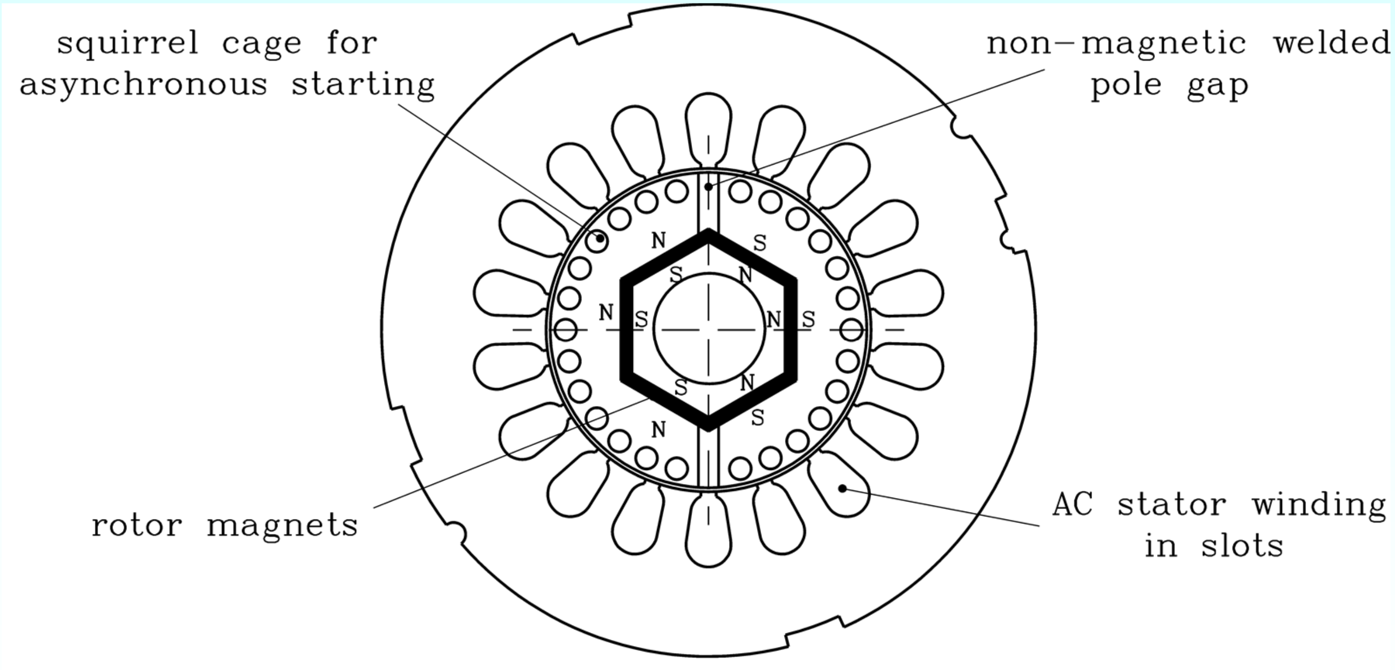
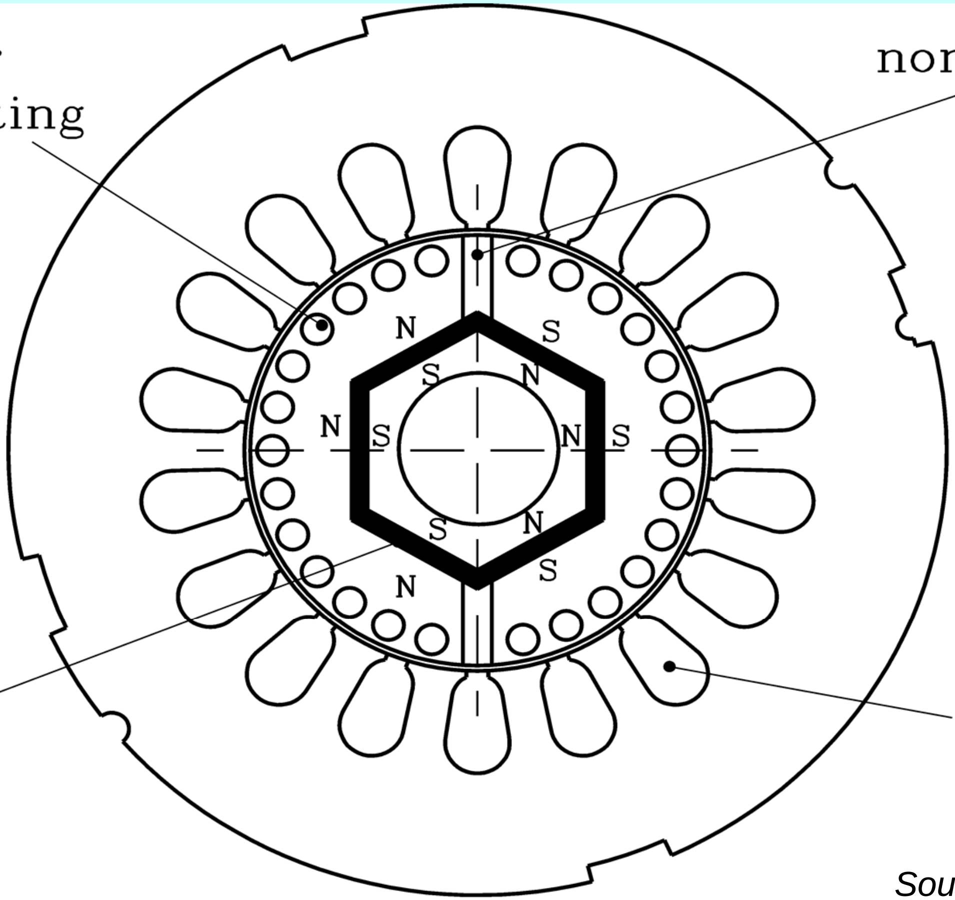
3. PM synchronous machines with rotor cage



Source: Siemens AG, Germany

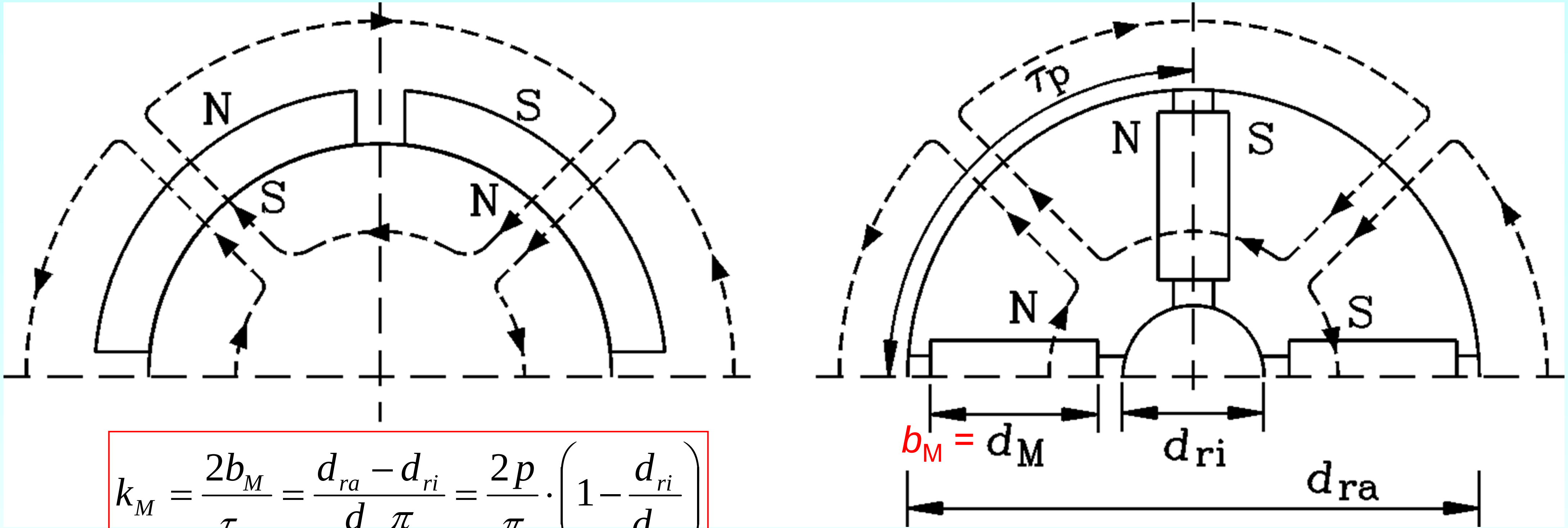
Two-pole PM synchronous machines with rotor cage



Source: Siemens AG, Germany

Two pole PM synchronous machine with cage rotor and rare earth permanent magnets. Note non-magnetic (welded with non-magnetic steel) gap between N- and S-pole to reduce PM stray flux

PM rotor flux concentration

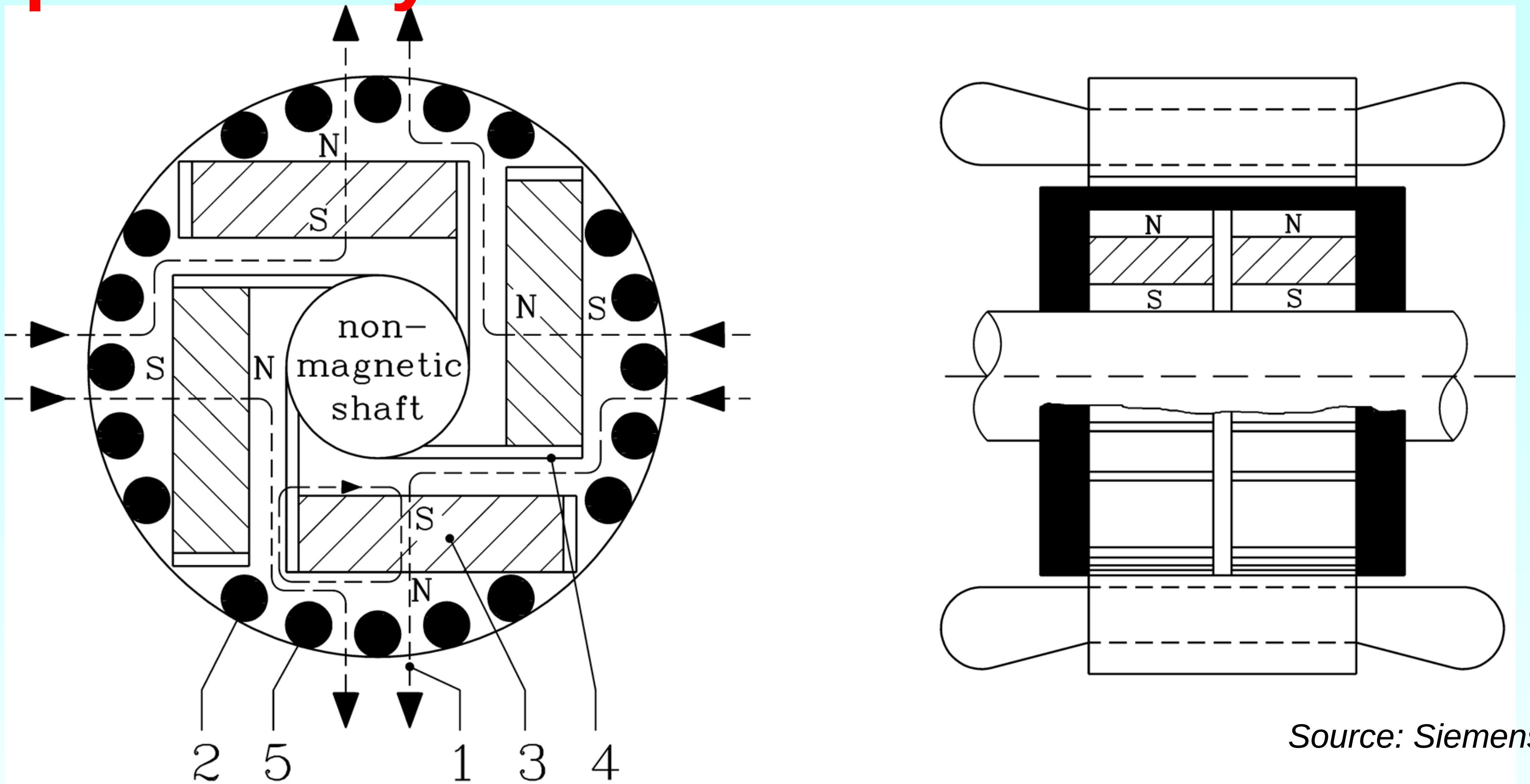


$$k_M = \frac{2b_M}{\tau_p} = \frac{d_{ra} - d_{ri}}{\frac{d_{ra}\pi}{2p}} = \frac{2p}{\pi} \cdot \left(1 - \frac{d_{ri}}{d_{ra}}\right)$$

Comparison of rotor PM arrangement for (left) surface mounted and (right) buried magnets; flux concentration factor k_M depends on pole count $2p$

Pole count $2p$	2	4	6	8
k_M	0.45 < 1	0.9 < 1	1.34 > 1	1.78 > 1

Four-pole PM synchronous machines with rotor cage

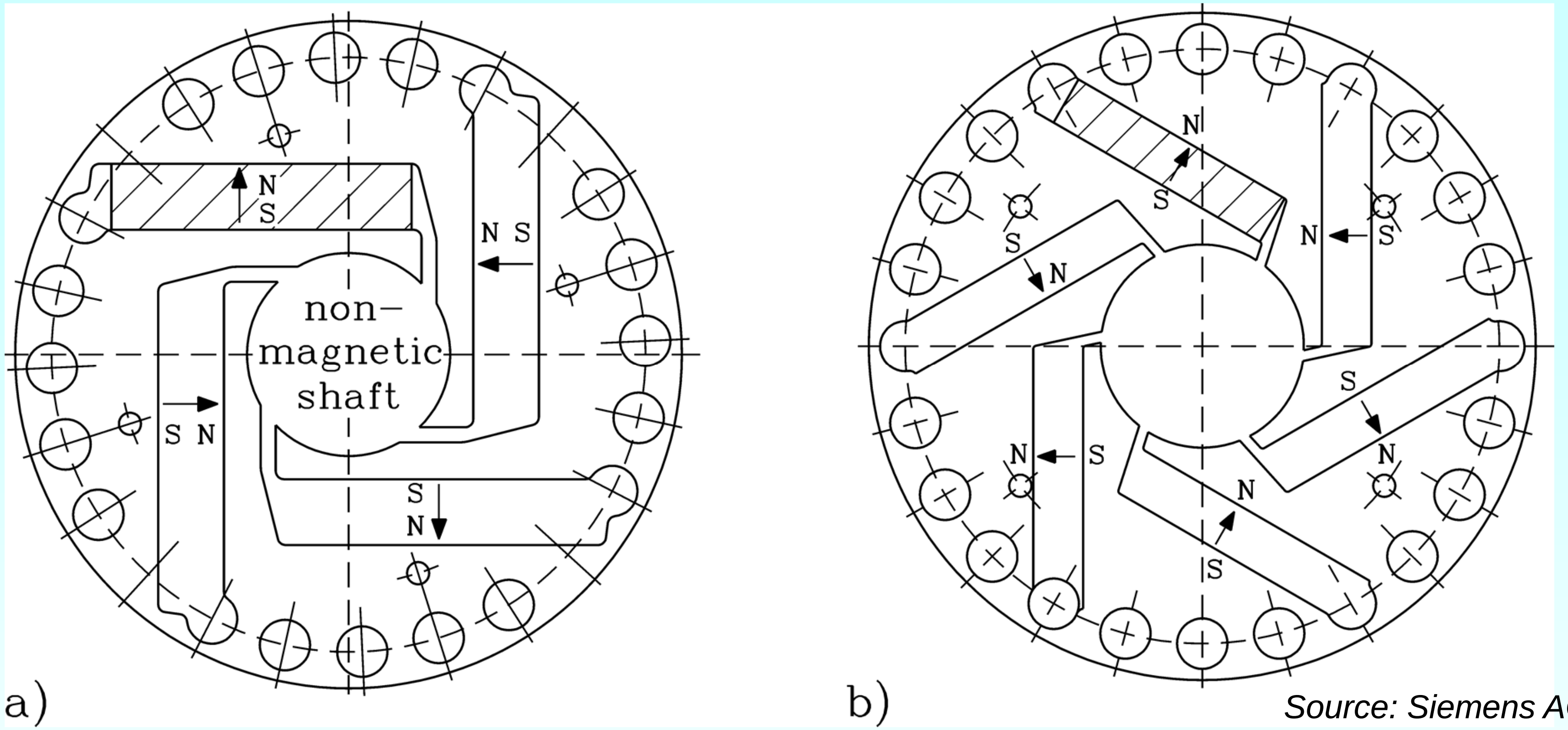


Source: Siemens AG, Germany

Flux concentration with special arrangement of rotor magnets is also possible for 4 pole machines: a) Axial cross section: 1: PM main flux, 2: PM stray flux, 3: permanent magnets (PM), 4: non-magnetic flux barrier to reduce PM stray flux, 5: rotor cage, b) side view.

Pole count $2p$	4	6
k_M	$1.27 > 1$	$1.9 > 1$

4- and 6-pole PM synchronous machines with rotor cage



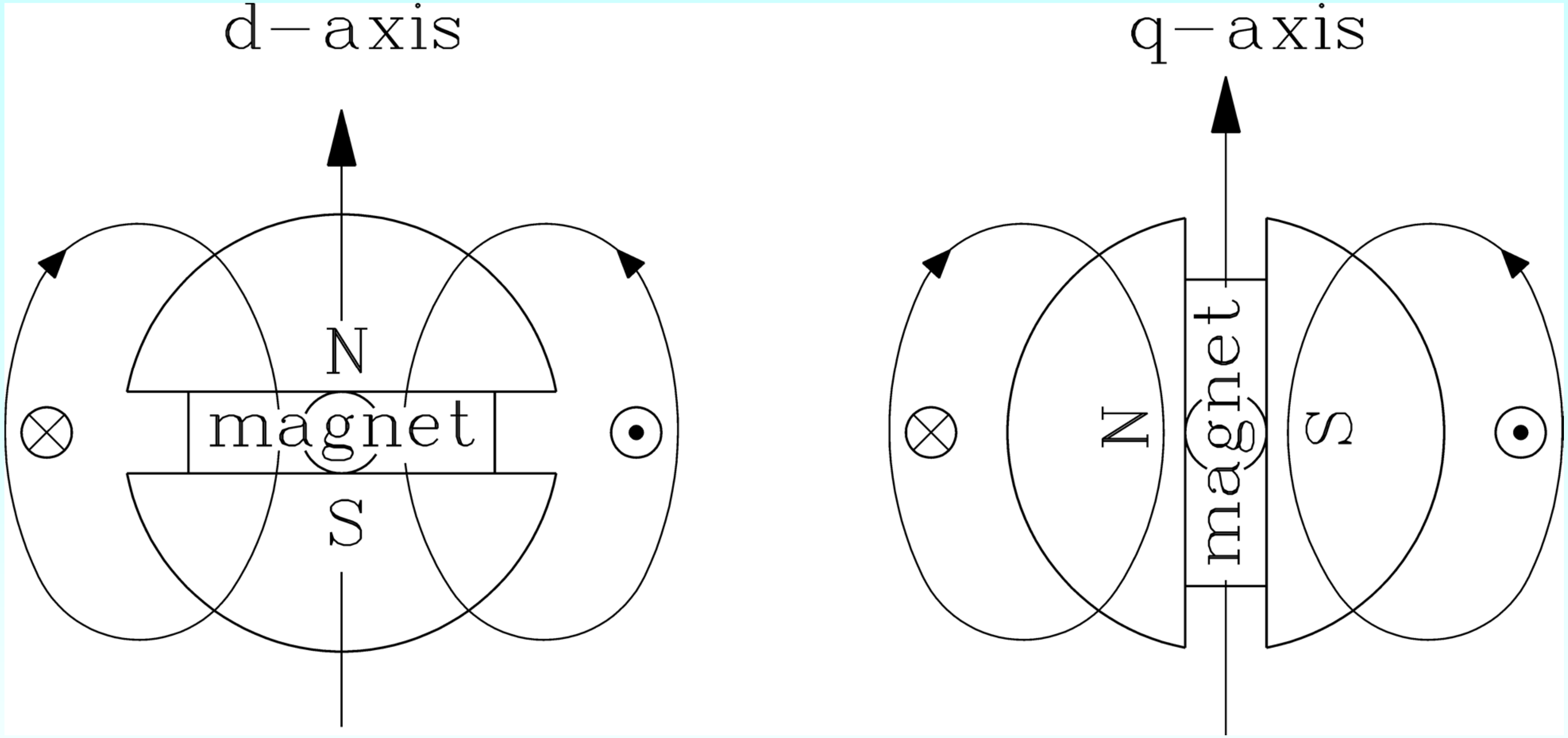
Source: Siemens AG, Germany

4- and 6-pole machine with buried **ferrite** magnets: B_R = 0.4 T, H_C = 300 kA/m at 20°C, magnet height 15 mm, air gap 1mm. Air gap flux density at no-load is calculated, assuming ideal iron. The closed loops of flux lines pass one magnet (height h_M) and two times the air gap δ.

Pole count $2p$	4	6
k_M	1.27	1.9
B_δ / T	0.43	0.6

$$B_\delta = \frac{B_R}{\frac{1}{k_M} + \frac{\mu_M \cdot 2\delta}{\mu_0 \cdot h_M}}$$

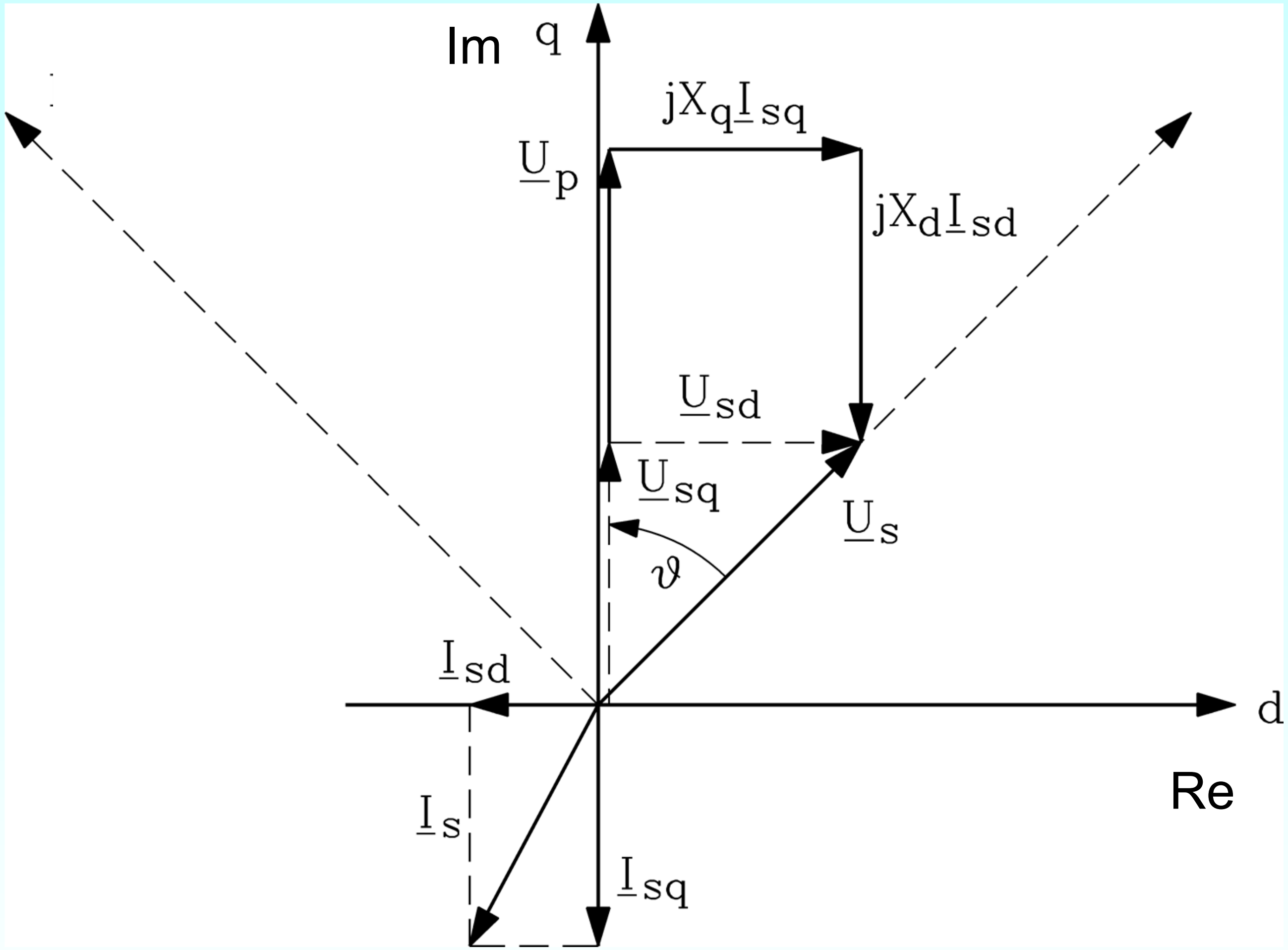
d- and *q*-axes of PM synchronous machine rotor



Simplified 2-pole arrangement with buried rotor magnets:

The stator *d*-flux has to cross the rotor magnets, thus yielding a lower *d*-inductance (left). The stator *q*-flux can avoid the rotor magnets, which results in a higher *q*-inductance (right).

PM synchronous machine phasor diagram at the grid



$L_d \neq L_q$

Simplification:
$R_s = 0$

Phasor diagram of PM machine with buried magnets ($L_d \neq L_q$) with neglected stator resistance for generator operation. At positive U_{sd} and U_{sq} the current components I_{sd} and I_{sq} are negative. The load angle ϑ is positive.

Torque of a salient pole synchronous machine at U_s= const. & R_s=0

- **OPERATION at "rigid" grid:** $\underline{U}_s$ = constant

  We choose: *d*-axis = Re-axis, *q*-axis = Im-axis of complex plane:

$$\underline{U}_s = U_{sd} + jU_{sq} \qquad \underline{I}_s = I_{sd} + jI_{sq} \qquad \underline{U}_p = jU_p$$

$R_s = 0$:

$$\underline{U}_s = jX_d \underline{I}_{sd} + jX_q \underline{I}_{sq} + \underline{U}_p \quad \Rightarrow \quad \underline{U}_s = jX_d I_{sd} - X_q I_{sq} + jU_p$$

- **Active power P_e :**

$$P_e = m_s U_s I_s \cos\varphi = m_s \cdot \mathrm{Re}\{\underline{U}_s \underline{I}_s^*\} = m_s (U_{sd} I_{sd} + U_{sq} I_{sq})$$

$$P_e = m_s (-X_q I_{sq} I_{sd} + X_d I_{sd} I_{sq} + U_p I_{sq})$$

- **Electro-magnetic torque:**

$$M_e = \frac{P_m}{\Omega_{syn}} = \frac{P_e}{\Omega_{syn}} = \frac{m_s}{\Omega_{syn}} \cdot \left(U_p \cdot I_{sq} + (X_d - X_q) \cdot I_{sd} \cdot I_{sq} \right)$$

- **Two torque components:**

  a) prop. U_p as with round rotor machines

  b) **"Reluctance"torque** due to $X_d \neq X_q$. NO rotor excitation is necessary !

Torque as function of load angle ϑ

$$\underline{U}_s = jX_d I_{sd} - X_q I_{sq} + jU_p \quad \Rightarrow \begin{cases} U_{sd} = -X_q I_{sq} & \Rightarrow I_{sq} = -\dfrac{U_{sd}}{X_q} \\ jU_{sq} = jX_d I_{sd} + jU_p & \Rightarrow I_{sd} = \dfrac{U_{sq} - U_p}{X_d} \end{cases}$$

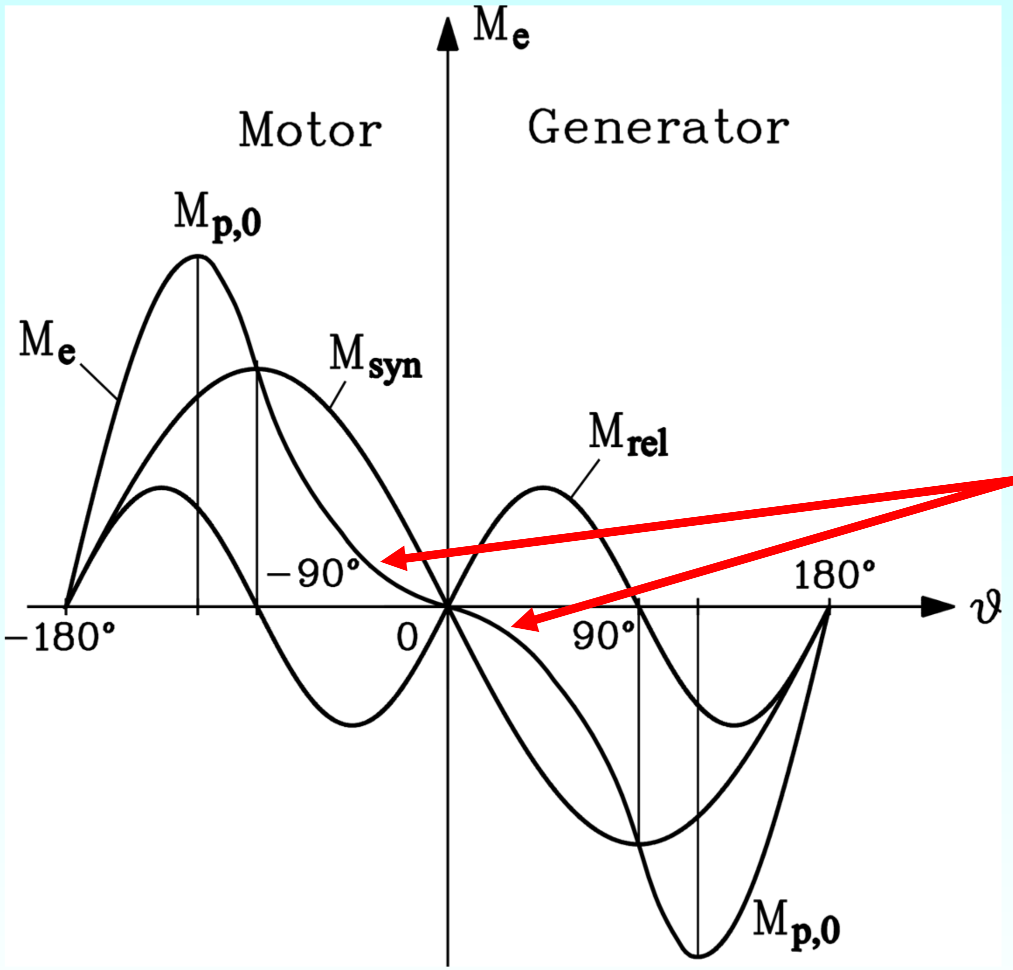
$$\underline{U}_s = U_{sd} + jU_{sq} \quad \begin{cases} U_{sd} = U_s \sin\vartheta \\ U_{sq} = U_s \cos\vartheta \end{cases}$$

$$M_e = \frac{m_s}{\Omega_{syn}} \cdot \left(U_p \cdot I_{sq} + (X_d - X_q) \cdot I_{sd} \cdot I_{sq} \right) =$$

$$= \frac{m_s}{\Omega_{syn}} \cdot \left(-\frac{U_p U_s \sin\vartheta}{X_q} - \frac{X_d - X_q}{X_d X_q} \cdot U_s \sin\vartheta \cdot (U_s \cos\vartheta - U_p) \right)$$

$$M_e = -\frac{p \cdot m_s}{\omega_s} \left(\frac{U_s U_p}{X_d} \sin\vartheta + \frac{U_s^2}{2} \left(\frac{1}{X_q} - \frac{1}{X_d} \right) \sin 2\vartheta \right)$$

Torque-load angle characteristic at the grid $L_d < L_q$



$$M_e = -\frac{p \cdot m}{\omega_s}\left(\frac{U_s U_p}{X_d}\sin\vartheta + \frac{U_s^2}{2}\left(\frac{1}{X_q} - \frac{1}{X_d}\right)\sin 2\vartheta\right)$$

Demand:

$$\left.\frac{dM_e}{d\vartheta}\right|_{\vartheta=0} \geq 0: \quad \frac{U_p}{X_d} + U_s \cdot \left(\frac{1}{X_q} - \frac{1}{X_d}\right) \geq 0$$

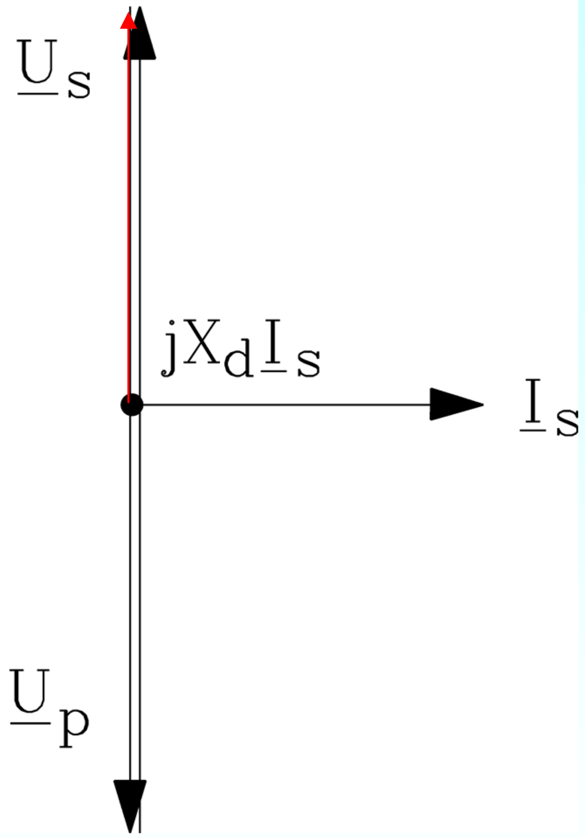
At $X_q = 2X_d$ a minimum back EMF of $U_p \geq U_s / 2$ is necessary!

The torque M_e of synchronous PM machine, operated from grid with fixed voltage and frequency, is determined by permanent magnet torque M_{syn}, which depends linear on stator voltage, and by reluctance torque M_{rel}, which depends on square of stator voltage **($L_d < L_q$).**

Asynchronous operation due to pull-out

$$\mathrm{U_p + U_s = X_d I_s}$$

$$\underline{U}_p + jX_d \underline{I}_s = \underline{U}_s$$



- When the machine is loaded higher than the maximum electromagnetic torque M_{p0} (pull-out torque), the rotor is pulled out of synchronism by the load torque.

- In the asynchronously running rotor the rotor currents are induced in rotor cage, generating an asynchronous torque.

- In motor operation the stator field, running at synchronous speed, is now faster than the turning rotor.

- So the stator field will oppose the rotor permanent magnet flux at certain instants (**"phase opposition"**), causing danger of irreversible demagnetization of rotor magnets.

- At phase opposition a large current is consumed:

$$I_s = \frac{U_s + U_p}{X_d} > I_N$$

- The rotor cage self-field opposes the stator field and reduces the inner rotor field, hence shielding inner PM against demagnetization.

Asynchronous starting

- Three-phase AC stator current system I_s with frequency f_s: causes field wave

$$B_s(x,t) = \hat{B}_s \cos\left(\frac{x_s \pi}{\tau_p} - \omega_s t\right)$$

- Induces rotor cage: AC rotor current system I'_r with frequency $f_r = s \cdot f_s$: yields asynchronous starting torque M_{asyn}

- Rotor permanent magnet field rotates with rotor speed $n = (1-s) n_{syn}$:

$$B_p(x,t) = \hat{B}_p \cos\left(\frac{x_s \pi}{\tau_p} - (1-s)\cdot \omega_s t\right)$$

- Induces stator winding: Causes three-phase AC stator current system I_p with frequency $f = (1-s) \cdot f_s$: yields asynchronous braking torque M_p

- Interaction between B_s and B_p causes pulsating torque M_P with ω_p

$$M_P(t) = \hat{M}_P(s) \cdot \sin(s \cdot \omega_s t)$$

$$\omega_p = \omega_s - (1-s)\cdot \omega_s = s \cdot \omega_s$$

Asynchronous braking torque M_p

-Voltage equation for induced additional current system I_p in the stator winding:

$$0 = R_s \underline{I}_p + j\omega \underline{\Psi}_s / \sqrt{2} = R_s \underline{I}_p + j\omega L_d \underline{I}_p + j\omega \underline{\hat{\Psi}}_p / \sqrt{2}$$

Assumption: $L_d = L_q$

- Additional current system I_p in the stator winding with $\omega = (1-s)\cdot\omega_s$

$$\underline{I}_p = -\frac{j\omega \underline{\hat{\Psi}}_p / \sqrt{2}}{R_s + j\omega L_d}$$

- Losses in the stator winding due to current system I_p: $P_{Cu,p} = 3R_s I_p^2 = -\Omega_m M_p$

$$M_p = -\frac{P_{Cu,p}}{\Omega_m} = -\frac{p\cdot m\cdot R_s}{(1-s)\cdot\omega_s}\cdot\frac{((1-s)\cdot\omega_s)^2\cdot\hat{\Psi}_p^2/2}{R_s^2 + (\omega_s(1-s)\cdot L_d)^2} = -\frac{(1-s)\cdot\omega_s\cdot p\cdot m\cdot R_s\cdot\hat{\Psi}_p^2/2}{R_s^2 + (\omega_s(1-s)\cdot L_d)^2}$$

$$M_p = -\frac{\omega\cdot p\cdot m\cdot R_s\cdot\hat{\Psi}_p^2/2}{R_s^2 + (\omega L_d)^2}$$

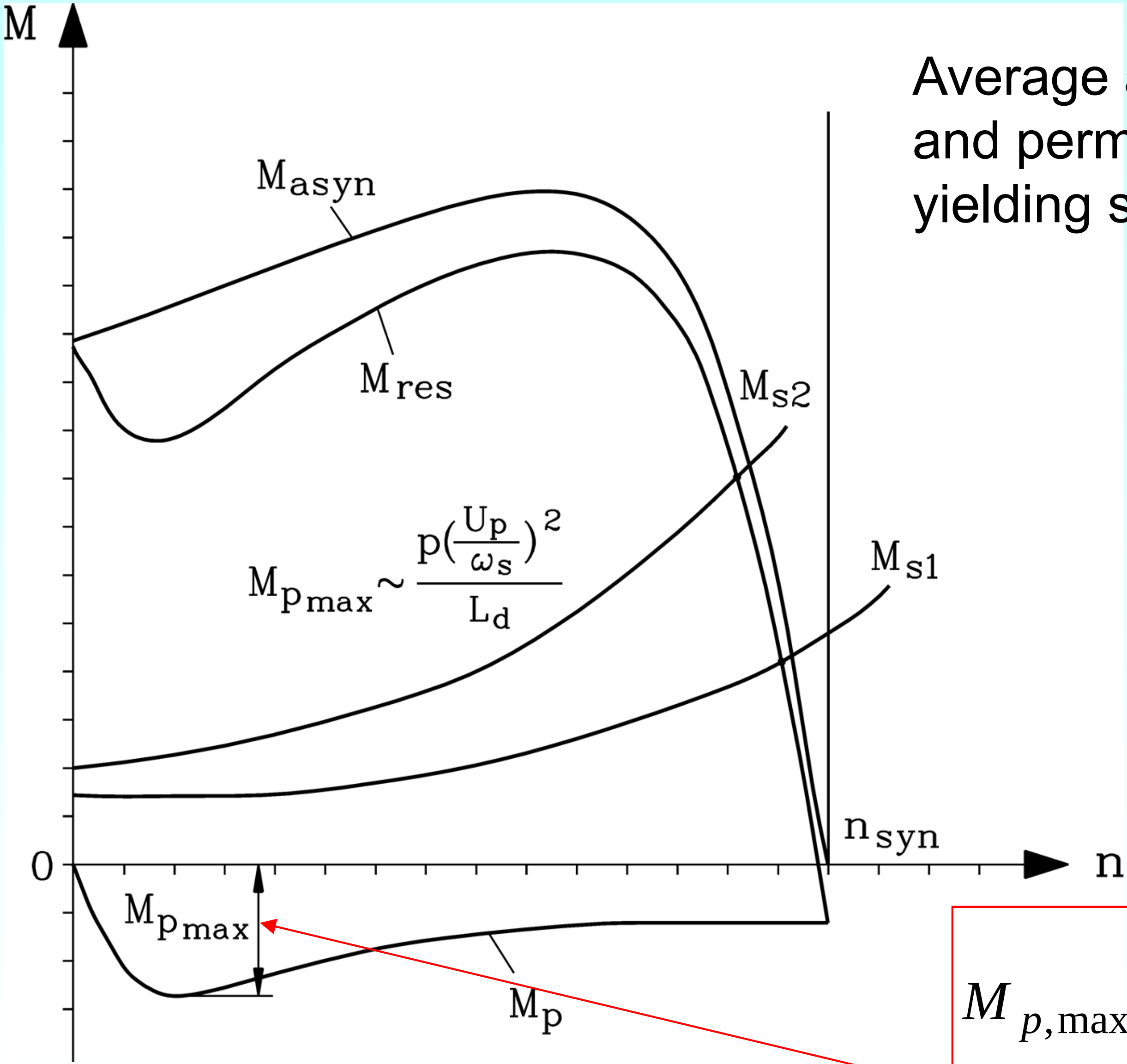
-Maximum braking torque (m = 3):

$$dM_p / d\omega = 0 : n^* = \omega^*/(2\pi p) = R_s/(2\pi p\cdot L_d)$$

$$M_{p,\max} = M_p(\omega^*) = \frac{3p}{2}\cdot\frac{\hat{\Psi}_p^2}{2L_d} = \frac{3p}{2}\cdot\frac{U_{pN}^2}{\omega_N^2 L_d}$$

Torque at asynchronous starting

Average asynchronous starting torque M_{asyn} and permanent magnet braking torque M_p, yielding saddle shaped resulting torque M_{res}



$$M_{p,\max} = M_p(\omega^*) = \frac{3p}{2}\cdot\frac{\Psi_p^2}{2L_d} = \frac{3p}{2}\cdot\frac{U_{pN}^2}{\omega_N^2 L_d}$$

Pulsating torque amplitude

- Pulsating torque amplitude due to stator field $B_s(x,t) = \hat{B}_s \cos\left(\frac{x_s\pi}{\tau_p} - \omega_s t\right)$

and additional field $B_p(x,t) = \hat{B}_p \cos\left(\frac{x_s\pi}{\tau_p} - (1-s)\cdot\omega_s t\right)$

- Additional field amplitude B_p is proportional to induced current I_p.

$$\hat{M}_P(s) = F\cdot z\cdot\frac{d_{si}}{2} = \frac{\hat{B}_s}{\sqrt{2}}\cdot\frac{\hat{I}_p(s)}{\sqrt{2}}\cdot l_{Fe}\cdot z\cdot\frac{d_{si}}{2} = \frac{\hat{B}_s}{\sqrt{2}}\cdot I_p(s)\cdot l_{Fe}\cdot 2mN_s k_{ws}\cdot\frac{p\tau_p}{\pi}$$

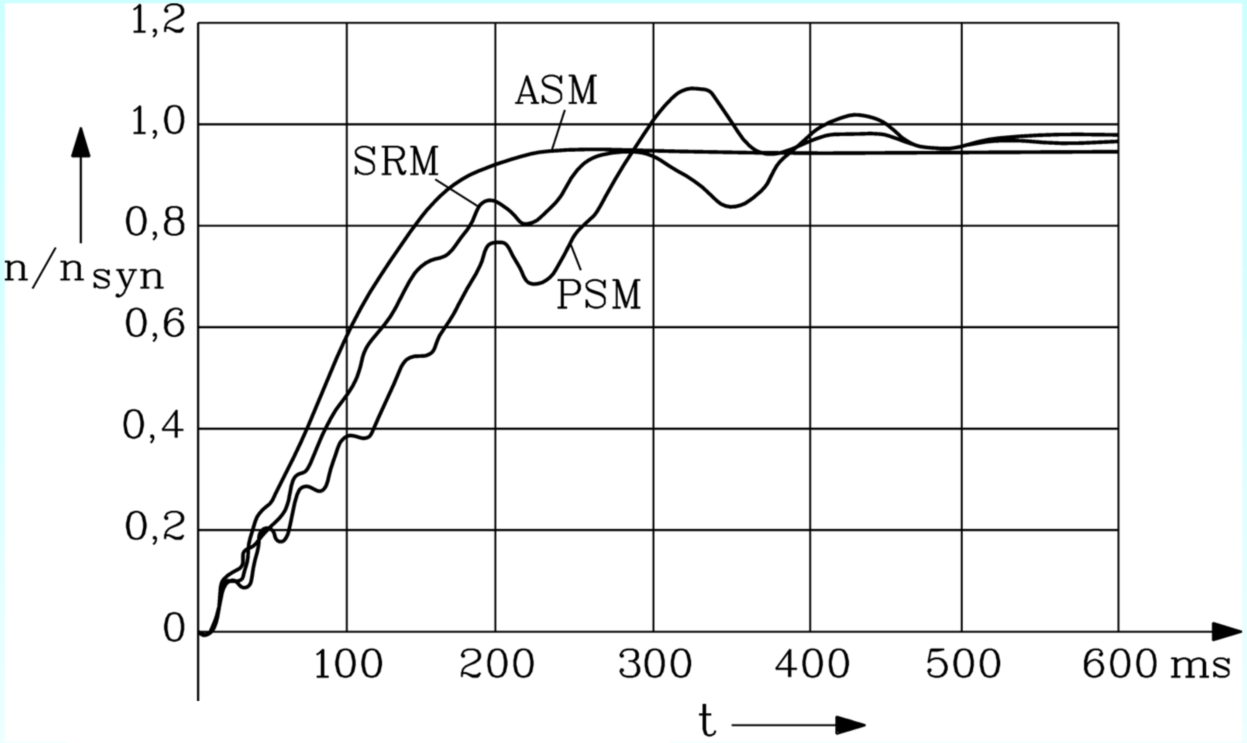
$$\underline{I}_p(s) = -\left.\frac{j\omega(s)\underline{\hat{\Psi}}_p/\sqrt{2}}{R_s + j\omega(s)L_d}\right|_{R_s\approx 0} \approx -\left.\frac{\omega(s)\underline{\hat{\Psi}}_p/\sqrt{2}}{\omega(s)L_d}\right|_{s<<1} = -\frac{\omega_s\underline{\hat{\Psi}}_p/\sqrt{2}}{\omega_s L_d} = \frac{\underline{U}_p}{X_d}$$

$$\hat{M}_P(s) = m\cdot p\cdot N_s k_{ws}\cdot\frac{(2/\pi)\tau_p l_{Fe}\hat{B}_s}{\sqrt{2}}\cdot\frac{U_p}{X_d} = m\cdot p\cdot\frac{\omega_s}{\omega_s}N_s k_{ws}\cdot\frac{\Phi_h}{\sqrt{2}}\cdot\frac{U_p}{X_d}$$

$$\boxed{\hat{M}_P(s) = \frac{m\cdot p}{\omega_s}\cdot U_h\cdot\frac{U_p}{X_d} \approx \frac{m\cdot p}{\omega_s}\cdot U_s\cdot\frac{U_p}{X_d}}$$

$$U_h = \omega_s N_s k_{ws}\cdot\Phi_h/\sqrt{2} \approx U_s$$

Calculated asynchronous starting



Comparison of

- Induction machines (ASM),
- Synchronous reluctance machine (SRM)
- Permanent magnet synchronous machines with rotor cage (PSM)

Source: Bunzel, E.; Elektrie, 1987

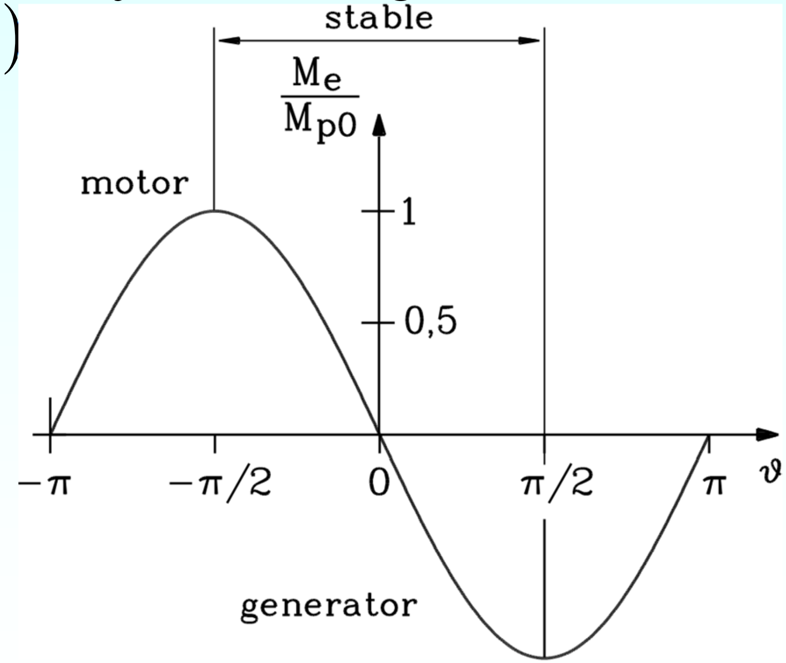
- ASM has highest starting torque and no pulsating torque (apart from the pulsation due to the switching transient at the start)
- SRM has a lower average starting torque due to the GOERGES-saddle and a pulsating torque with $|f_s - f_3| = 2sf_s$
- PSM has the lowest average starting torque due to the braking torque M_p and a pulsating torque with $s \cdot f_s$. In case of buried magnets the SRM-torque effect (GOERGES) adds!

Synchronization after asynchronous start-up

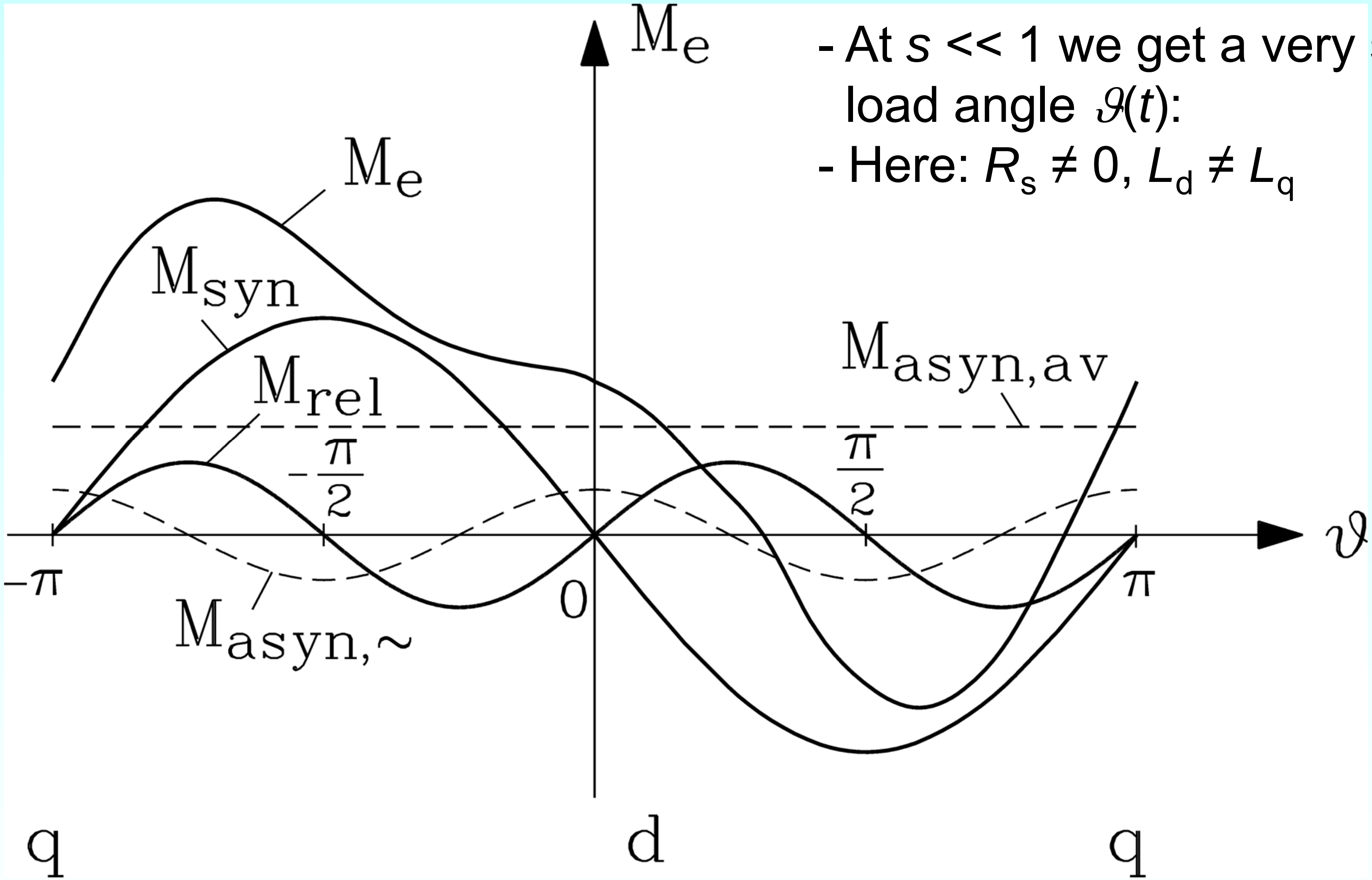
- At synchronous speed the slip is zero: The asynchronous torque of the cage is zero.
- The speed of PM rotor is synchronous speed, so the frequency of the stator current system I_p is: $f_3 = (1-s)\,f_s = f_s$
- Hence current I_s and I_p unite as the total stator current I_s at synchronous speed.
- **The pulsating torque becomes the constant synchronous torque!**

- At $s \ll 1$ the frequency $s \cdot f_s$ corresponds with a very slowly increasing load angle $\vartheta(t)$: $M_p(t) = \hat{M}_p(s) \cdot \sin(s \cdot \omega_s t + \vartheta) \Rightarrow \hat{M}_e \cdot \sin(\vartheta)$
- With the assumption $R_s = 0$, $L_d = L_q$ we get:

$$M_e = -\frac{p \cdot m}{\omega_s} \cdot \frac{U_s U_p}{X_d} \cdot \sin(\vartheta)$$



Torque components shortly before synchronization



- At $s << 1$ we get a very slowly changing load angle $\vartheta(t)$:
- Here: $R_s \neq 0$, $L_d \neq L_q$

Torque at low slip, rotor passing from –180° to 180°, when slipping by one pole pair

Synchronization - Critical slip (1)

- Slipping rotor with small slip, passing the load angle from –180° to 180°:

$$\Delta\gamma = \gamma_s - \gamma_r = \vartheta \qquad \Delta\dot{\gamma} = \dot{\gamma}_s - \dot{\gamma}_r = \Omega_{syn} - \Omega_m(t) = s(t)\cdot(\omega_s / p)$$

$$\Delta\ddot{\gamma} = \ddot{\gamma}_s - \ddot{\gamma}_r = \dot{s}(t)\cdot(\omega_s / p) \qquad \ddot{\gamma}_s = \dot{\Omega}_{syn} = 0$$

$$(J_M + J_L)\cdot\frac{d^2\gamma_r}{dt^2} = M_e - M_s \quad \Rightarrow \quad -J\cdot\frac{\omega_s}{p}\cdot\frac{ds}{dt} = M_e(\vartheta(t)) - M_s$$

Assumptions:

(1) No load torque: $M_s = 0$,

(2) No reluctance effect: $L_d = L_q$: $M_{rel} = 0$, $M_{asyn,\sim} = 0$.

(3) At small slip $s << 1$: Asynchronous torque is nearly zero $M_{asyn,av} \approx 2s\cdot M_b / s_b \cong 0$

Hence the torque during synchronization is only:

$$M_e = M_{syn} + M_{rel} + M_{asyn} \approx M_{syn} = -M_{p0}\cdot\sin\vartheta$$

Synchronization - Critical slip (2)

- At $t = 0$: Rotor position relative to stator field is: $\vartheta_1 = -\pi$; rotor slip $s_1 \ll 1$.

- Rotor passes from $\vartheta_1 = -\pi$ to $\vartheta_2 = 0$ during the time $t_s = 1/(2 \cdot s_{av} \cdot f_s)$, being accelerated by M_{syn}, to the smaller slip $s_2 < s_1$.
- Average slip during that time is $s_{av} = (s_1 + s_2)/2$.

- The final chance to be pulled into synchronism is at $t = t_s$, because afterwards M_{syn} becomes negative. So if $s_2 = 0$, the rotor synchronizes.
- The corresponding slip s_1 is then the maximum admissible slip s_{cr} for synchronization.

$$\int_0^{t_s} -J \cdot \frac{\omega_s}{p} \cdot \frac{ds}{dt} \cdot dt = \int_{s_1}^{s_2} -J \cdot \frac{\omega_s}{p} \cdot ds = J \cdot \frac{\omega_s}{p} \cdot (s_1 - s_2) = J \cdot \frac{\omega_s}{p} \cdot s_1 = \int_0^{t_s} M_{syn}(\vartheta(t)) \cdot dt =$$

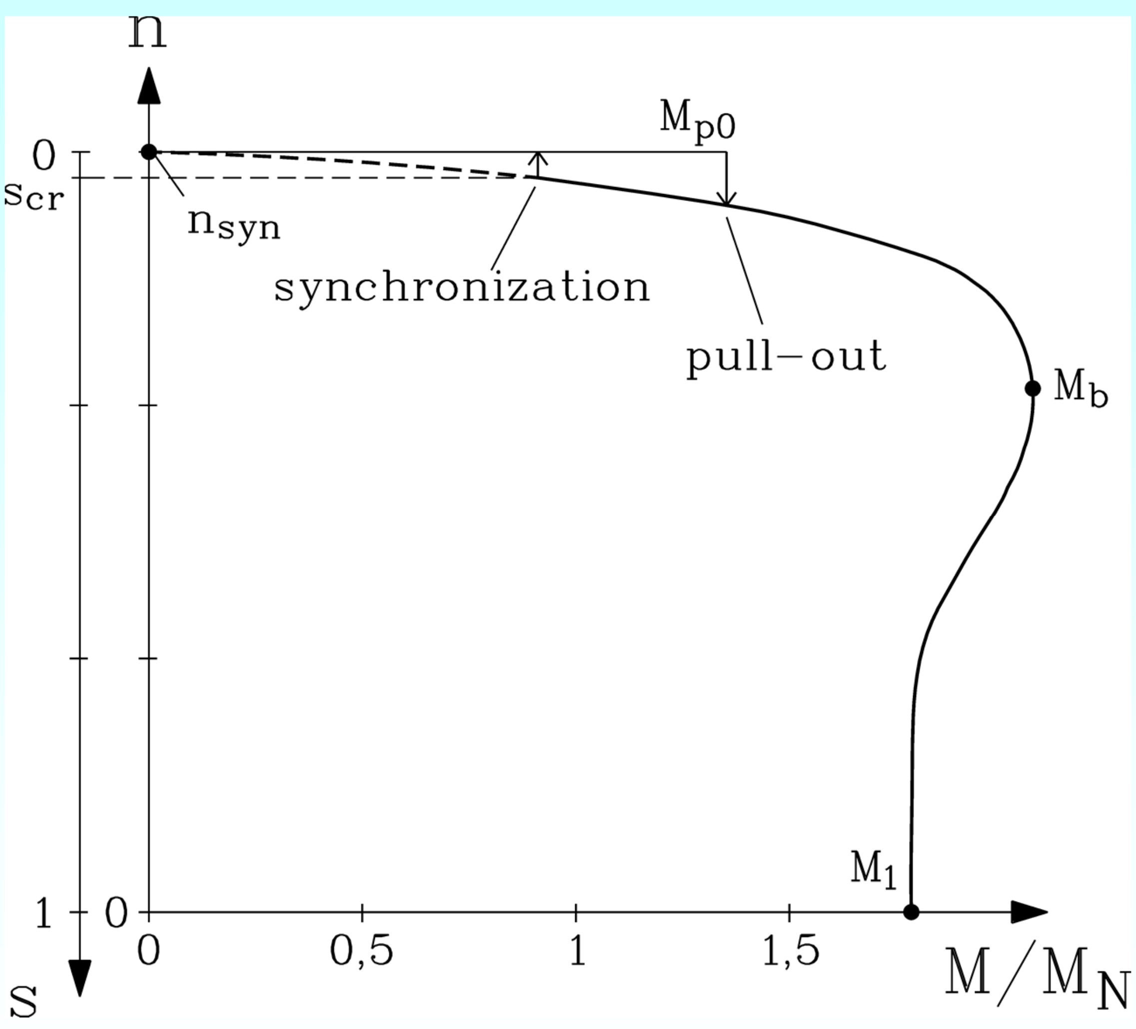
$$= \frac{t_s}{\pi} \int_{-\pi}^{0} -M_{p0} \cdot \sin(\vartheta) \cdot d\vartheta = \frac{t_s}{\pi} \cdot 2M_{p0} = \frac{1}{2\pi \cdot s_{av} f_s} \cdot 2M_{p0} = \frac{1}{2\pi \cdot (s_1/2) \cdot f_s} \cdot 2M_{p0}$$

- Critical slip s_{cr} for synchronization: $\sqrt{2/\pi} = 0.8 > 0.5$

$$s_1 = s_{cr} = \frac{1}{\Omega_{syn}} \cdot \sqrt{\frac{4M_{p0}}{J \cdot p}} = \frac{1}{\omega_s / p} \cdot \sqrt{\frac{2P_{p0}}{\pi \cdot (J_M + J_L) \cdot f_s}}$$

$$s_{cr} \cong \frac{0.5}{\omega_s / p} \cdot \sqrt{\frac{P_{p0}}{(J_M + J_L) \cdot f_s}}$$

Pull-in and pull-out of PM synchronous machines with rotor cage



Condition for synchronization is therefore: Slip is less than CRITICAL SLIP

$$s_{cr} \cong \frac{0.5}{\omega_s / p} \cdot \sqrt{\frac{P_{p0}}{(J_M + J_L) \cdot f_s}}$$

Example:

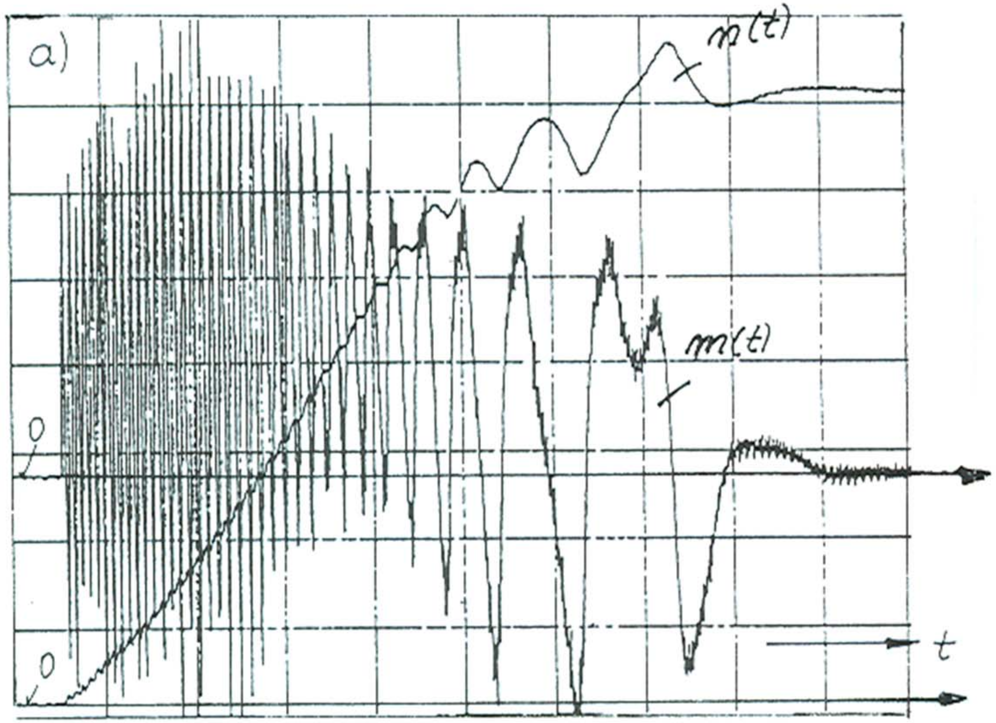
Barium ferrite six-pole synchronous permanent magnet motor, shaft height 160mm, totally enclosed, fan-cooled:
50 ... 150 Hz, 1000 ... 3000/min, 125 ... 380 V, Y, 42 A, rated torque 49.7 Nm, overload capability 150%. total momentum of inertia (motor and load): 1.5 kgm²

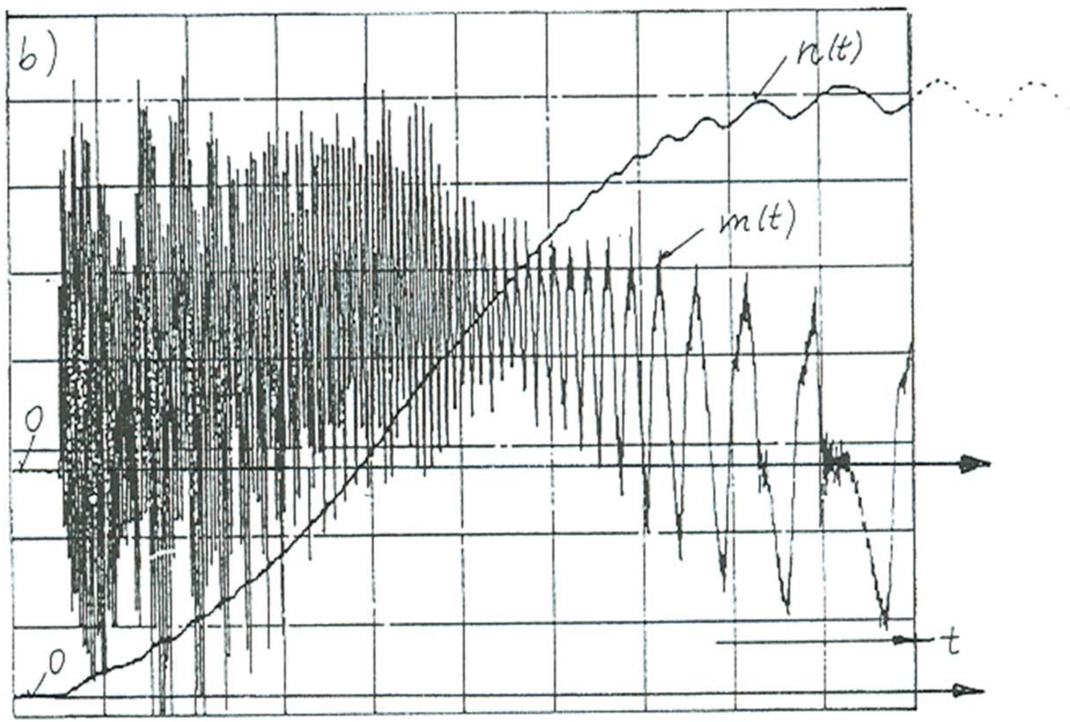
$$1.5 \cdot 49.7 \cdot (2\pi 50/3) Nm/s = 7806W$$

$$s_{cr} = \frac{0.5}{2\pi \cdot 50/3} \cdot \sqrt{\frac{7806}{50 \cdot 1.5}} = 0.049 = 4.9\%$$

Measured torque and speed at asynchronous starting

Load inertia 100% Load inertia 240%





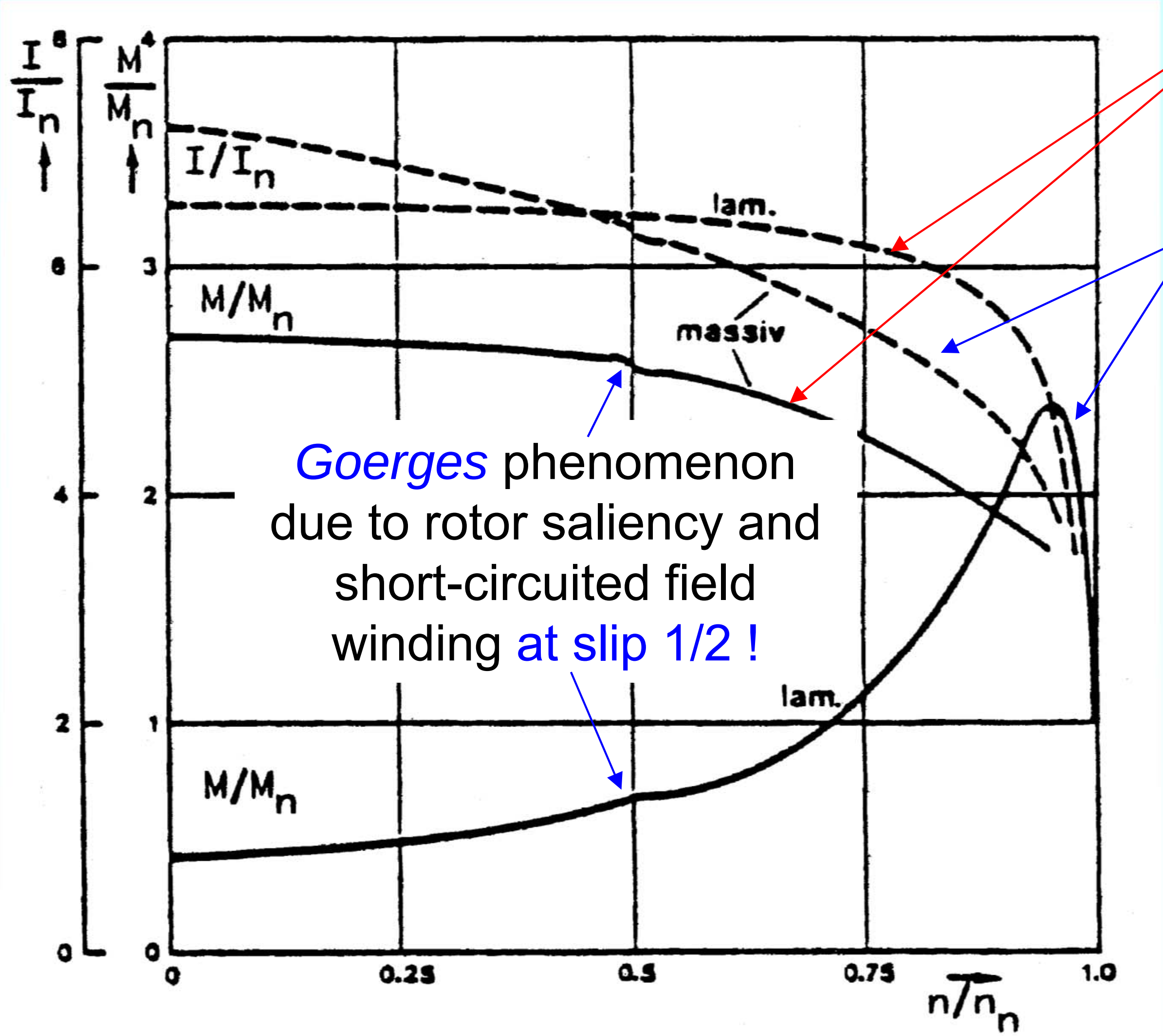
Measured time function of starting torque of PM synchronous machine with squirrel cage rotor at fixed stator voltage and frequency:

a) Synchronisation of motor visible,

b) Increased load inertia by factor 2.4: No synchronisation is possible!

The pulsating torque with $s \cdot f_s$ is clearly visible !

Asynchronous start-up of electrically excited (big) synchronous motors



- Laminated rotor poles need starting copper cage (= heat sensitive)

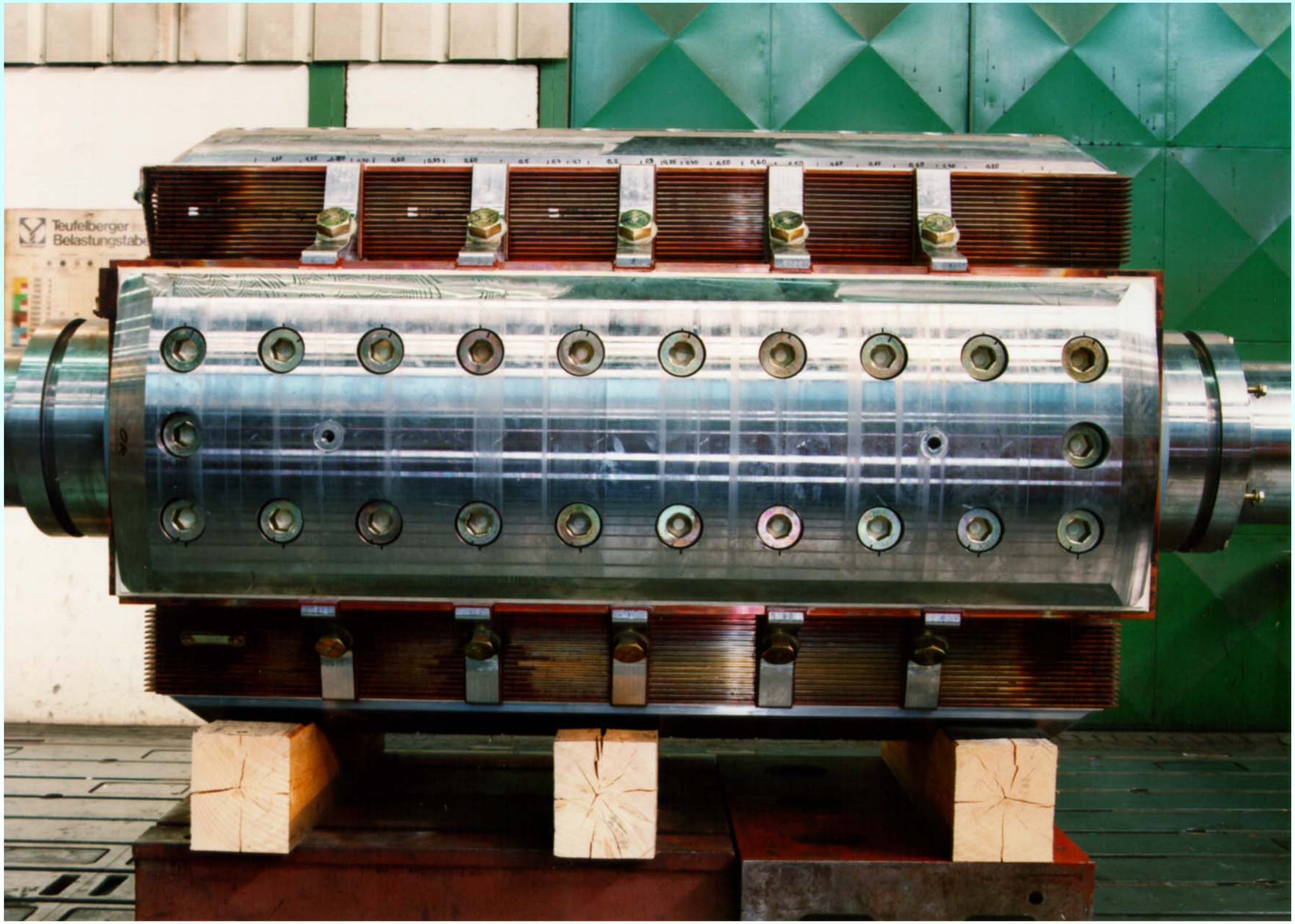
- Massive rotor poles doe not need rotor cage: massive pole surface (conductive iron) is carrying eddy currents

Advantages:

a) no heat expansion problem of cage !

b) 10 ... 20 times bigger iron rotor resistance shifts break down slip to nearly unity = much bigger starting torque.

Massive pole synchronous motors



- 4 pole motor
- Screwed poles
- During start-up the field winding is short-circuited:

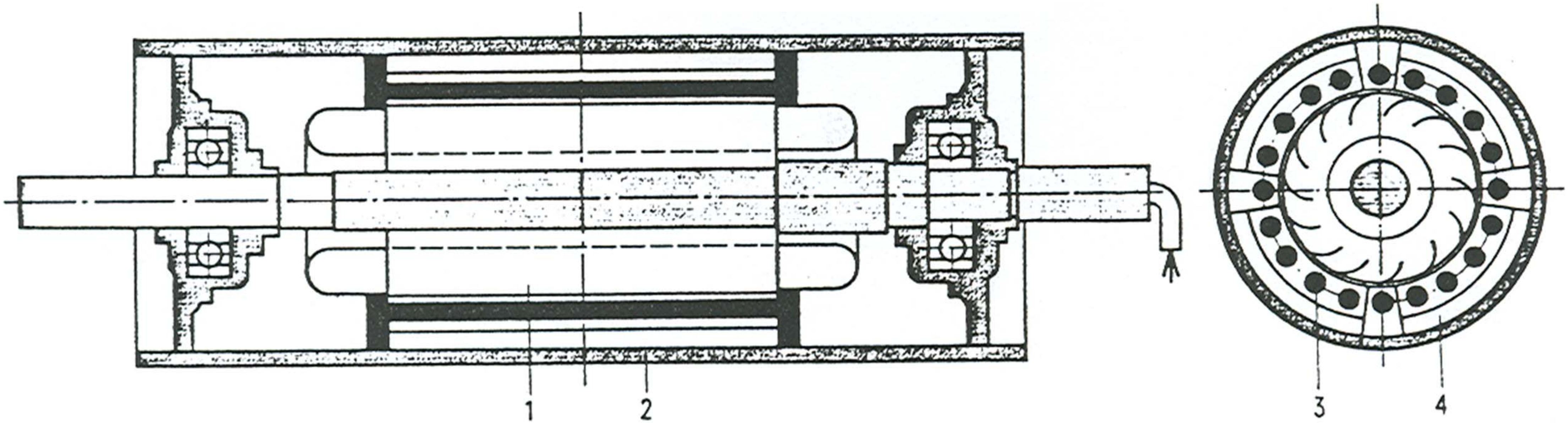
a) to avoid induced over-voltage, which is deadly for the power electronics of the excitation rectifier

b) to avoid a braking torque M_p

c) to add a small asynchronous torque of the field winding as a second “cage”

Source: Andritz Hydro, Austria

Outer-rotor PM synchronous machines with rotor cage

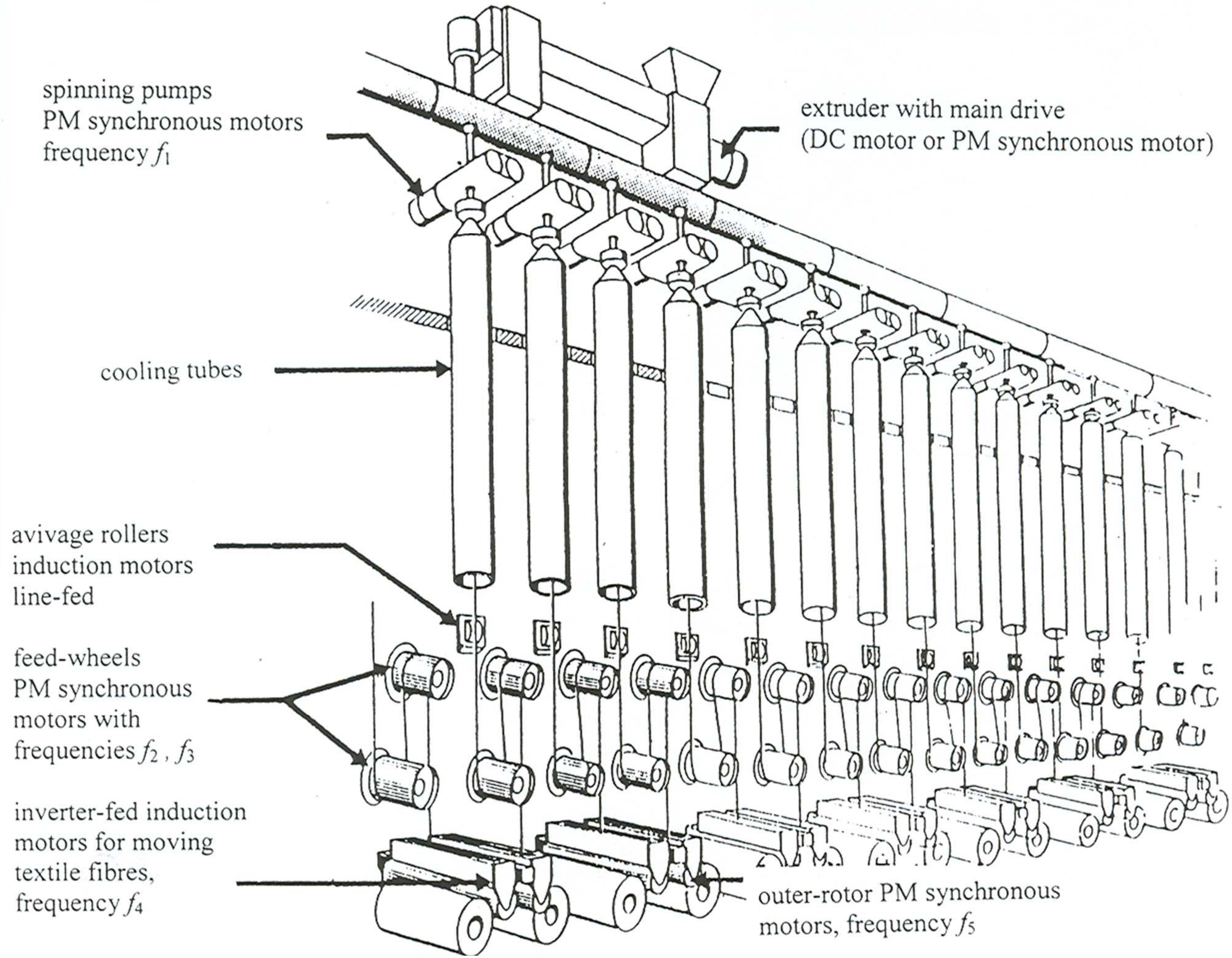


Cross section of outer rotor 4-pole PM synchronous motor with ferrite surface mounted magnets and squirrel cage: 1: inner stator AC three-phase winding, 2: outer rotor iron back, 3: squirrel cage, 4: ferrite magnets

Application: Textile fibre fabrication

Source: Siemens AG, Germany

PM synchronous motor group drives for synthetic thread fabrication



Source: Siemens AG, Germany