominal point.

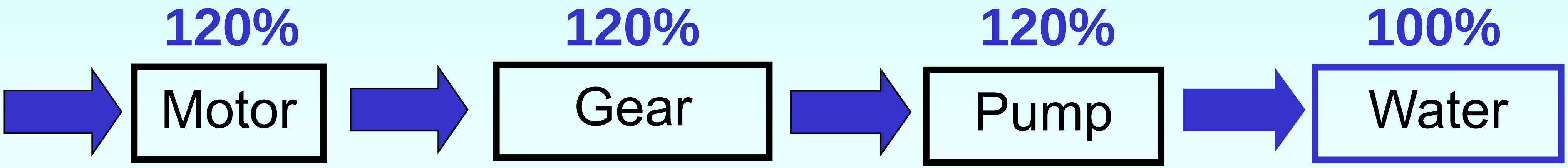
Energy waste by over-sized drives

Design with too high safety margins yields too high no-load losses!

Example: Drive chain: E-Motor, gear, pump:

Safety margin per component +20%: Leads to an oversized motors by 72%

$$1.2 \cdot 1.2 \cdot 1.2 = 1.73$$

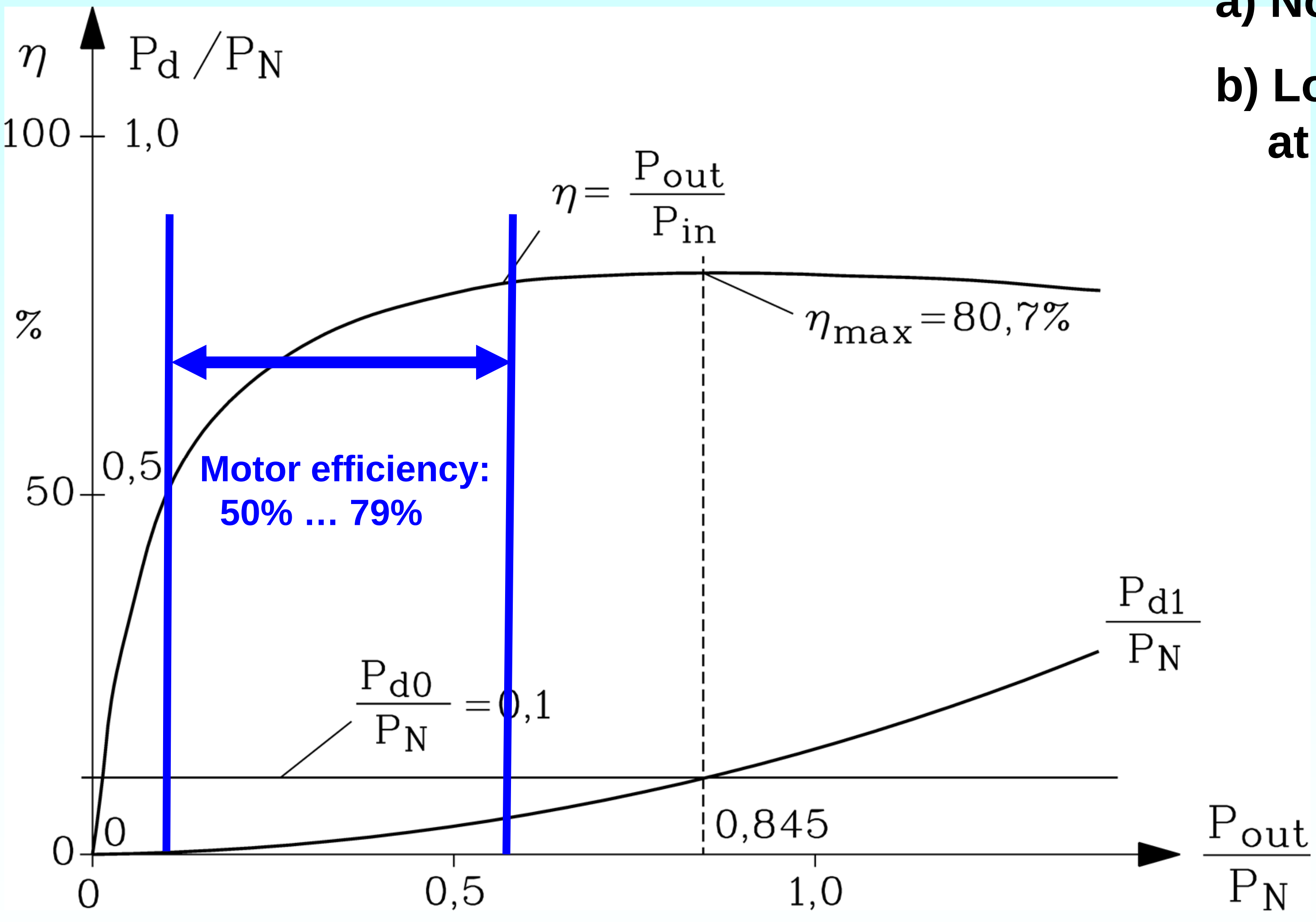


The motor is operated always at very small partial load:

Too big no-load losses of the over-sized motor = too high energy consumption

Low efficiency at very low partial-load operation

Max. motor loading only: $1/1.73 = 58\%$



Example:

a) No-load losses 0.1P_N

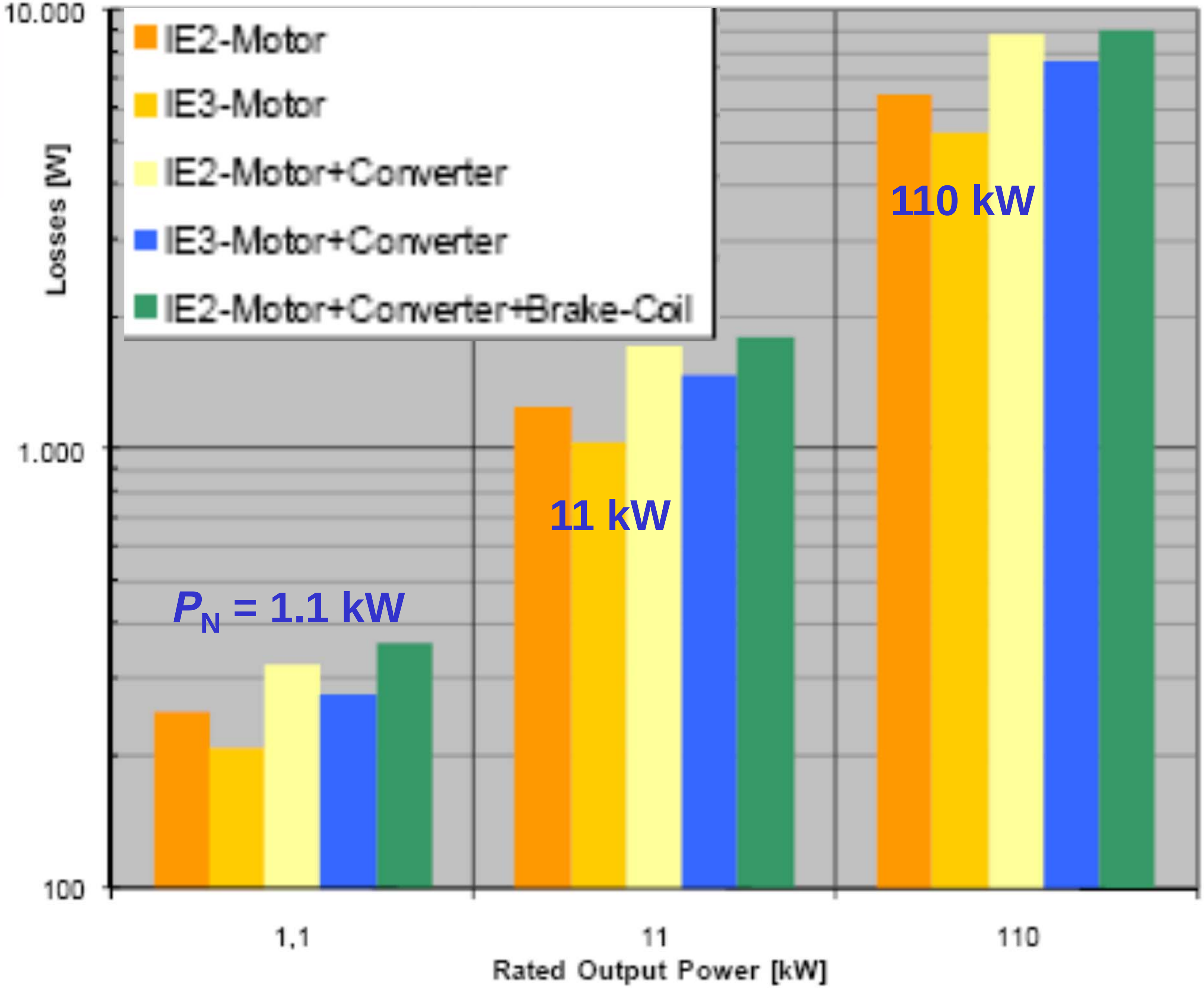
b) Load losses ~ P^2
at rated power: 0.14P_N

Used power range:

- In the hydraulic circuit (water):
  20% …. 100%

- E-Motor:
  9% … 58 %

Motor efficiency:
50% … 79%



Typical losses of energy-efficient motors

Cage induction motors are fed from
- the grid and
- via converter
- with converter, using an electric safety brake, which must be energized for being opened

Source:

IEC-TS 60034-31-1

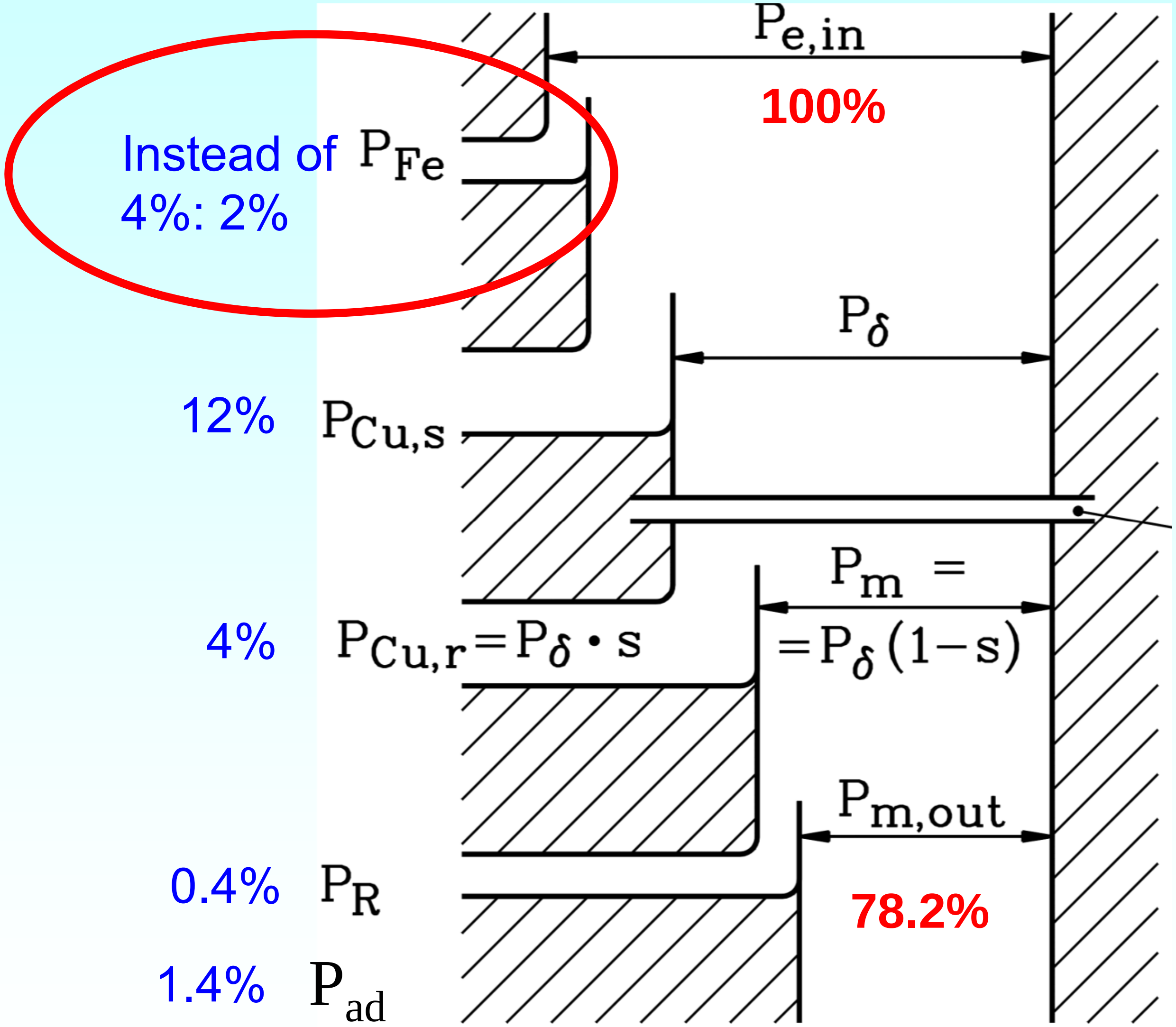
How can *efficiency* be increased for a certain motor power ?

- **Using low loss iron sheets**:
  Reduction of iron losses: 1.7 W/kg instead of 2.3 W/kg.
- **Increase of slot fill factor (copper vs. slot area)**:
  From 0.38 to 0.44 (single layer winding): Reduction of stator resistance by larger cross section per turn: Lower I^2R losses.
- **Using copper instead of aluminium cage rotor:**
  Conductivity rises by 57/34 = 167%: Reduction of rotor cage losses.
- **Increasing slot number per pole and phase *q*:** e.g. from 3 to 4:
  a) air gap flux density distribution is more sinusoidal: Additional losses are reduced.
  b) Increase of winding cooling surface: lower temperature, lower I^2R losses.
- **Increase of iron stack length:** Motor utilization “power/volume” is reduced to reach the maximum efficiency point.

A lot of rules exist to increase efficiency, but usually all these measures increase motor manufacturing costs (e. g. efficiency increase +1% = material mass increase +5%). So motors with increased efficiency are <u>usually more expensive.</u>

Use of low loss iron sheets

Loss structure:



Low loss iron sheets –

- Reduction of iron losses P_{Fe}!

e.g. M270-50A instead of M530-50A

(2.7 W/kg instead of 5.3 W/kg)

- Si-content in the iron sheets increases electric resistance – reduces the eddy current losses: BUT: reduces saturation limit !
- Magnetizing current and so $P_{Cu,s}$ are increased !
- Trade-off demands careful motor design!
- **Efficiency: 78.2% → 80.2%**

Copper cast cage for Premium Efficiency IE3 motors

Low loss cage:

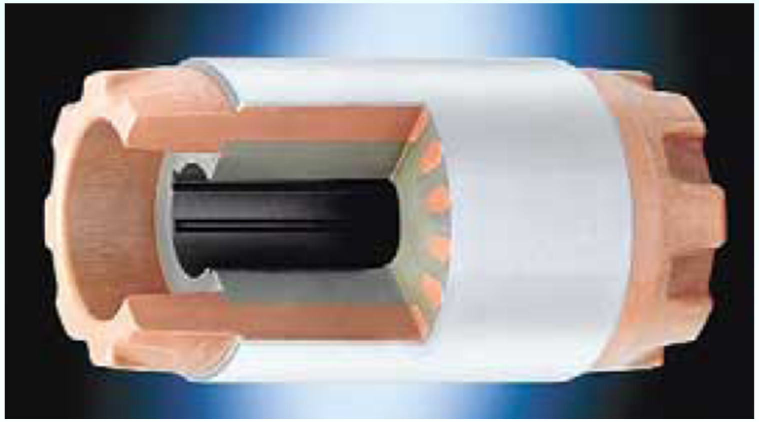
Introduction of copper die cast technology instead of aluminum die cast cages!: $P_{Cu,s}$, $P_{Cu,r}$ and additional losses are reduced ! $\underline{I}_s \sim -\underline{I}'_r \Rightarrow \underline{I}'_r \downarrow \Rightarrow \underline{I}_s \downarrow$

BUT: Much higher melting temperature: Expensive casting!

Melting temperature: Alu: 670°C, Cu: 1080°C

Alternative: Composite cage: Alu bars and copper end rings!

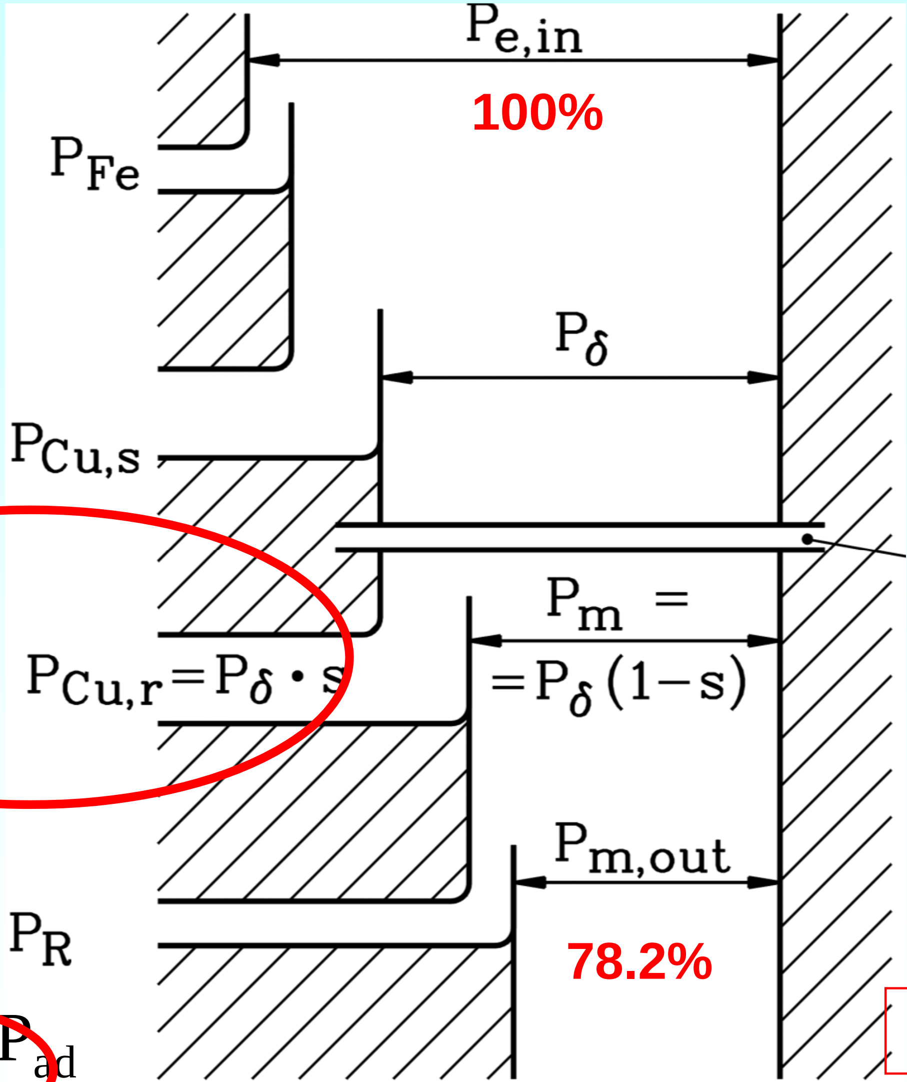
Sources: SEW-Eurodrive


Sources: Siemens AG

Copper cast cage

Sources: SEW-Eurodrive, Siemens

Loss balance:



4% (P_{Fe})

12% ($P_{Cu,s}$)

Instead of 4%: 2.3% ($P_{Cu,r}$)

0.4% (P_R)

Instead of 1.4%: 0.9 % (P_{ad})

$P_\delta \sim M_e$ at Ω_{syn}

Efficiency: 78.2% → 80.4%

$P_m \sim M_e$ at Ω_m

$P_{m,out} \sim M_s < M_e$ at Ω_m

$M_s = M_e - M_d$

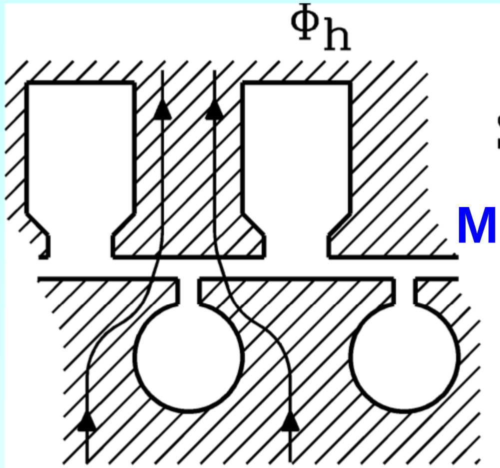
Shaft torque = Air gap torque – loss torque

Lower winding temperature - higher efficiency

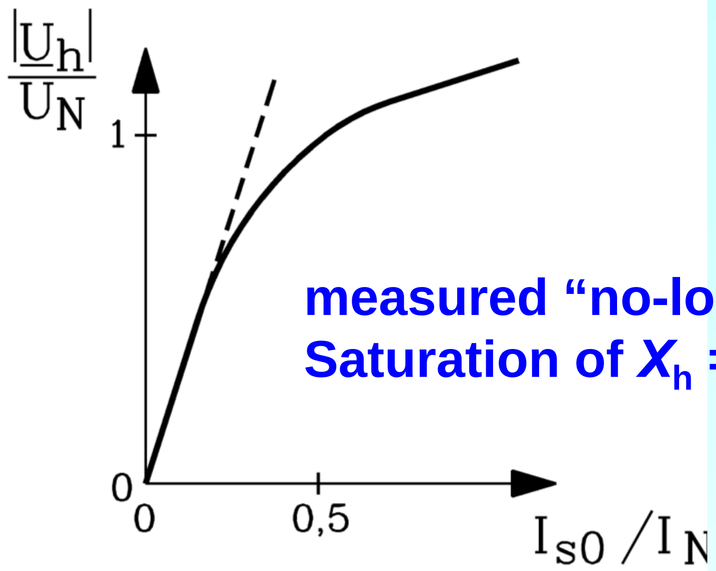
Temperature rise	60 K	105 K
Warm phase resistance	4.11 Ohm	4.7 Ohm
Slip at rated power 2.55 kW	4.44 %	5.05 %
Speed / torque	860 /min / 28.4 Nm	854 /min / 28.6 Nm
Input power	3198 W	3268 W
Stator copper losses	380 W	434 W
Iron losses	133 W	133 W
Rotor cage losses	121 W	137 W
Friction, windage & add. losses	61 W	61 W
Output power	2503 W	2503 W
Efficiency	**78.27 %**	**76.59 %**

Influence of winding temperature on efficiency for a 8-pole motor, 2.5 kW, 60 Hz, 440 V, ambient temperature 20°C

Saturation of teeth and yokes by main flux

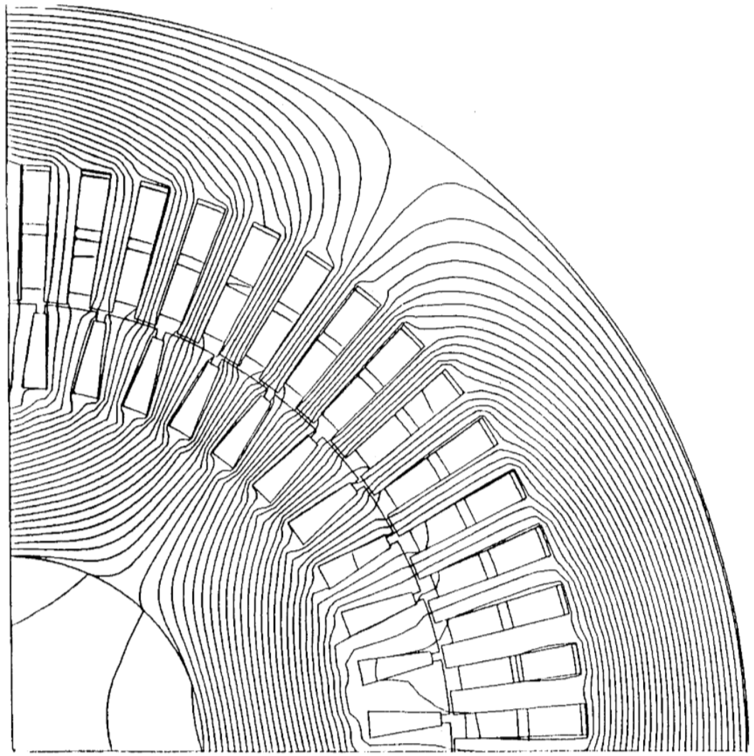


Main flux path



$= I_m$

measured "no-load"-characteristic:
Saturation of $X_h = U_h/I_0$ is measured!



Numerically calculated two-dimensional magnetic flux density B of a three-phase, 4-pole high voltage cage induction machine with wedge rotor slots at no-load (s = 0, rotor current zero)

(Q_s / Q_r = 60/44) at rated voltage

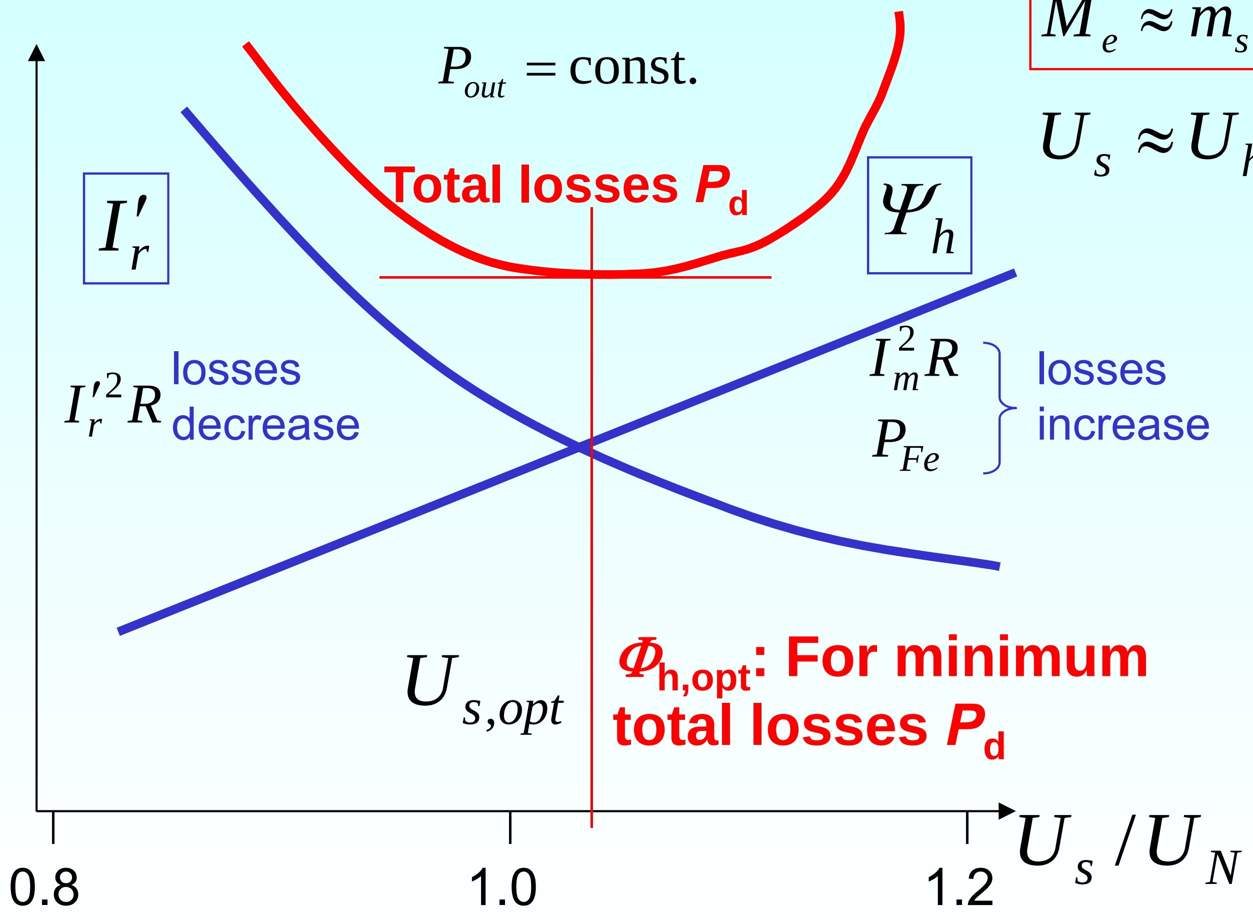
Variation of losses with stator voltage

- Grid-fed cage induction at constant output power P_{out}

$$P_{out} = 2\pi \cdot n \cdot M_s \approx 2\pi \cdot n \cdot M_e \sim M_e \qquad n \approx \text{const. for } s \approx s_N$$

$$M_e \approx m_s \cdot p \cdot I'_r \cdot \Psi_h / \sqrt{2} = \text{const.}$$

$$U_s \approx U_h = \omega_s \cdot \Psi_h / \sqrt{2} = \omega_s \cdot L_h \cdot I_m$$



For minimum losses at nominal voltage: Adjusting of $U_s = U_{s,opt}$ via number of turns of stator winding $\Phi_h = \Psi_h / (N_s k_{ws})$

$$\frac{U_N}{U_{s,opt}} = \frac{N_{s,new}}{N_{s,old}}$$

$$N_{s,new} = N_{s,old} \cdot (U_N / U_{s,opt})$$

With $N_{s,new}$ we get minimum losses at U_N !

Flux adjustment by choosing number of turns per phase

$$U_s \approx U_h = \omega_s \cdot \Psi_h / \sqrt{2} = \omega_s \cdot N_s \cdot k_{ws} \cdot \Phi_h / \sqrt{2}$$

$$U_{sN} \sim N_{s,old} \cdot \Phi_{hN}$$

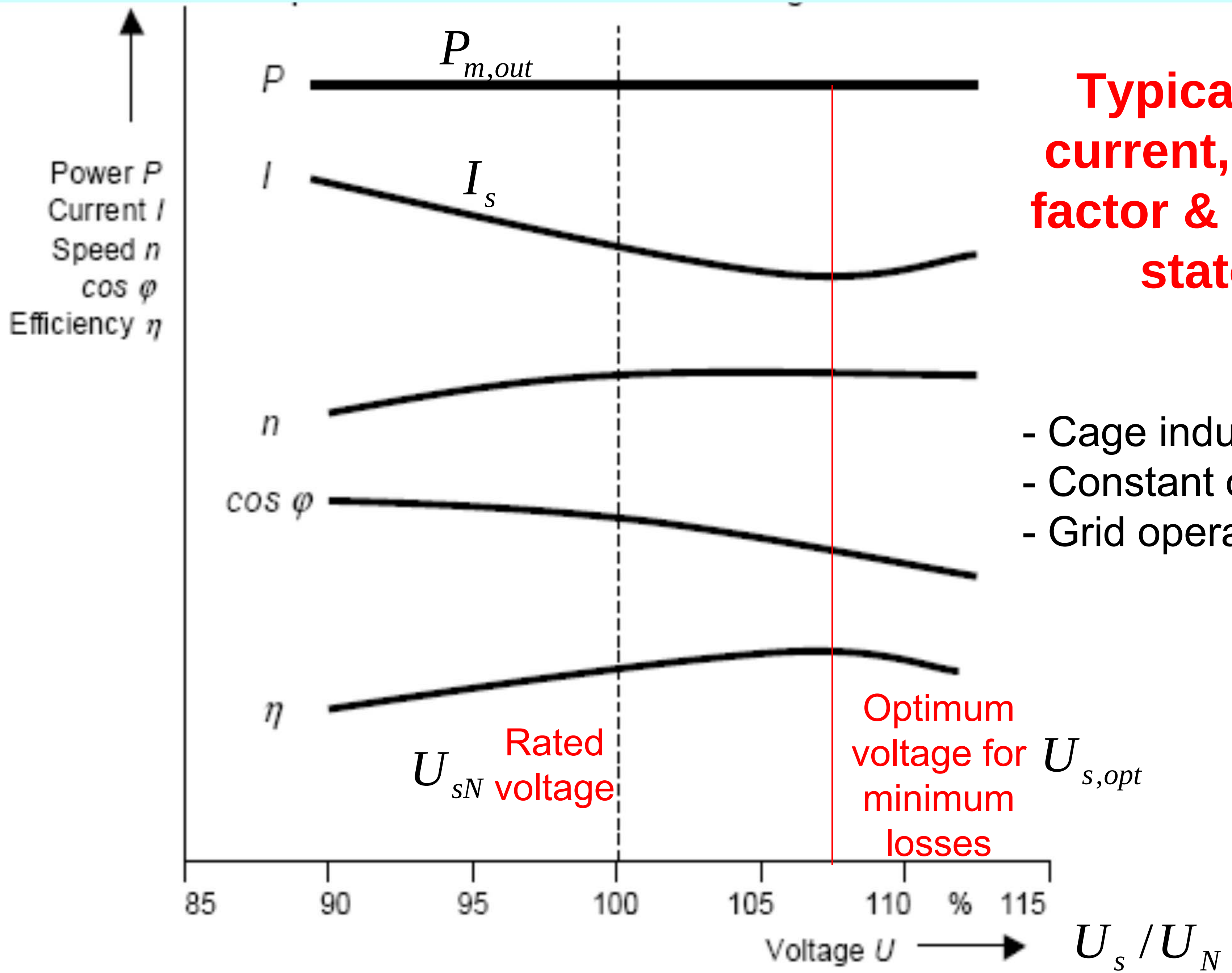
$$\left.\begin{aligned} U_{s,opt} &\sim N_{s,old} \cdot \Phi_{h,opt} \\ U_{s,N} &\sim N_{s,new} \cdot \Phi_{h,opt} \end{aligned}\right\} \quad \frac{U_{s,N}}{U_{s,opt}} = \frac{N_{s,new}}{N_{s,old}}$$

$$\boxed{N_{s,new} = N_{s,old} \cdot (U_N / U_{s,opt})}$$

With $N_{s,new}$ we get minimum losses at U_N !

- A re-winding of the motor is necessary.

- Therefore this "optimum flux"-test is done in the prototyping stage!



Typical variation of current, speed, power factor & efficiency with stator voltage

- Cage induction machines
- Constant output power
- Grid operation

Source:

IEC-TS 60034-31-1

Pay back of motors with increased efficiency

Example: 22 kW-Motor, Operating time 10 h per day = 2500 h/year
Motor with increased efficiency by 185,-- Euro more expensive

Motor	**A**	**B**
Efficiency at 86% load	**92.6%**	91%
Power consumption	20.43 kW	20.79 kW
Difference	- 0.36 kW	
Energy consumption/year	51.3 MWh	52.2 MWh
Energy savings/year	- 900 kWh	

Costs:
Energy: 9 ct/kWh, Power: 40,-- Euro/(kW & year)
Reduced costs: $0.36 \cdot 40 + 0.09 \cdot 900 = 14.4 + 81.0 = 95.4$ Euro

Pay-back time: $185 / 95.4 = 1.9 =$ ca. 2 years

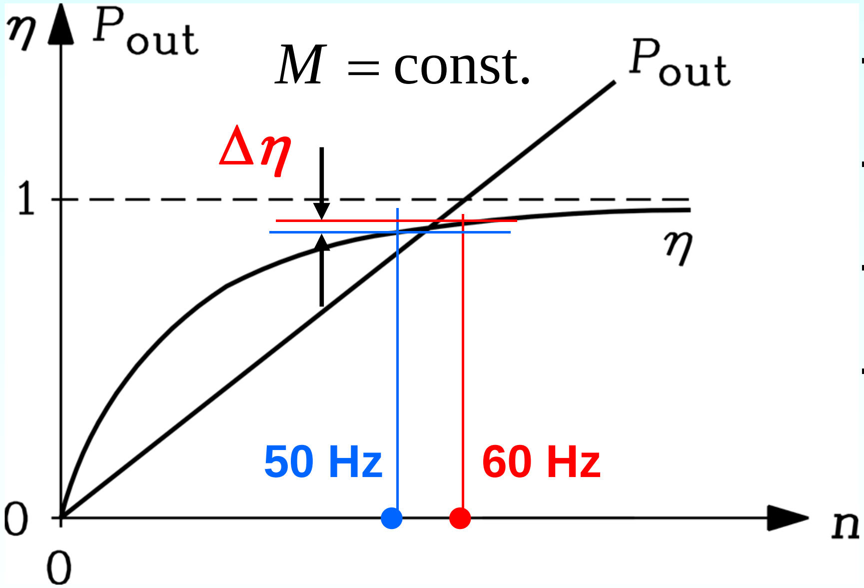
Source: SEV Bulletin, 2005

Why is the efficiency at 60 Hz higher than at 50 Hz?

Efficiency η of a motor at 60 Hz is higher than at 50 Hz at the same torque *M*, because

- speed $n = (1-s)\cdot f_s/p$ and therefore power P_{out} is increased by 20% at the same slip
- whereas the losses stay more or less constant!

$$M \sim \Psi_h \cdot I'_r \approx \Psi_h \cdot I_s : \quad U_s = U_N = \text{const.} : M \sim I$$

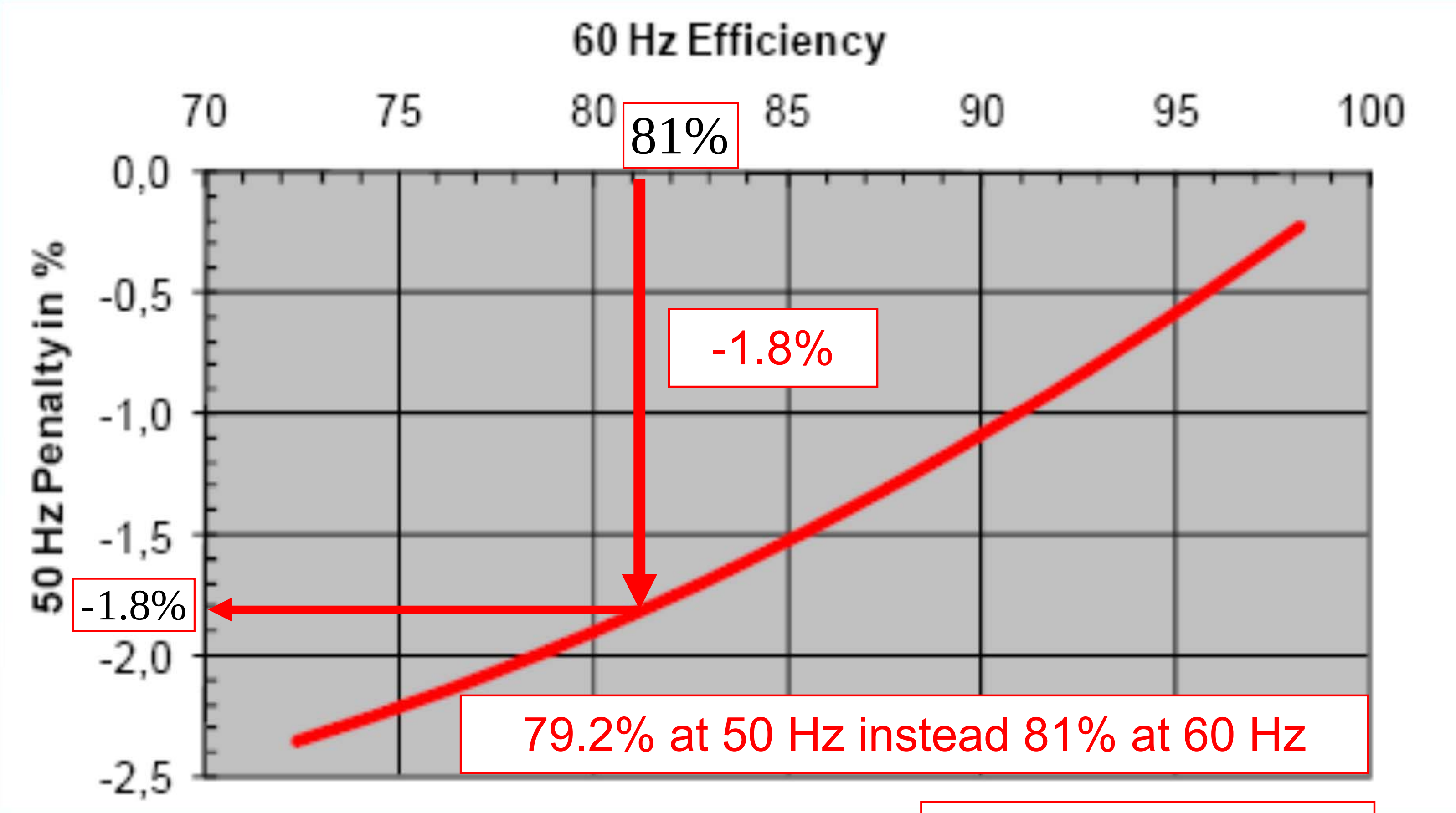


$\Delta\eta$: Efficiency increase at 60Hz

- Torque is proportional to Current: $M \sim I$
- Losses P_d are prop. to (Current)^2 ~ $R \cdot I^2$
- Power P_{out} is prop. to Speed x Torque
- Speed *n* is prop. to stator frequency

$$\eta = \frac{P_{out}}{P_{out} + P_d} = \frac{2\pi \cdot n \cdot M}{2\pi \cdot n \cdot M + k_d M^2}$$

Typical reduction of efficiency in %-points from 60 Hz to 50 Hz grid operation



$81\% - 1.8\% = 79.2\%$

- 4-pole low voltage cage induction motors
- Same rated torque, BUT power at 60 Hz by 20% increased

Source:
IEC-TS 60034-31-1

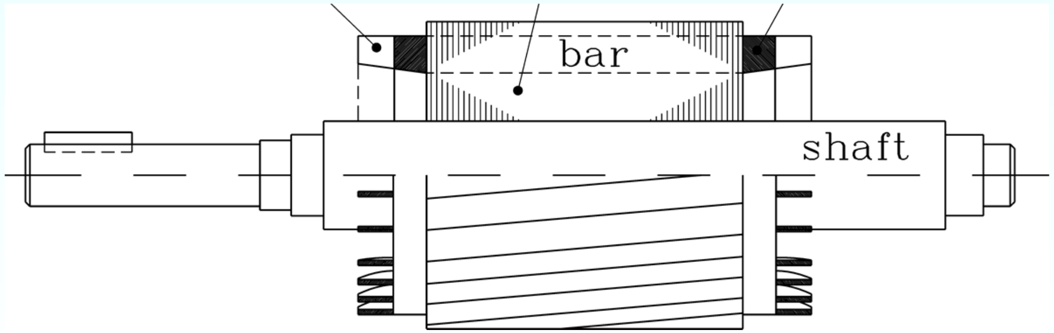
Grid-operated cage induction motor vs. inverter-operated PM synchronous motors

Stators with distributed 3-phase AC windings

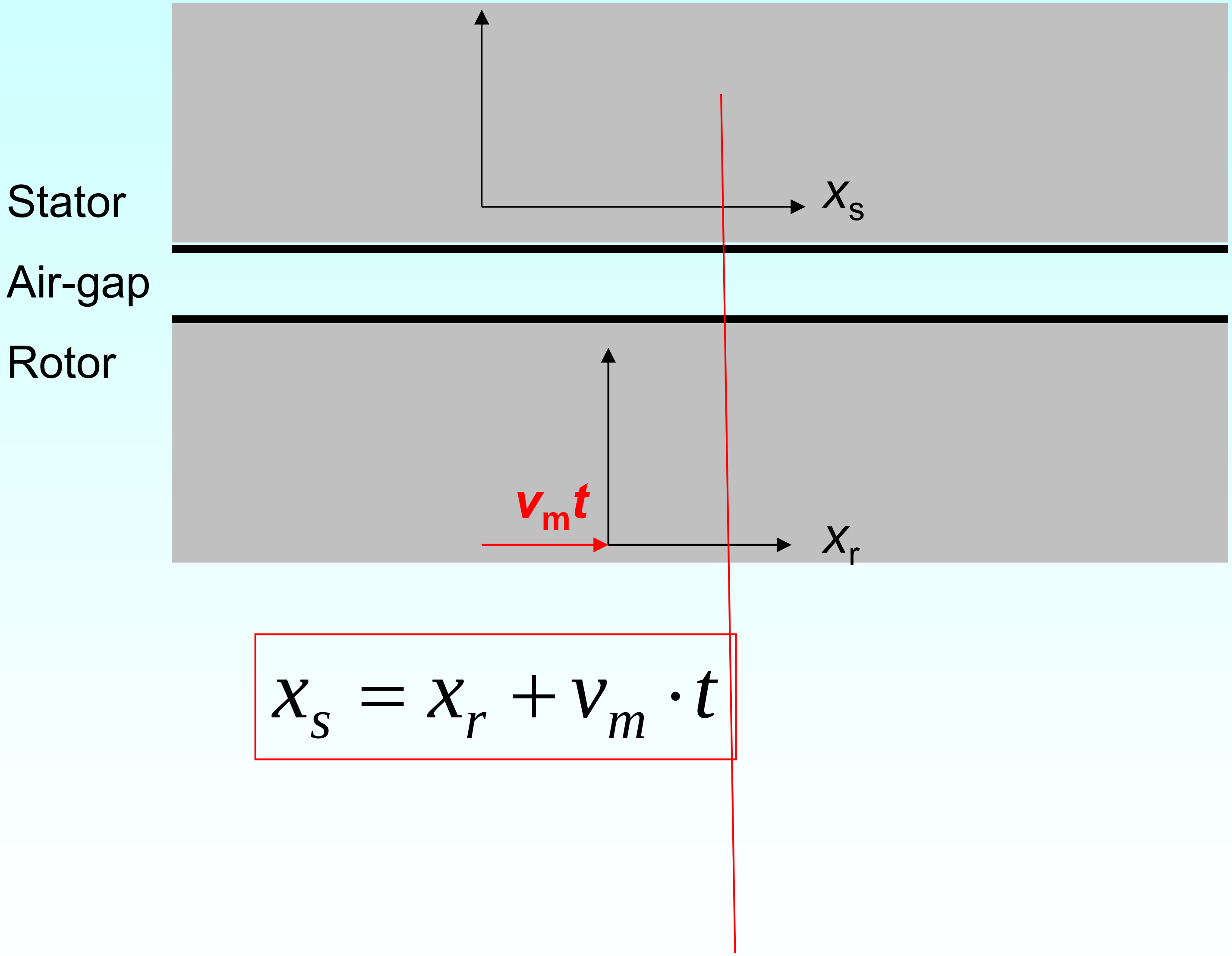
	Induction motor, eff2 ≈ IE2	PM synchronous motor	PM synchronous motor
Cooling	Shaft-mounted fan	Shaft-mounted fan	No fan
Motor operation	Grid	Inverter	Inverter
Frame size	132 mm	100 mm	132 mm
Frequency	50 Hz	100 Hz	75 Hz
Rot. speed	1450/min	1500/min	1500/min
Pole count	4	8	6
Active mass	40.4 kg	26.6 kg	50.5 kg
Power rating	**7500 W**	**8950 W**	**8640 W**
Nomin. Efficiency	**89.0%**	**91.0%**	**94.3%**

→ Efficiency increases !

4.4 Space harmonic effects in induction machines

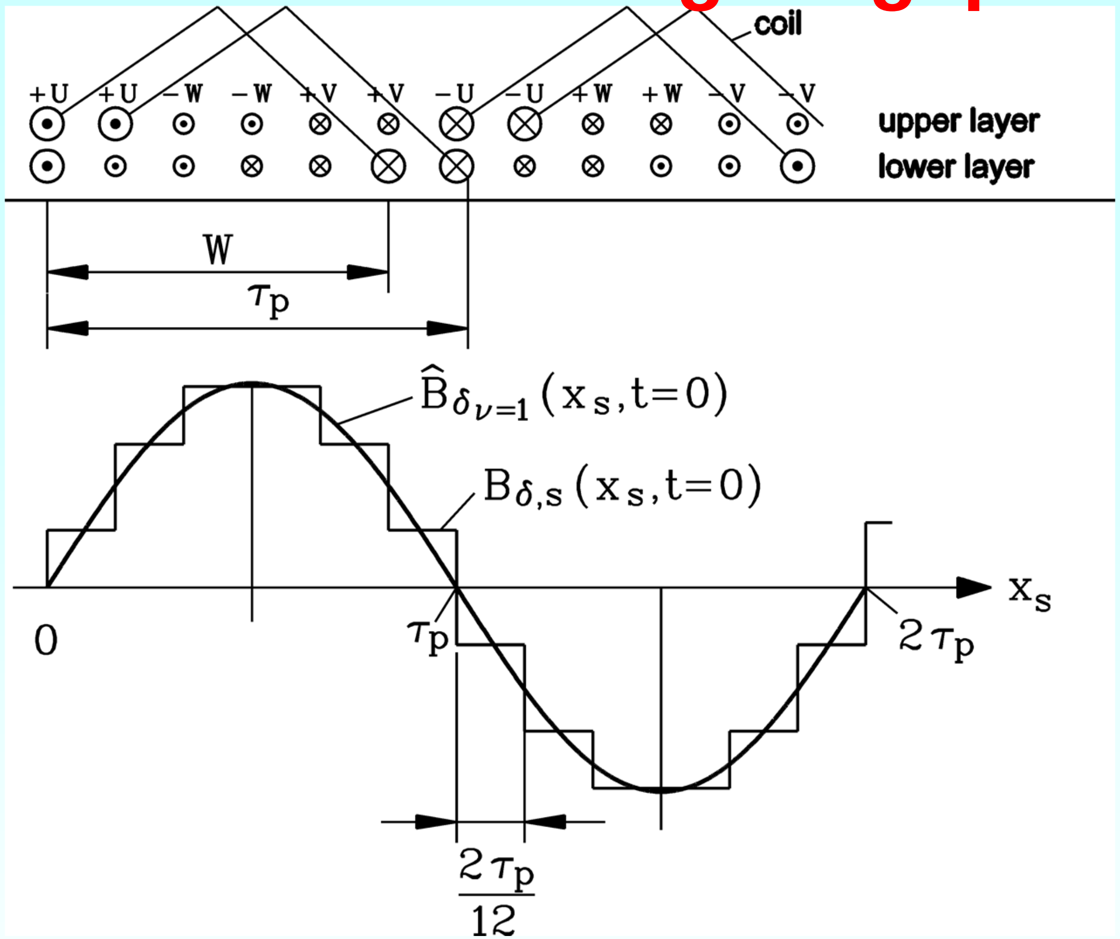


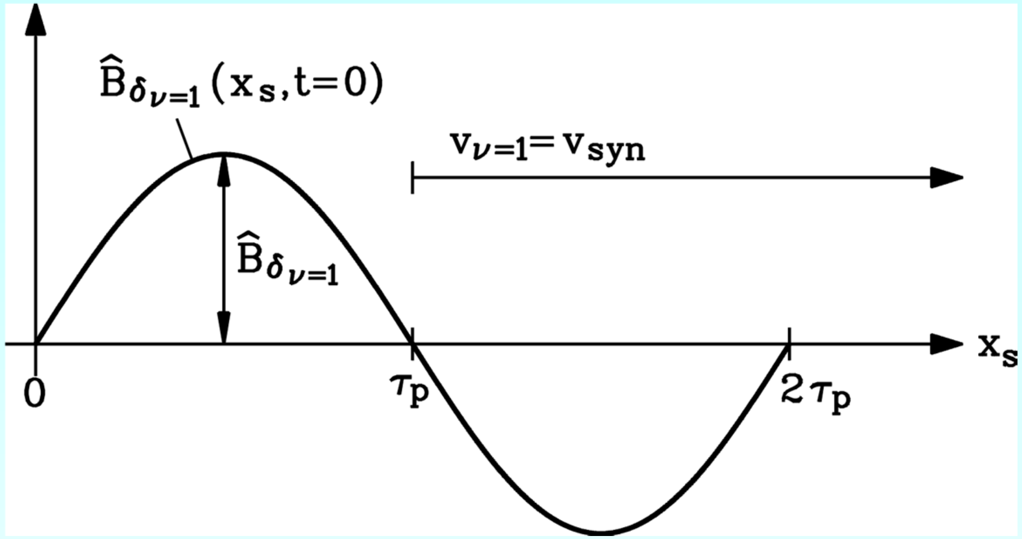
Stator and rotor coordinate frame



$$x_s = x_r + v_m \cdot t$$

Stator winding air gap flux density distribution

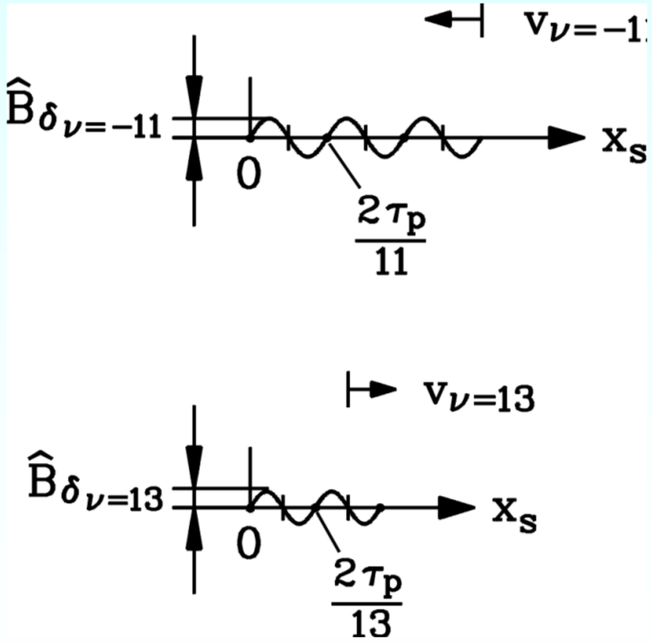




Example:

m = 3, *q* = 2

coil pitch 5/6



Step air gap field function contains *FOURIER* fundamental and harmonic waves with

- smaller amplitude, smaller wave length, smaller speed

- determined by ordinal number ν

FOURIER analysis of stator step air gap field function

Wave function as *FOURIER* sum $$B_{\delta,s}(x_s,t) = \sum_{\nu=1,...}^{\infty} B_{\delta,\nu} \cdot \cos(\frac{\nu\pi x_s}{\tau_p} - \omega_s t)$$ $\omega_s = 2\pi f_s$

Ordinal number at m_s = 3: $\nu = 1 + 2m_s \cdot g$ = 1, -5, 7, -11, 13, -17,...

(g: integer number: $g = 0, \pm1, \pm2, \pm3, ...$)

Wave amplitude $$B_{\delta,\nu} = \frac{\mu_0}{\delta} \cdot \frac{\sqrt{2}}{\pi} \cdot \frac{m_s}{p} \cdot N_s \frac{k_{w,\nu}}{\nu} \cdot I_s$$ (no iron saturation considered)

Wave length $\lambda_\nu = 2\tau_p / |\nu|$

Wave velocity: $v_\nu = \lambda_\nu \cdot f_s$

Sign of **ordinal numbers** ν: direction of wave velocity:

+: with fundamental, -: opposite to fundamental

Winding factor: $k_{w,\nu} = k_{p,\nu} \cdot k_{d,\nu}$ **pitch factor x distribution factor**

Fourier analysis of stator step air gap field function

Example:

Three phases, four poles, two-layer winding: q = 2, W/τ_p = 5/6, Q_s = 24 slots:

	Relative amplitudes	*winding factor*			*wave speed at f_s = 50 Hz*
ν	$\lvert B_{\delta\nu} / B_{\delta 1} \rvert$ (%)	$k_{p,\nu}$	$k_{d,\nu}$	$k_{w,\nu}$	v_ν (m/s)
1	100	0.966	0.966	0.933	6.28
-5	1.4	0.259	0.259	0.067	- 1.26
7	1.0	0.259	-0.259	-0.067	0.9
-11	9.1	0.966	-0.966	-0.933	- 0.6
13	7.7	-0.966	-0.966	0.933	0.5
-17	0.4	-0.259	-0.259	0.067	- 0.37
19	0.38	-0.259	0.259	-0.067	0.33

"Slot harmonics": $\nu = 1 + \frac{Q_s}{p} \cdot g$

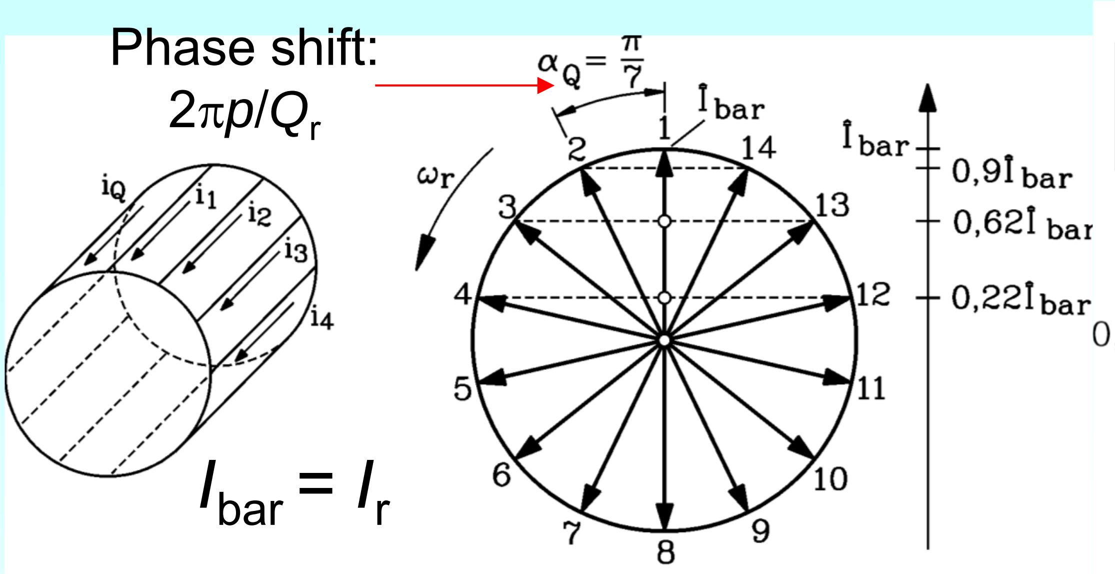
Q_s/p = 24 / 2 = **12**

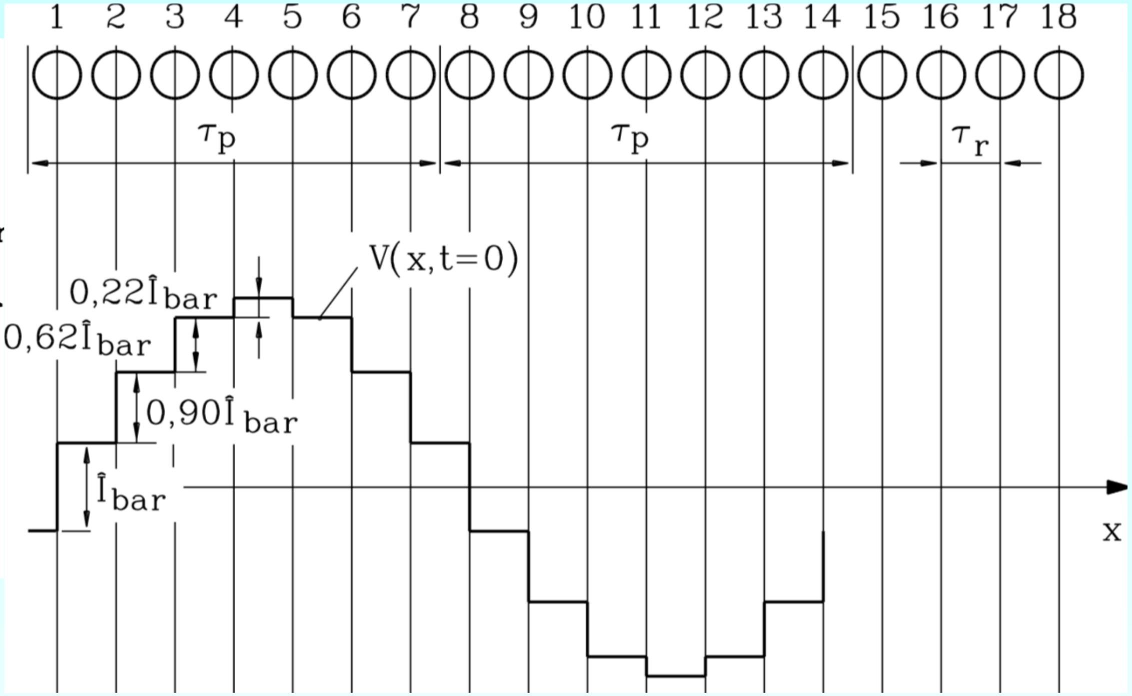
First pair of slot harmonics: ν = -11, 13

Wave length : $\lambda_{-11} = 2\tau_p / 11, \lambda_{13} = 2\tau_p / 13$

Average: $(\lambda_{-11} + \lambda_{13}) / 2 \cong 2\tau_p / 12$

Rotor current distribution and rotor field





$f_r = s \cdot f_s$

Squirrel-cage winding with Q_r = 28 bars, $2p$ = 4: Rotor magnetic field calculated form m.m.f.: Determination here for unsaturated case: $B_\delta = \mu_0 V / \delta$

- Velocity of rotor field: $v = v_m + v_{r,syn} = 2pn\tau_p + 2f_r\tau_p = 2p \cdot n_{syn}(1-s)\cdot\tau_p + 2\cdot sf_s\cdot\tau_p =$

$$= 2p\cdot\frac{f_s}{p}\cdot(1-s)\cdot\tau_p + 2\cdot sf_s\cdot\tau_p = 2f_s\tau_p = v_{syn}$$

Rotor fundamental field rotates synchronously with stator field = electromagnetic torque is constant !

Rotor cage analogy to a poly-phase winding

- Ordinal number μ instead of ν
- Each bar = a rotor phase: $m_r = Q_r$
-1/2 turn per phase: N_r = ½
- Winding factor is unity: $k_{wr,\mu} = 1$
- Formula for ordinal numbers: Stator: $\nu = 1 + 2m_s \cdot g$

$2m_s$: Number of phase belt per pole pair

Rotor:

Q_r/p: Number of phase belt per pole pair

$$\mu = 1 + (Q_r / p) \cdot g$$

$$B_{\delta,s}(x_s, t) = \sum_{\nu=1,-5,7,\ldots}^{\infty} B_{\delta,\nu} \cdot \cos\left(\frac{\nu \pi x_s}{\tau_p} - \omega_s t\right)$$

↓

$$B_{\delta,r}(x_r, t) = \sum_{\mu=1}^{\infty} B_{\delta,\mu} \cdot \cos\left(\frac{\mu \pi x_r}{\tau_p} - \omega_r t\right)$$

Rotor cage air gap field harmonics

Rotor cage air gap field harmonics, excited by rotor current I_r:

- Ordinal number μ !
- 1/2 turn per phase: $N_r = 1/2$
- Each bar = a rotor phase: $m_r = Q_r$!
- Winding factor is unity: $k_{wr,\mu} = 1$.

Wave function as *FOURIER* sum $$B_{\delta,r}(x_r,t) = \sum_{\mu=1}^{\infty} B_{\delta,\mu} \cdot \cos\left(\frac{\mu\pi x_r}{\tau_p} - \omega_r t\right) \qquad \omega_r = 2\pi f_r$$

Ordinal number: $\mu = 1 + (Q_r / p) \cdot g$ (g: integer number: $g = 0, \pm 1, \pm 2, \pm 3, \ldots$)

Wave amplitude $$B_{\delta,\mu} = \frac{\mu_0}{\delta} \cdot \frac{\sqrt{2}}{\pi} \cdot \frac{Q_r}{p} \cdot \frac{1}{2} \cdot \frac{1}{\mu} \cdot I_r$$ (no iron saturation considered)

Wave length $\lambda_\mu = \frac{2\tau_p}{|\mu|}$ **Wave velocity** $v_{\mu,r} = \lambda_\mu \cdot f_r = 2 \cdot s \cdot f_s \cdot \tau_p / \mu = s \cdot v_{syn} / \mu$

Stator winding induced by rotor field harmonics

- The stator winding is only induced by those rotor space harmonics, that have the same pole count as the stator space harmonics !

- *Example:* Stator: Single layer inter slot winding: q = integer:

  Stator ordinal numbers: $\nu = 1, -5, 7, -11, 13, -17, 19, -23, 25, -29, 31, -35, 37, ...$

  Rotor: 4-pole motor, 28 rotor bars: $\mu = 1, -13, 15, -27, 29, ...$

Facit:

Only the rotor space harmonics with 13 and 29 pole pairs will induce the stator winding.

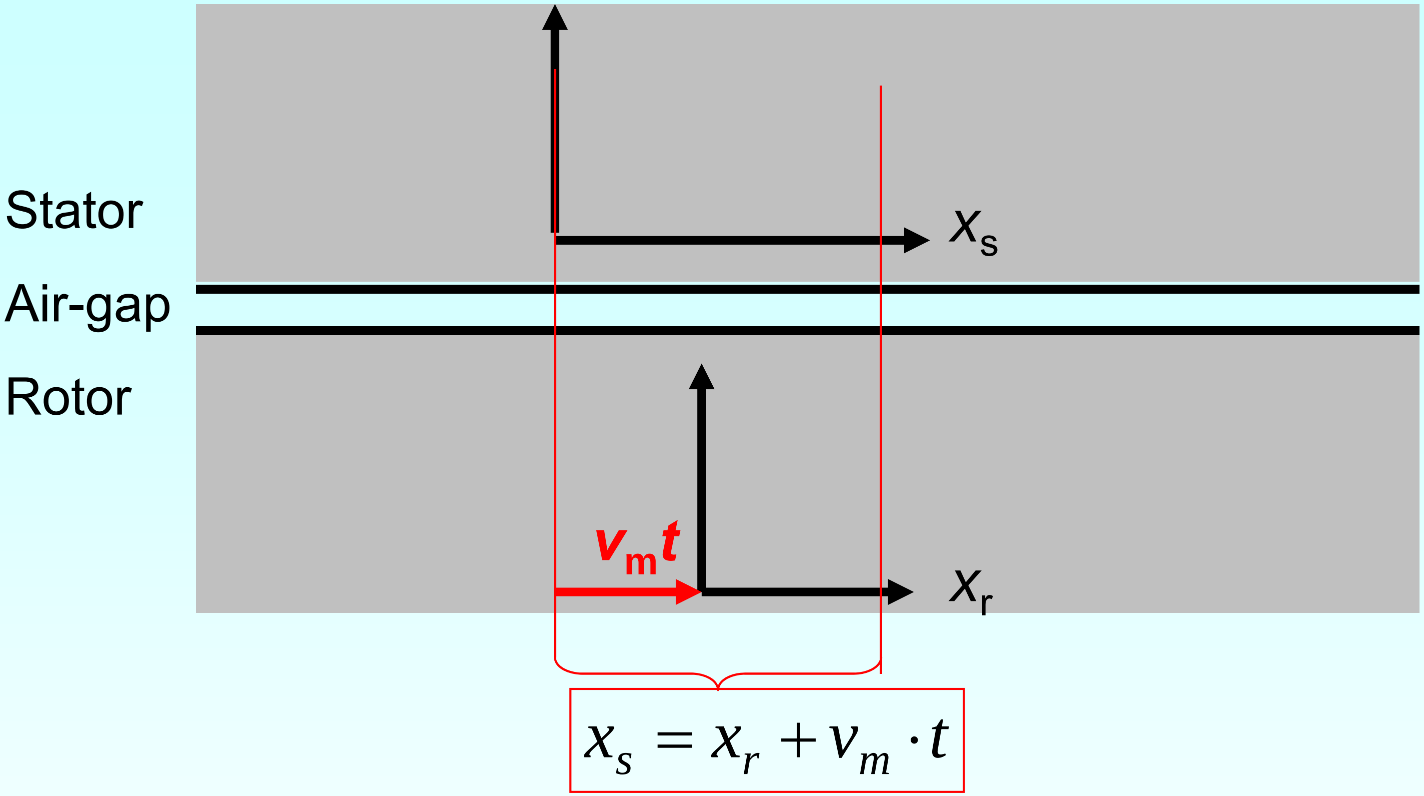
Explanation:

- Stator ordinal numbers ν are odd and not divisible by 3:

  a) Rotor field waves with even pole pair number: e.g.: Flux per coil is zero.

  b) Rotor field waves with 3g as pole pair number: Voltage in all three phases identical, cannot drive a current in Y-connected winding.

Rotor field harmonics in stator coordinate frame



$$x_r = x_s - v_m t = x_s - (1-s)\cdot v_{syn}\cdot t = x_s - (1-s)\cdot 2 f_s \tau_p \cdot t$$

$$B_{\delta r\mu}(x_r,t) = B_{\delta r\mu}\cdot\cos\left(\frac{\mu\pi x_r}{\tau_p} - 2\pi\cdot s\cdot f_s t\right) \qquad B_{\delta r\mu}(x_s,t) = B_{\delta r\mu}\cdot\cos\left(\frac{\mu\pi x_s}{\tau_p} - 2\pi f_s t\cdot(s+\mu(1-s))\right)$$

- Rotor field harmonics induce stator winding with frequency $f_{r,\mu} = f_s\cdot|\mu(1-s)+s|$
- Speed of rotor field harmonics relative to stator: $v_\mu = \dot{x}_s = v_{syn}\cdot(1-s+s/\mu)$

Stator harmonic currents, induced by rotor field harmonics

- Speed of rotor field harmonics relative to stator: $v_\mu = v_m + v_{\mu,r} = v_{syn} \cdot (1 - s + s/\mu)$
- Rotor field harmonics induce stator winding with frequency $f_{r,\mu} = f_s \cdot |\mu(1-s) + s|$
- Stator harmonic currents are generated, causing additional losses !

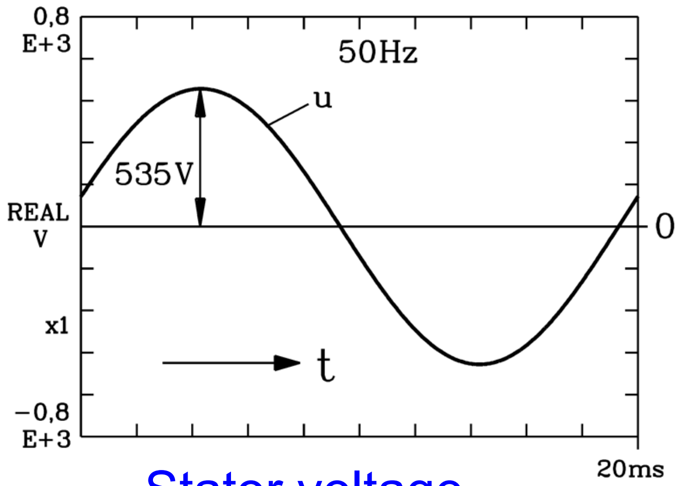
	relative amplitudes	*wave speed with respect* *to rotor*		*to stator*
μ	$\lvert B_{\delta\mu} / B_{\delta 1}\rvert$	$v_{\mu,r}$	(m/s)	v_μ
1	100 %	0.31		6.28
-13	7.6 %	- 0.024		5.95
15	6.7 %	0.02		6.0
-27	3.7 %	- 0.011		5.96
29	3.4 %	0.01		5.98

	frequency of stator current harmonics
μ	$f_{r,\mu}$ / Hz at $s_N = 5\%$
1	50
-13	615
15	Not induced
-27	Not induced
29	1380

Example: 4-pole motor,28 rotor bars, stator frequency: 50 Hz, slip $s_N = 0.05 \approx 0$:

$$f_{r,\mu} = f_s \cdot |\mu| = f_s \cdot |1 + g \cdot Q_r / p| \approx f_s \cdot |g| \cdot Q_r / p$$ **~ rotor slot frequency**

Measured stator harmonic currents

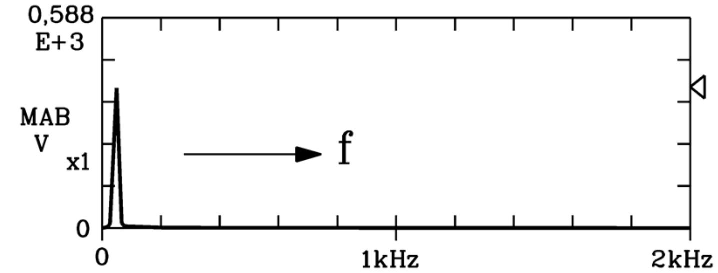


Stator voltage sinusoidal

voltage SPECTRUM

		RMS
1	50,0 Hz	0,377E+3 V
2	255,0	0,1912E+1
3		
4		
5		
6		
7		
8		
9		
10		

U_{LL}=377V

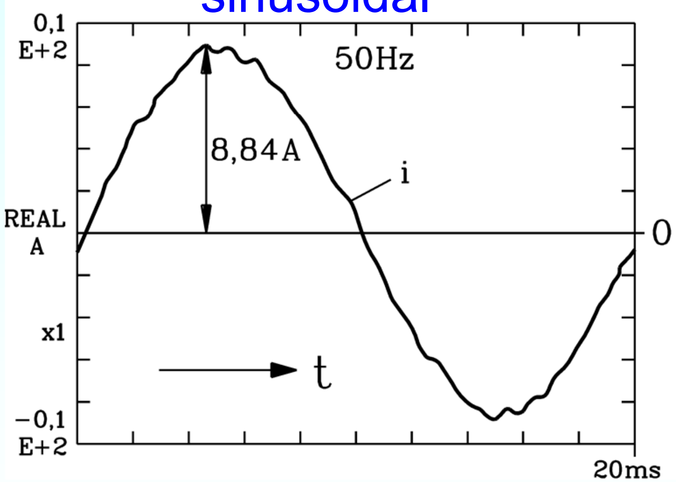


current SPECTRUM

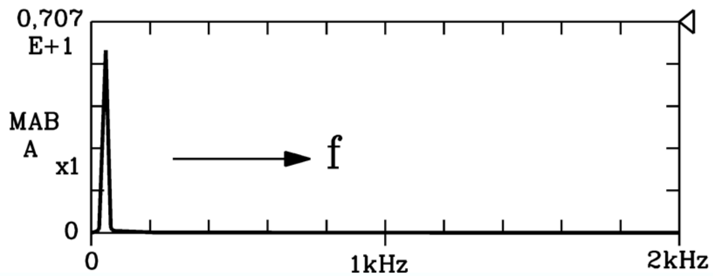
		RMS
1	50,0 Hz	0,615E+1 A
2	355,0	0,778E−1
3	910,0	0,795E−1
4	255,0	0,458E−1
5	1110,0	0,490E−1
6	555,0	0,358E−1
7	610,0	0,250E−1
8	555,0	0,233E−1
9	610,0	0,248E−1
10	1210,0	0,280E−1

I: 6,16A

Harmonic currents



Stator current contains harmonics



FOURIER spectra

Loaded 2-pole 3 kW cage induction motor, 22 rotor slots, at sinusoidal voltage supply, rated slip 0.05 :

Measured stator line-to-line voltage

Measured stator phase current:

showing stator harmonic currents also with roughly 22x50 = 1110 Hz ≈1095 Hz.

$$f_{r,\mu} = f_s \cdot |\mu(1-s)+s| =$$

$$= 50 \cdot |23 \cdot (1-0.05)+0.05| =$$

$$= 1095 Hz$$

Calculation of measured stator harmonic currents

Motor data:

2-pole motor, 3 kW, 380 V Y, 50 Hz, 6.2 A, skewed cage with 22 rotor bars, rated slip 0.05.

Stator line-to-line peak voltage value: $\sqrt{2}\cdot 380 = \underline{\underline{537}}$ V (measured: 535 V)

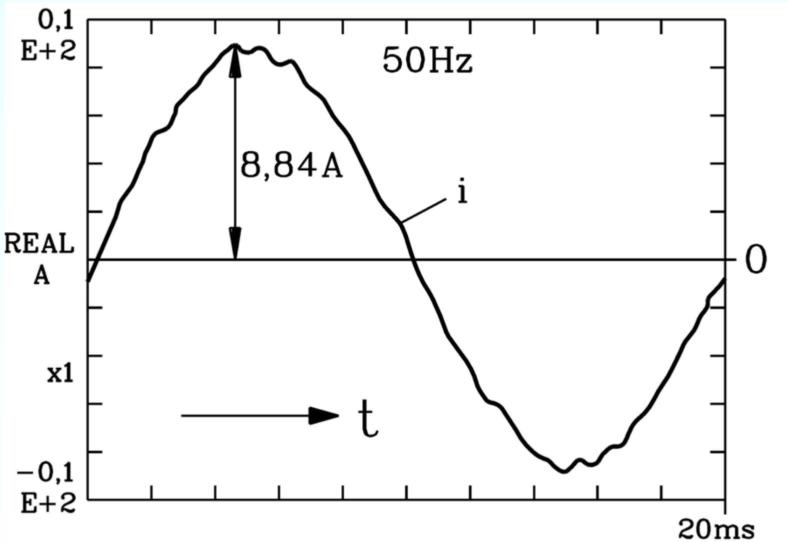
Stator current peak value: $\sqrt{2}\cdot 6.2 = \underline{\underline{8.77}}$ A (measured: 8.84 A)

Ordinal numbers of rotor space harmonics: $\mu = 1 \pm Q_r / p = 1 \pm 22/1 = \underline{\underline{-21, 23}}$

Corresponding stator current harmonic frequencies:

$\mu = -21:$ Not induced, as 21 is no stator harmonic ordinal number!

$\mu = 23: \quad f_{r,\mu} = 50\cdot\left|23\cdot(1-0.05)+0.05\right| = \underline{\underline{1095}}$ Hz



current SPECTRUM

measured

	Frequency	RMS
1	50,0 Hz	0,615E+1 A
2	355,0	0,778E−1
3	910,0	0,795E−1
4	355,0	0,458E−1
5	1110,0	0,490E−1
6	555,0	0,358E−1
7	610,0	0,250E−1
8	555,0	0,233E−1
9	610,0	0,248E−1
10	1210,0	0,280E−1

I: 6,16A

Harmonic rotor bar currents, induced by stator field harmonics

- Rotor slip with respect to stator field harmonics (**harmonic slip**):

$$s_\nu = \frac{n_{syn,\nu} - n}{n_{syn,\nu}} \quad \Rightarrow \quad s_\nu = 1 - \nu \cdot (1 - s) \quad \leftarrow \begin{cases} n = (1-s) \cdot n_{syn,\nu} \\ n_{syn,\nu} = n_{syn} / \nu \end{cases}$$

- Stator field harmonics induce rotor cage with frequency $f_{r,\nu}$, causing currents $I_{r,\nu}$

$$f_{r,\nu} = s_\nu \cdot f_s = f_s \cdot \left| 1 - \nu \cdot (1 - s) \right|$$

- **Rotor time-harmonic bar currents $I_{r,\nu}$:** Phase shift between adjacent harmonic bar currents is $2\pi \cdot \nu \cdot p / Q_r$

- Rotor cage is induced by ALL stator field harmonic waves !

Example: Stator frequency 50 Hz, no-load s = 0, 28 rotor bars.

Induced by stator field harmonic ν = -5.

$$s_{\nu=-5} = 1 - \nu \cdot (1 - s) = 1 + 5 \cdot (1 - 0) = \underline{\underline{6}}, \quad f_{r,\nu=-5} = s_\nu \cdot f_s = 6 \cdot 50 = \underline{\underline{300}}\,\text{Hz}$$

Phase shift between bar currents: $\varphi_\nu = 360° \cdot \nu \cdot p / Q_r = 360 \cdot (-5) \cdot 2 / 28 = -128.6°$.

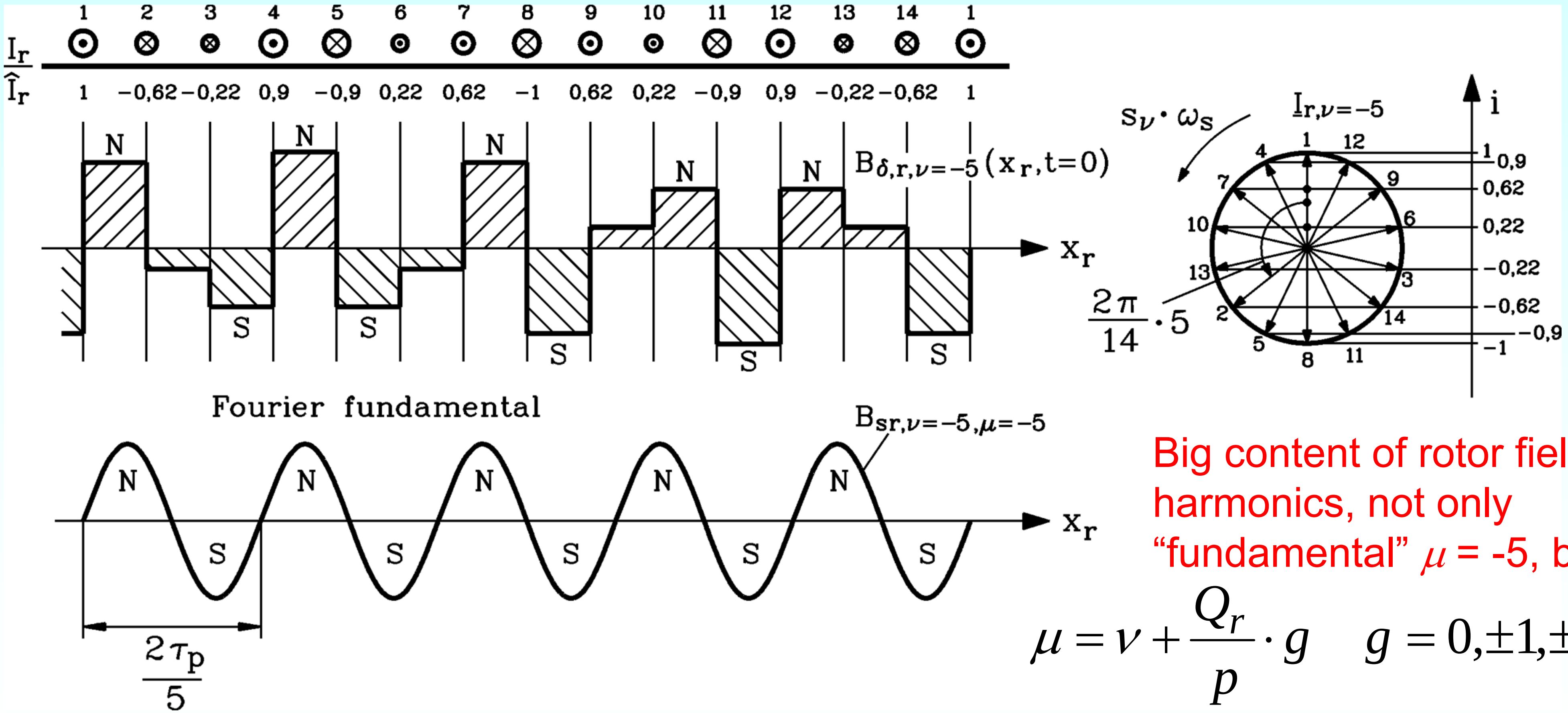
Rotor bar currents: $\hat{I}_{r\nu} \cos(\omega_{r\nu} t - \varphi_\nu) = \hat{I}_{r\nu} \cos(\omega_{r\nu} t + 128.6°)$

Bar 1	Bar 2	Bar 3	Bar 4	etc.
$i = \hat{I}_{r\nu} \cos(\omega_{r\nu} t)$	$\hat{I}_{r\nu} \cos(\omega_{r\nu} t - \varphi_\nu)$	$\hat{I}_{r\nu} \cos(\omega_{r\nu} t - 2\varphi_\nu)$	$\hat{I}_{r\nu} \cos(\omega_{r\nu} t - 3\varphi_\nu)$	...

Example: Rotor harmonic current due to 5th stator field harmonic

Stator frequency 50 Hz, no-load s = 0, 28 rotor bars. Rotor bar currents, **induced by stator field harmonic ν = -5.** $i/\hat{I}_{r\nu}$ at t = 0:

bar 1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	-0.6	-0.2	0.9	-0.9	0.2	0.6	-1	0.6	0.2	-0.9	0.9	-0.2	0.6



Big content of rotor field harmonics, not only "fundamental" μ = -5, but:

$$\mu = \nu + \frac{Q_r}{p} \cdot g \qquad g = 0, \pm 1, \pm 2, \ldots$$

Rotor cage air gap field due to harmonic rotor current $I_{r,\nu}$

Rotor cage air gap field harmonics, excited by HARMONIC rotor current $I_{r,\nu}$:

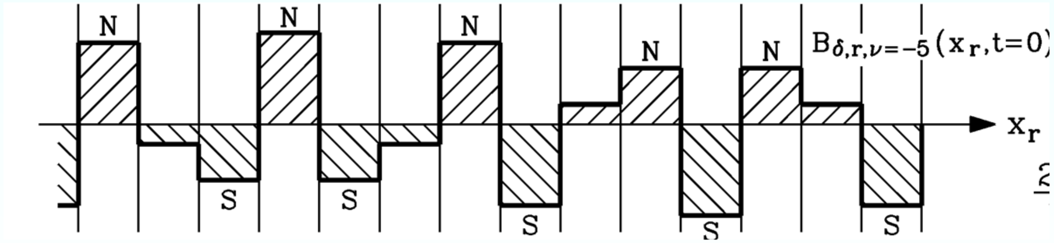
- Ordinal number μ !
- 1/2 turn per phase: $N_r = 1/2$
- Each bar = a rotor phase: $m_r = Q_r$!
- Winding factor is unity: $k_{wr,\mu} = 1$.

Wave function: $$B_{\delta,r}(x_r,t) = \sum_{\mu=\nu}^{\infty} B_{\delta,\mu} \cdot \cos\left(\frac{\mu\pi x_r}{\tau_p} - \omega_r t\right) \qquad \omega_r = 2\pi f_r = s_\nu \omega_s$$

Ordinal number: $\mu = \nu + (Q_r / p) \cdot g$ (g: integer number: $g = 0, \pm1, \pm2, \pm3, \ldots$)

Wave amplitude $$B_{\delta,\mu} = \frac{\mu_0}{\delta} \cdot \frac{\sqrt{2}}{\pi} \cdot \frac{Q_r}{p} \cdot \frac{1}{2} \cdot \frac{1}{\mu} \cdot I_{r,\nu}$$ (no iron saturation considered)

Wave length $\lambda_\mu = \dfrac{2\tau_p}{|\mu|}$



Wave velocity $v_{\mu,r} = \lambda_\mu \cdot f_{r,\nu} = 2 \cdot s_\nu \cdot f_s \cdot \tau_p / \mu = s_\nu \cdot v_{syn} / \mu$

Determination of rotor harmonic currents $I_{r,\nu}$

Rotor bar induced voltage due to ν^{th} stator field harmonic:

$$u_{i,rs\nu} = -k_{wr} N_r \cdot \frac{d\Phi_{s,\nu}(t)}{dt} = M_{rs\nu} \cdot di_s / dt \Rightarrow M_{rs\nu} = \mu_0 \cdot N_r k_{wr} N_s k_{ws,\nu} \cdot \frac{2m_s}{\pi^2 \cdot \nu^2 \cdot p} \cdot \frac{\tau_p l_{Fe}}{\delta}$$

$$\underline{U}_{i,rs\nu} = js_\nu \omega_s M_{rs\nu} \cdot \underline{I}_s \qquad M_{rs\nu} = N_s k_{ws,\nu} m_s / (N_r k_{wr} m_r) \cdot L_{rh\nu}$$

Rotor bar self-induced voltage $U_{i,rr\nu}$:

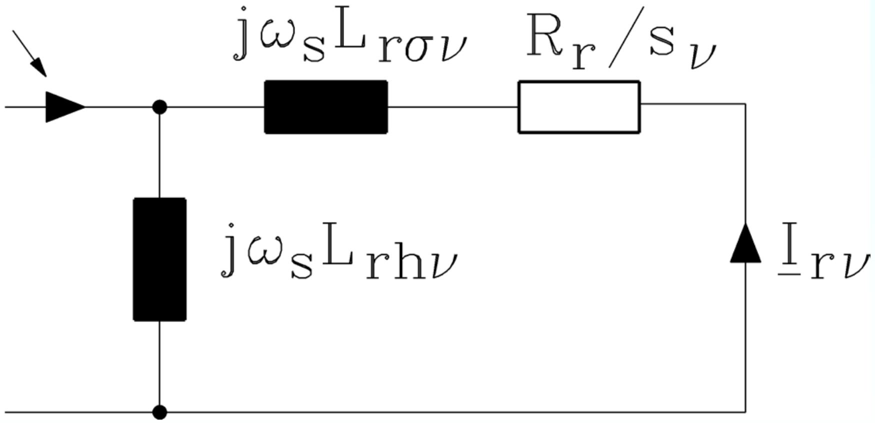
$$\underline{U}_{i,rr\nu} = js_\nu \omega_s L_{hr\nu} \cdot \underline{I}_{r\nu} \quad \Rightarrow \quad L_{hr\nu} = \mu_0 (N_r k_{wr})^2 \cdot \frac{2m_r}{\pi^2 \cdot \nu^2 \cdot p} \cdot \frac{\tau_p l_{Fe}}{\delta}$$

Rotor voltage equation:

$$0 = R_r \underline{I}_{r\nu} + js_\nu \omega_s L_{r\sigma\nu} \underline{I}_{r\nu} + js_\nu \omega_s L_{rh\nu} \underline{I}_{r\nu} + js_\nu \omega_s M_{rs\nu} \underline{I}_s$$

$$\frac{2m_s N_s k_{ws\nu}}{Q_r} \cdot \underline{I}_s \qquad N_s k_{ws,\nu} m_s / (N_r k_{wr} m_r) = N_s k_{ws,\nu} m_s \cdot 2 / Q_r$$

Equivalent circuit for rotor voltage equation for ν^{th} rotor harmonic current !



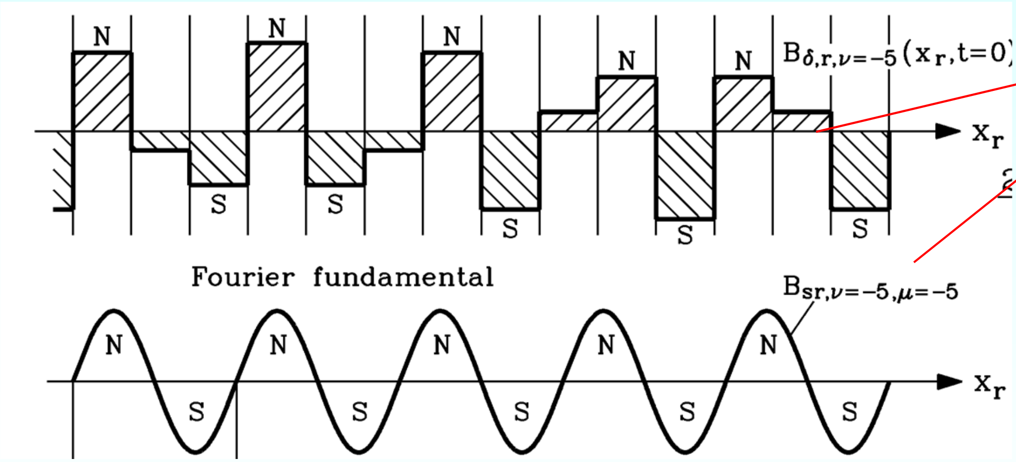
$$\underline{I}_{r\nu} = -\frac{(2m_s N_s k_{ws\nu} / Q_r) \cdot j\omega_s L_{rh\nu}}{R_r / s_\nu + j\omega_s \cdot (L_{r\sigma\nu} + L_{rh\nu})} \cdot \underline{I}_s$$

Rotor leakage inductance

a) Rotor bar excites slot stray field and stray field of winding overhang (end rings):

$$js_\nu \omega_s L_{r\sigma} \underline{I}_{r\nu}$$

b) Self-induction of rotor air-gap field harmonics with ordinal numbers $\mu \neq \nu$, which is called “harmonic” leakage



$$\Delta \underline{U}_{i,rr\nu} = js_\nu \omega_s L_{hr\nu} \cdot (1/\eta_\nu^2 - 1) \cdot \underline{I}_{r\nu}$$

Sum of a) and b) = Total rotor leakage inductance (of an unskewed machine)

$$L_{r\sigma\nu} = L_{r\sigma} + L_{hr\nu} \cdot (1/\eta_\nu^2 - 1)$$

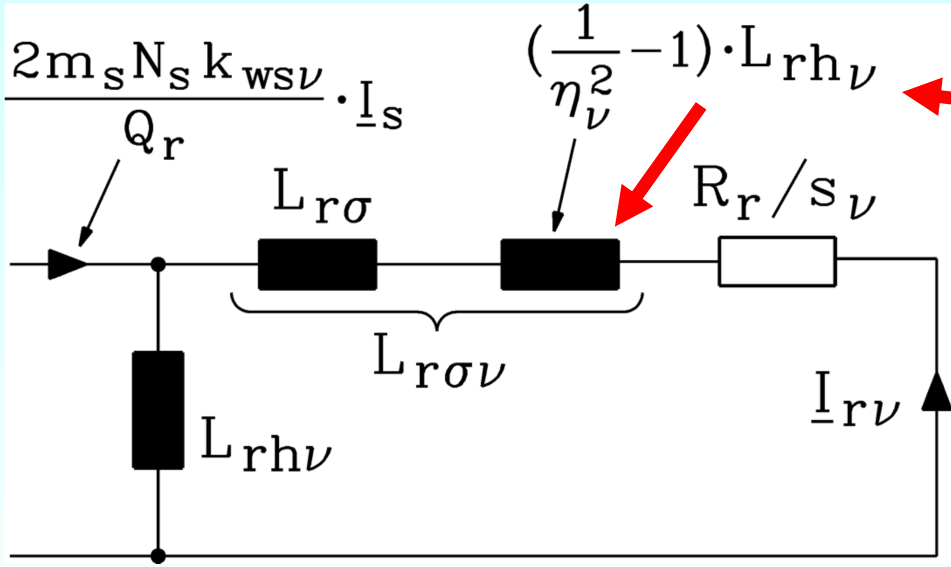
With skewing: Skewing factor χ_ν ($|\chi_\nu| < 1$). Leakage inductance increases due to increased decoupling of stator and rotor winding: $L_{r\sigma\nu} = L_{r\sigma} + L_{hr\nu} \cdot (1/(\eta_\nu^2 \chi_\nu^2) - 1)$ ↑

Influence of rotor field harmonics on rotor harmonic current

Big content of rotor field harmonics, not only "fundamental" μ = -5, but with ordinal numbers

$\mu = \nu + \dfrac{Q_r}{p} \cdot g \quad g = 0, \pm 1, \pm 2, \ldots$ lead to additional self-induced voltage:

$$\Delta \underline{U}_{i,rr\nu} = j s_\nu \omega_s L_{hr\nu} \cdot (1/\eta_\nu^2 - 1) \cdot \underline{I}_{r\nu} \qquad \text{with} \qquad \eta_\nu = \frac{\sin(\nu \cdot p \cdot \pi / Q_r)}{\nu \cdot p \cdot \pi / Q_r}$$



This influence is **dominant** and limits the harmonic rotor currents for increased ordinal number of stator field harmonics !

Example:

Four-pole motor, 28 bars: Increase of rotor self induction voltage due to all rotor field harmonics:

ν	1	-5	7	-11	13
$1/\eta_\nu^2$ -1	0.017	0.55	1.47	14.67	170.87

Rough calculation of rotor harmonic currents

$$\eta_\nu = \frac{\sin(\nu \cdot p \cdot \pi / Q_r)}{\nu \cdot p \cdot \pi / Q_r} \qquad \underline{I}_{r\nu} = -\frac{(2m_s N_s k_{ws\nu} / Q_r) \cdot j\omega_s L_{rh\nu}}{R_r / s_\nu + j\omega_s \cdot (L_{r\sigma\nu} + L_{rh\nu})} \cdot \underline{I}_s$$

$$L_{r\sigma\nu} = L_{r\sigma} + L_{hr\nu} \cdot (1/\eta_\nu^2 - 1)$$

$$\underline{I}_{r\nu}\big|_{s_\nu \neq 0} \cong -\frac{(2m_s N_s k_{ws\nu} / Q_r) \cdot L_{rh\nu}}{L_{r\sigma\nu} + L_{rh\nu}} \cdot \underline{I}_s \approx -\frac{2m_s N_s k_{ws\nu} / Q_r}{1/\eta_\nu^2} \cdot \underline{I}_s \sim \eta_\nu^2 k_{ws\nu} \cdot \underline{I}_s$$

Rotor harmonic currents are bigger (1) at $Q_r > Q_s$, (2) at low pole count p at given slot numbers.

Example:

Two-pole motor, Q_s = 36 slots: p = 1, a) $Q_s > Q_r$: 28 bars Q_r/p = 28 b) $Q_s < Q_r$: 44 bars Q_r/p = 44

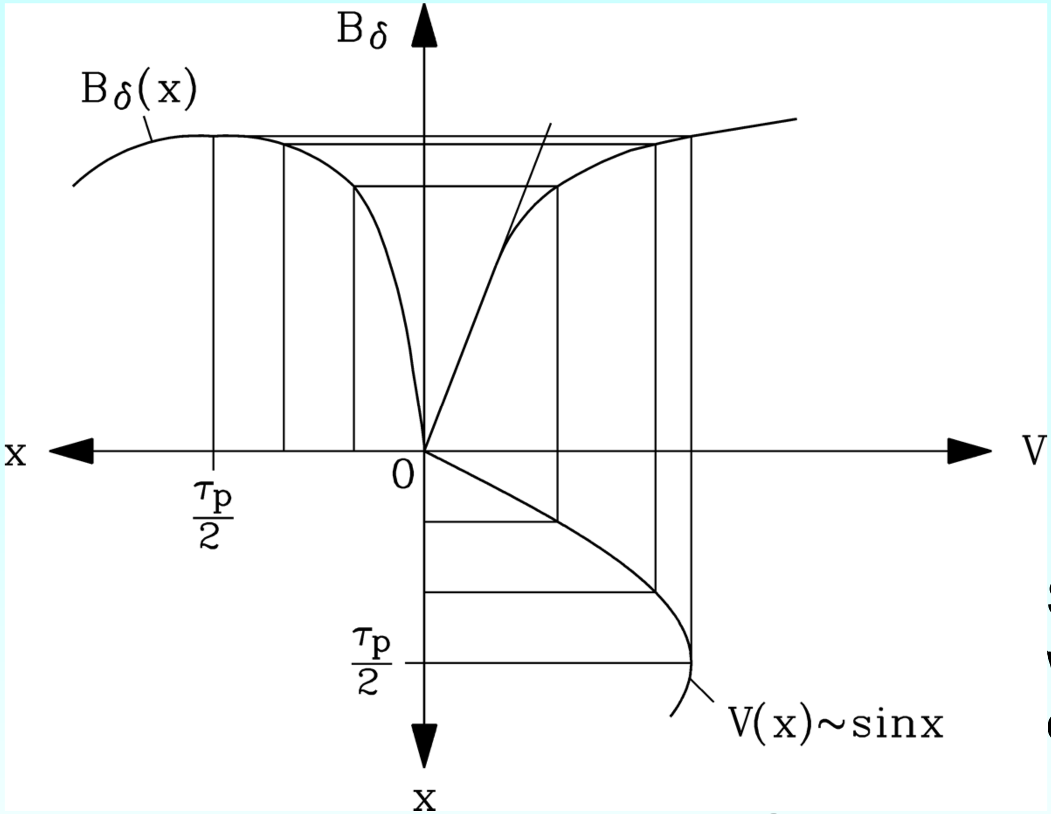
ν	a)	-5	7	b)	-5	7
$I_{r\nu} \sim \eta_\nu^2$		0.899	0.81		0.958 (+6.5%)	0.92 (+13%)

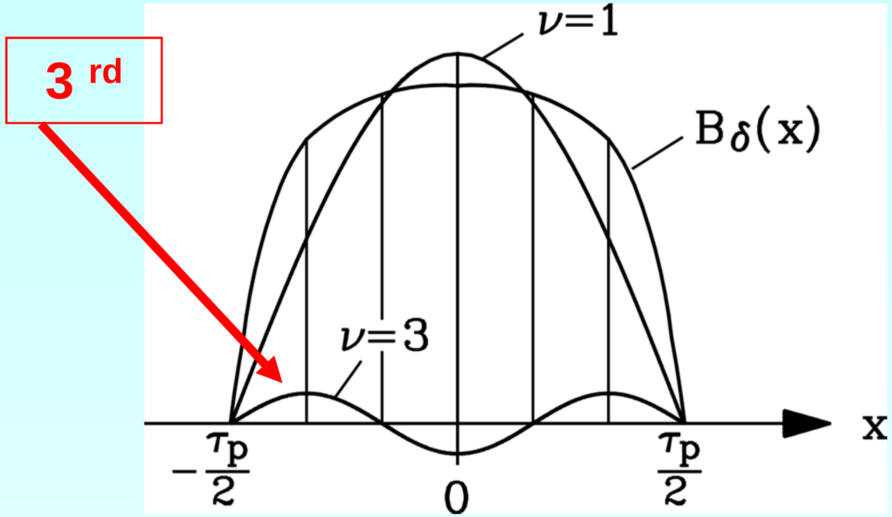
+39%

Four-pole motor, Q_s = 36 slots: p = 2, a) $Q_s > Q_r$: 28 bars Q_r/p = 14, b) $Q_s < Q_r$: 44 bars Q_r/p = 22

ν	a)	-5	7	b)	-5	7
$I_{r\nu} \sim \eta_\nu^2$		0.645	0.405		0.841 (+30%)	0.9708 (+75%)

Iron saturation causes additional 3rd air gap field harmonic





Sinusoidal m.m.f. distribution of stator and rotor winding (I_m) causes non-sinusoidal air gap flux density, which contains ***3rd field harmonic !***

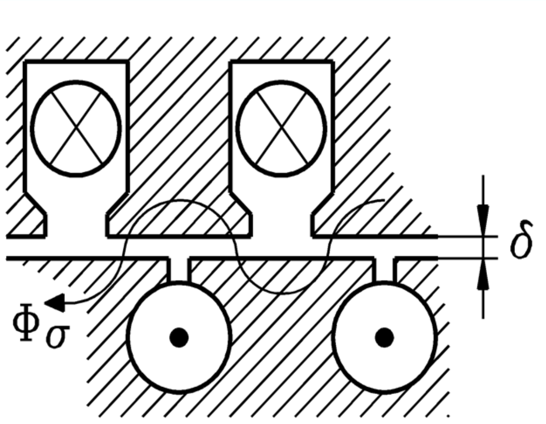
$$B_{\delta s,\nu=3}(x_s,t) = B_{\delta s,\nu=3} \cdot \cos\left(\frac{3x_s\pi}{\tau_p} - 3\omega_s t\right)$$

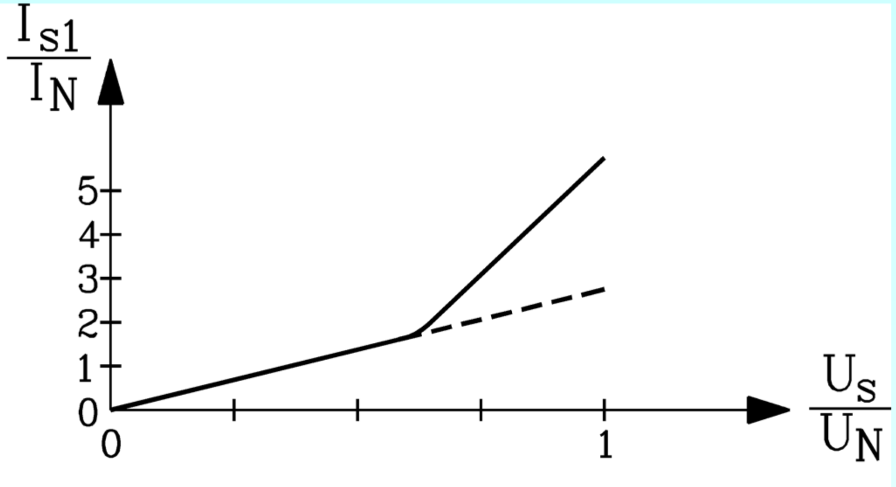
3rd field harmonic moves with n_{syn} !

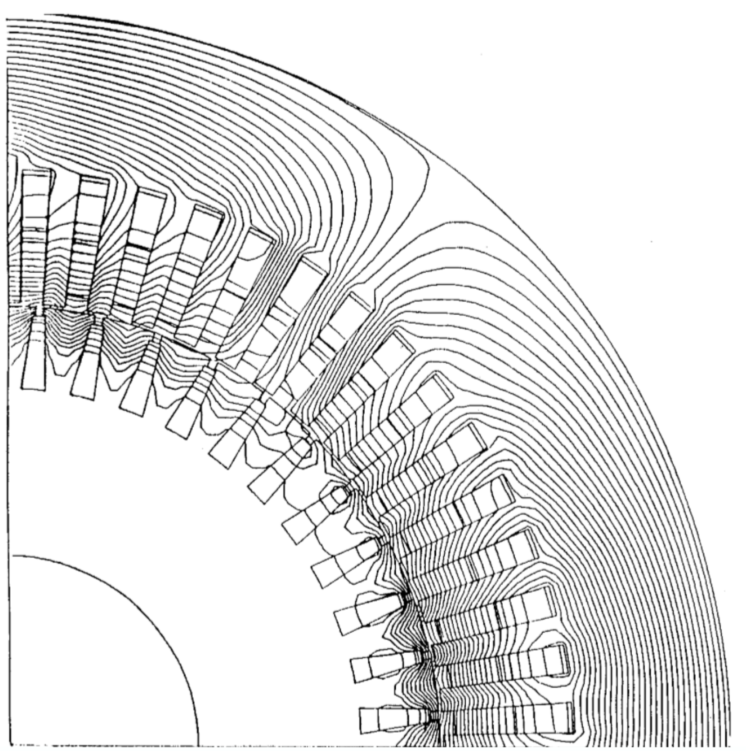
inducing the rotor with $s \cdot 3f_s$, causing additional rotor harmonic current $I_{r\nu=3}$ with additional losses.

At high slip - tooth tip saturation by zig-zag stray flux

Zig-zag stray flux path







Measured "locked rotor"-characteristic at s = 1

Saturation of $X_\sigma = X_{s\sigma} + X'_{r\sigma}$: $X_\sigma \cong U_s/I_{s1}$!

Numerically calculated two-dimensional magnetic flux density B of a three-phase, 4-pole high voltage cage induction machine with wedge rotor slots

at stand still (locked rotor) s = 1

(Q_s / Q_r = 60/44) at rated voltage

Effect of fundamental and harmonic fields in induction machines

Stator air gap field:

$$B_{\delta,s}(x_s,t) = \sum_{\nu=1,-5,7,\ldots}^{\infty} B_{\delta,\nu} \cdot \cos\left(\frac{\nu\pi x_s}{\tau_p} - \omega_s t\right) \qquad \nu = 1 + 2m_s g$$

Fundamental field $\nu = 1$

Harmonic fields $\nu > 1$

Stator

Rotor

Rotor fundamental current $\underline{I}_{r,1} \quad \omega_r = s \cdot \omega_s$

Rotor harmonic currents $\underline{I}_{r,\nu} \quad \omega_{r,\nu} = s_\nu \cdot \omega_s$

$$B_{\delta,r}(x_r,t) = \sum_{\mu=1}^{\infty} B_{\delta,\mu} \cdot \cos\left(\frac{\mu\pi x_r}{\tau_p} - \omega_r t\right)$$

$$B_{\delta,r,\nu}(x_r,t) = \sum_{\mu=1}^{\infty} B_{\delta,\nu,\mu} \cdot \cos\left(\frac{\mu\pi x_r}{\tau_p} - s_\nu \omega_r t\right)$$

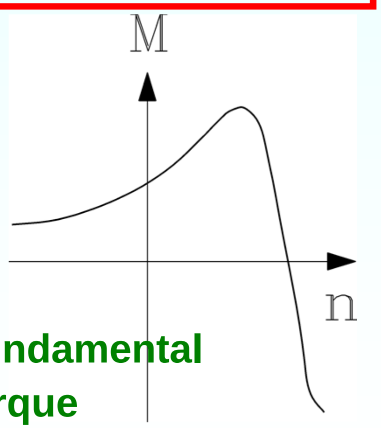
$\mu = 1 + (Q_r / p) \cdot g$ ***Rotor air gap fields*** $\mu = \nu + (Q_r / p) \cdot g$

Fundamental field $\mu = 1$

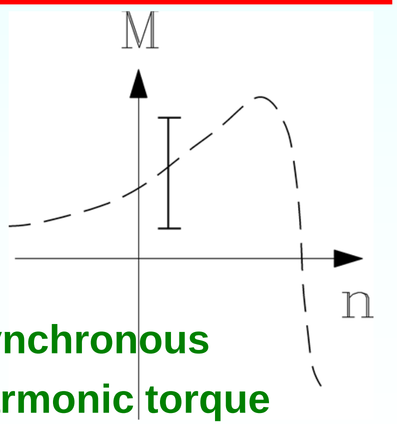
Harmonic fields $|\mu| \neq 1$

Fundamental field $\mu = \nu$

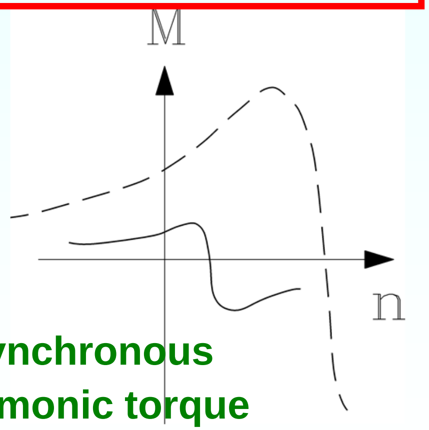
Harmonic fields $|\mu| \neq 1$



Fundamental torque



Synchronous harmonic torque



Asynchronous harmonic torque

Negligible effect

Asynchronous harmonic torque

- *Lorentz* forces of $I_{r\nu}$ with stator field harmonic $B_{\delta s\nu}$ yield an **"asynchronous harmonic torque"**: $M_{e\nu}$

Special case $\nu = 1$: this is the asynchronous torque of the *Kloss* function.

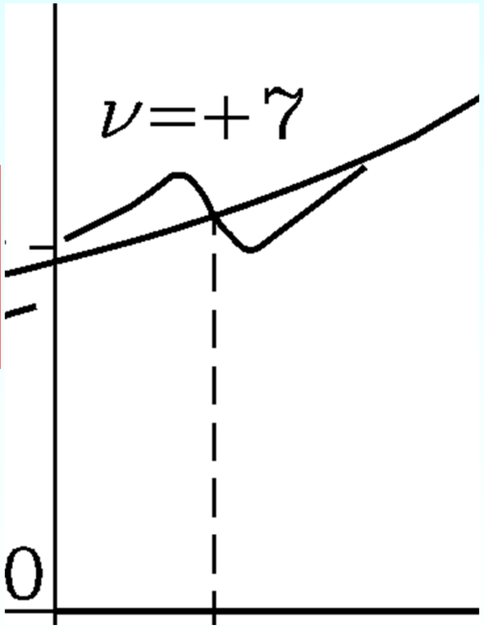
- Rotor harmonic currents $I_{r\nu}$ produce additional cage losses $P_{Cu,r\nu} = Q_r \cdot R_r \cdot I_{r\nu}^2$
- Air gap power of ν^{th} stator field harmonic: $P_{\delta\nu} = Q_r \cdot (R_r / s_\nu) \cdot I_{r\nu}^2$
- Mechanical power: $P_{m\nu} = \Omega_m M_{e\nu} = (1 - s_\nu)\Omega_{syn,\nu} M_{e\nu} = (1 - s_\nu) \cdot P_{\delta\nu}$

$$\Rightarrow M_{e\nu} = \frac{Q_r R_r I_{r\nu}^2}{s_\nu \cdot \Omega_{syn,\nu}}$$

$$M_{e\nu} = \frac{(2m_s \cdot N_s k_{ws\nu})^2 \cdot (\omega_s L_{rh\nu})^2}{Q_r \cdot \Omega_{syn,\nu}} \cdot \frac{s_\nu \cdot R_r}{R_r^2 + (s_\nu \cdot \omega_s (L_{r\sigma\nu} + L_{rh\nu}))^2} \cdot I_s^2$$

Harmonic break down slip: $s_{\nu b} = \pm \frac{R_r}{\omega_s (L_{r\sigma\nu} + L_{rh\nu})}$

Harmonic KLOSS function, depending on slip: $s_\nu = \frac{n_{syn,\nu} - n}{n_{syn,\nu}}$



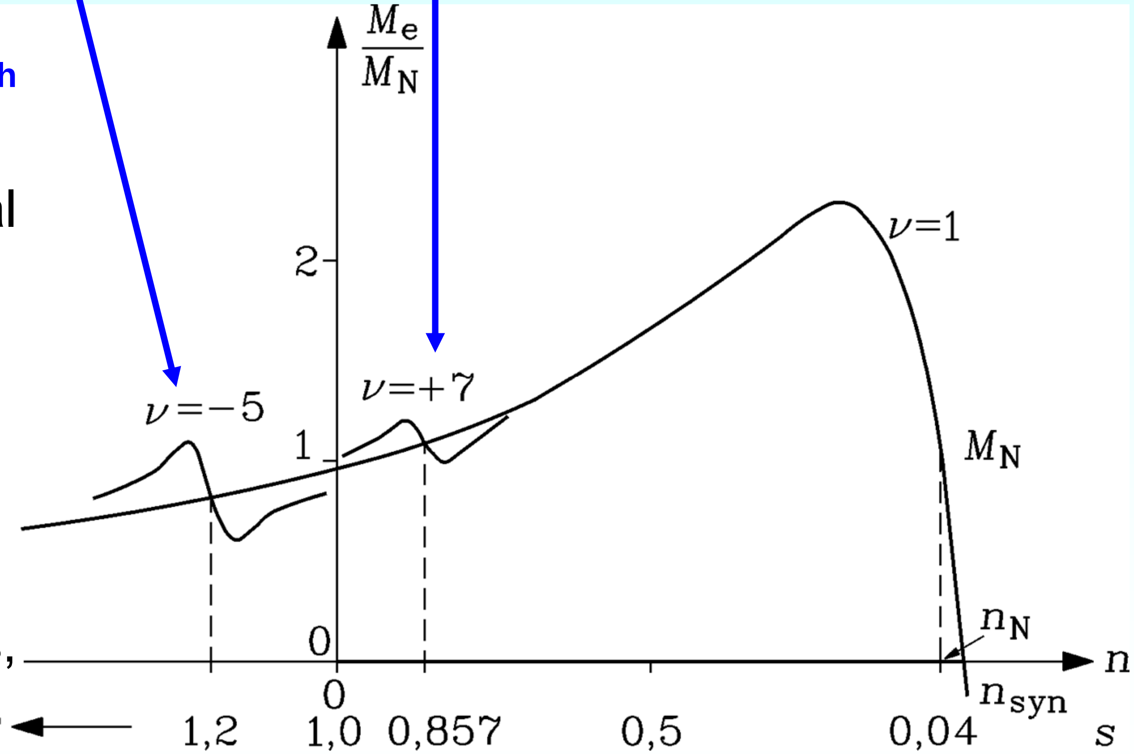
Asynchronous torque in torque-speed curve

At a certain rotor speed n the rotor cage is NOT induced by ν-th stator field harmonic = no asynchronous torque !

$$f_{r,\nu} = s_\nu \cdot f_s = f_s \cdot \left|1-\nu\cdot(1-s)\right| = 0 \quad \Rightarrow \quad s = 1-1/\nu \quad \Rightarrow \quad n = n_{syn}/\nu$$

ν	1	-5	7	-11	13
s	0	1.2	0.86	1.09	0.92
n /1/min	1500	- 300	214	- 136	115

Asynchronous harmonic of 5th and 7th stator field harmonic:
Superimposed on fundamental asynchronous torque.



Example:

R_s/X_s =1/100, R_r/X_r = 1.3/100,
σ = 0.067, $X_s = X'_r = 3Z_N$,
36 stator & 28 rotor slots,
4 poles, 3 phases, full-pitched coils,
unskewed slots

Synchronous harmonic torque

- Rotor field harmonic μ of rotor fundamental current I_r produces a **"synchronous harmonic" torque** with an arbitrary stator field harmonic ν of the stator fundamental current I_s.
- *Condition for constant torque generation:*

(i) same wave length, (ii) same velocity of stator and rotor field wave.

Only fulfilled at a certain rotor slip *s* = *s:**

Stator harmonic field (excited by I_s) :

$$B_{\delta,\nu} \cdot \cos(\frac{\nu \pi x_s}{\tau_p} - \omega_s t) \qquad B_{\delta,\nu} \sim I_s$$

Rotor harmonic field (excited by I_r) :

$$B_{\delta,\mu} \cdot \cos(\frac{\mu \pi x_r}{\tau_p} - s \cdot \omega_s t) \qquad B_{\delta,\mu} \sim I_r$$

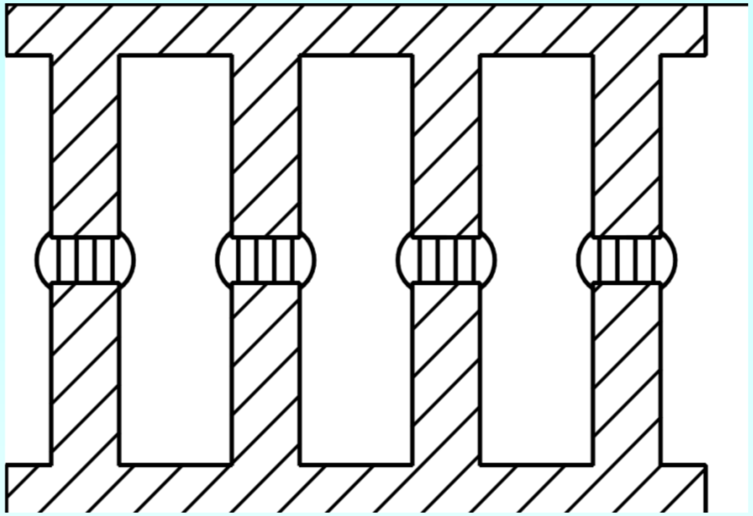
(i) Identical wave lengths: $\lambda_\nu = \lambda_\mu \Rightarrow |\nu| = |\mu| \Rightarrow \underline{\underline{\nu = \mu}}$ or $\underline{\underline{\nu = -\mu}}$

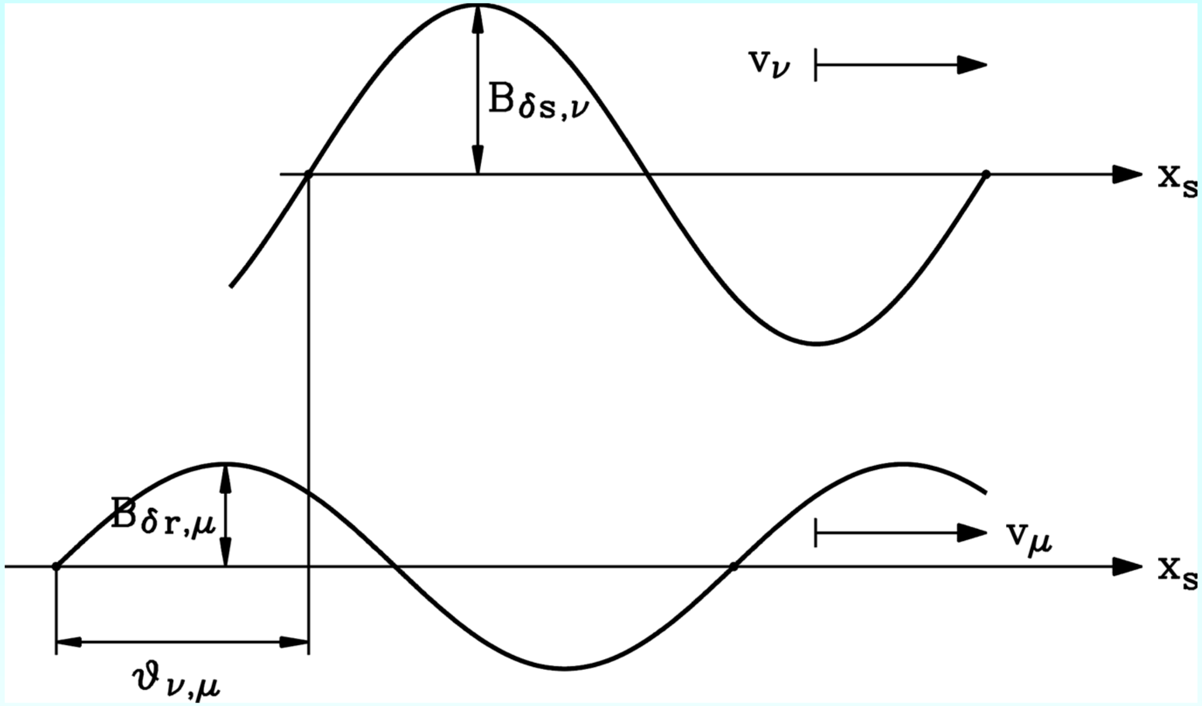
(ii) Identical velocity: $v_\nu = v_{syn} / \nu \qquad v_\mu = v_{syn} \cdot (1 - s + s/\mu)$

For $v_\nu = v_\mu \; : \; v_{syn} / \nu = v_{syn} \cdot (1 - s^* + s^*/\mu) \Rightarrow s^* = \frac{1/\nu - 1}{1/\mu - 1}$

$\nu = \mu :$	$s^* = 1$
$\nu = -\mu :$	$s^* = \frac{\nu - 1}{\nu + 1}$

Synchronous harmonic torque





Cogging = synchronous torque $\nu = \mu$:

$s^* = 1$: At stand still!

- If stator and rotor teeth number is the same, cogging will occur at $n = 0$, representing a synchronous torque $\nu = \mu$
- Generation of synchronous harmonic torque by an arbitrary ν-th stator and μ-th rotor field harmonic, travelling in the air gap with same speed and having same wave length:

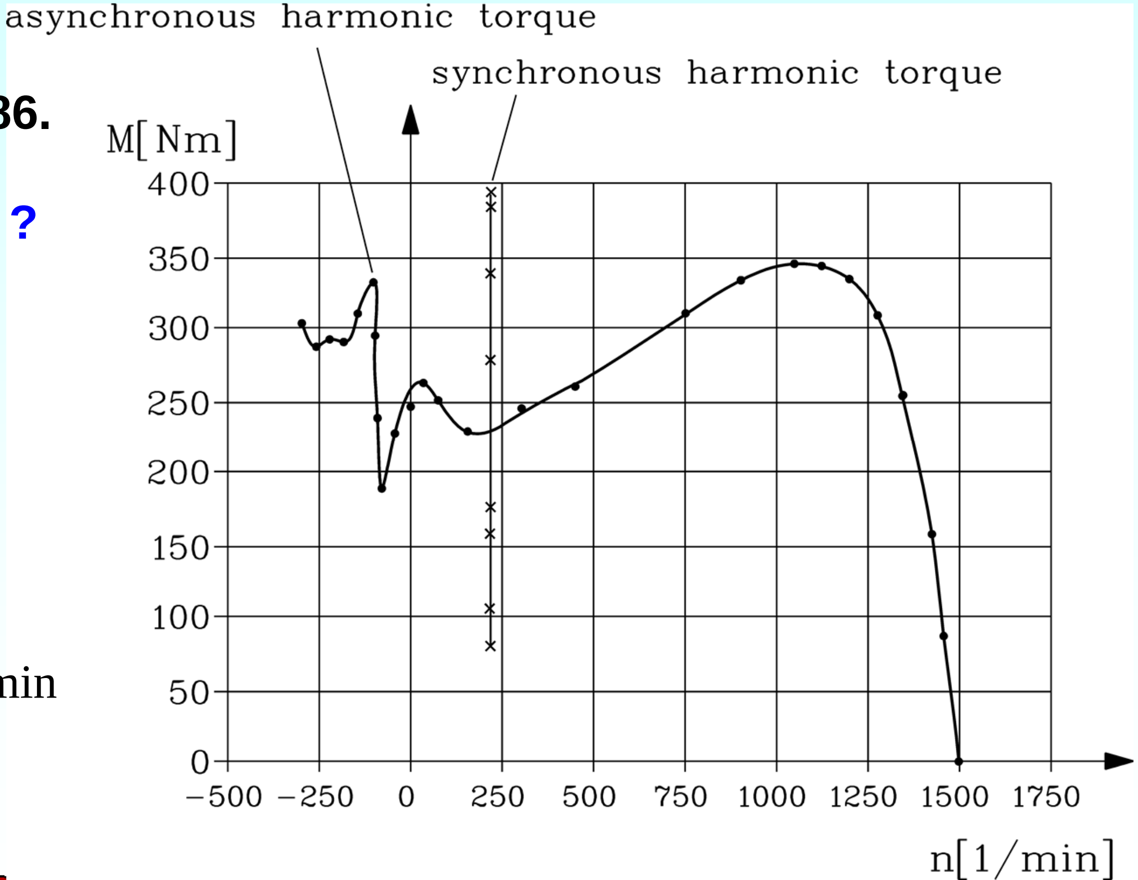
$$M_{e,\nu\mu} \sim B_{\delta s,\nu} \cdot B_{\delta r,\mu} \cdot \sin(\vartheta_{\nu\mu})$$

Example: Synchronous harmonic torque

Data: 4-pole cage induction motor, 380 V, D, 50 Hz, 15 kW, rated torque M_N = 100 Nm, unskewed slots, Q_s/Q_r = 36/28, air gap 0.45 mm, iron stack length 195 mm, stator bore diameter 145 mm, two-layer winding.

(i) Asynchronous harmonic torque due to ν = -11 stator field harmonic with synchronous harmonic slip $s_\nu = 0$ at slip $s = 1 - 1/\nu = 1 + 1/11 = 1.09$, corresponding with speed $n = (1-s) \cdot n_{syn} = (1-1.09) \cdot 1500 = \underline{\underline{-136/\text{min}}}$.

(ii) Synchronous harmonic torque at slip 0.86.



Which field harmonics generate this torque ?

Stator ordinal numbers:

$\nu = 1 + 2m_s g = 1 + 6g = 1, -5, 7, -11, 13, -17, 19, ...$

Rotor ordinal numbers:

$\mu = 1 + (Q_r / p)g = 1 + 14g = 1, -13, 15, -27, 29, ...$

Condition fulfilled for

$\nu = -\mu = 13$:

$$s^* = \frac{\nu - 1}{\nu + 1} = \frac{12}{14} = 0.857 \quad n^* = (1 - s^*) \cdot 1500 = 215/\text{min}$$

Saturation causes additional harmonic torque

The **3rd stator harmonic saturation wave** causes a rotor harmonic current $I_{r\nu=3}$ to flow with frequency $s\cdot 3f_s$, which excites a rotor field with harmonic field waves with ordinal numbers $\mu = 3 + \frac{Q_r}{p}\cdot g\,,\quad g = 0,\pm 1,\pm 2,\ldots$

These harmonic waves also generate with the stator field harmonics a synchronous harmonic torque:

Stator harmonic field (excited by I_s):

$$B_{\delta,\nu}\cdot\cos\left(\frac{\nu\pi x_s}{\tau_p}-\omega_s t\right)\qquad B_{\delta,\nu}\sim I_s$$

Rotor harmonic field (excited by $I_{r,\nu=3}$):

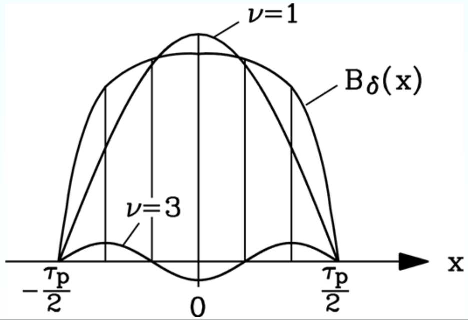
$$B_{\delta,\mu}\cdot\cos\left(\frac{\mu\pi x_r}{\tau_p}-s\cdot 3\omega_s t\right)\qquad B_{\delta,\mu}\sim I_{r,\nu=3}$$

(i) Identical wave lengths: $\lambda_\nu = \lambda_\mu \Rightarrow |\nu| = |\mu| \Rightarrow \underline{\underline{\nu = \mu}}$ or $\underline{\underline{\nu = -\mu}}$

(ii) Identical velocity: $v_\nu = v_{syn}/\nu \qquad v_\mu = v_{syn}\cdot(1 - s + 3s/\mu)$

Synchronous torque occurs at $v_\nu = v_\mu$:

$$\nu = \mu,\ \nu = -\mu:\qquad s^* = \frac{1/\nu - 1}{3/\mu - 1}$$



Example: Synchronous torque due to iron saturation

Data:

2-pole cage induction motor, 380 V, D, 50 Hz, 11 kW, rated torque M_N = 37 Nm, skewed slots, Q_s/Q_r = 36/28, insulated copper cage to avoid flow of inter-bar currents, two-layer stator winding, winding pitch 1/2.

(i) Asynchronous harmonic torque due to ν = -5 stator field harmonic:

Synchronous harmonic slip $s_\nu = 0$ **at slip** $s = 1 - 1/\nu = 1 + 1/5 = 1.2$**, corresponds to speed:**

$$n = (1-s)\cdot n_{syn} = (1-1.2)\cdot 3000 = \underline{\underline{-600/\min}}.$$

(ii) Synchronous harmonic torque at slip 1.07 and 0.86.

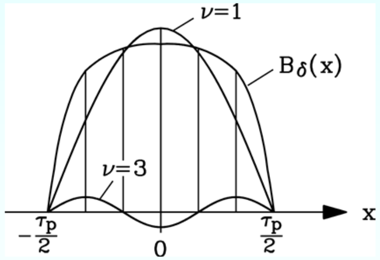
Which field harmonics generate these torque components ?

Stator ordinal number: $\nu = 1 + 2m_s g = 1 + 6g = 1, -5, 7, -11, 13, -17, 19, -23, 25, -29, 31, -35, 37, ...$

Rotor ordinal numbers of I_r: $\mu = 1 + (Q_r/p)g = 1 + 28g = 1, -27, 29, ...$

Rotor ordinal numbers of $I_{r,\nu=3}$: $\mu = 3 + (Q_r/p)g = 3 + 28g = 3, -25, 31, ...$

(saturation effect)



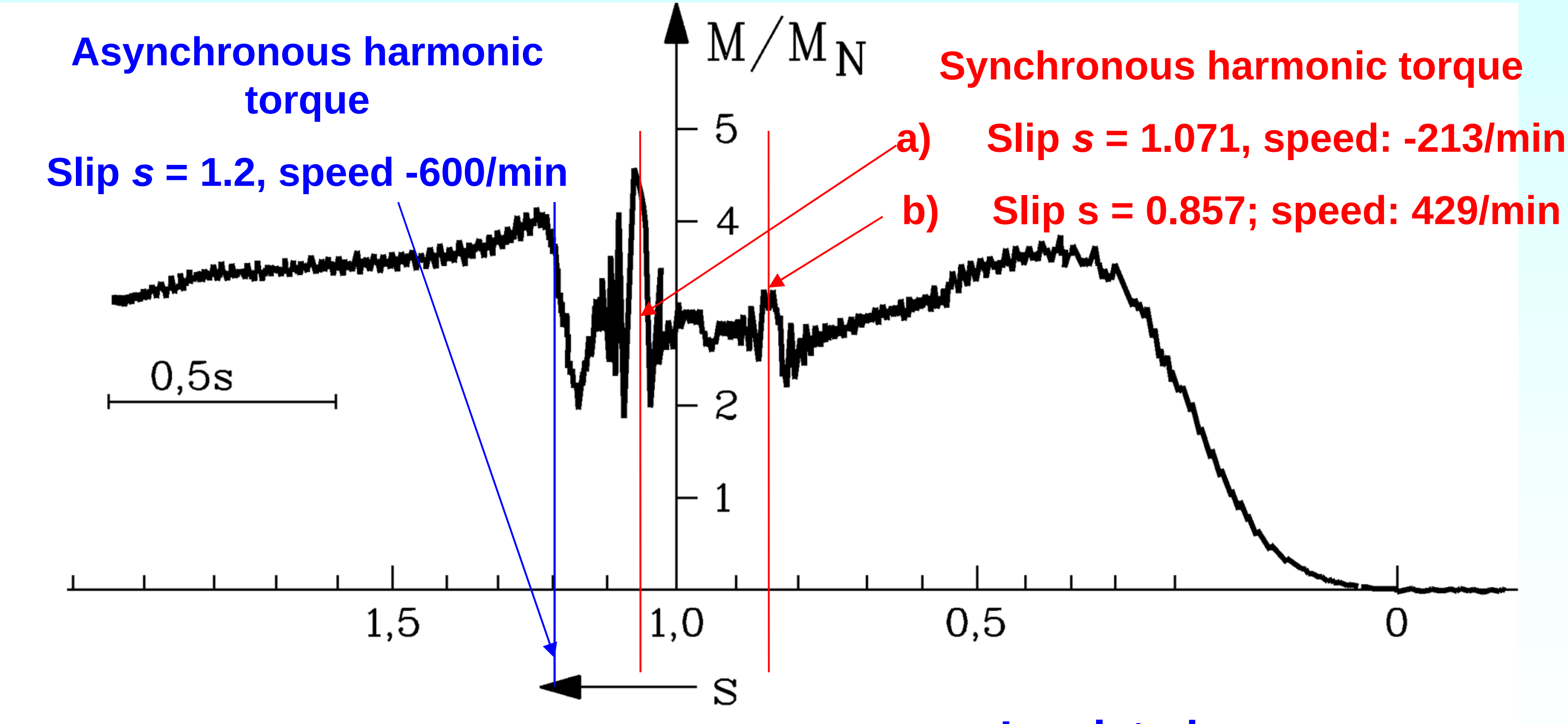
Condition fulfilled: $\nu = -\mu = 25: s^* = \frac{1-1/25}{1+3/25} = \underline{\underline{0.857}}$, $\nu = -\mu = -29: s^* = \frac{1+1/29}{1-1/29} = \underline{\underline{1.071}}$

$$\nu = \mu = 31: s^* = \frac{1-1/31}{1-3/31} = \underline{\underline{1.071}}$$

Example: Measured torque with harmonic torque (1)

Data: 2-pole cage induction motor, 380 V, D, 50 Hz, 11 kW, rated torque M_N = 37 Nm

Shaft torque measured with accelerometer; motor with additional inertia mounted to shaft was reversed from –3000/min to 3000/min by changing two phase connections, thus allowing to measure the motor torque in slip range 2 ... 0.

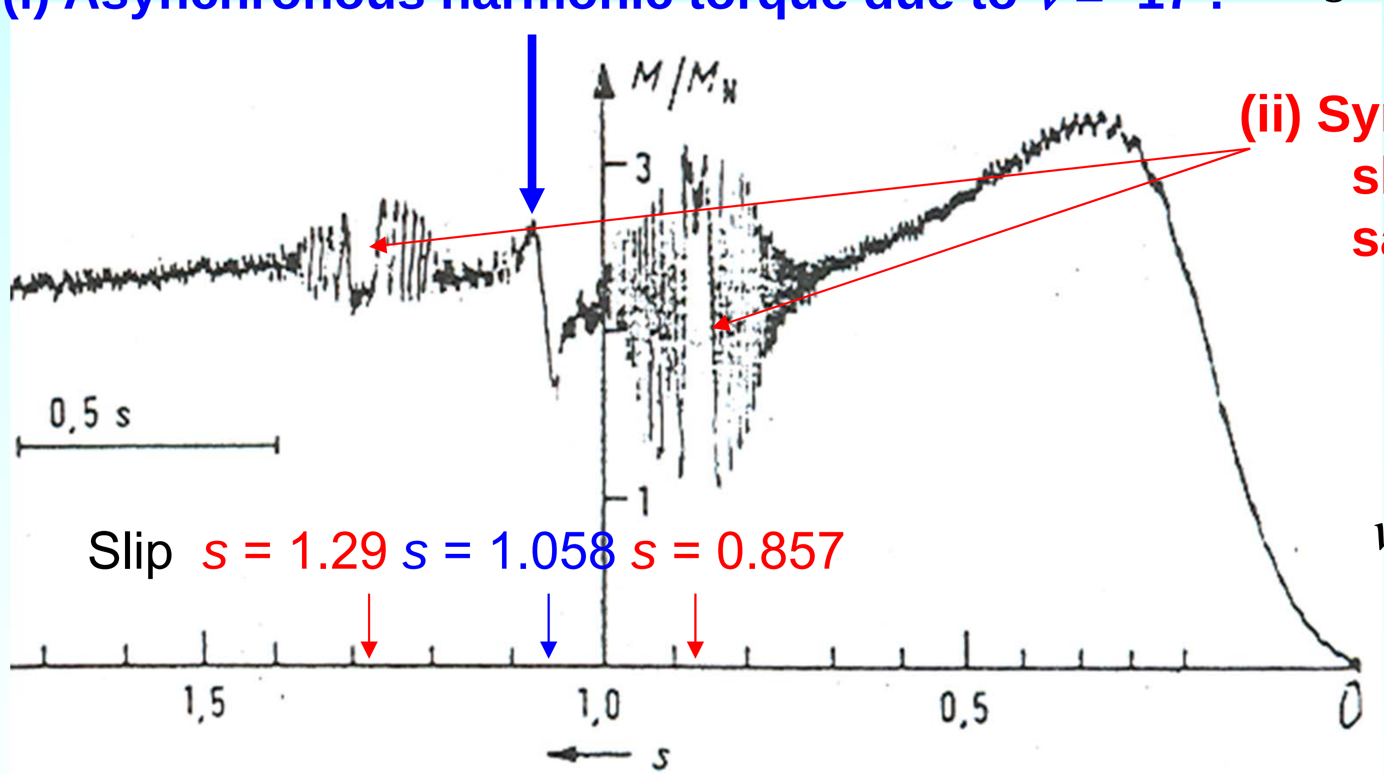


Insulated copper cage

Example: Measured synchronous harmonic torque (2)

Data: 4-pole cage induction motor, 380 VD, 50 Hz/9.5 kW, rated torque M_N = 64 Nm, skewed slots, Q_s/Q_r = 36/28, insulated copper cage (= no inter-bar currents), unchorded stator winding.

(i) Asynchronous harmonic torque due to ν = -17 : $s = 1 - 1/\nu = 1 + 1/17 = \underline{\underline{1.058}}$



(ii) Synchronous harmonic torque at slip 1.29 and 0.86 due to saturation harmonics:

$$\nu = \mu = -11 : s^* = \frac{1+1/11}{1+3/11} = \underline{\underline{0.857}}$$

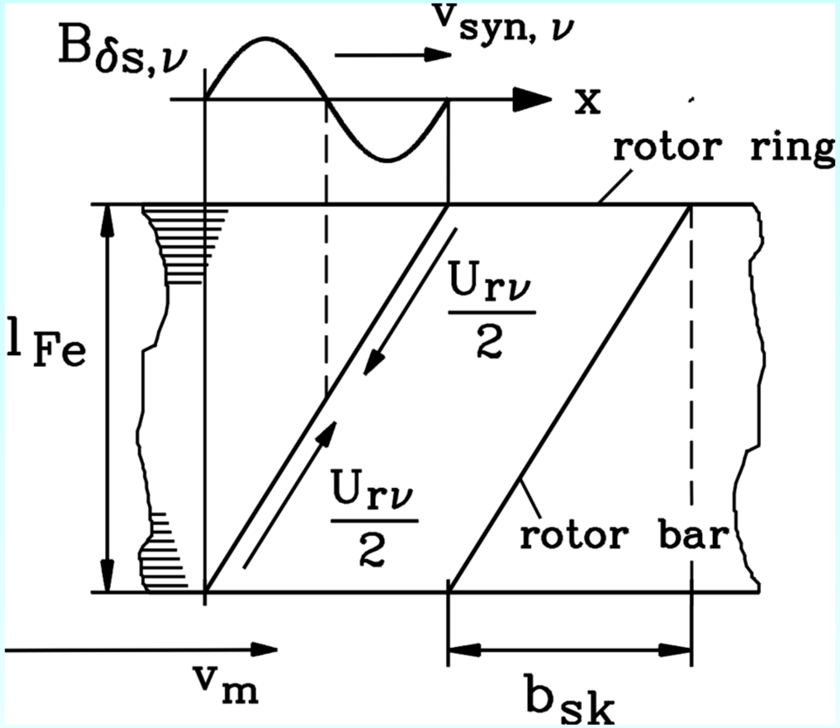
$$\nu = -\mu = -17 : s^* = \frac{1+1/17}{1-3/17} = \underline{\underline{1.29}}$$

$$\nu = 1 + 2m_s g = 1 + 6g = 1, -5, 7, -11, 13, -17, 19, -23, 25, -29, ...$$

$$\mu = 1 + (Q_r/p)g = 1 + 14g = 1, -13, 15, -27, 29, ...$$

$$\mu = 3 + (Q_r/p)g = 3 + 14g = 1, -11, 17, -25, 31...$$

Skewing of rotor cage reduces harmonic bar currents



Due to skew of rotor bar b_{sk} a certain stator field harmonic cannot induce the rotor cage.

$$U_i = \Delta v \cdot B \cdot l \qquad \Delta v = v_{syn,\nu} - v_m$$

Thus no harmonic current $I_{r\nu}$ for that ν-th harmonic will be generated. This may be expressed by **skewing factor**

$$\chi_\nu = \frac{\sin(S_\nu)}{S_\nu} \quad , \quad S_\nu = \frac{\nu \pi b_{sk}}{2\tau_p}$$

Reduction of rotor harmonic current:

$$\underline{I}_{r\nu} = -j \frac{(2m_s \cdot N_s k_{ws\nu} / Q_r) \cdot \omega_s L_{rh\nu}}{R_r / s_\nu + j \cdot \omega_s (L_{r\sigma\nu} + L_{rh\nu})} \cdot \chi_\nu \cdot \underline{I}_s$$

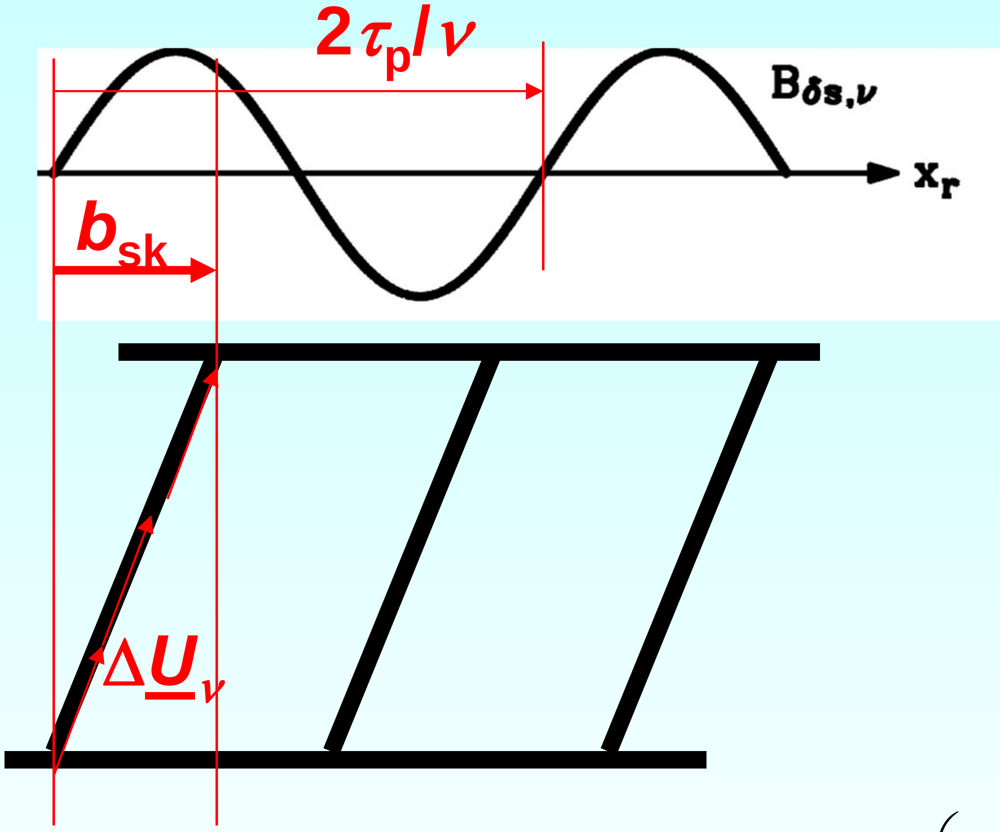
Example: **4-pole induction motor, 36/28 stator/rotor slots.**

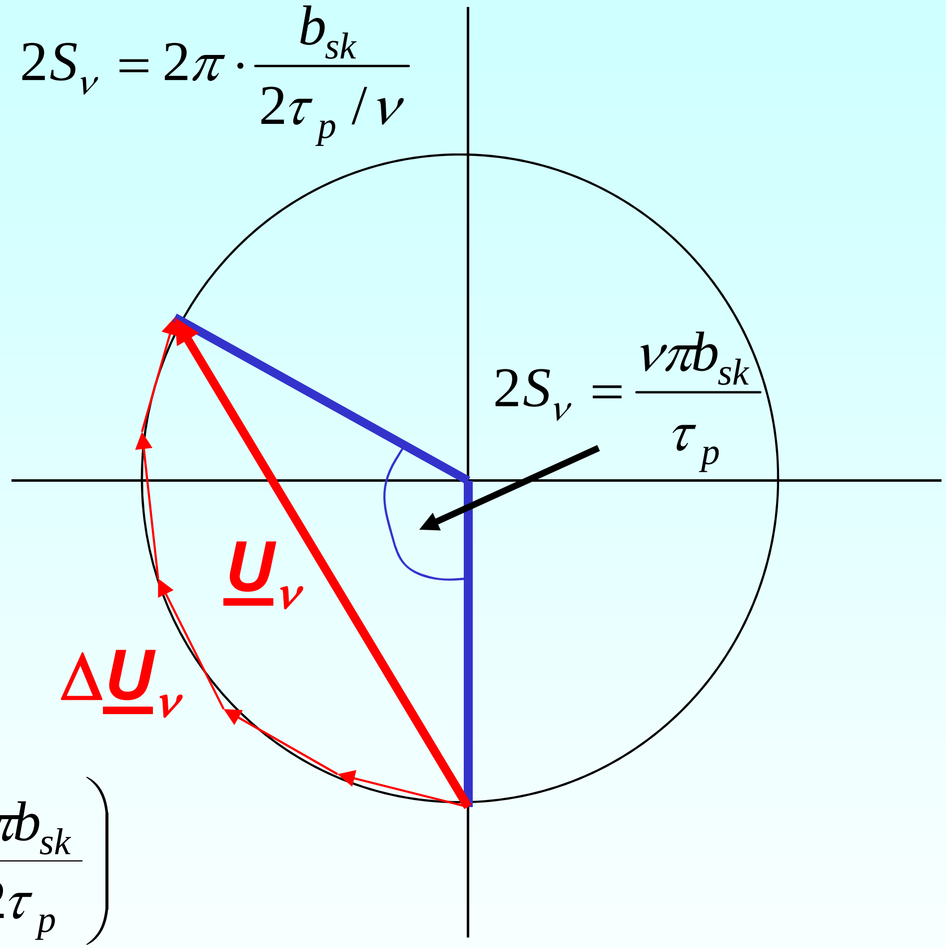
Cage bars skewed by one stator slot pitch: $b_{sk} = \tau_p / 9$

Stator field harmonics:

ν	1	-17	19	-35	37
χ_ν	0.9949	0.0585	-0.0523	-0.0284	0.0267

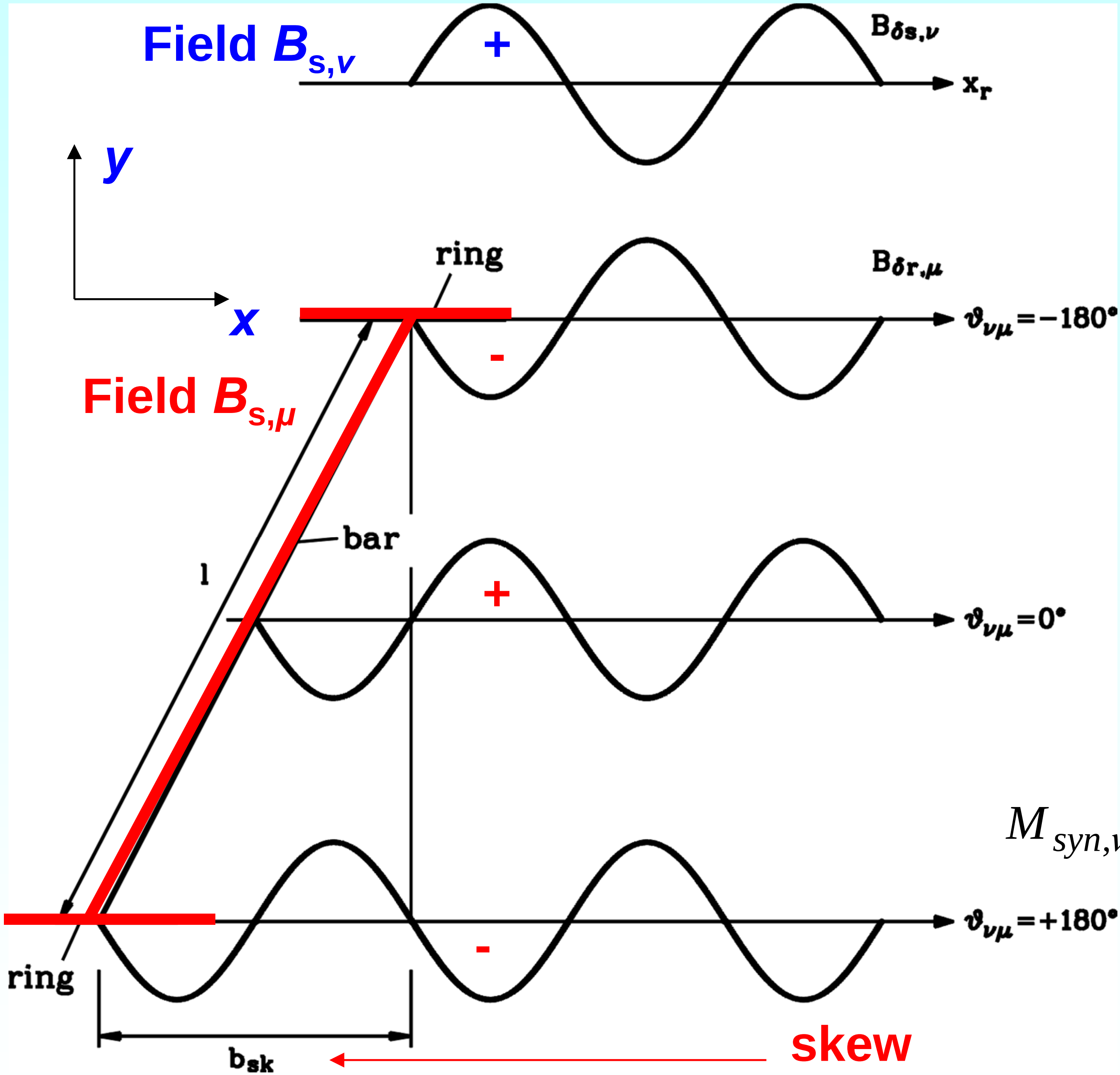
Skewing factor χ_ν for rotor cage





$$\chi_\nu = \frac{|\underline{U}_\nu|}{\left|\sum \Delta \underline{U}_\nu\right|} = \frac{2\sin(S_\nu)}{2S_\nu} = \frac{\sin(S_\nu)}{S_\nu} = \frac{\sin\left(\frac{\nu\pi b_{sk}}{2\tau_p}\right)}{\frac{\nu\pi b_{sk}}{2\tau_p}}$$

Skewing reduces synchronous harmonic torque



- **Skewing of rotor bars b_{sk} leads also to skew of rotor field harmonics, excited by rotor current I_r.**

- **So phase shift between stator and rotor field harmonic varies along bar length.**

- **This leads to reduction or cancelling of synchronous slot harmonic torque:**

$$M_{syn,\nu\mu} \sim \int_0^{2p\tau_p} \int_0^{l} B_{s\nu}(x,y)\cdot B_{r\mu}(x,y)\cdot dy\cdot dx$$

$$M_{syn,\nu\mu} = 0$$

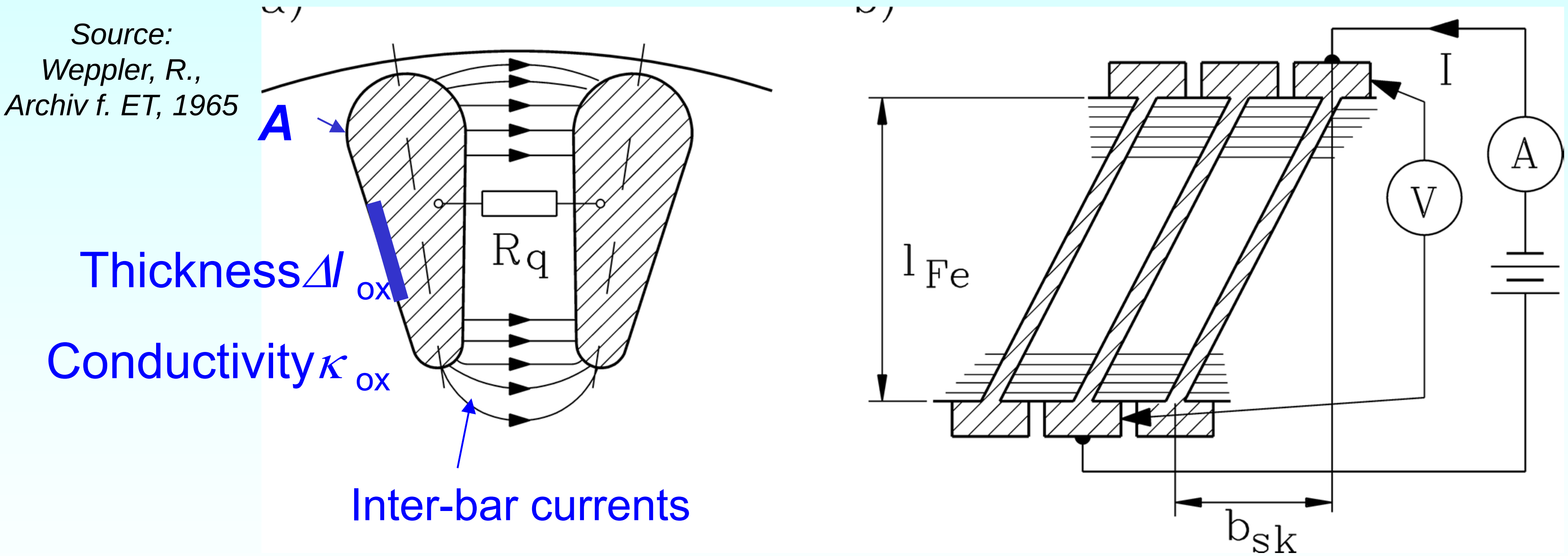
Inter-bar currents

Inter-bar resistance $R_q = \dfrac{\Delta l_{ox}}{\kappa_{ox} \cdot A}$ is determined by :

- thickness of oxidation layer Δl_{ox} between bar and iron
- conductivity of this oxide κ_{ox}.

Typical value for aluminium die cast cages: $r_q = R_q \cdot A = \Delta l_{ox} / \kappa_{ox} = 10^{-6}\,\Omega \cdot m^2$

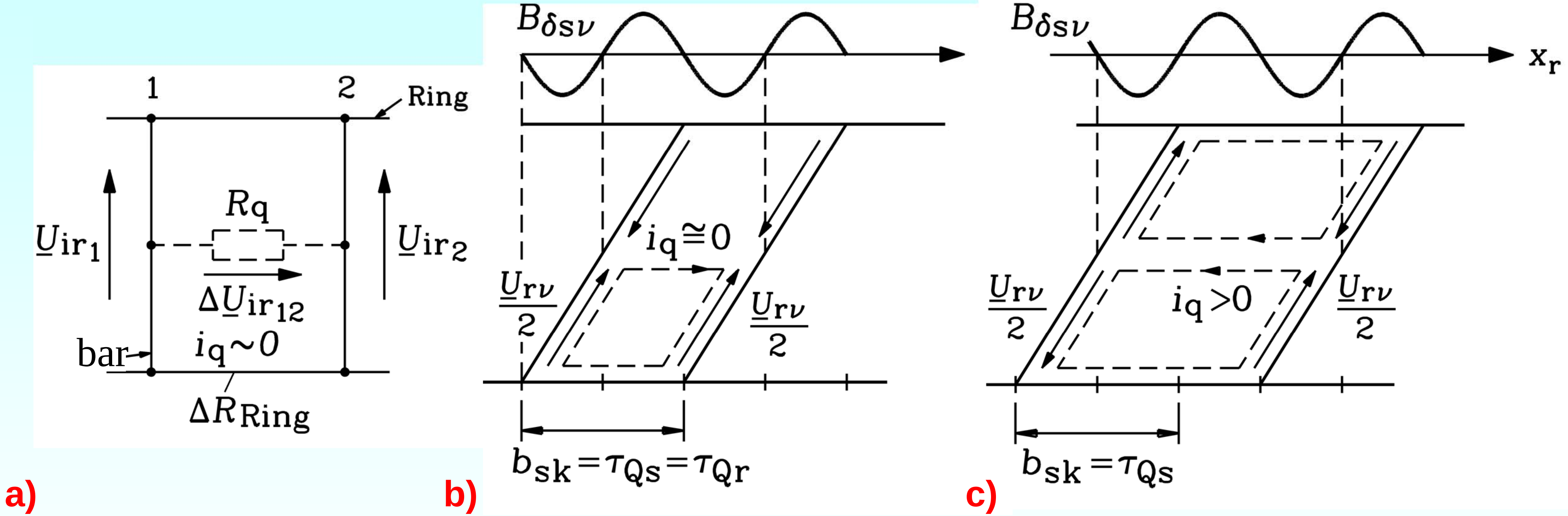
Source: Weppler, R., Archiv f. ET, 1965



Measurement set-up for inter-bar resistance $R_q = U/I - R_{bar} - \Delta R_{ring}$

Inter-bar resistance R_q is much bigger than bar or ring resistance !

Losses due to inter-bar current depend
- on skewing
- on slot number ratio Q_s/Q_r

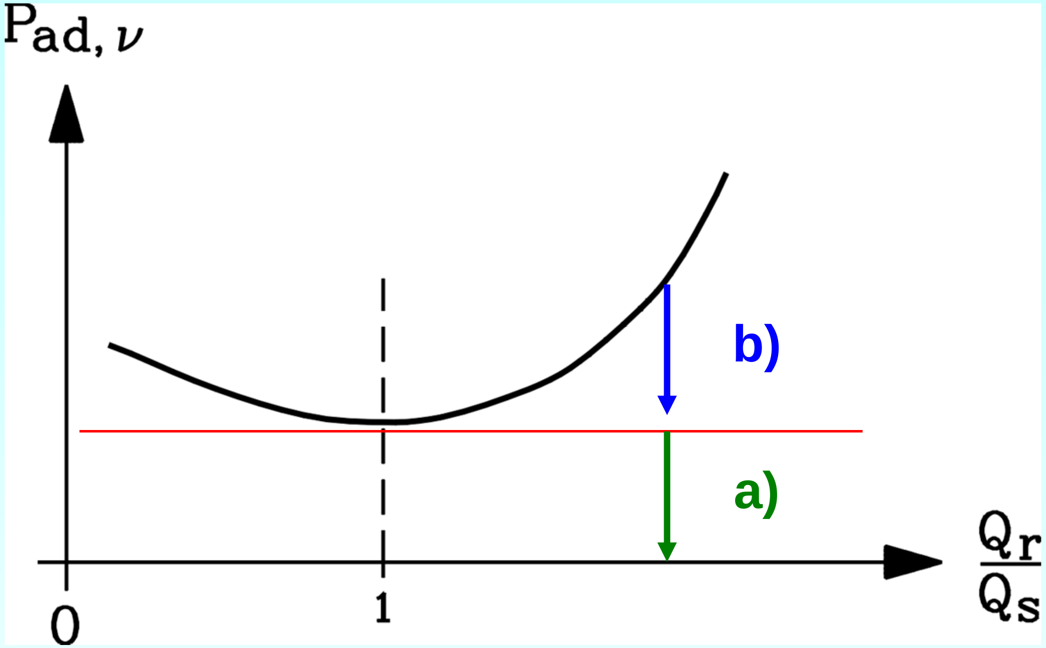


a) Unskewed cage: No inter-bar current, because $\Delta R_{Ring} << R_q$

b) Skewed cage: slot numbers equal $Q_r = Q_s$: no inter-bar current

c) $Q_r = Q_s/1.5$; BIG harmonic inter-bar current flows, as harmonic voltages add up.

Slot number ratio influences inter-bar current losses



Skewing in non-insulated rotor cages may give rise to inter-bar currents, which cause additional losses and may increase asynchronous harmonic torque.

$$P_{ad,r} = Q_r \cdot \sum_{\nu \neq 1}^{\infty} (R_r I_{r\nu}^2 + R_q I_{q\nu}^2)$$

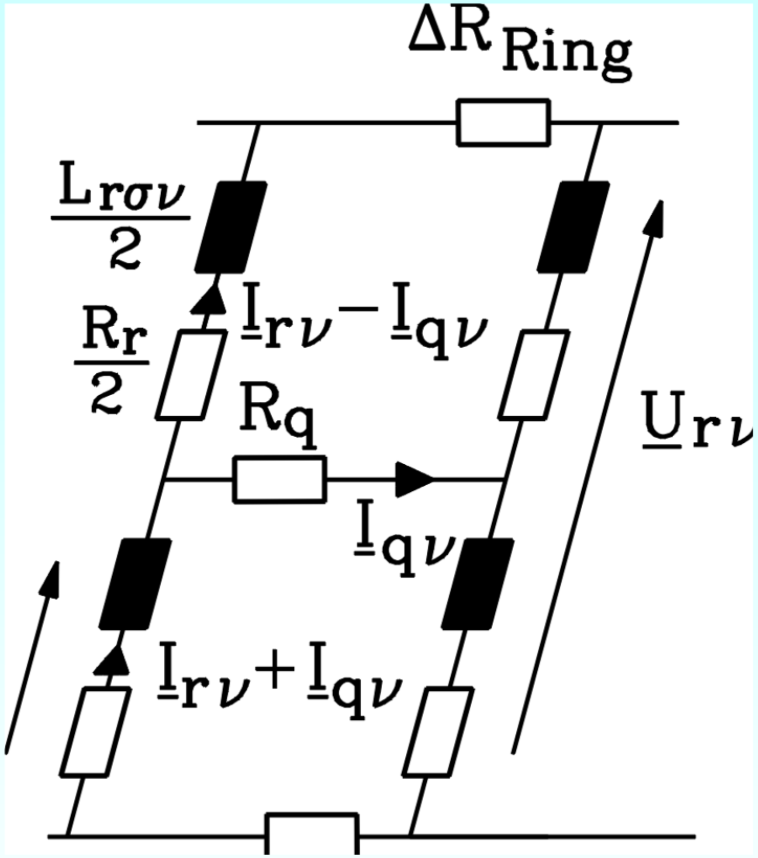
a) Harmonic losses **b) Inter-bar losses**

***Q*$_s$ = *Q*$_r$:** At $Q_r/Q_s = 1$ inter-bar losses are minimum.

***Q*$_s$ > *Q*$_r$:** Low rotor slot number ⇒ big deviation of step-like rotor flux density distribution from sine wave fundamental ⇒ big “harmonic leakage” inductance $L_{r\sigma\nu}$, which limits rotor harmonic and inter-bar current: SMALL LOSSES $P_{ad,r}$.

***Q*$_s$ < *Q*$_r$:** High rotor slot number ⇒ SMALL “harmonic leakage” inductance $L_{r\sigma\nu}$, big rotor harmonic and inter-bar current: BIG LOSSES $P_{ad,r}$.

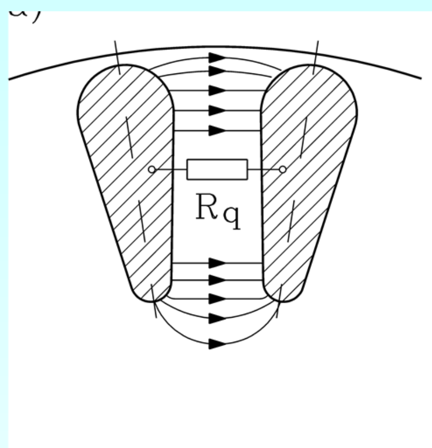
Inter-bar resistance and losses



Rotor mesh of two adjacent bars, ring segments and "concentrated" inter-bar resistance

Example:

Motor 200 kW, 50 Hz, 2 poles
stator/rotor slot number 36/28
closed rotor slots, Aluminium cage
skewed by one stator slot pitch.
Half-surface of rotor bar:
$A = 28570\ \text{mm}^2$,
"nominal" inter-bar resistance:



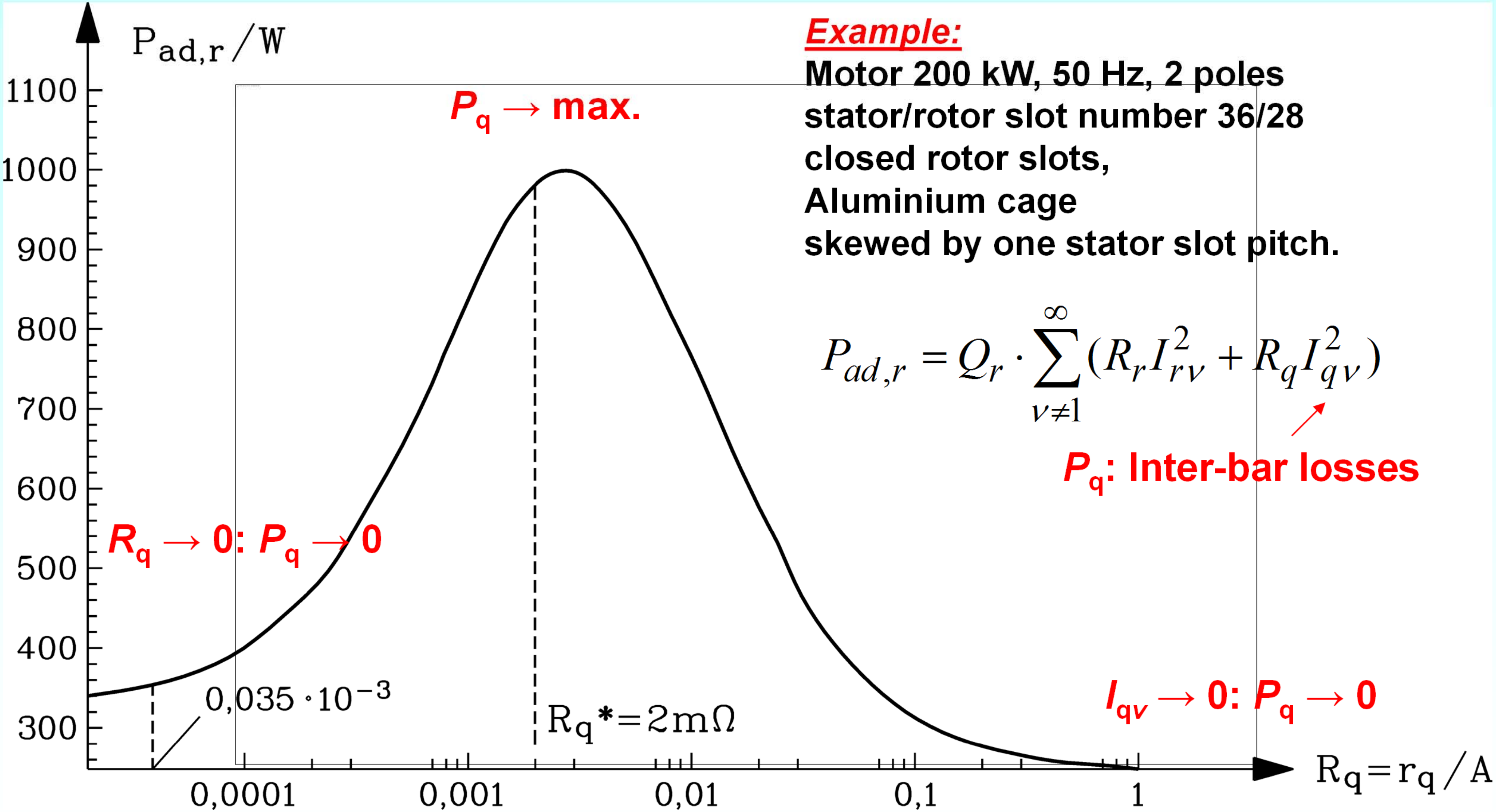
$$r_q = 10^{-6}\,\Omega\cdot m^2:$$

$$R_{qN} = r_q / A = 10^{-6} /(28570\cdot 10^{-6}) = \underline{\underline{0.035}}\,\text{m}\Omega$$

Inter-bar losses are 350 W (= 0.18% rated power).

In worst case at $R^*_q = 2\ \text{m}\Omega$ losses may reach 1 kW (= 0.5% of rated power).

Dependence of inter-bar current losses with R_q



Example:

Motor 200 kW, 50 Hz, 2 poles
stator/rotor slot number 36/28
closed rotor slots,
Aluminium cage
skewed by one stator slot pitch.

$$P_{ad,r} = Q_r \cdot \sum_{\nu \neq 1}^{\infty} (R_r I_{r\nu}^2 + R_q I_{q\nu}^2)$$

P_q: Inter-bar losses

Influence of inter-bar currents on asynchronous harmonic torque

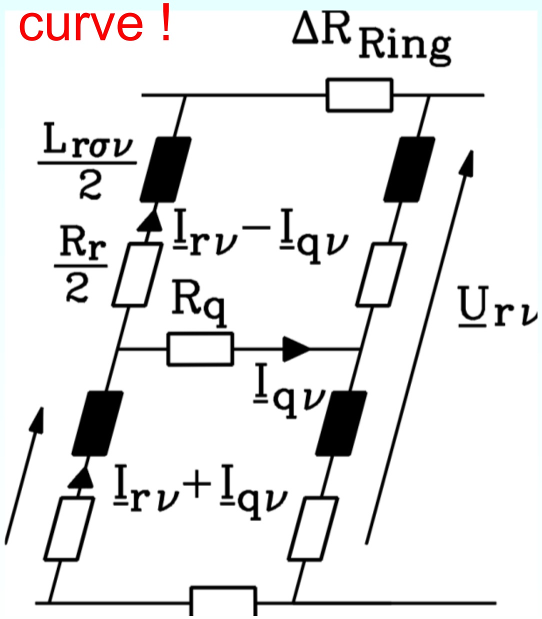
- Skewing effect: Harmonic rotor current $I_{r\nu}$ is reduced, but additional inter-bar current $I_{q\nu}$ flows.

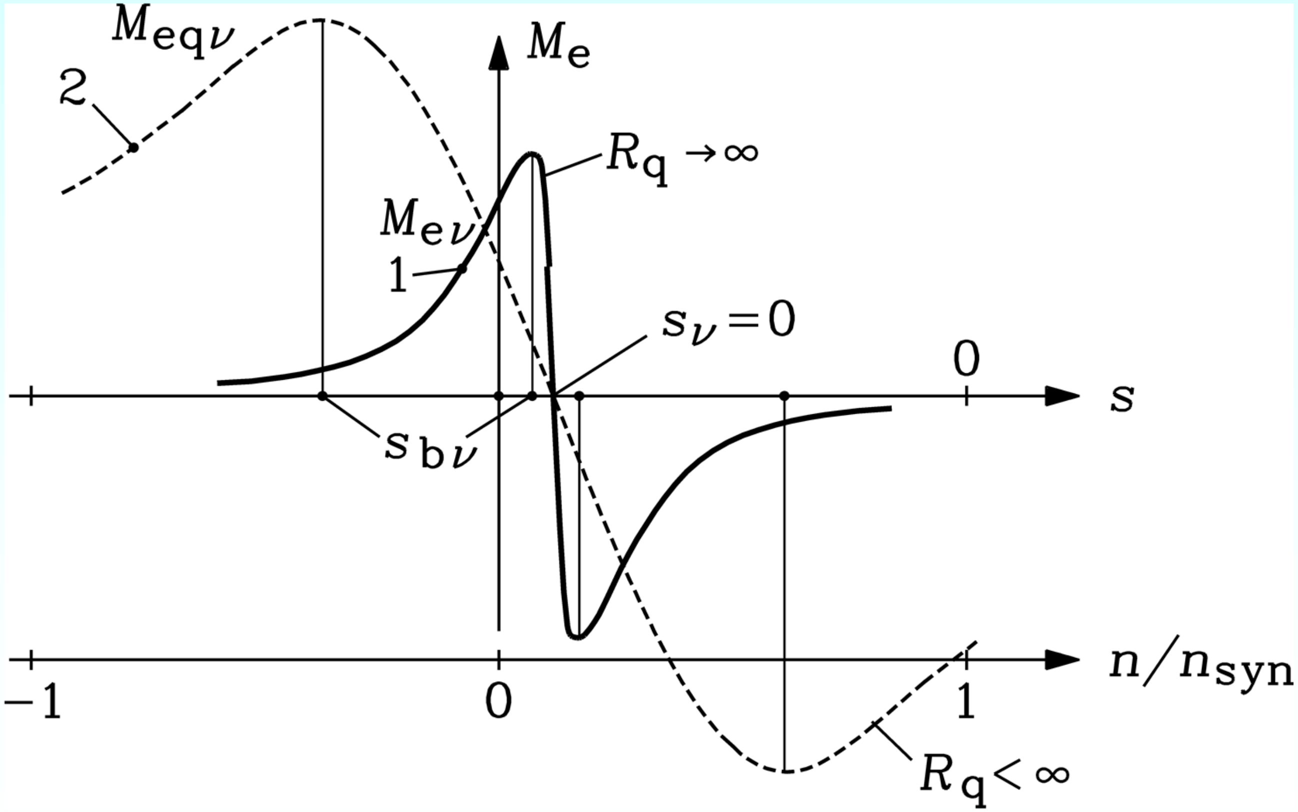
- Asynchronous harmonic torque due to $I_{r\nu}$ is reduced, but additional torque due to $I_{q\nu}$ occurs.

- R_q is an additional rotor resistance, INCREASING the harmonic break down slip:

$$s_{\nu b} = \pm \frac{R_r + 4R_q}{\omega_s (L_{r\sigma\nu} + L_{rh\nu})}$$ instead of $$s_{\nu b} = \pm \frac{R_r}{\omega_s (L_{r\sigma\nu} + L_{rh\nu})}$$.

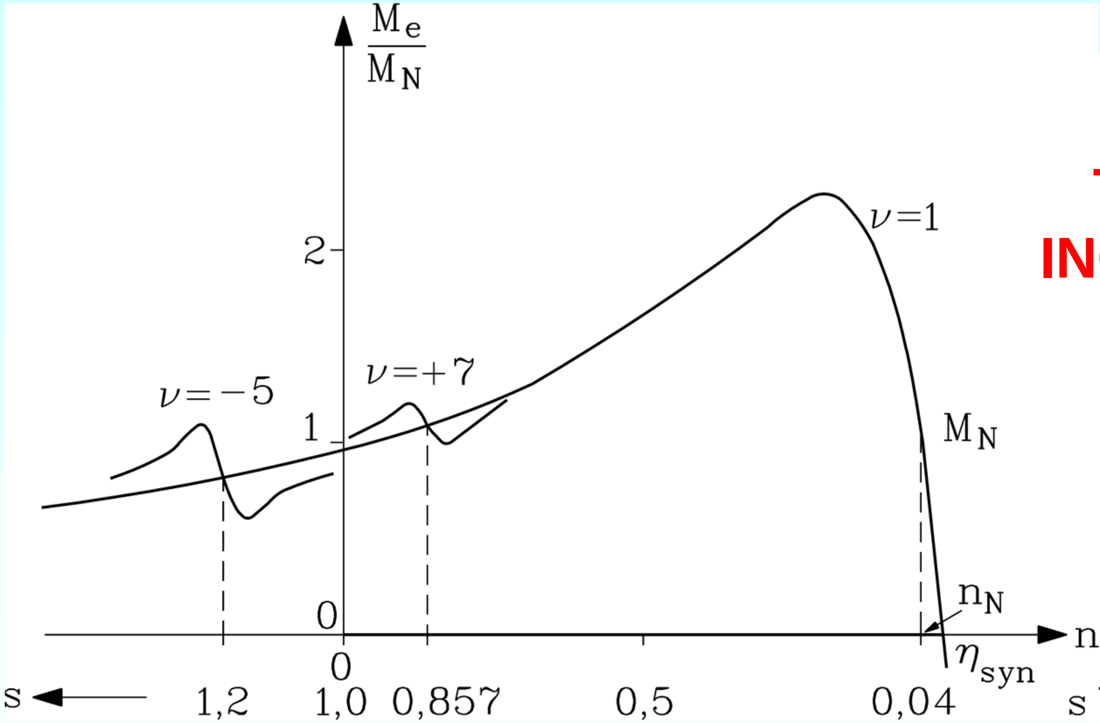
So we get a broad „saddle“ shaped distortion of *M(n)*-curve !

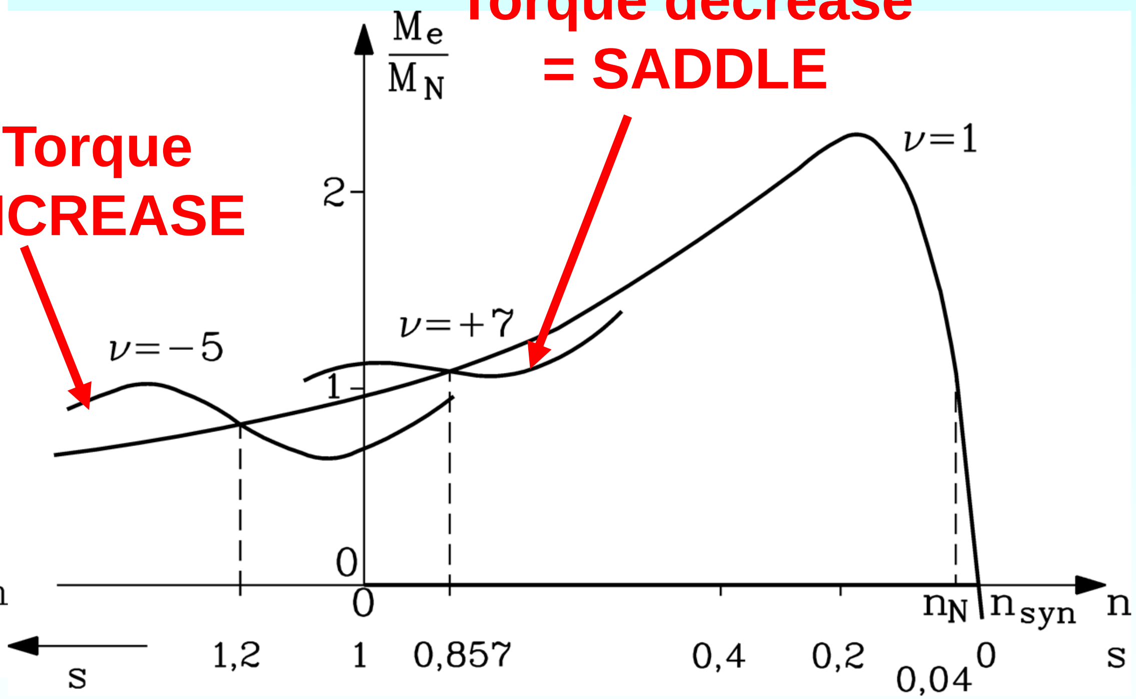




Broad „saddle“ shaped distortion of *M(n)*- curve due to inter-bar currents

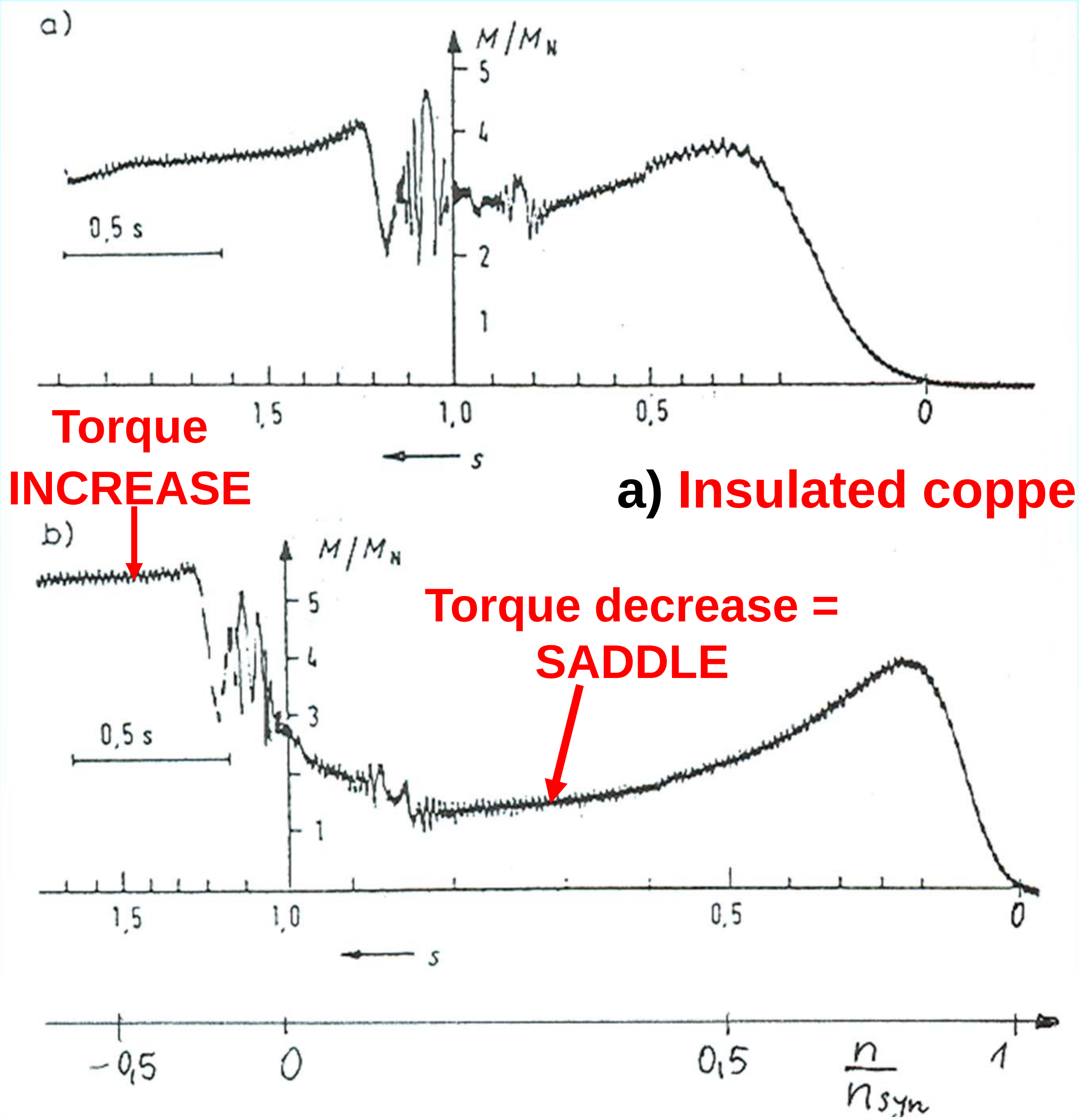
Here: Example of harmonic asynchronous torque of 5th and 7th harmonic





Without inter-bar currents → **With inter-bar currents**

Measured distortion of *M(n)*-curve due to inter-bar currents (1)



Data: **2-pole cage induction motor, 380 V, D, 50 Hz, 11 kW, rated torque M_N = 37 Nm**

skewed slots, Q_s/Q_r = 36/28, two-layer stator winding, winding pitch 1/2.

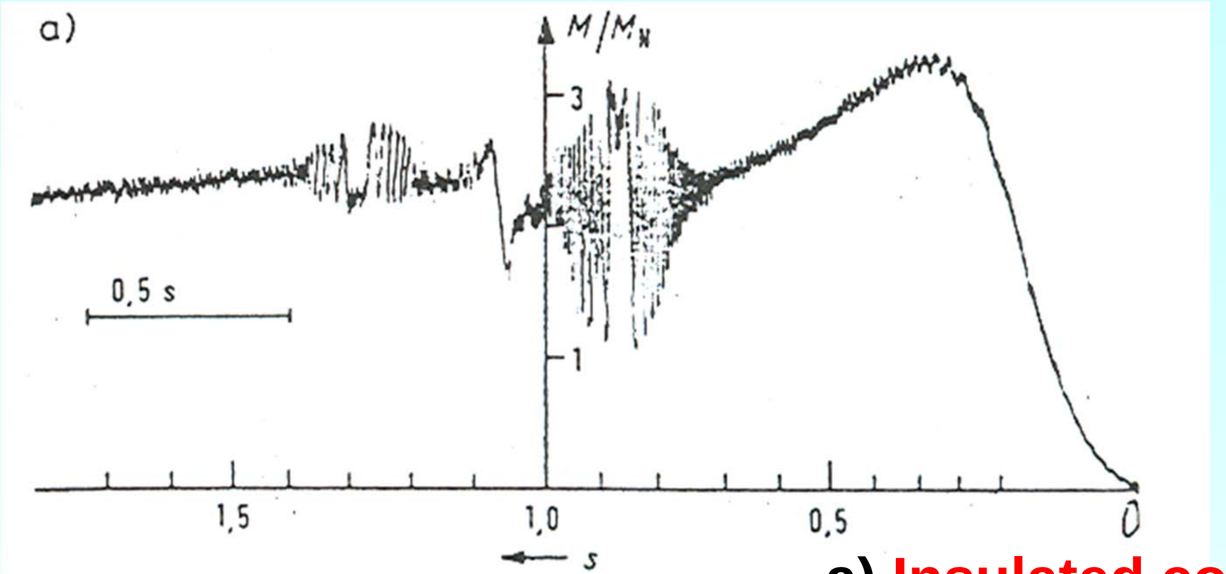
a) Insulated copper cage: NO inter-bar currents

Harmonic torque components at the same slip values, but asynchronous torque amplitudes changed

Synchronous harmonic torques nearly unchanged

b) Die-cast aluminium cage: flow of inter-bar currents

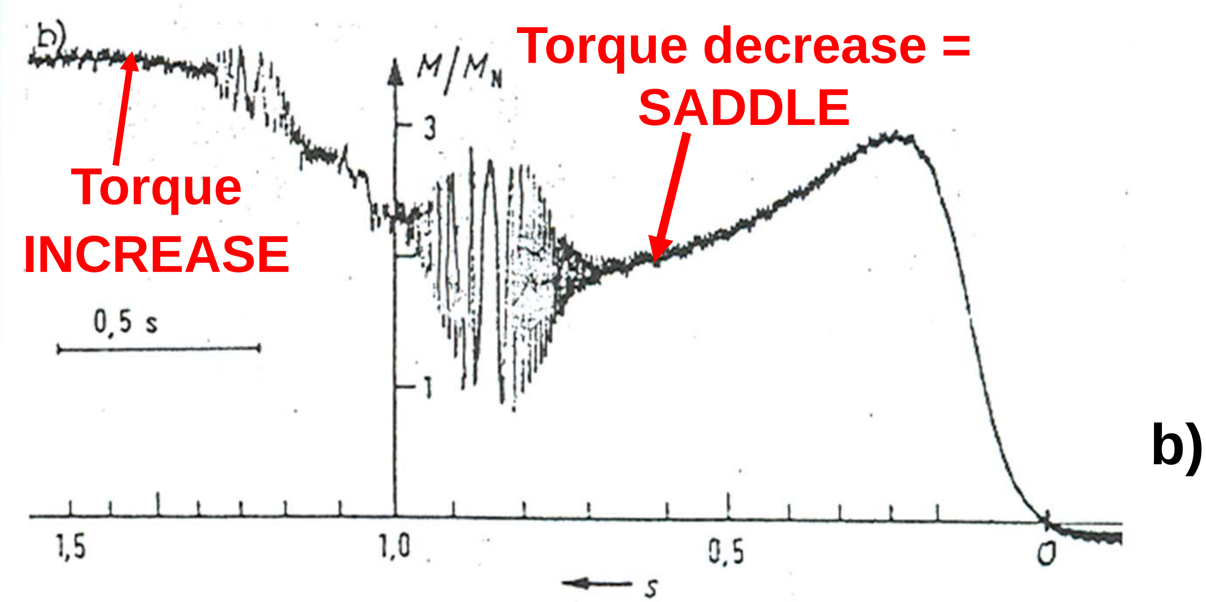
Measured distortion of *M(n)*-curve due to inter-bar currents (2)



<u>*Data:*</u> 4-pole cage induction motor, 380 V, D, 50 Hz, 11 kW, rated torque M_N = 37 Nm

skewed slots, Q_s/Q_r = 36/28, two-layer stator winding, winding full pitched.

a) Insulated copper cage: NO inter-bar currents

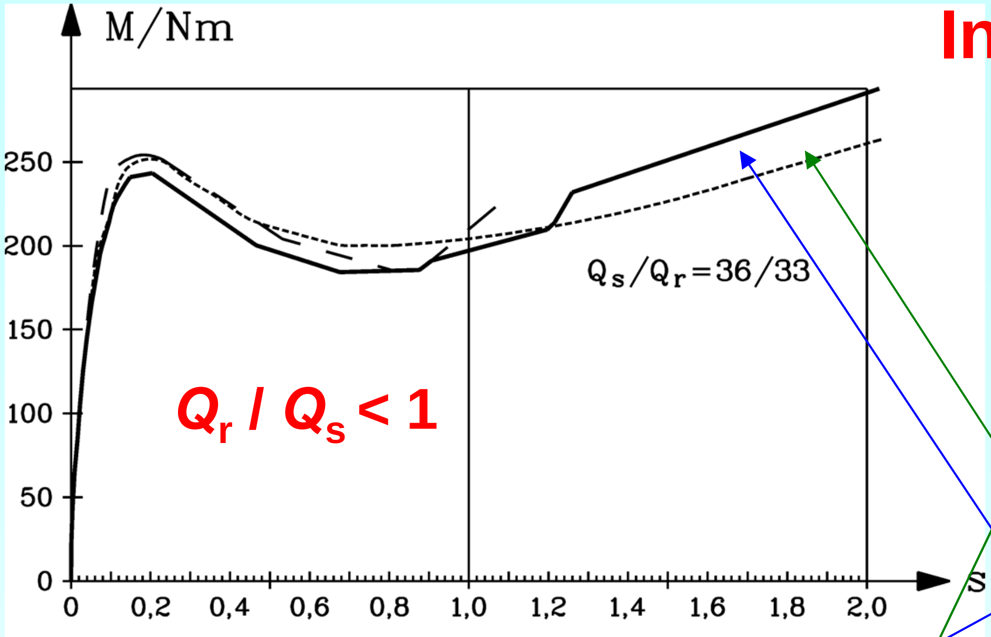


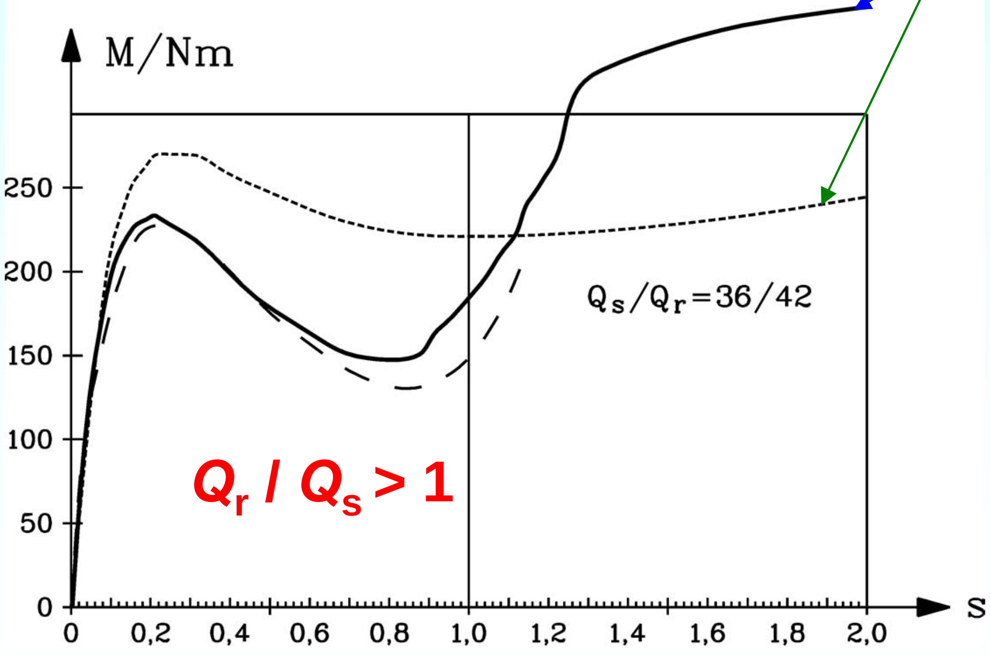
Harmonic torque components at the same slip values, but asynchronous torque amplitudes changed

Synchronous harmonic torques nearly unchanged

b) Die-cast aluminium cage: flow of inter-bar currents

Influence of slot number ratio on *M(n)*-curve due to inter-bar currents





Data: **6-pole induction motor, f_s = 50 Hz, 380 V, 11 kW, M_N = 110 Nm, 36 stator slots, die-cast aluminium cage:**

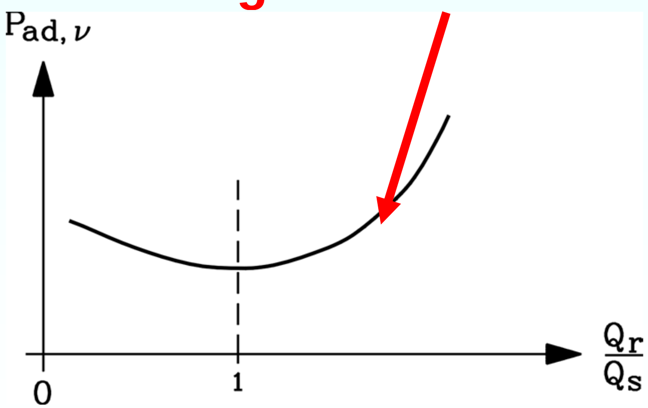
calculated fundamental asynchronous torque

calculated total torque

- - - - - - - - - - measured total torque

- Big influence on torque at Q_r / Q_s > 1

- Corresponds with big additional losses !



Basics on acoustics sound

Audible frequency range of the human ears: 16Hz … 20kHz.

Different types of sound:

- Tone (Oscillation of the air density with a fixed frequency),
- Sound (Superposition of several different tones with different amplitudes),
- Noise (statistically distributed amplitude and frequency spectrum),
- Sonic boom (discontinuous increase of sound pressure).

Centre frequencies of the octave bands in the audible range in Hz:

63 125 250 500 1000 2000 4000 8000

Octave band = doubling of frequency!

Radiation of sound – sound waves

Speed of sound: $c_s = \lambda \cdot f$ (f: frequency, λ: wave length)

Sound wave: Longitudinal wave as longitudinal oscillation of the air density: The air molecules oscillate with the **particle velocity of sound *v*(*t*)** around the particle orbit, which is determined by the thermal movement of the air molecules

Sound wave propagation: The wave front is propagating spherical. At a far distance from the source of sound it may be approximated by a plane wave ("far field").

Far field of the sound wave (without absorption): Sound pressure $p(t)$:

$$p(t) = \rho_{air} \cdot c_s \cdot v(t) = Z_s \cdot v(t) \qquad v(t) = v \cdot \sin(2\pi \cdot f \cdot t)$$

$\rho_{air} = 1.29\ \mathrm{kg/dm^3}$, $c_s = 343\ \mathrm{m/s}$ at 20°C, 1 bar = 10^5 Pa

$Z_s = 443\ \mathrm{Ns/m^2}$: specific acoustic impedance

Electric analogy:

$U \Leftrightarrow p(t) \qquad I \Leftrightarrow v(t)$

Intensity of sound = Power of sound / cross-section area: $I(t) = P(t)/A = F(t) \cdot v(t)/A = p(t) \cdot v(t)$

Average intensity per period $1/f$ in a planar sound wave: $I = p_{rms} \cdot v_{rms} = p \cdot v/2 = p^2/(2Z_s)$

Human ear: Audible threshold at f = 1 kHz: $I_0 = 10^{-12}\ \mathrm{W/m^2}$, $p_0 = 2 \cdot 10^{-5}$ Pa

Limit of pain at f = 1 kHz: $I_{lim} = 10\ \mathrm{W/m^2}$, $p_{lim} = 65$ Pa

Human sense for loudness - Sound level

According to the physiological (empirical) law of ***Weber*** and ***Fechner*** the human ear is sensing the **loudness *L* of pure tones** in dependence of the logarithm of the sound intensity I.

$$L \sim \lg(I / I_0)$$ L (Unit: phon)

Therefore the logarithmic ***Sound intensity level*** is defined:

$L_I = \lg(I/I_0)$ (Unit: Bel, B) or $L_I = 10 \cdot \lg(I/I_0)$ (dB, Dezibel)

The sound intensity level L_I in dB is at 1000 Hz equal to the ***loudness*** L, which is detected by the human ear.

At other frequencies than 1000 Hz the curves of equal loudness in dependence of the sound intensity were determined empirically by ***Fletcher*** and ***Munson***. They found, that the human ear hears

a) below 1 kHz the same sound intensity less loud than L_I ,

b) between 1 … 6 kHz the same sound intensity louder than L_I ,

c) above 6 kHz the same sound intensity less loud than L_I .

Sound pressure level

Sound pressure level: $L_p = 10 \cdot \lg(p^2/p_0^2) = 20 \cdot \lg(p/p_0) = L_I = 10 \cdot \lg(I/I_0)$ (dB)

Sound power level: in a planar sound wave with the wave front area *A*:

$P = I \cdot A$: $L_W = 10 \cdot \lg(P/P_0)$ (dB) $P_0 = 10^{-12}$ W, $A_0 = 1$ m^2

The real *measuring area A* must be considered for the determination of L_W from L_p.

$$L_W = 10 \cdot \lg(P/P_0) = 10 \cdot \lg(I \cdot A/(I_0 \cdot A_0)) = 20 \cdot \lg(p/p_0) + 10 \cdot \lg(A/A_0)$$

$$\boxed{L_W = L_p + 10 \cdot \lg(A/A_0)}$$

Example:

The acoustic noise of a motor is measured with microphones at 1 m distance from the motor, placed in the corners of a hexahedral measuring surface (edge length 2 m, side area 2 x 2 m^2) around the motor.

$A = 5 \times 2 \times 2 = 20$ m^2 $$L_W = L_p + 10 \cdot \lg(20/1) = L_p + 13dB$$

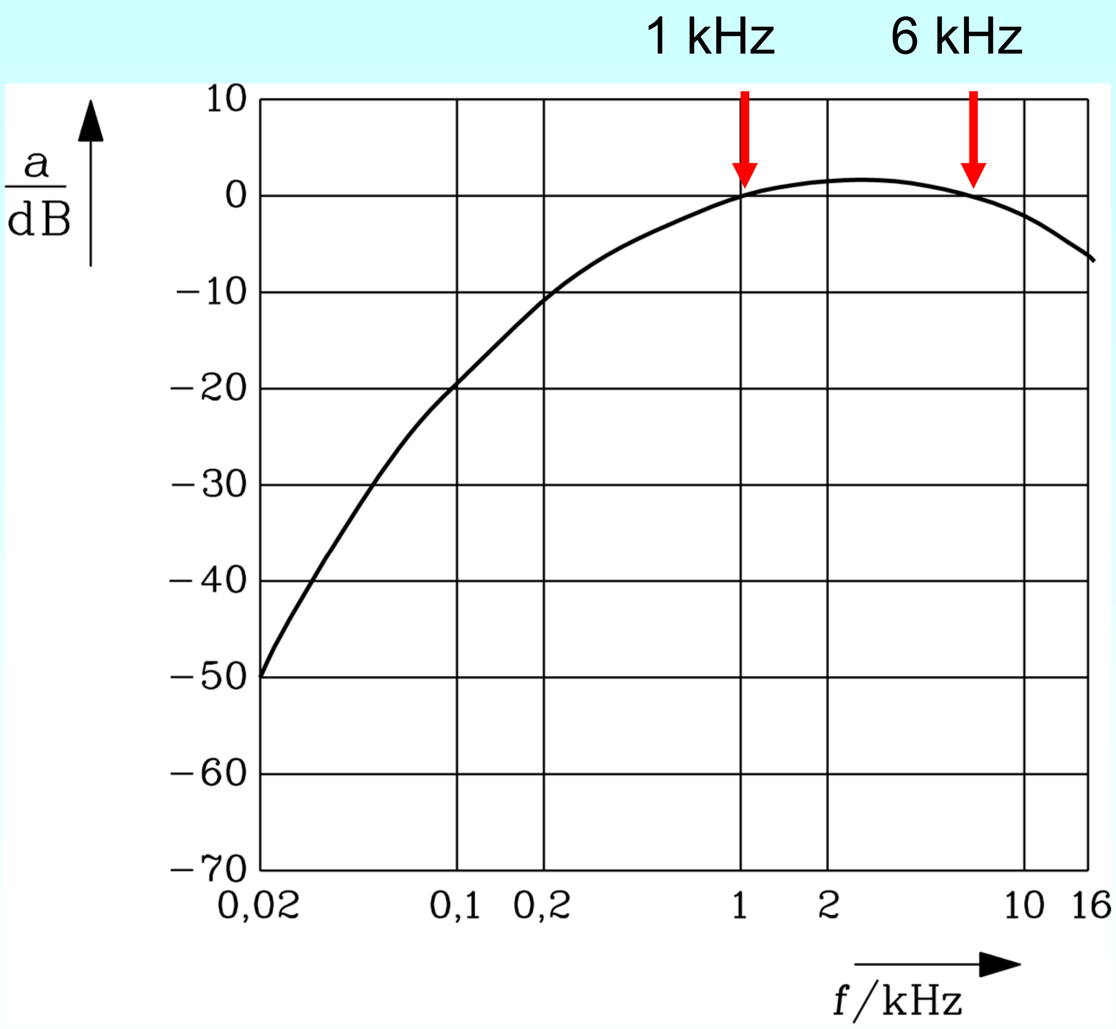
A-weighting of the sound pressure level

According to IEC-651/1979 the sound pressure level L_p is weighted with a factor ***a(f)*** according to the results of *Fletcher-Munson*, in order to consider the human hearing in a simple way.

$$L_I\big|_{dB(A)} = L_I\big|_{dB} + a(f)$$

$$L_{pA} = L_p\big|_{dB(A)} = L_p\big|_{dB} + a(f)$$

Unit: dB(A)



Examples for sound pressure levels

a) Increase of sound pressure level from one to two equally loud motors:

$$I_{\mathrm{Sres}} = 2I_{\mathrm{S1}} \qquad L_{\mathrm{Ires}} = 10 \cdot \lg(2I_{\mathrm{S1}} / I_{\mathrm{S0}}) = 10 \cdot \lg(I_{\mathrm{S1}} / I_{\mathrm{S0}}) + 10 \cdot \lg 2 = L_{\mathrm{I1}} + 3\mathrm{dB}$$

$$\boxed{L_{\mathrm{p1}} = L_{\mathrm{I1}} \quad L_{\mathrm{p,res}} = L_{\mathrm{Ires}} = L_{\mathrm{p1}} + 3\mathrm{dB}}$$

A doubling of the sound intensity increases the sound pressure level by 3 dB!

b) Resulting sound pressure level for three different loud motors:

$L_{\mathrm{p1}} = 85$ dB, $L_{\mathrm{p2}} = 83$ dB und $L_{\mathrm{p3}} = 82$ dB

$L_{\mathrm{I1}} = L_{\mathrm{p1}} = 85$ dB, $L_{\mathrm{I2}} = L_{\mathrm{p2}} = 83$ dB, $L_{\mathrm{I3}} = L_{\mathrm{p3}} = 82$ dB

$$I_{\mathrm{S}} = I_{\mathrm{S0}} \cdot 10^{L_{\mathrm{I}}/10}$$

$$L_{\mathrm{Ires}} = 10 \cdot \lg\left(10^{L_{\mathrm{I1}}/10} + 10^{L_{\mathrm{I2}}/10} + 10^{L_{\mathrm{I3}}/10}\right)$$

$$\boxed{L_{\mathrm{Ires}} = 10 \cdot \lg\left(10^{85/10} + 10^{83/10} + 10^{82/10}\right) = 88.3\mathrm{dB} = L_{\mathrm{p,res}}}$$

Influence of distance on the sound pressure level

The sound radiation wave is a spherical wave.

The constant sound power ***P* = const.** is acting on an increasing area: $A = 2\pi r_S^2$

Hence the sound intensity *I* = *P*/*A* is decreasing with increasing distance r_s from the source.

$$I \sim 1/r_S^2$$

$$L_I(r_{S1}) = 10 \cdot \lg(I_1/I_0) = 10 \cdot \lg\left((P/P_0) \cdot (A_0/A_1)\right) \qquad A_0 = 2\pi r_{S0}^2$$

$$L_I(r_S) = 10 \cdot \lg\left(\frac{P}{P_0} \cdot \frac{A_0}{2\pi r_S^2}\right) = 10 \cdot \lg\left(\frac{P}{P_0} \cdot \frac{A_0}{A_1}\right) + 10 \cdot \lg\left(\frac{A_1}{2\pi r_S^2}\right)$$

$$L_I(r_S) = L_I(r_{S1}) + 20 \cdot \lg(r_{S1}/r_S) \qquad A_1 = 2\pi r_{S1}^2 \qquad L_p(r_S) = L_p(r_{S1}) + 20 \cdot \lg(r_{S1}/r_S)$$

Example:

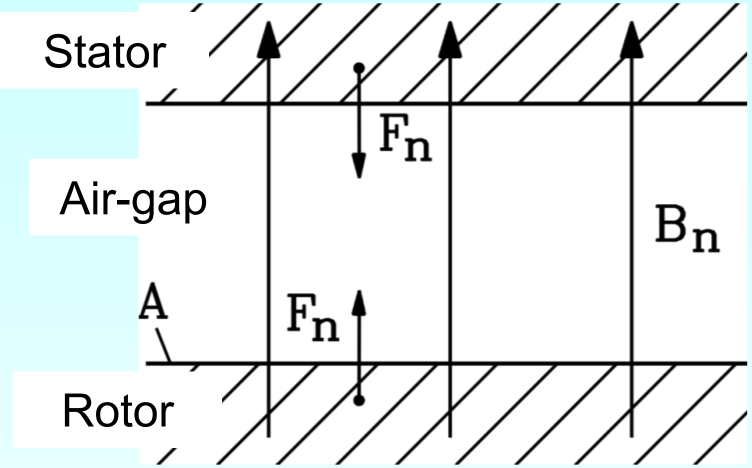
L_p at r_{S1} = 1 m distance is 75 dB. Determine the value at r_S = 2 m distance!

$$L_p(r_S = 2\text{m}) = L_p(r_S = 1\text{m}) + 20 \cdot \lg(1/2) = L_p(r_S = 1\text{m}) - 6\text{dB}$$

Normal force in electric machines

- Magnetic field B_n crossing the air gap between two parallel iron surfaces (surface A) leads to an attracting force, the **magnetic pull F_n:**

$$f_n = \frac{F_n}{A} = \frac{B_n^2}{2\mu_0}$$



- The space harmonic air gap field waves of stator and rotor must be considered as the total radial magnetic field, which exerts a time-varying magnetic pull on stator and rotor iron surface.

$$B_{\delta s \nu}(x_s, t) = B_{\delta s \nu} \cdot \cos\left(\frac{\nu \pi x_s}{\tau_p} - 2\pi f_s t\right)$$

a) Stator harmonic field wave, excited by stator current I_s with stator frequency f_s
b) Rotor harmonic field wave, excited by rotor fundamental bar current I_r with rotor frequency f_r

$$B_{\delta r \mu}(x_r, t) = B_{\delta r \mu} \cdot \cos\left(\frac{\mu \pi x_r}{\tau_p} - 2\pi \cdot s \cdot f_s t\right) \quad B_{\delta r \mu}(x_s, t) = B_{\delta r \mu} \cdot \cos\left(\frac{\mu \pi x_s}{\tau_p} - 2\pi f_s t \cdot (s + \mu(1-s))\right)$$

Rotor wave in stator fixed reference frame:

$$x_r = x_s - v_m t = x_s - (1-s) \cdot v_{syn} \cdot t = x_s - (1-s) \cdot 2 f_s \tau_p \cdot t$$

Magnetic pull due the harmonic waves

$$f_n(x_s,t)=\frac{B^2(x_s,t)}{2\mu_0}\sim\left(\sum_\nu B_{\delta s\nu}+\sum_\mu B_{\delta r\mu}\right)^2 \quad\Rightarrow\quad f_{n,\mu\nu}\sim\sum_{\nu,\mu}B_\nu^2+2B_\nu B_\mu+B_\mu^2$$

Mainly the mixed products $2B_\nu B_\mu$ result in radial forces, whose pulsating frequencies are in the audible region between 100 Hz and 16 kHz:

$$\alpha=\frac{\nu\pi x_s}{\tau_p}-2\pi f_s t \qquad \beta=\frac{\mu\pi x_s}{\tau_p}-2\pi f_s t\cdot(s+\mu(1-s))$$

$$B_{\delta s\nu}\cos\alpha\cdot B_{\delta r\mu}\cos\beta=B_{\delta s\nu}B_{\delta r\mu}\cdot\frac{1}{2}\cdot[\cos(\alpha+\beta)+\cos(\alpha-\beta)]$$

$$\alpha+/-\beta=2(\nu+/-\mu)p\cdot\frac{\pi x_s}{2p\tau_p}-2\pi f_s t[(\mu-1)\cdot(1-s)+2/-0]$$

As a result, **radial force density waves** are derived, which exert an oscillating pull on stator and rotor iron stack. **Number of nodes 2*r*** of force wave: $2r=2p\cdot|\nu\pm\mu|$

$$f_{n,\nu\mu}(x_s,t)=\frac{B_{\delta s\nu}B_{\delta r\mu}}{2\mu_0}\cdot\cos\left(2r\cdot\frac{\pi x_s}{2p\tau_p}-2\pi f_{Ton}t\right)$$

$$f_{Ton}=f_s\cdot|(\mu-1)\cdot(1-s)+2/-0|$$

Radial force wave parameters

$$\alpha+\beta=2(\nu+\mu)p\cdot\frac{\pi x_s}{2p\tau_p}-2\pi f_s t\left[1+s+\mu\cdot(1-s)\right]$$

$$\alpha+\beta=2p\cdot(\nu+\mu)\cdot\frac{\pi x_s}{2p\tau_p}-2\pi f_s t\cdot\left[(\mu-1)\cdot(1-s)+2\right]$$

$$\alpha-\beta=2(\nu-\mu)p\cdot\frac{\pi x_s}{2p\tau_p}-2\pi f_s t\left[1-s-\mu\cdot(1-s)\right]$$

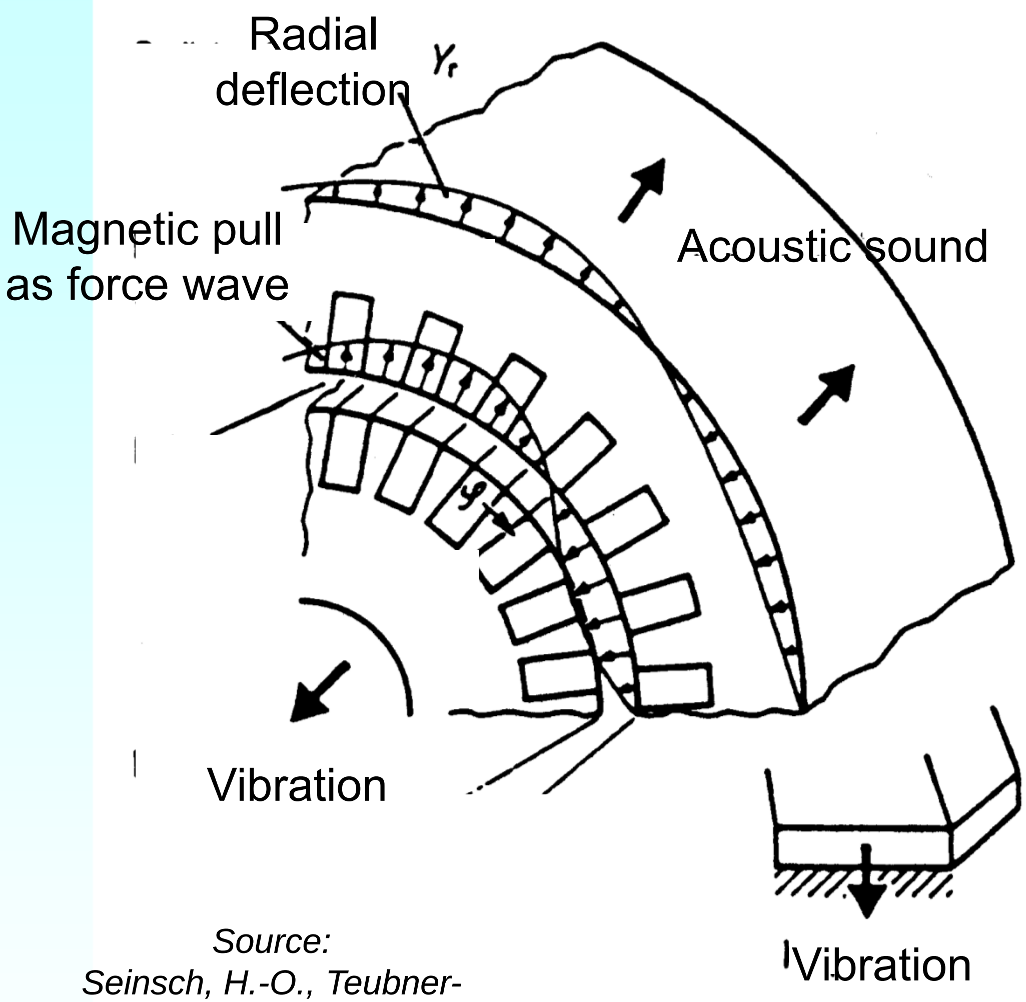
$$\alpha-\beta=2p(\nu-\mu)\cdot\frac{\pi x_s}{2p\tau_p}-2\pi f_s t\left[(1-s)\cdot(1-\mu)\right]$$

$$\boxed{\alpha\pm\beta=2r\cdot\frac{\pi x_s}{2p\tau_p}-2\pi f_{Ton}t}$$

$$2r=2p\cdot(\nu\pm\mu) \qquad 2r>0: 2r=2p\cdot\left|\nu\pm\mu\right|$$

$$f_{Ton}>0: \quad f_{Ton}=f_s\cdot\left|(\mu-1)\cdot(1-s)+2/-0\right|$$

Electromagnetic acoustic noise



Source: Seinsch, H.-O., Teubner-Verlag, Stuttgart

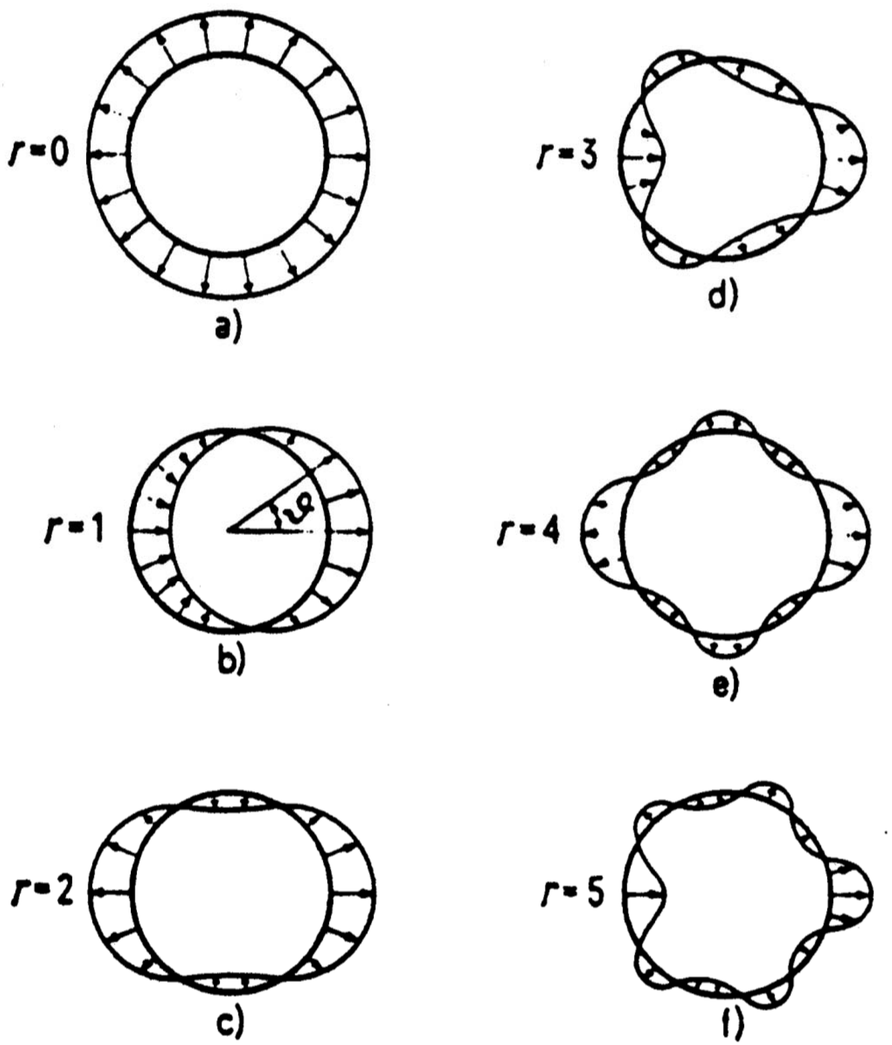
- The stator iron may be regarded as a steel ring, whereas the rotor is a steel cylinder.

- Therefore the stator is less stiff than the rotor and is bent by the radial force waves.

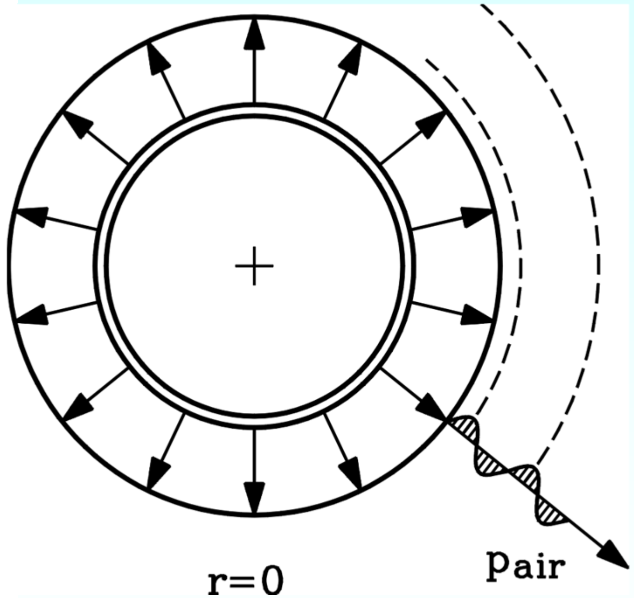
- As the iron surface is shaken with this frequency, the surrounding air is compressed and de-compressed with frequency f_{Ton}.

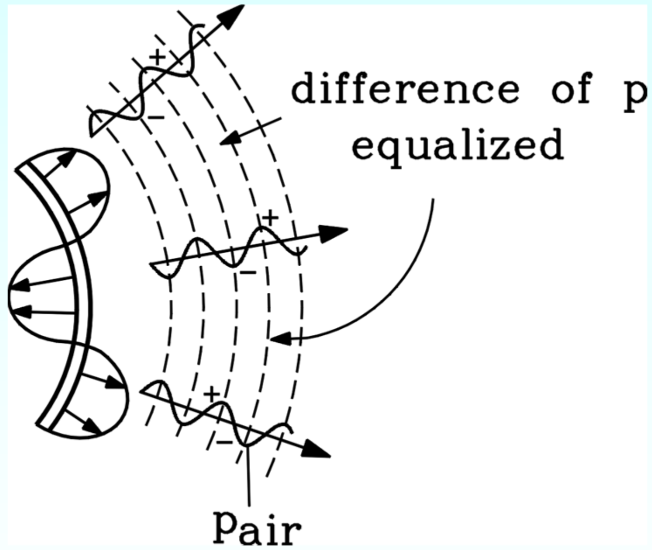
- So acoustic sound waves are generated with that tonal frequency f_{Ton} to be heard by e.g. human beings.

Deformation of the stator yoke



- $2r = 0$: Stator surface oscillates in phase along the stator circumference, so a far reaching sound pressure wave p_{air} is generated.
- $2r > 0$: With increased node number the sound pressure p is equalized better along the circumference.

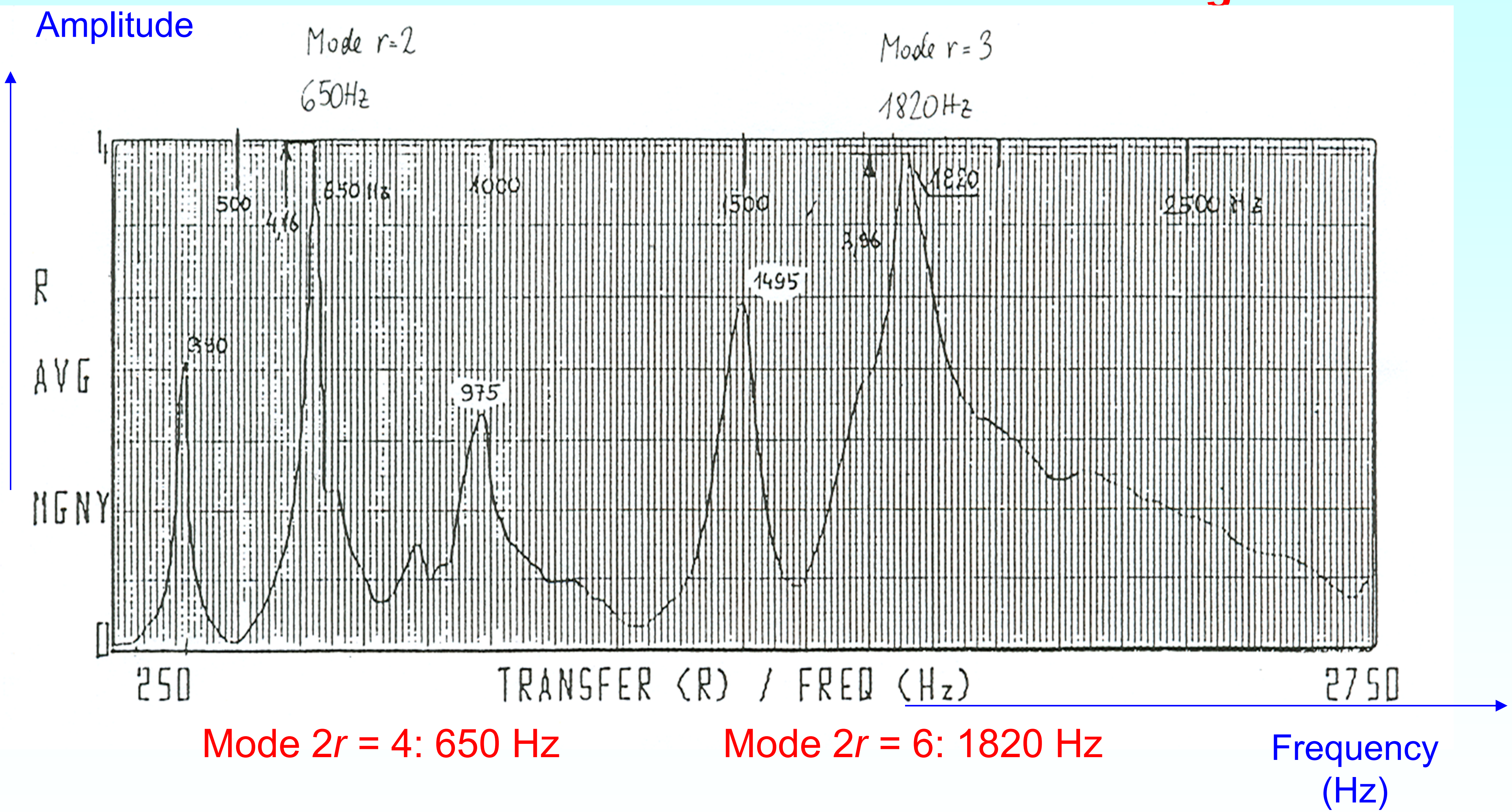




Source: Jordan, H.;
Der geräuscharme Elektromotor,
Verlag Girardet, Essen, 1957

Stator is approximated for the "far pressure field" as a vibrating sphere.

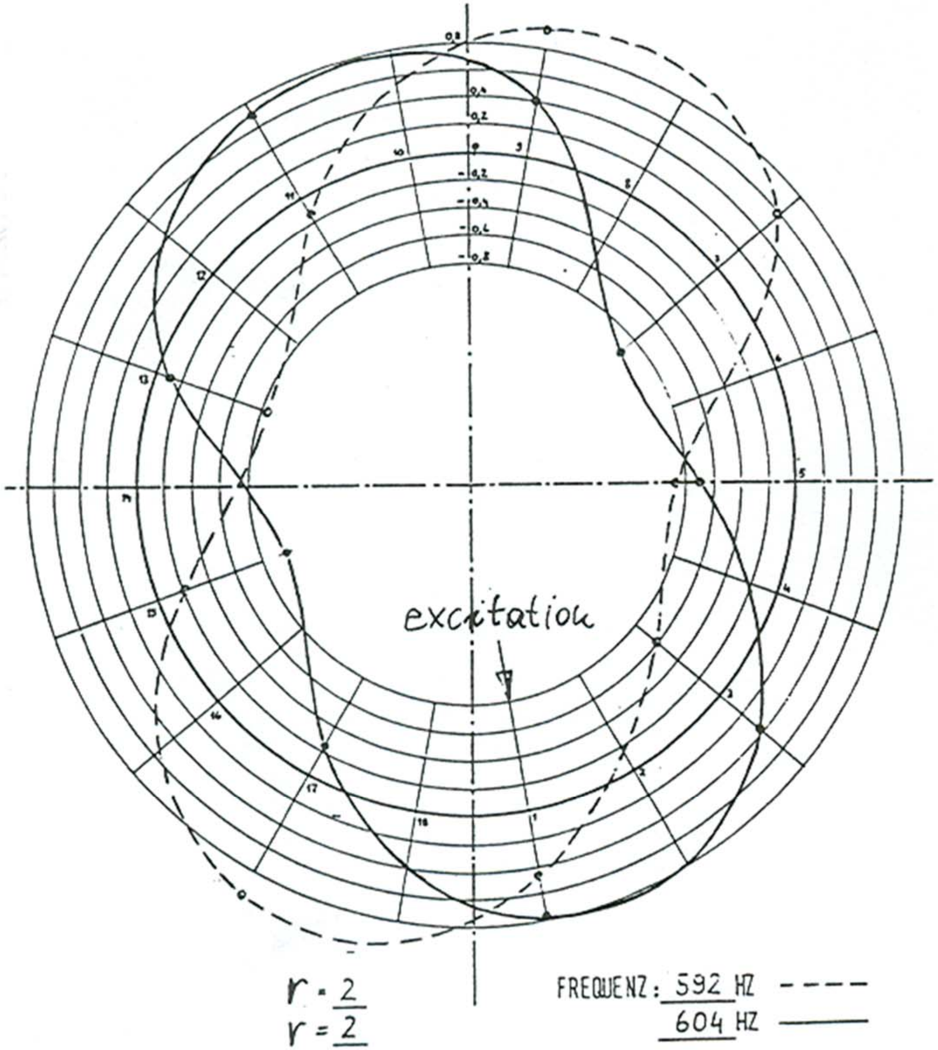
Example: Measured natural vibration frequencies of a 6-pole standard stator iron stack with winding

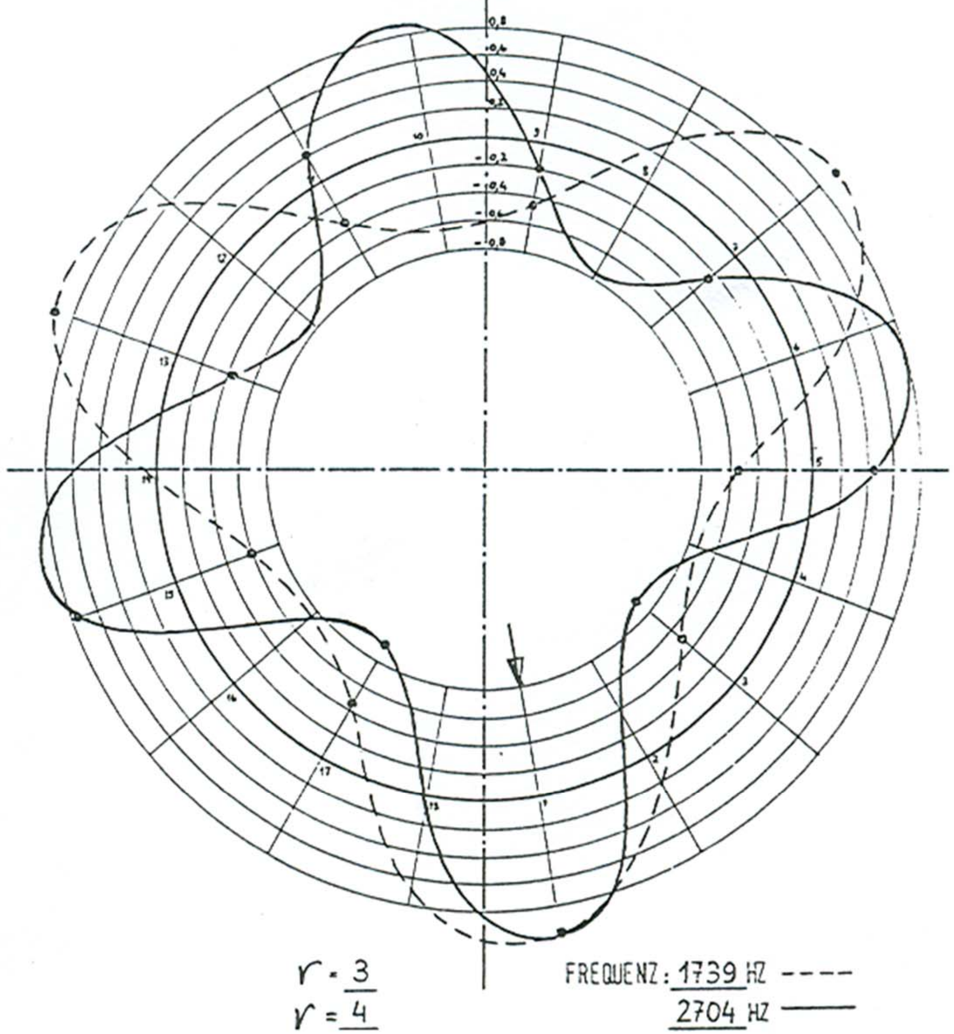


Mode 2*r* = 4: 650 Hz

Mode 2*r* = 6: 1820 Hz

Measured natural yoke bending modes





2*r* = 4: 592 Hz & 604 Hz

2*r* = 6: 1739 Hz   2*r* = 8: 2704 Hz

6-pole standard stator iron stack with winding

Example: Tonal frequencies, Rotor a) (1)

Motor 11 kW, 380 V, D, 50 Hz, $2p = 6$, air gap 0.35 mm, iron stack $l_{Fe} = 170$ mm

$Q_s = 36$, Single layer winding, semi-closed slots, rotor aluminium cage; slip $s = 3\%$

Rotor a): $Q_s > Q_r = 33$, rotor slot skew: 1 rotor slot pitch.

Frequency calculation:

Stator slot harmonics

- Stator field harmonics: 1,-5,+7,-11,+13,-17,+19,-23,+25,-29,+31,-35,+37,..
- Ordinal numbers of rotor field harmonics, excited by rotor current I_r under load:

$$\mu = 1 + (Q_r / p) g = 1 + 11g : = 1, -10, +12, -21, +23, -32, +34, ...$$

$$\underline{\nu = -11,\ \mu = -10:}\ r = |p(\nu - \mu)| = |3(-11+10)| = 3,\quad f_{Ton} = 50|(-10-1)(1-0.03)+0| = \underline{\underline{533.5}}\ \mathrm{Hz}$$

$$\underline{\nu = 13,\ \mu = 12:}\ r = |p(\nu - \mu)| = |3(13-12)| = 3,\quad f_{Ton} = 50|(12-1)(1-0.03)+0| = \underline{\underline{533.5}}\ \mathrm{Hz}$$

$$\underline{\nu = -23,\ \mu = 23:}\ r = |p(\nu + \mu)| = |3(-23+23)| = 0,\quad f_{Ton} = 50|(23-1)(1-0.03)+2| = \underline{\underline{1167}}\ \mathrm{Hz}$$

$$\underline{\nu = -35,\ \mu = 34:}\ r = |p(\nu + \mu)| = |3(-35+34)| = 3,\quad f_{Ton} = 50|(34-1)(1-0.03)+2| = \underline{\underline{1700.5}}\ \mathrm{Hz}$$

Stator slot harmonics create a rather big force wave at 533.5 Hz !

Tonal frequencies, Rotor a) (2)

From modal analysis natural bending modes and frequencies are known:
***r* = 2, *f* = 592Hz**

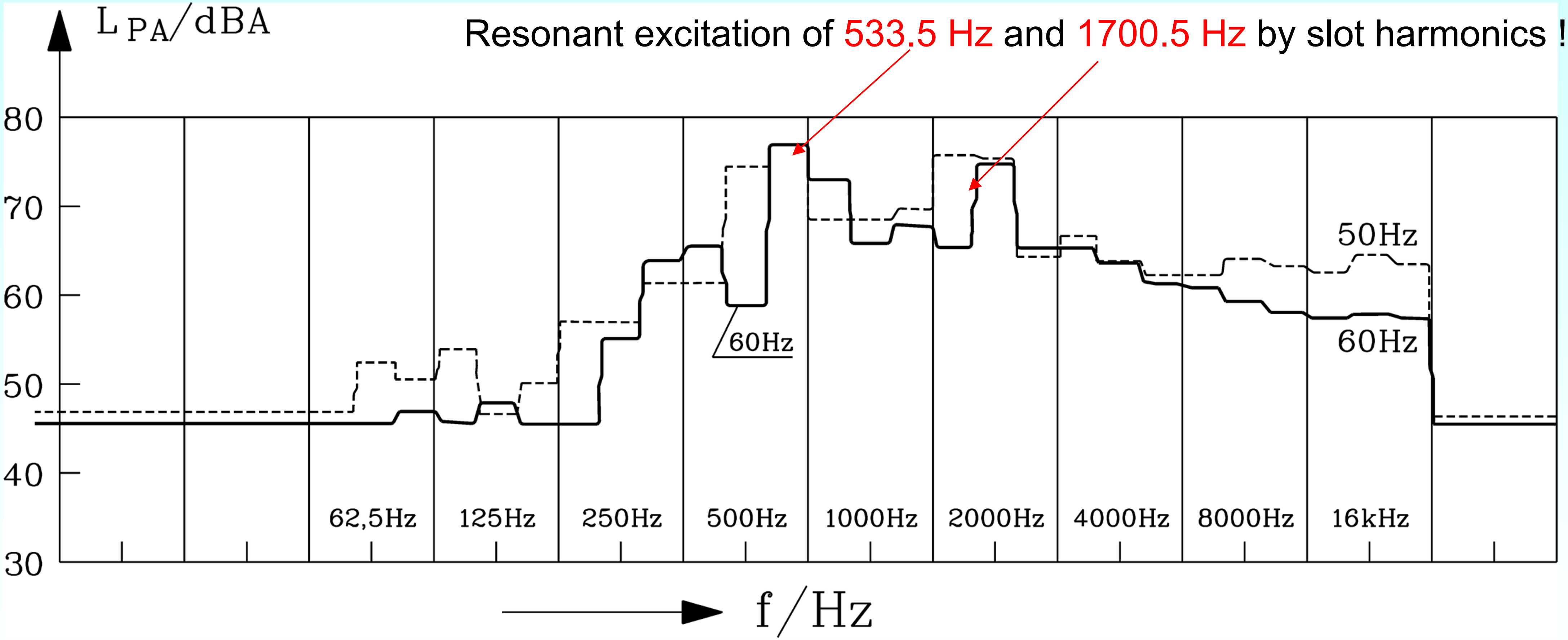
***r* = 3, *f* = 1739 Hz**

Rotor a):
- Oscillation with 533.5 Hz and 1700.5 Hz are amplified.
- Measured sound pressure level L_{pA} showed noise peaks at the resonance at 592 Hz and 1739 Hz, which are excited by the force waves 533.5 Hz and 1700.5 Hz.

Total sound pressure level: 78 dB(A).

Measured sound pressure level (1 m distance)

Motor 11 kW, 380 V, D, 50 Hz, $2p$ = 6, air gap 0.35 mm, iron stack l_{Fe} = 170 mm
Q_s = 36, Single layer winding, semi-closed slots, rotor aluminium cage; slip s = 3%
Rotor: a) $Q_s > Q_r = 33$, skew: 1 rotor slot pitch.



Loud machine: 78 dB(A) due to resonant excitation of vibration eigen-modes by slot harmonics !

Tonal frequencies, Rotor b)

Example:

Motor 11 kW, 380 V, D, 50 Hz, $2p = 6$, air gap 0.35 mm, iron stack $l_{Fe} = 170$ mm

$Q_s = 36$, Single layer winding, semi-closed slots, rotor aluminium cage; slip $s = 3\%$

Rotor b): $Q_s < Q_r = 42$; skew: 1 rotor slot pitch.

Frequency calculation:

Stator slot harmonics

- Stator field harmonics: 1,-5,+7,-11,+13,-17,+19,-23,+25,-29,+31,-35,+37,..
- Ordinal numbers of rotor field harmonics, excited by rotor current I_r under load:

$$\mu = 1 + (Q_r / p)g = 1 + 14g \; : = 1,-13,+15,-27,+29,-41,+43,...$$

$$\underline{\nu = 13,\ \mu = -13:}\ r = |p(\nu+\mu)| = |3(13-13)| = 0,\quad f_{Ton} = 50|(-13-1)(1-0.03)+2| = \underline{\underline{579.5}}\ \text{Hz}$$

$$\underline{\nu = -29,\ \mu = 29:}\ r = |p(\nu+\mu)| = |3(-29+29)| = 0,\quad f_{Ton} = 50|(29-1)(1-0.03)+2| = \underline{\underline{1458}}\ \text{Hz}$$

From modal analysis natural bending modes and frequencies are known:

$r = 2$, $f = 592$Hz; **$r = 3$, $f = 1739$ Hz.**

Rotor b):

- Exciting vibration modes differ considerably from the natural vibration modes ($r = 0$ instead of $r = 2$ or 3).
- Thus no resonance excitation occurs.

Low total sound pressure level: 62dB(A)).