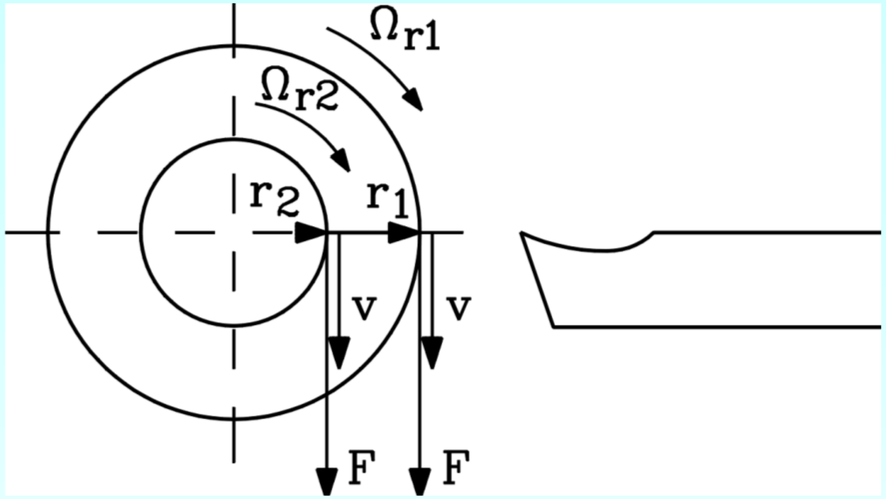
1. Permanent magnet synchronous machines as “brushless DC drives”

1.3 High speed PM machines

Source: TU Darmstadt

High speed applications: Tools machinery



High speed cutting:

- Constant cutting force *F* and velocity *v* demanded
- For smaller diameters 2*r* increased speed Ω_r necessary, but reduced torque $M = F.r$,
- Power is constant power $P = F.v$

Motor	Maximum speed	Constant Power
A	30 000/min	4.5 kW
B	12 000/min	27 kW

Typical rated data of hi-speed PM spindle drives

Utilization of electrical machines

- **Electromagnetic utilization *C*** ("***Esson´s number***") of electric machines is (inner) apparent machine power *S* versus speed and "bore volume" (factor $\pi/4$ neglected in that definition).

$$C = \frac{(S/n)}{d_{si}^2 l_{Fe}}$$

$$C = \frac{\pi^2}{\sqrt{2}} \cdot k_w \cdot A \cdot B_\delta$$

A: rms current loading

B_δ: Fundamental air gap field amplitude

k_w: Fundamental winding factor

- For raising power output S of a given motor volume ~ $d_{si}^2 l_{Fe}$ *, either speed can be raised or current load and flux density: S ~ n · C.*

- Due to **iron saturation** the air gap flux density B_δ amplitude cannot be raised much above 1 T.

- **Current loading *A*** can be increased by increasing the current (or the number of conductors), but this also means increased losses in the stator. Therefore high current loading is only possible for intensive cooling.

“Power from speed” $S \sim C \cdot n \cdot V$ - “Torque from size” $M \sim S/n \sim C \cdot V$

Low speed machines:

a) Totally enclosed PM servo motor with self cooling (without any fan):

40.5 Nm, 1000/min, A = 145 A/cm, B_δ = 0.65 T,

d_{si} = 154 mm, l_{Fe} = 175 mm, k_w = 0.933, C = 1.0 kVAmin/m^3, P_δ = 4.24 kW

b) High torque PM motor with water jacket cooling:

737 Nm, 600/min, A = 611 A/cm, B_δ = 0.8 T,

d_{si} = 280 mm, l_{Fe} = 200 mm, k_w = 0.866, C = 4.9 kVAmin/m^3, P_δ = 46.32 kW

High speed machine: PM motor with water jacket cooling:

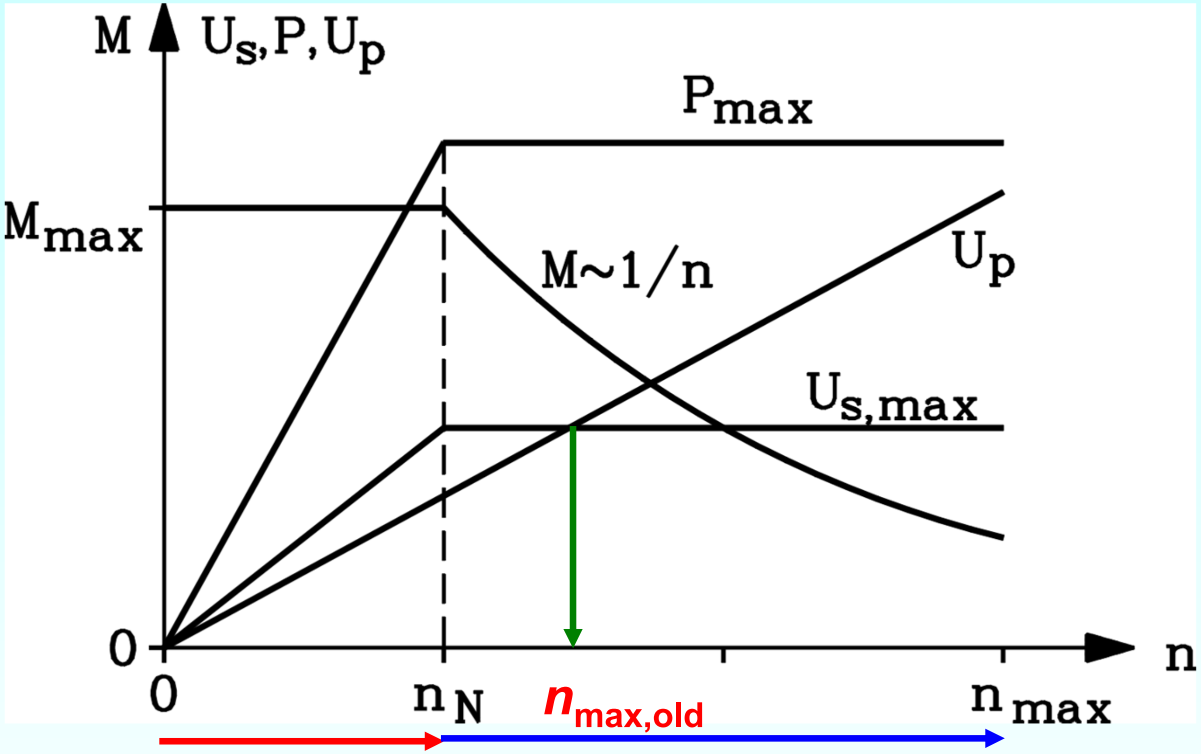
12 Nm, 24000/min, A = 225 A/cm, B_δ = 0.7 T

d_{si} = 90 mm, l_{Fe} = 90 mm, k_w = 0.933, C = 1.7 kVAmin/m^3, P_δ = 30.0 kW

Facit:

With only 70% higher electromagnetic utilization C the motor c) has about 7 times higher output power than motor a), as speed is increased by factor 24.

Flux weakening with negative *d*-current



Constant torque **Flux weakening range**

- At rated speed n_N the inverter voltage limit $U_{s,max}$ is reached.

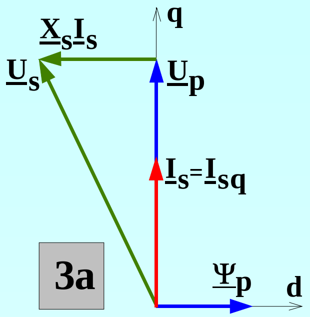
- By introducing a negative d-current a voltage component opposite to the back EMF U_p is induced, so that U_s remains constant.

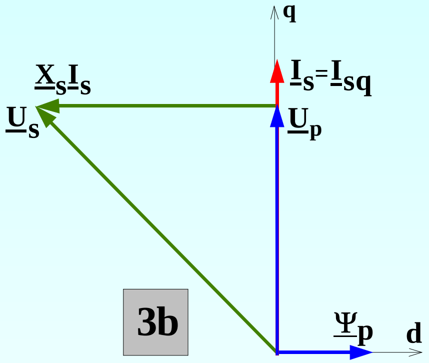
- This d-current does not generate any torque with the rotor permanent flux !

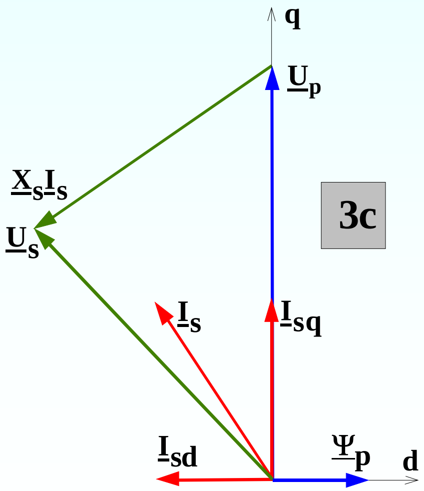
- At constant resulting current due to the required d-current the q-current must be reduced. Hence torque M decreases ! („Flux weakening range“)

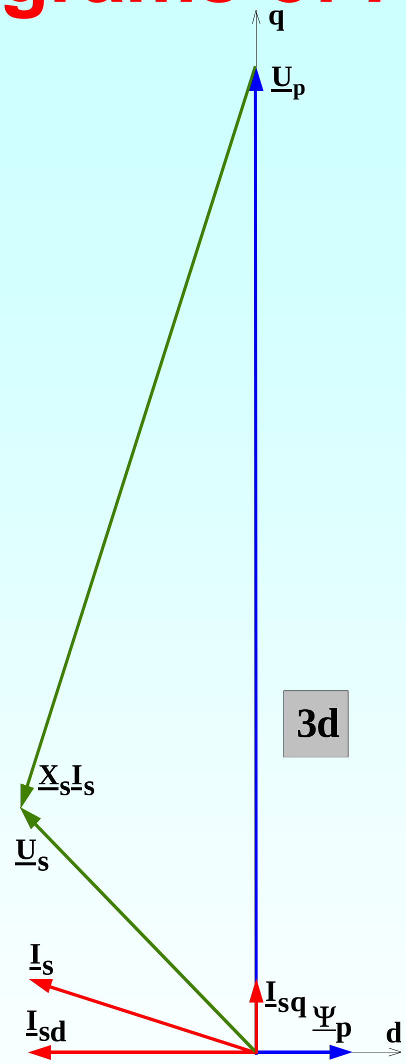
Instead of $n_{max,old}$ (at $U_s = U_p$) now a higher speed n_{max} is reached, but at a reduced torque, which is not any longer proportional to stator current I_s !

Phasor diagrams of PM machine with field weakening









- stator resistance R_s is neglected, $\underline{X}_s = jX_d = jX_q$

a) rated speed, rated torque,

b) rated speed, overload torque,

c) speed at 170% rated speed, decreased torque, flux weakening by negative I_d-current,

d) very high speed (400%) can only be reached by strong field weakening; nearly the whole current consists of flux weakening component I_d, whereas torque-producing component I_q is very small.

Demands for flux weakening

	Voltage u_s	Current i_s	d-axis i_{sd}	q-axis i_{sq}	Power	Speed n	$\cos\varphi$
a)	0.8	1.0	0	1.0	P_N	n_N	0.89 ind
b)	1.0	2.0	0	2.0	$2P_N$	n_N	0.7 ind
c)	1.0	1.5	**-0.8**	1.2	$2P_N$	$1.67n_N$	0.98 ind
d)	1.0	1.7	**-1.6**	0.5	$2P_N$	$4n_N$	0.89 **cap**

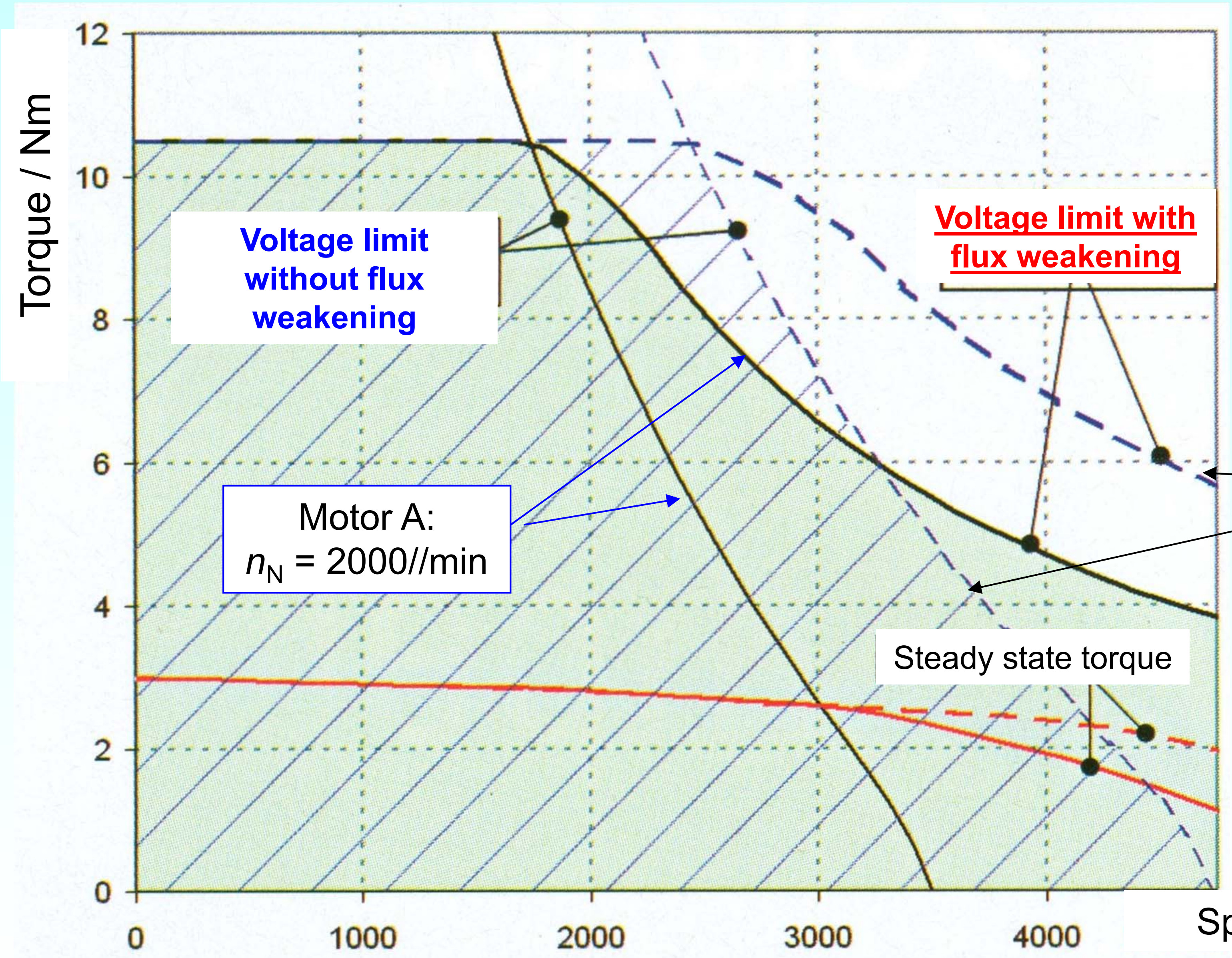
Big field weakening: $U_p >> U_{s,max}$, we may neglect $U_{s,max}$ and R_s.

$$I_{s,d} \cong U_p / X_s = \Psi_p / L_s = \Psi_p / L_d$$

The necessary field weakening current must be smaller than the inverter current limit !

$$\boxed{I_{s,d,\max} < I_{s,\max}}$$

Torque-speed curves without and with flux weakening



Voltage limit without flux weakening

Voltage limit with flux weakening

Motor A:
n_N = 2000//min

Steady state torque

Motor B:
n_N = 3000//min

Speed /min^{-1}

Example:

Two PM synchronous motors A & B with same size, but different number of turns per phase N

$N_A/N_B = 3/2$

Source: Reimann, H. Siemens AG, in: Der Konstrukteur 5/2012

Steady state short circuit current of PM machine

$$I_{s,k} = U_p / \sqrt{R_s^2 + X_s^2} \cong U_p / X_s = \Psi_p / L_s = \Psi_p / L_d < I_{s,\max}$$

- Demand for infinite field weakening:

  *If the **short-circuit current** $I_{s,k}$ of the PM machine is smaller than the inverter current limit $I_{s,max}$, an infinite field weakening is theoretically possible.*

- Motor and system design for high field weakening:
  - synchronous *d*-axis inductance L_d is big, e.g. big leakage inductance
  - permanent magnet flux linkage with stator winding Ψ_p is small
  - inverter current limit is high
  - inverter voltage limit is high.

$$\boxed{L_d = L_h + L_\sigma}$$

$$\boxed{L_h = \frac{U_{s,s}}{\omega_s \cdot I_s} = \mu_0 \cdot (N_s \cdot k_{ws})^2 \cdot \frac{2m_s}{\pi^2 \cdot p} \cdot \frac{\tau_p l_{Fe}}{\delta_{res}}}$$

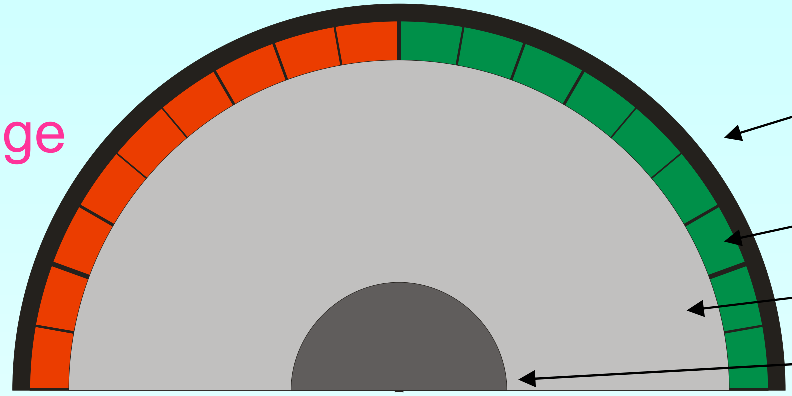
Rotor configurations of PM Synchronous Machines

Surface magnets:

small L_h, big $I_{s,k}$,
small flux weakening range

GFC: Glass fíber composite

CFC: Carbon fiber composite



GFC/CFC-Bandage

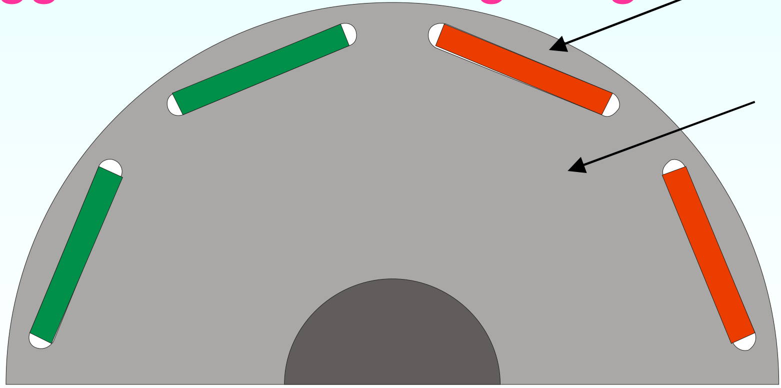
Magnet

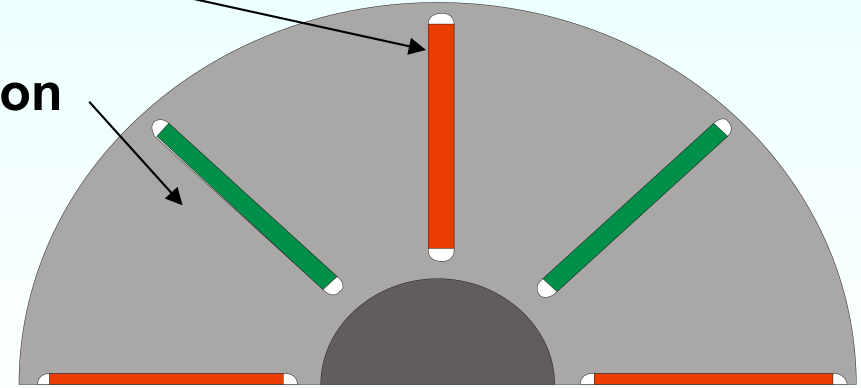
Rotor iron

Shaft

Buried magnets:

bigger L_h, smaller $I_{s,k}$,
bigger flux weakening range





Magnet

Rotor iron

No flux concentration

Applied flux concentration

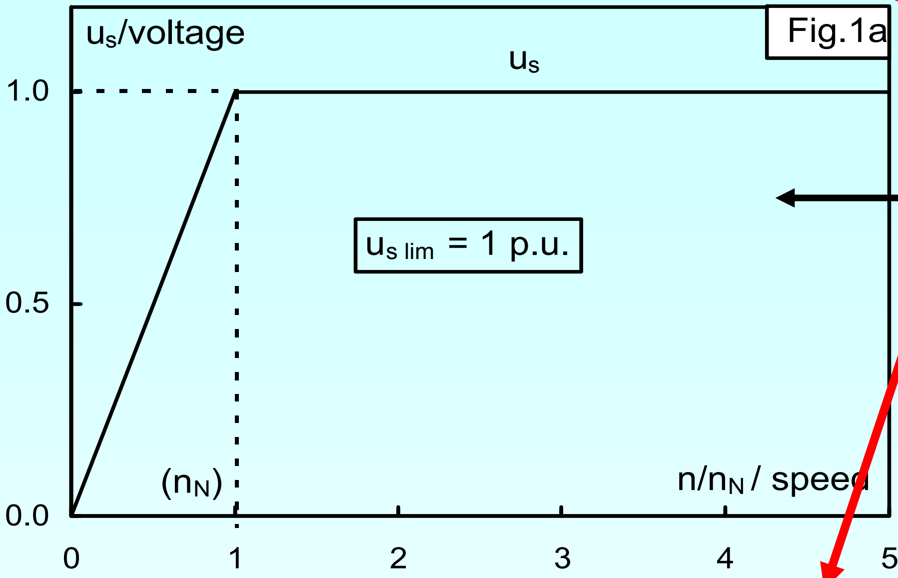
Example: Two different machines for flux weakening

Two PM motors A and B are compared with different voltage drop and different back EMF, but the same inverter voltage and current limit $U_{s,\lim}$ and $I_{s,\lim}$. Numbers are given in per unit-values of $U_{s,\lim}$ and $I_{s,\lim}$, e.g. $u_s = U_s / U_{s,\lim}$, $i_s = I_s / I_{s,\lim}$.

$$I_{s,k} / I_{s,\lim} = i_{s,k} = U_p / (X_s I_{s,\lim})$$

PM machine	A	B
$U_p/U_{s,\lim}$ at n_N	0.6	0.8
$X_s I_{s.\lim}/U_{s,\lim}$ at n_N	0.8	0.6
$U_{s,\lim}$ at n_N	$\sqrt{0.6^2 + 0.8^2} = 1$	$\sqrt{0.8^2 + 0.6^2} = 1$
Short circuit current $I_{s,k}$	$0.6/0.8 = 0.75 < 1$	$0.8/0.6 = 1.33 > 1$
Field weakening ?	Unlimited	Limited

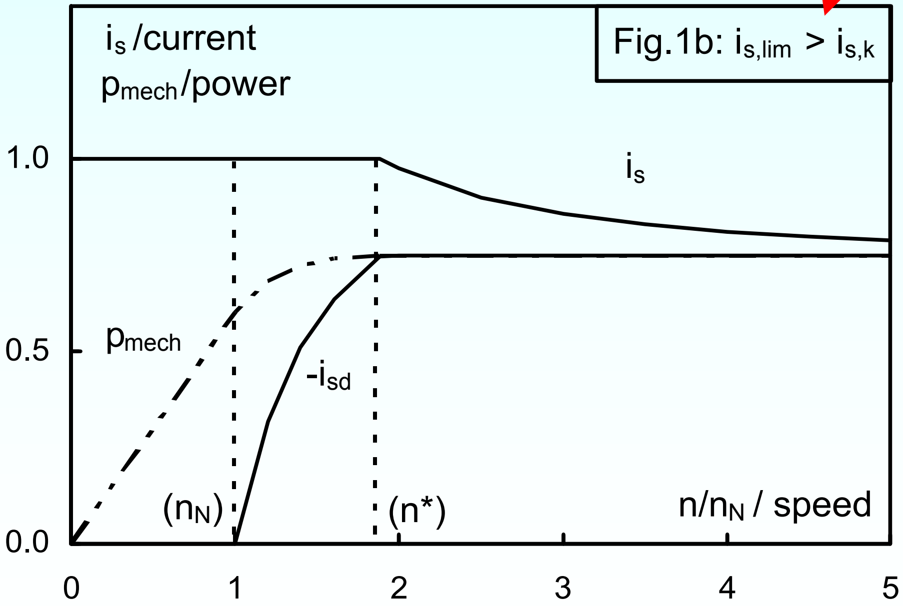
Comparison of two different PM machines concerning field weakening

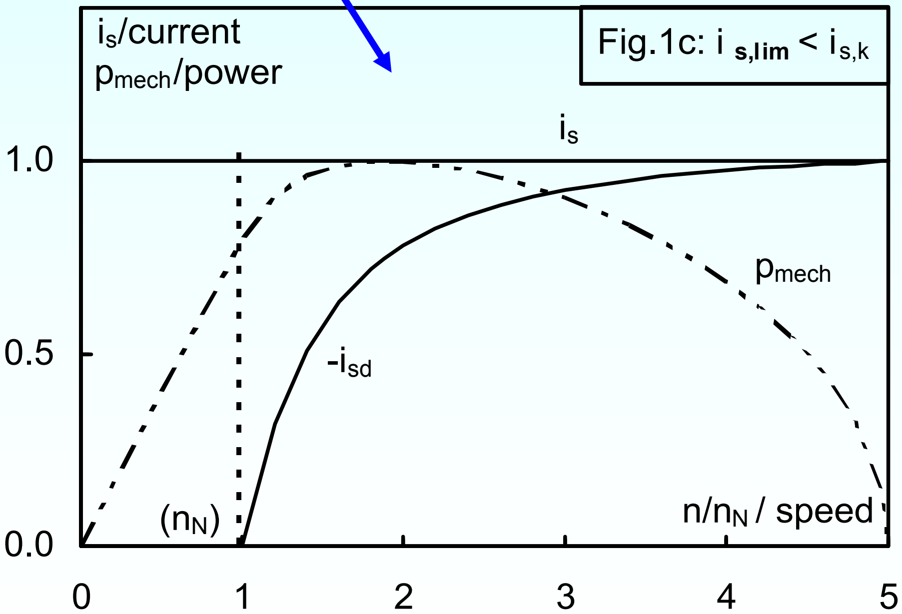


a) Voltage characteristic of inverter with voltage limit

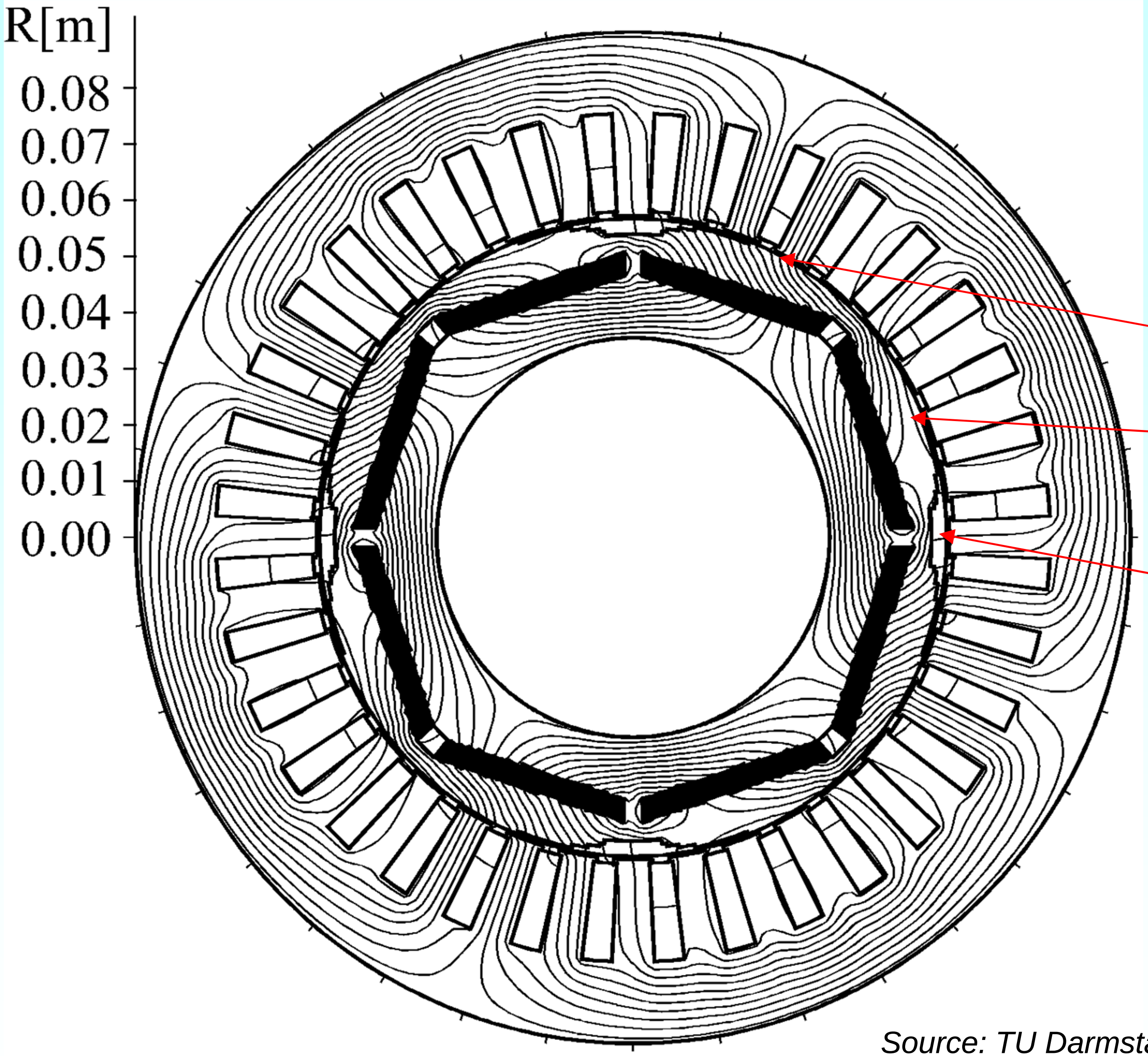
b) Motor A with inverter current limit $i_{s,\mathrm{lim}} > i_{s,k}$

c) Motor B with inverter current limit $i_{s,\mathrm{lim}} < i_{s,k}$





Buried magnet rotor at rated flux and current



4-pole machine

small magnetic air gap

sinusoidal air gap contour

rotor iron above magnets for big L_d

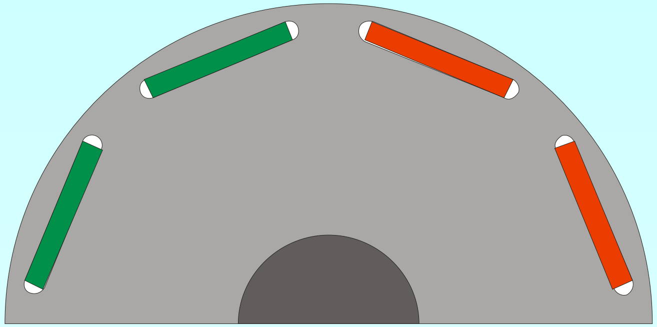
Inter-pole gaps to reduce L_q

Application:

High speed spindle drive for tooling machine (milling)

Source: TU Darmstadt

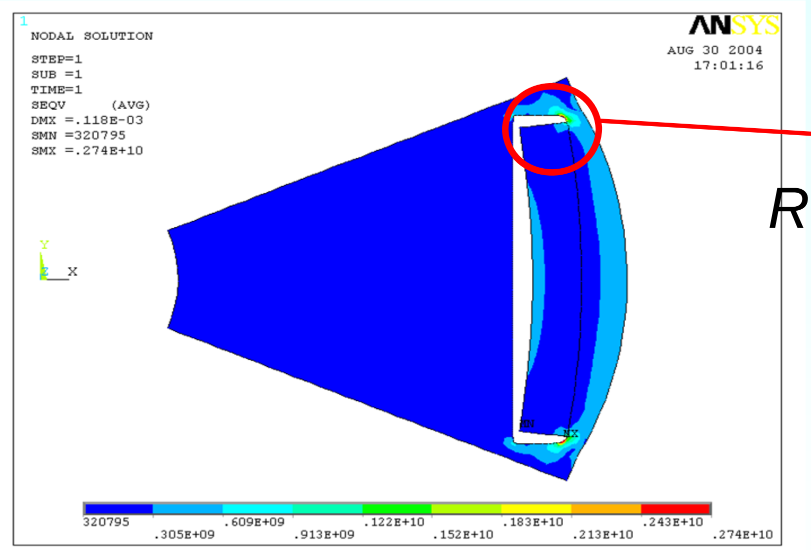
Fixing buried magnets at high speed via rotor sheet strength



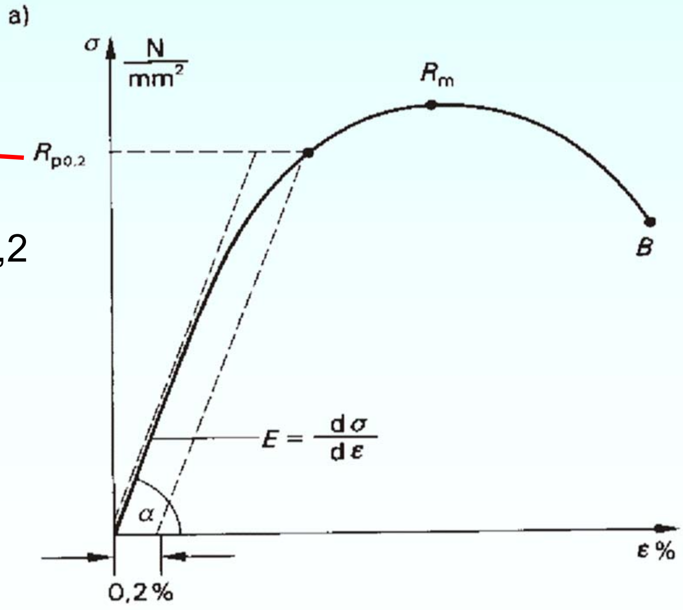
Magnets are fixed by rotors sheet:

- no bandage
- small magnetic air gap
- less magnet material in case of flux concentration

- Detailed mechanical calculation necessary, requires Finite Element Calculation
- max. tensile strength must stay below 0,2%-deformation limit of sheet $R_{p0,2}$



$R_{p0,2}$

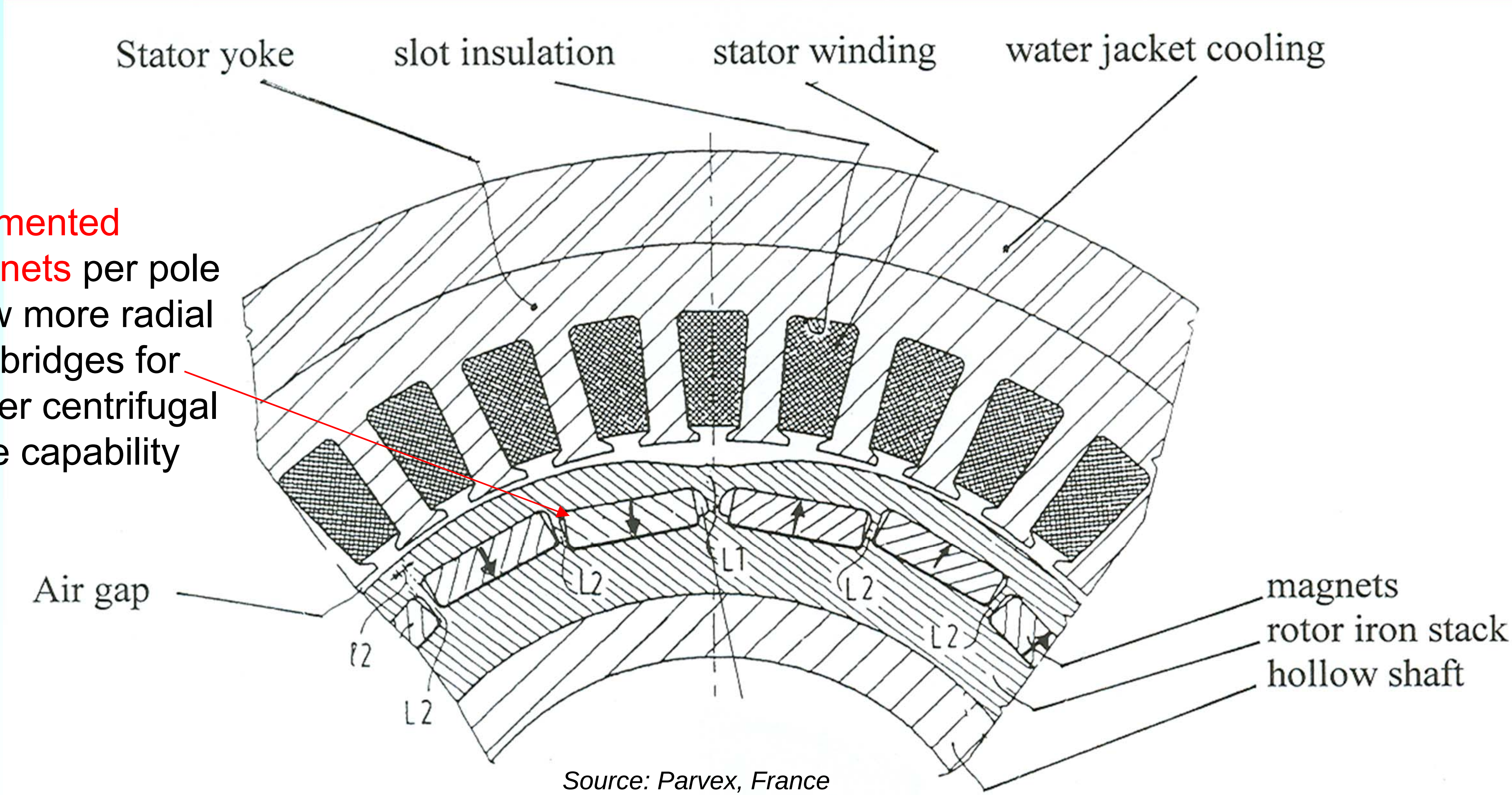


Stress σ versus strain ε diagram

Quelle: VDI Physik für Ingenieure

High speed buried magnet PM machine

Application: **Tools machinery – main drive with hollow shaft**



Segmented magnets per pole allow more radial iron bridges for higher centrifugal force capability

Source: Parvex, France

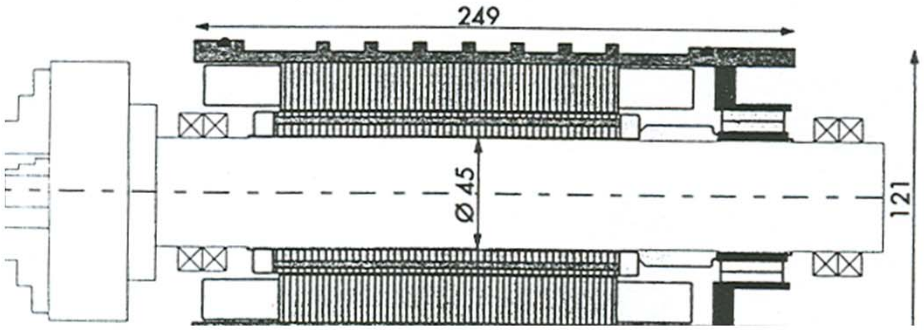
High speed PM main spindle drive with hollow shaft

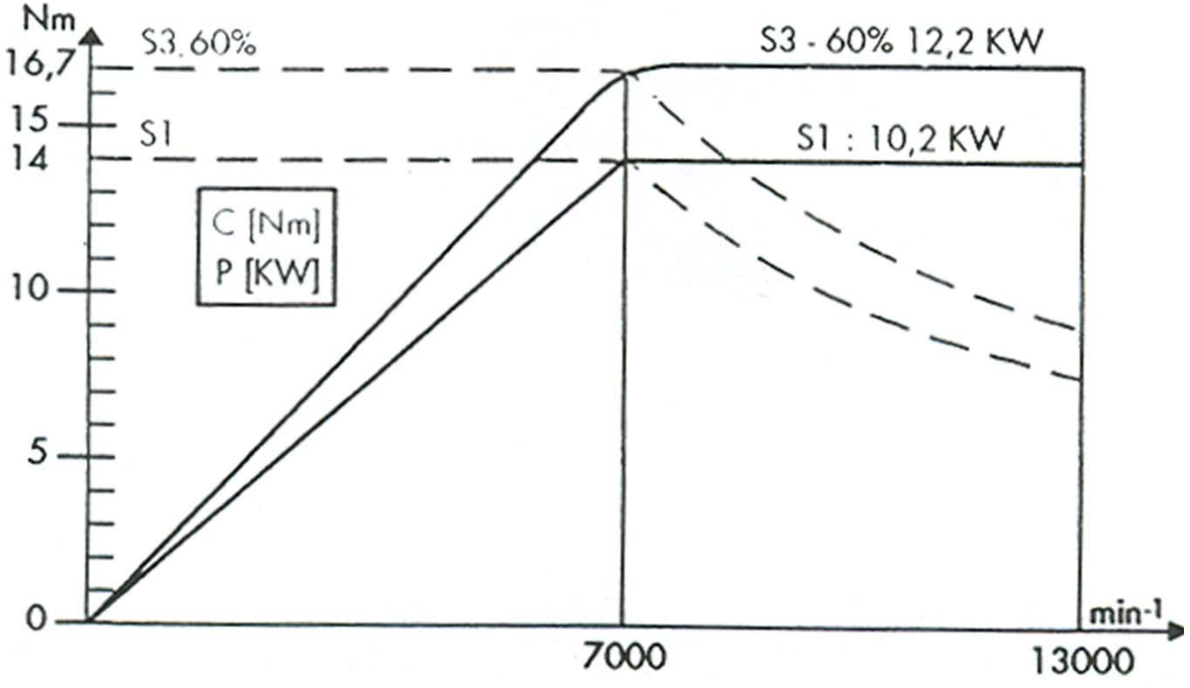
Application: **Tools machinery – main drive with hollow shaft**

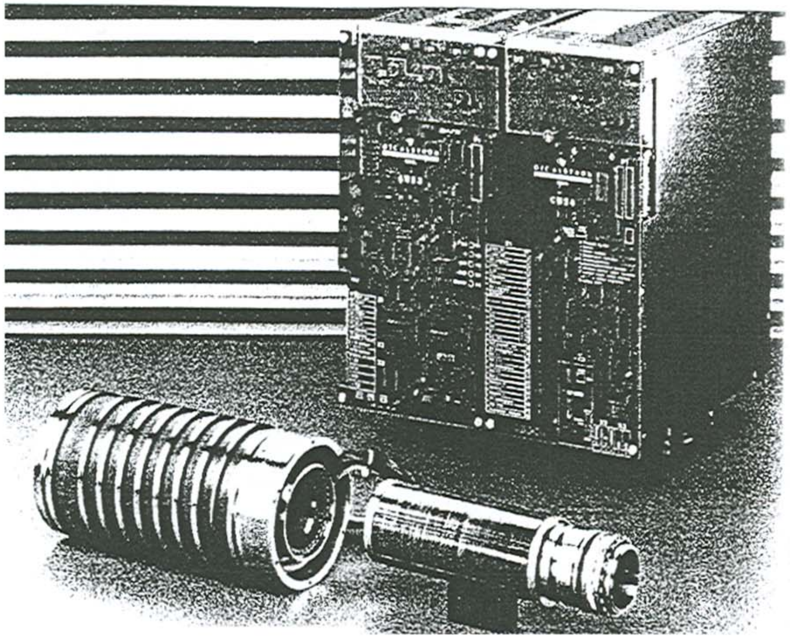
Antriebsbeispiel.
LW 635 CD + CMS4 60

Source: Parvex, France

Nennleistung : 10,2 KW
Nenndrehzahl : 7000 U/min
Max drehzahl : 13000 U/min
Rotor trägheit : 0,00182 kg m²
Nenndrehmoment : 14 Nm.







What means “High-Speed” ?

- NOT ONLY: high “rounds per second” *n* !
- BUT: High rotor circumference speed $v = d.\pi.n \cong 100 \dots 250$ m/s.
- High mechanical stress σ due to centrifugal forces !

σ proportional to mass density ρ and to *v*: $\sigma \sim \rho \cdot v^2$

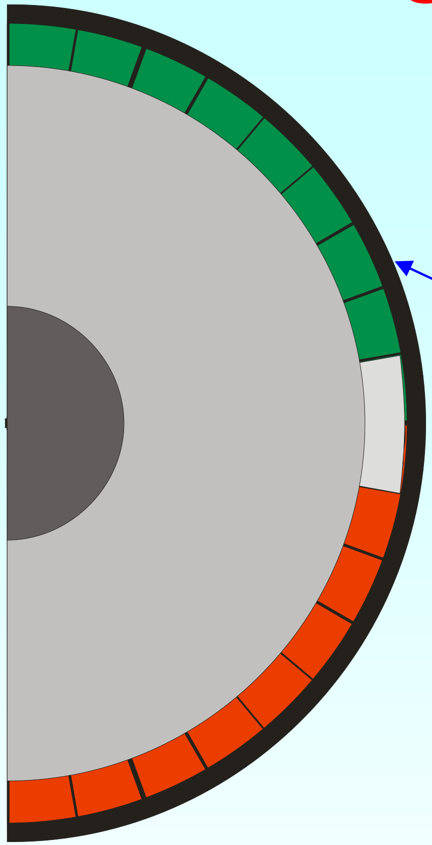
Example: Yield strength of steel sheet: $R_{p,0.2} = \sigma_{0.2} = 350\ \mathrm{N/mm^2}$

Thin steel ring (diameter *d*) at $v = 211$ m/s :

Tangential stress, $\rho = 7850\ \mathrm{kg/dm^3}$: $\sigma_t = \rho \cdot v^2 = 350\mathrm{N/mm}^2$

a) At $d = 1.3$ m: $n = 3000$ /min b) At $d = 90$ mm: $n = 45000$ /min.

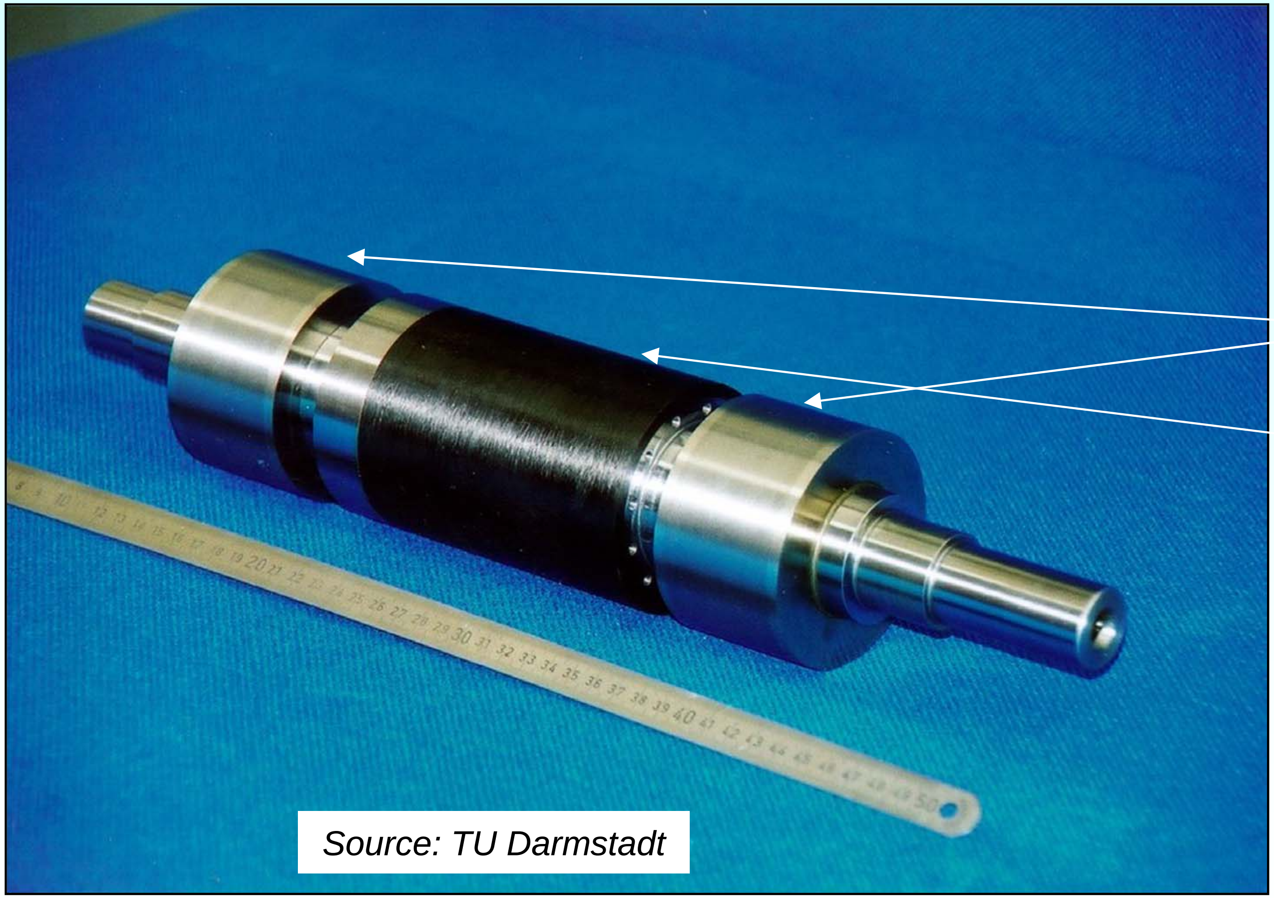
Surface mounted magnets $\alpha_e < 1$



Fixing of magnets by bandage:

Fiber and resin (“matrix“) = „Bandage“

Pre-fabricated bandage is pressed onto rotor with force.



Source: TU Darmstadt

PM-Rotor

40 kW, 40000/min

Magnetic bearings

Carbon fiber bandage

Outer diameter: ca. 90 mm

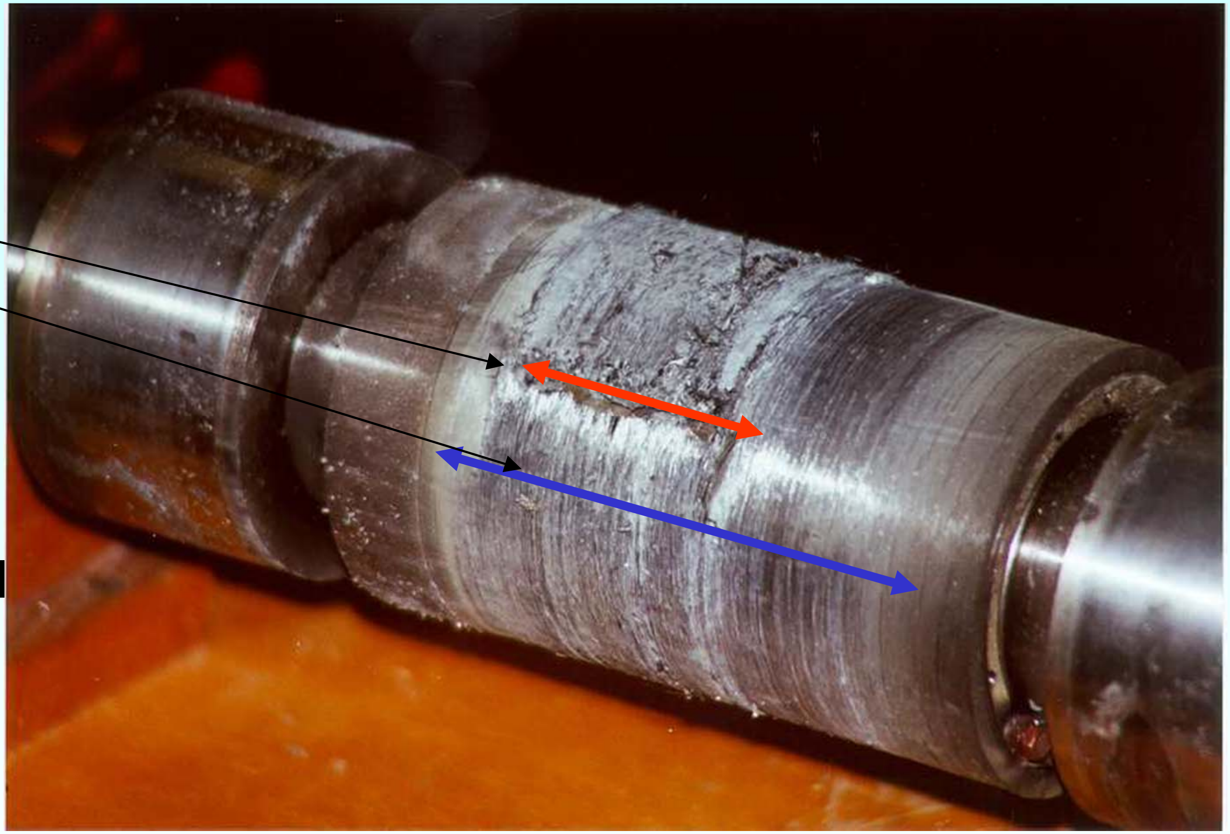
- 4-pole rotor

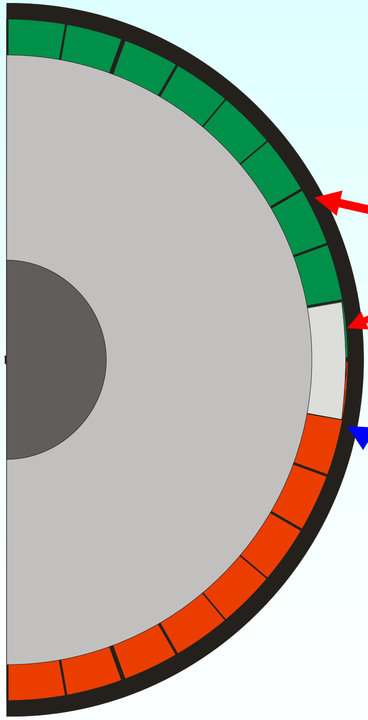
- Pole coverage ratio $\alpha_e < 1$

Example: Damaged Rotor due to Magnet Edge Pressure

Breaking of bandage at 35 000/min

- **Break length**
- **Total active length**





- Different centrifugal forces in magnets and inter-pole gap

- Magnet edges cut into carbon fiber bandage

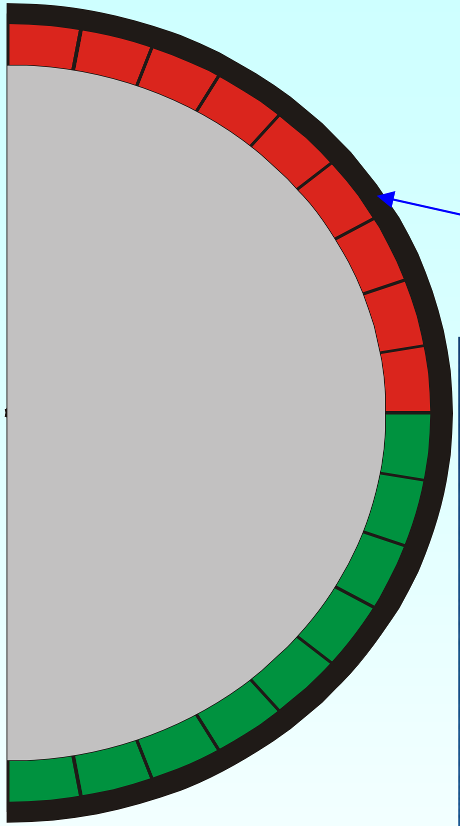
4-pole rotor with PM, carbon fiber bandage and magnetic bearings

Source: TU Darmstadt

Surface mounted magnets $\alpha_e = 1$

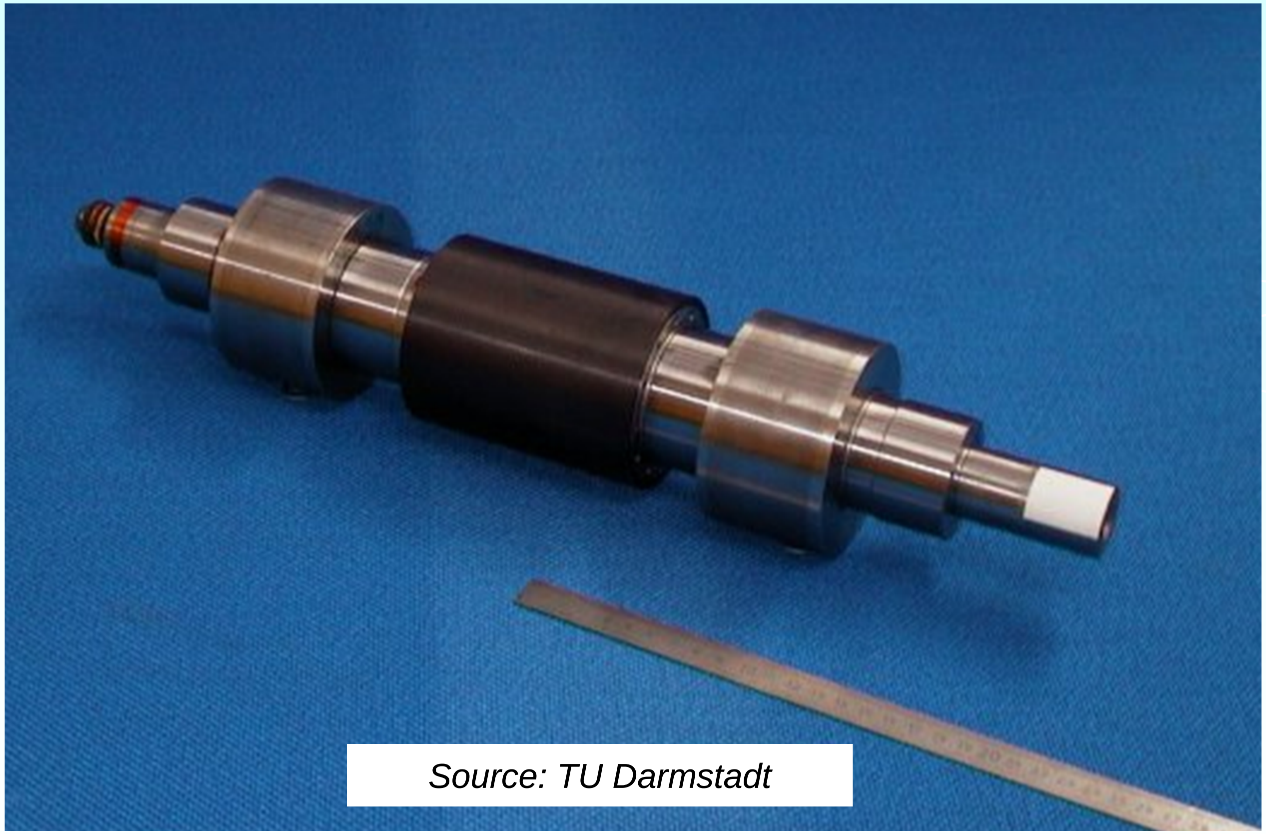
Fixing of magnets by bandage:

Fiber and resin (“matrix“) = „Bandage“



Pre-fabricated bandage is pressed onto rotor with force.

- **4-pole rotor**
- **Pole coverage ratio $\alpha_e = 1$**



Source: TU Darmstadt

PM-Rotor

40 kW, 40 000/min

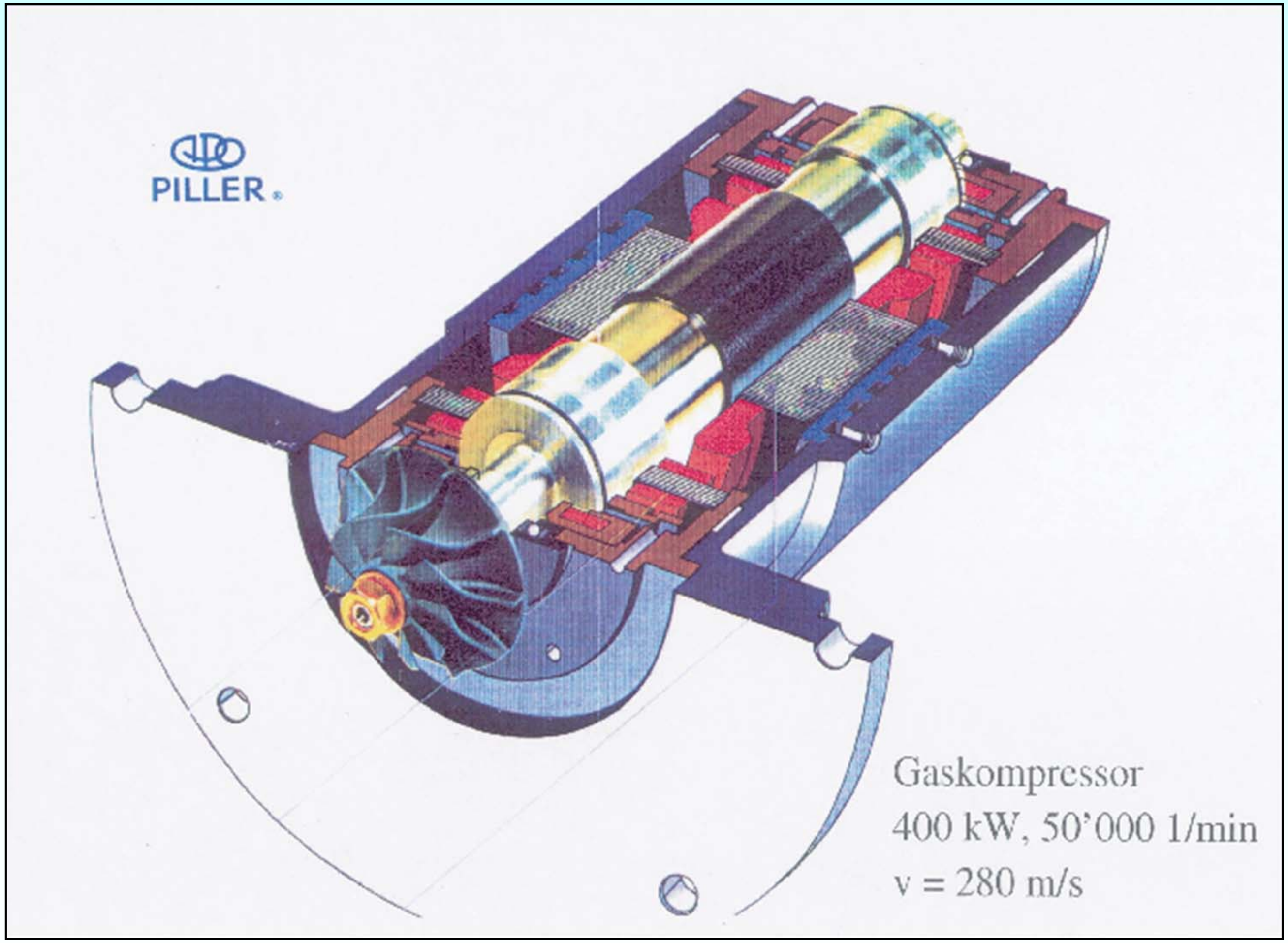
Magnetic bearings

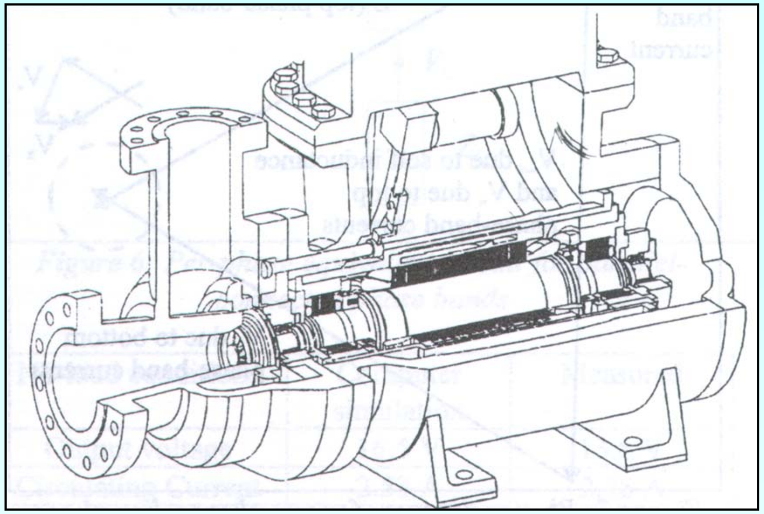
Carbon fiber bandage

Outer diameter: ca. 90 mm

Hi-speed application: Gas compressor

$$n = 50000/\min, v = d_{si}\pi n = 280\mathrm{m/s}, \quad d_{si} = 107\mathrm{mm}$$





- Magnetic bearings due to high speed
- 400 kW, 50000/min
- Small dimensions
- Carbon fiber bandage
- No gear box

Source: Piller, Germany

Example: Magnetically levitated PM drive 40 kW, 40 000/min

4-pole PM-Synchronous machine, surface magnets (Sm_2Co_{17})
16 magnet segments / pole, carbon fiber bandage, radial magnetic bearings

P_N = 40 kW
n_N = 40 000 min^{-1}
α_e = 0.87 (pole coverage ratio)

d_a = 88.6 mm rotor diameter
$v_{u,schl}$ = **222 m/s** rotor surface velocity at 120% over-speed

stator with water jacket cooling

Magnetic bearing stator

magnetic bearing rotor

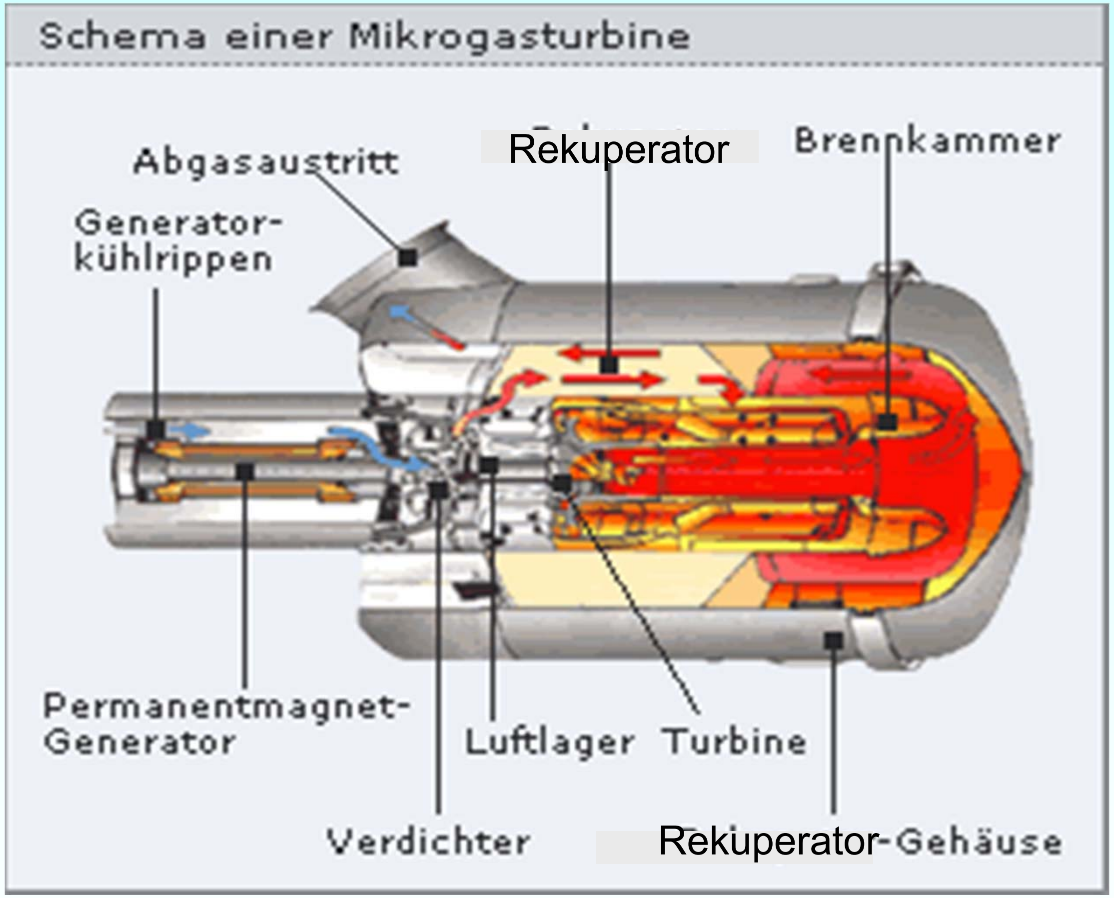
rotor with CFC-bandage

Radial distance sensors (eddy current principle)

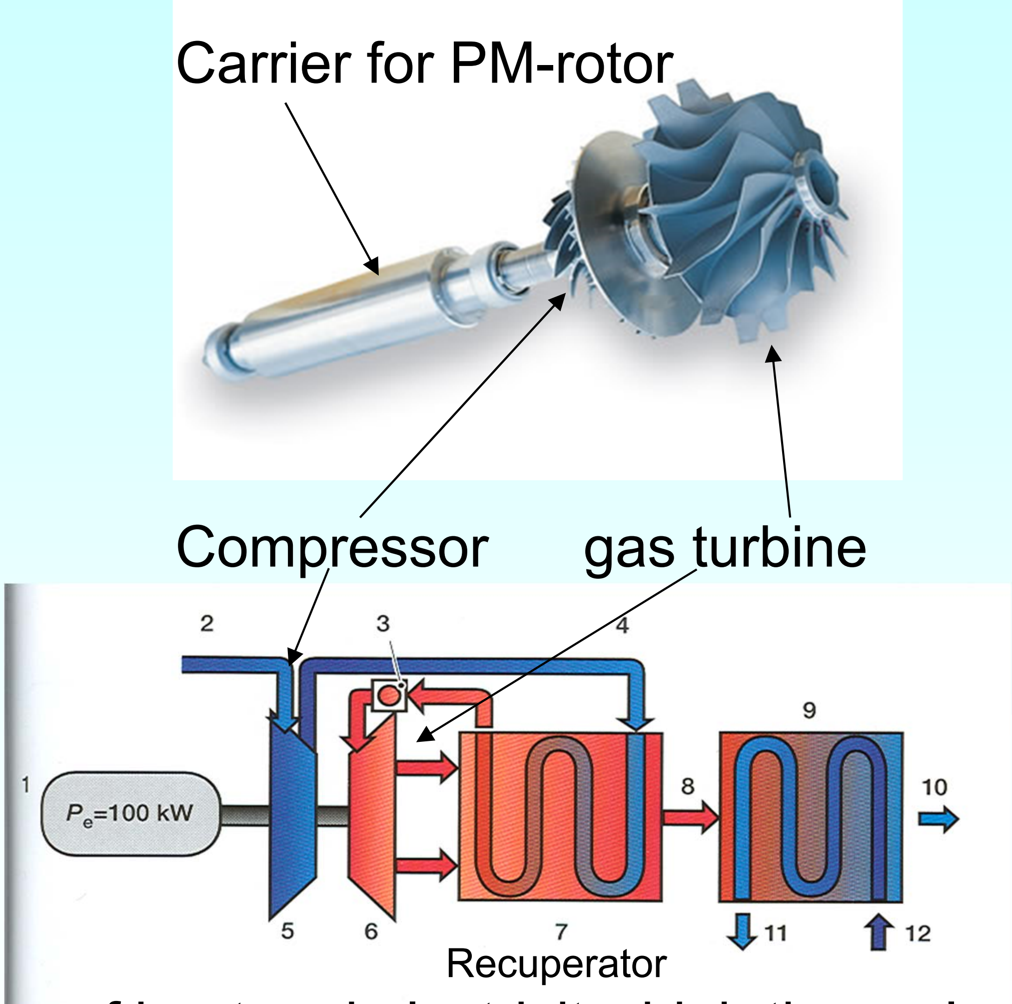
Source: TU Darmstadt

Application Hi-Speed: PM Generator for micro gas turbine

Micro gas turbine: gearless: high speed generator necessary (z. B. 100 kW)



Source: ABB, Sweden



„Micro“-gas turbine: Advantage: Co-generation of heat and electricity, high thermal efficiency - high utilization of gas energy

Cut-view of the stator part of a micro gas turbine

Blick ins Innere einer Mikrogasturbine: Brennkammer (obere Bildmitte, grünlich), Rekuperator (lamellenartige Struktur oben und unten, dort wird die komprimierte Verbrennungsluft durch heiße Turbinenabgase vorgewärmt) und Injektor (unten links). Foto: IFF/Dirk Mahler

Source: IFF, Dirk Mahler

Recuperator

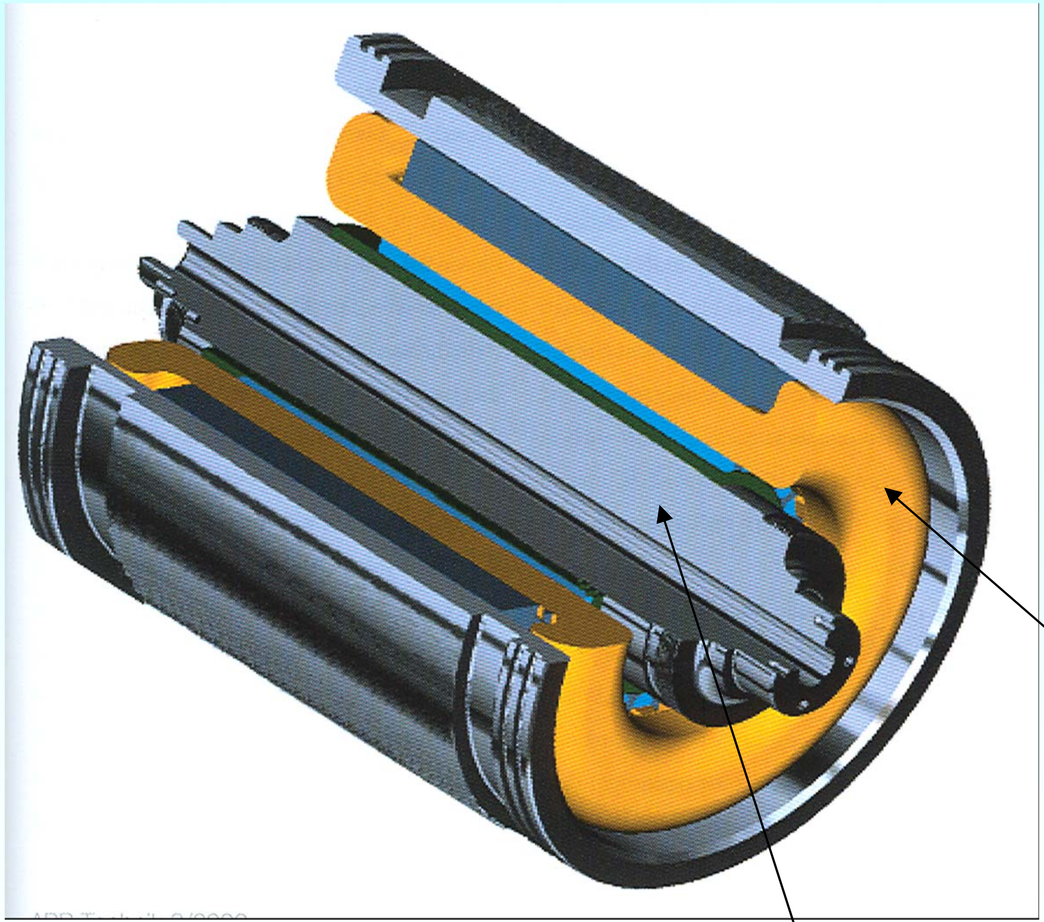
Combustion chamber

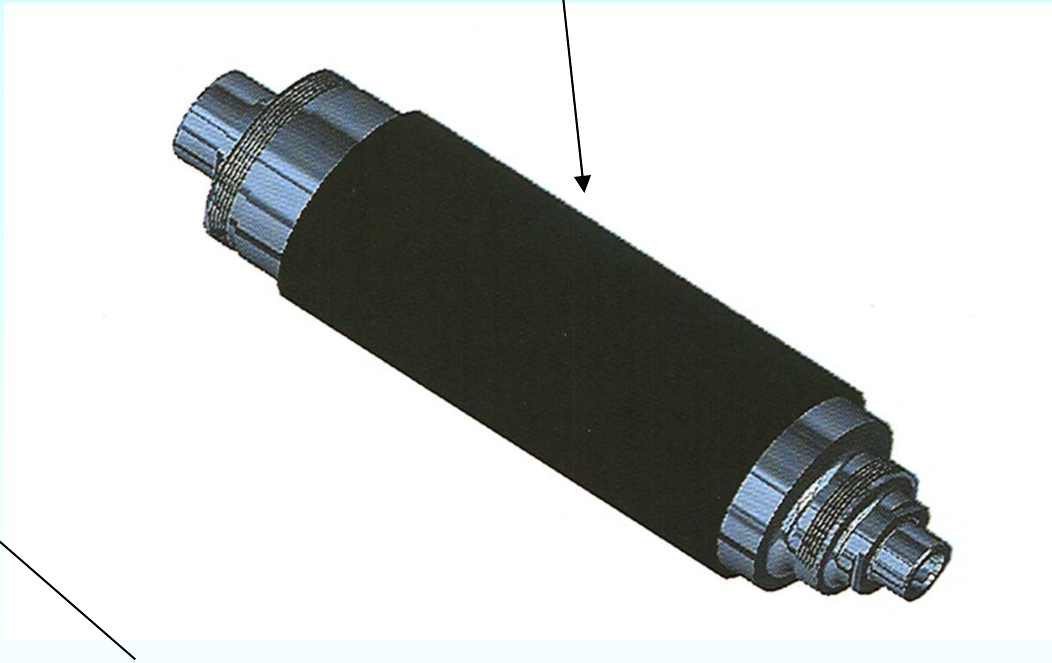
Gas injector

The recuperator is a heat exchanger, where the hot exhaust gasses pre-heat the compressed cold air for the burning process. Thus the efficiency increases.

Gearless high speed PM generator

70 000/min, 2300 Hz, four pole PM-rotor, Carbon fiber bandage



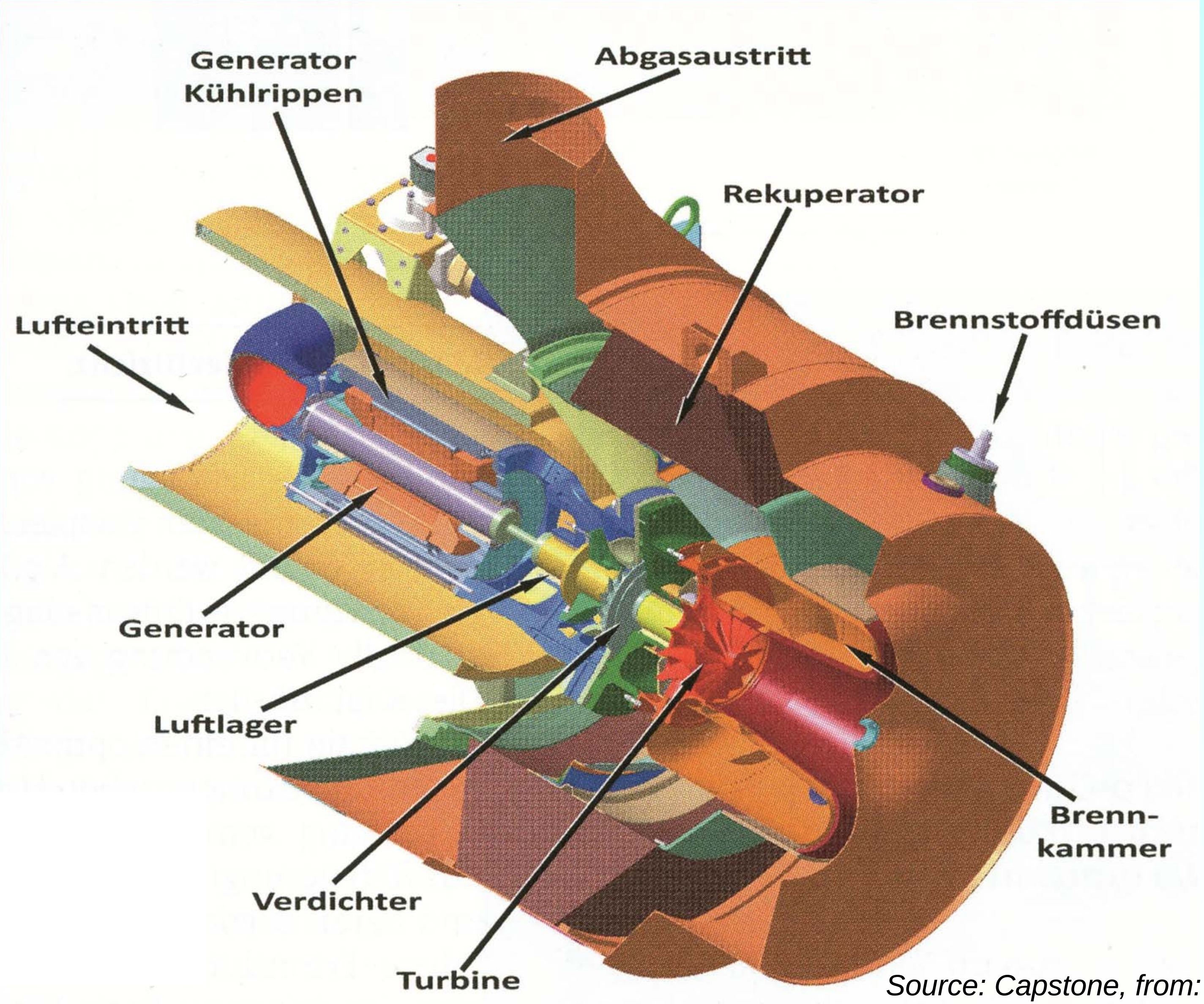


Stator with four pole three phase winding, fully encapsulated in resin for good heat transfer to iron

Massive rotor iron, special air bearings

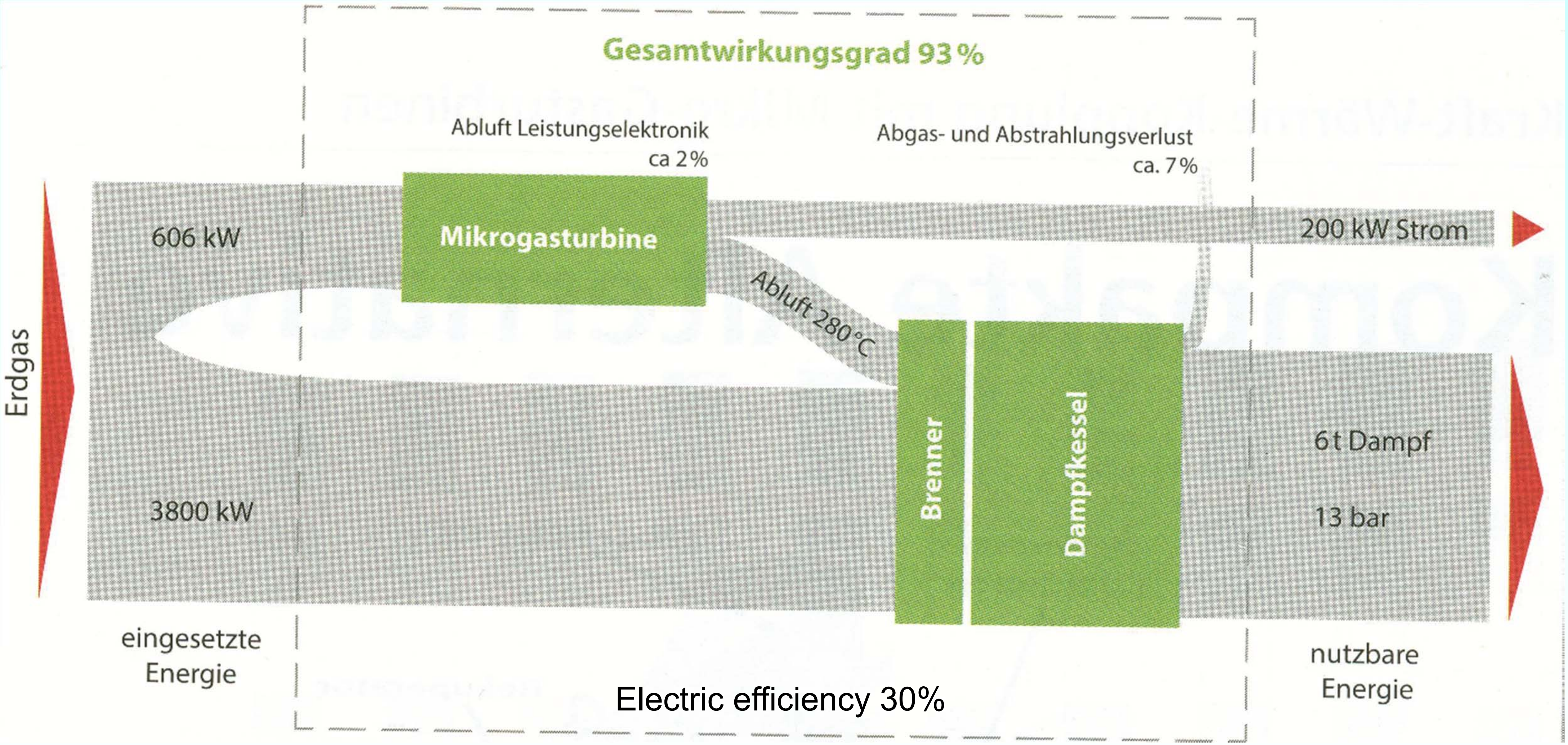
Source: ABB, Sweden

Micro gas turbine arrangement



Source: Capstone, from: BWK 64 (2012), no. 11

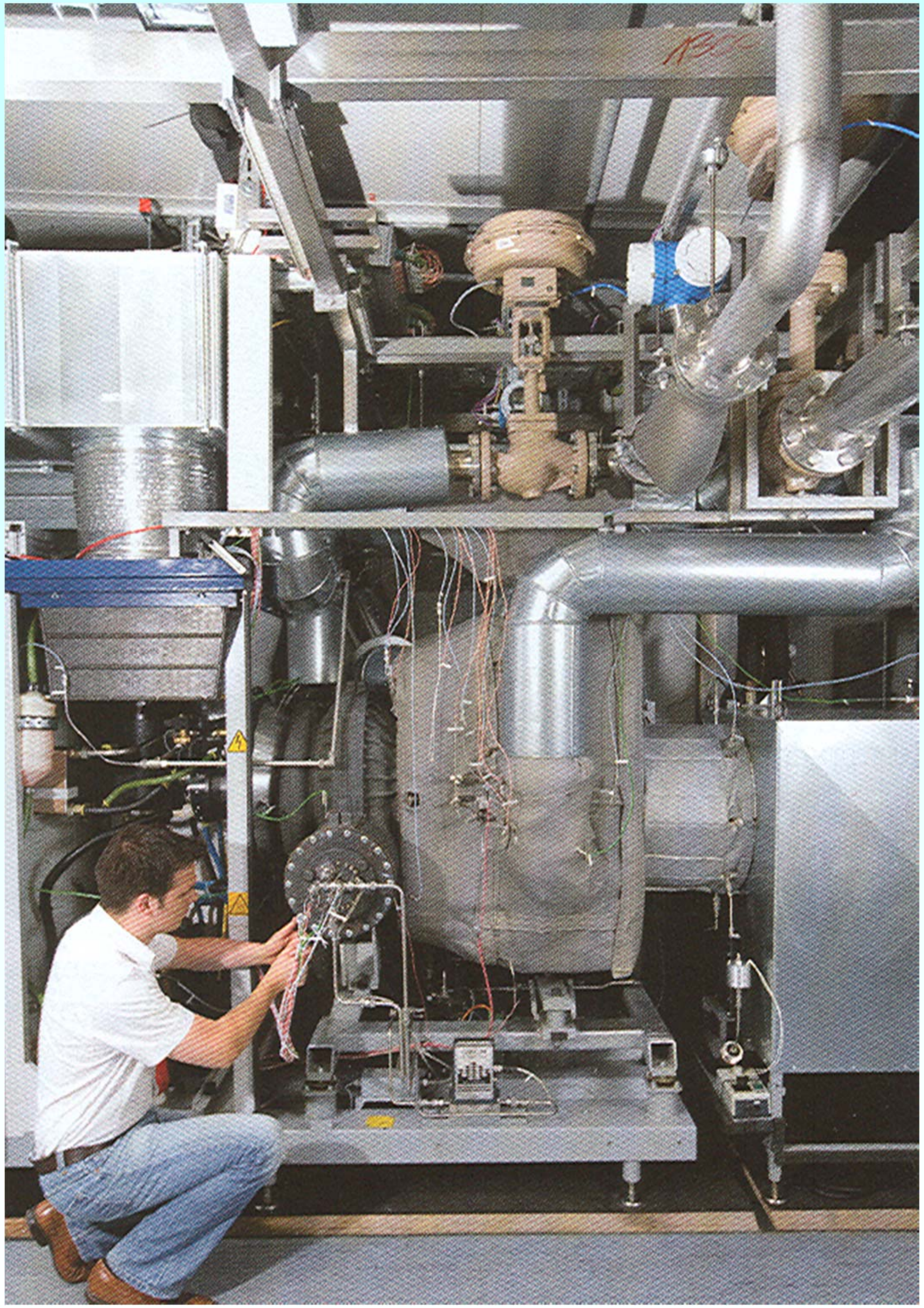
Example: 200 kW Micro gas turbine: Efficiency



Electric efficiency 30%

Co-generation of electric power and heating:
Total efficiency 93%

Source: Capstone, from: BWK 64 (2012), no. 11



Micro gas turbines

Typical ratings 50 … 300 kW (“micro”)

40000 … 240000 / min rotational speed

Acoustic noise ca. 65 dB(A) only in 1 m distance

Electric efficiency 28 … 33%

Co-generation of electric power and heating:
Thermal efficiency 75 … 90%

Life time for operation 80000 h

Maintenance intervals 6 000 h

General revision necessary after 30000 h

Source: BWK 62 (2010), no. 9, p. 65

Electric cars: Specification ACEA small & prototype motor

Speed	Base: 2200/min, Max.: 9000/min, Over-speed: 12000/min
Mech. Power	Peak: 30 kW, Continuous: 15 kW
Duty-Cycle EUROPED	30 kW: 2 min, @ 4500/min, 10 kW for 8 min.
Torque	Peak: 130Nm, Continuous: 75Nm
DC Supply	130...290V from Battery
Motor	Diameter < 250 mm, mass < 40 kg

Stator: 36 skewed (1 slot-pitch) slots, stack 225mm, bore 122.6mm, outer diameter of laminations: 180mm

Rotor: Surface NdFeB-magnets: remanence 1 T (20°C), magnet height 2.5 mm, magnetic air-gap (air gap + sleeve) δ_e = 1.05mm, pole coverage: 98%

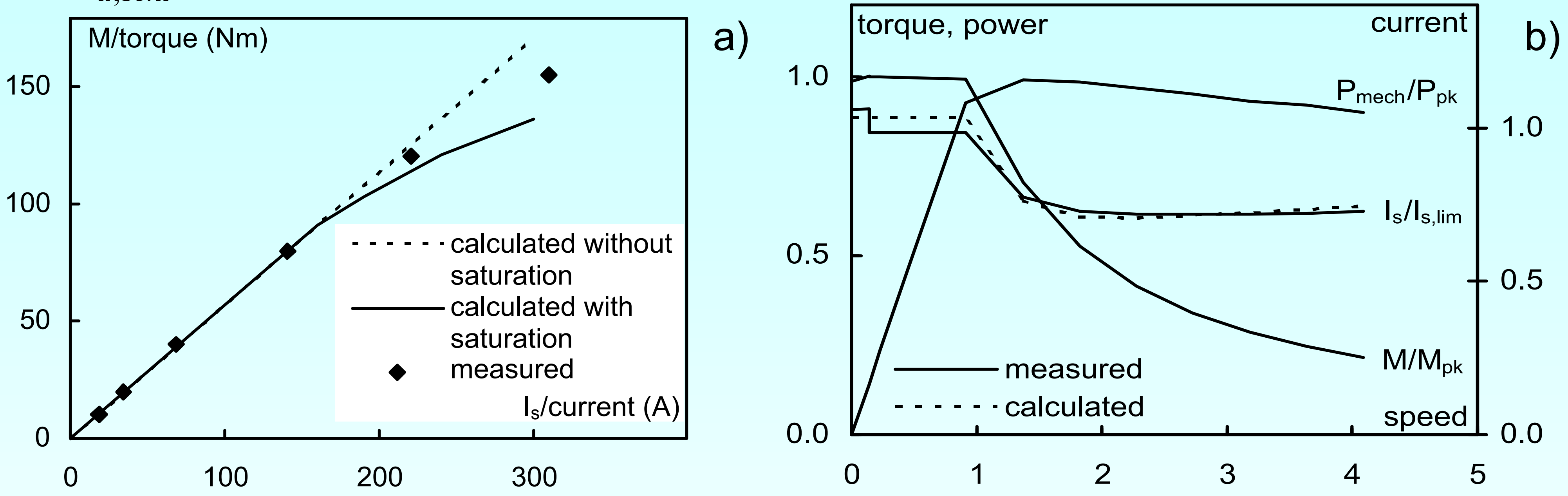
Armature: Single layer, round wire, three phase, star, 15 turns/phase, 10.55mOhm resistance/phase (20°C)

Cooling: Liquid jacket, coolant: 50%/50% water/glycol, 8 l/min flow rate, 50°C/55°C inverter/motor inlet temperature

Insulation: Thermal Class F (max.145°C plus 15 K in hot spots)

“Mild high speed”: Motors for electric or hybrid-electric cars

$v_{u,schl} = 77\text{m/s at } 120\% \text{ overspeed}$



Source: Siemens AG, Germany

c)

Source: Daimler, Stuttgart, Germany

Propulsion for electric car:

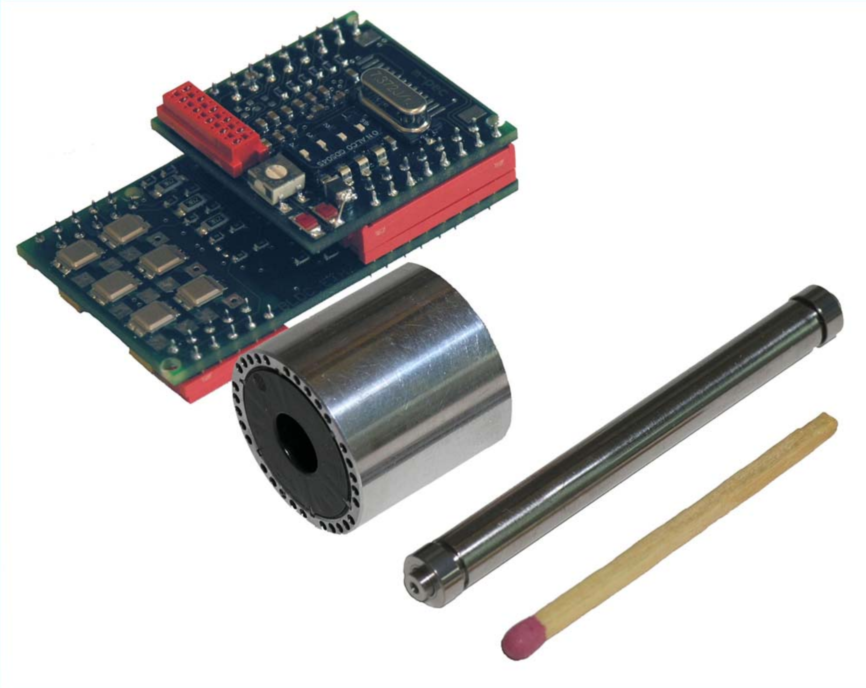
a) Torque-Current-Characteristic for propulsion motor,

b) Measured torque-speed drive characteristic at 132V DC link voltage = battery voltage,

$M_{pk} = 156Nm, P_{pk} = 35kW, I_{s,\lim} = 315A$

c) Prototypes of fuel cell powered cars NECAR (New Electric Car) of *DaimlerChrysler* with electric drive system

Ultra high-speed motor for a meso-scale gas turbine



Torque	M_N	**2 mNm**
Phase voltage	U_N	**11 V**
Current	I_N	**3 A**
Frequency	f_N	**8.3 kHz**
Rotor diameter	d_R	**6 mm**
Stack length	l_{Fe}	**15 mm**

100 W, 500 000/min

- Two-pole PM motor, six-step voltage feeding
- Titanium rotor sleeve
- Circumference speed: 157 m/s
- mechanical bearings $n \cdot d_m \cong 2 \cdot 10^6\,\mathrm{mm/min}$

d_m: average bearing diameter

Source: Swiss Federal Institute of Technology in *Zurich, Switzerland*

High power high-speed cage induction motors

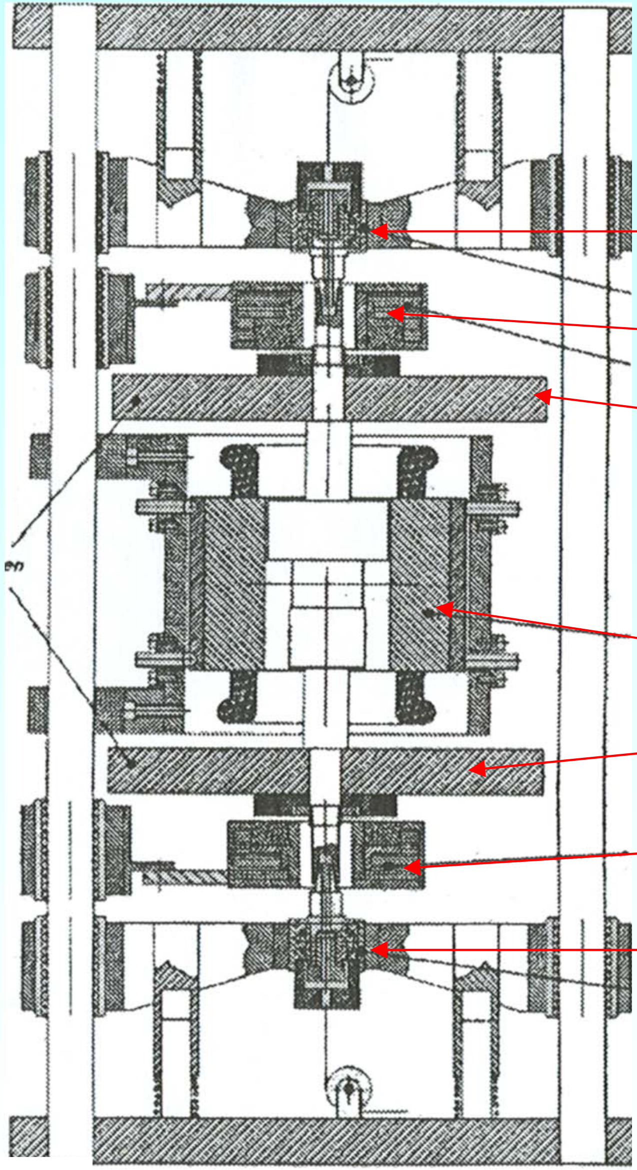
Source:

Siemens AG

15000/min, 4 MW cage induction motor

- **Gas compressor drives**
- **Rating 4 MW, 15000/min, 2.5 kNm … 16 MW, 6000 /min, 25.5 kNm**
- **Copper cage two-pole induction motors, massive rotor iron, ca. 240 m/s**
- **Active magnetic bearings, operation above first bending mode**
- **Medium voltage IGBT PWM inverter operation**

Homo-polar generator for flywheel



10 kW, 50000/min
Vacuum operation

Aux. bearing

HTSC magnetic bearing

Flywheel disc

Homo-polar generator

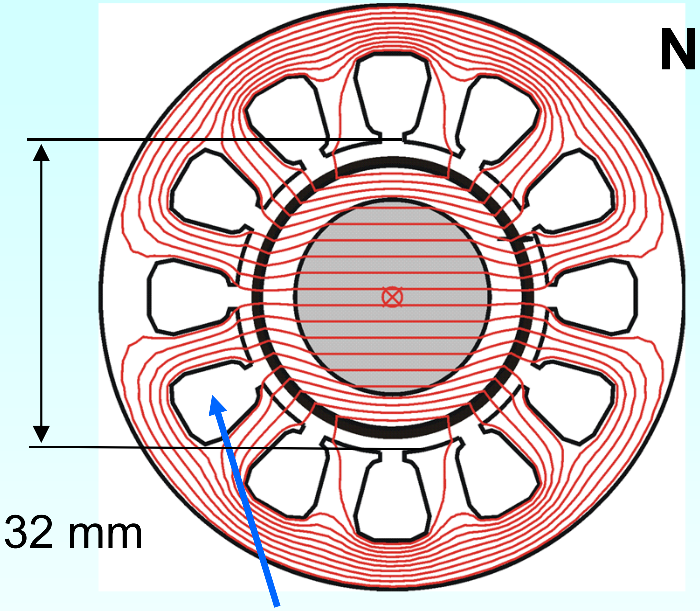
Flywheel disc

HTSC magnetic bearing

Auxiliary bearing

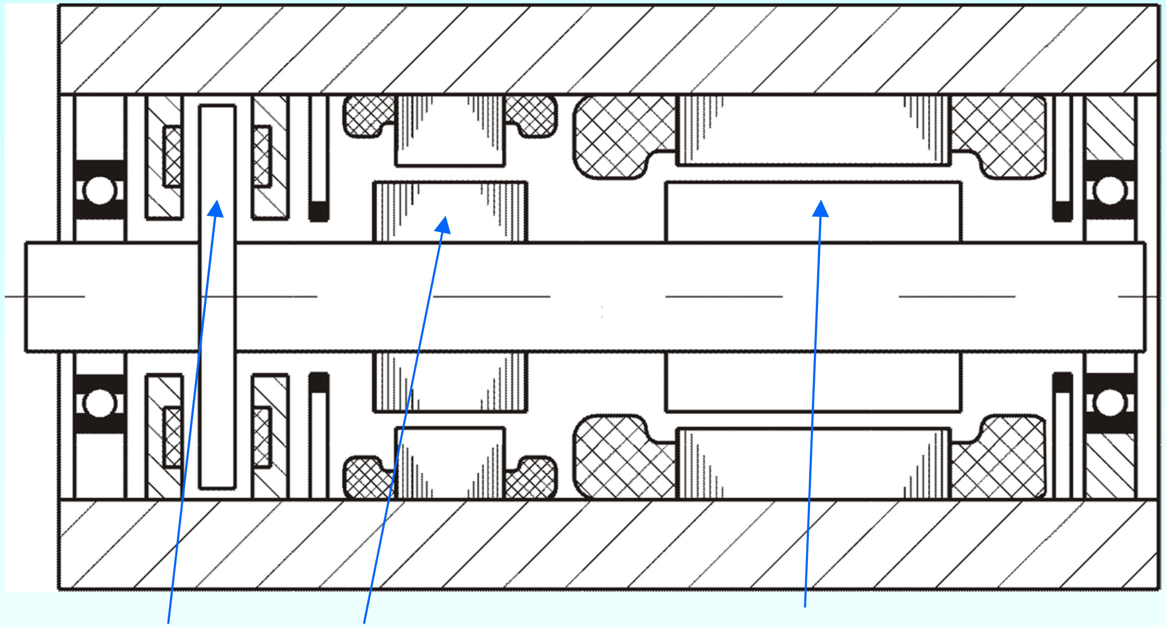
Source: Uni. Stuttgart & FSZ Karlsruhe, Germany

Bearingless high-speed PM motor 500 W, 60000/min



No-load flux plot

Program FEMAG



Axial & radial active magnetic bearing

Combination of PM motor and radial magnetic bearing = = BEARINGLESS

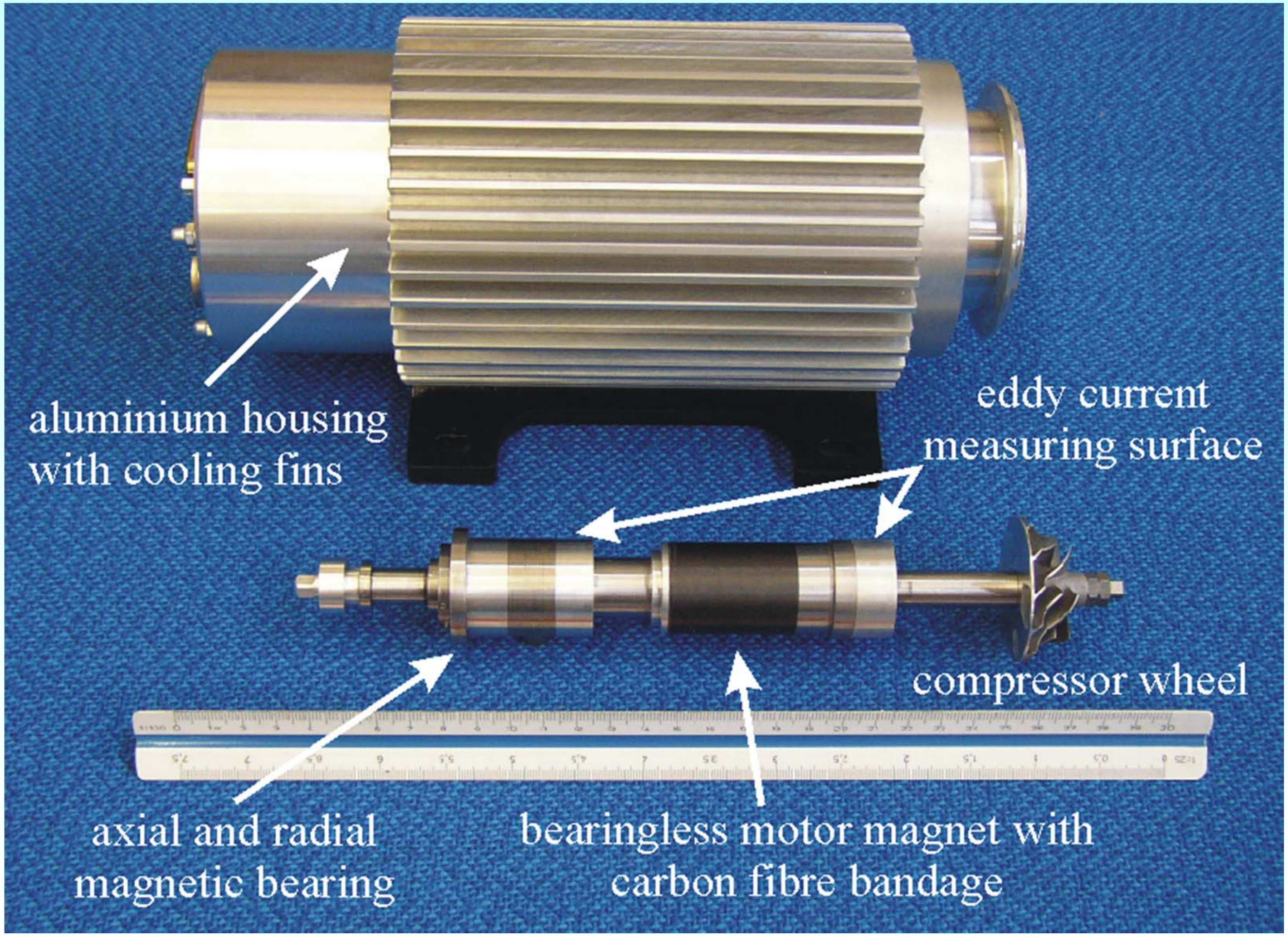
BEARINGLESS concept:

***Two* 3- phase stator windings:**

Pole count difference MUST be 2 !

- **Two-pole drive winding $2p_1 = 2$**
- **Four-pole levitation winding $2p_2 = 4$**

Bearingless high-speed PM motor 500 W, 60000/min



60000/min

500 W

Two-pole

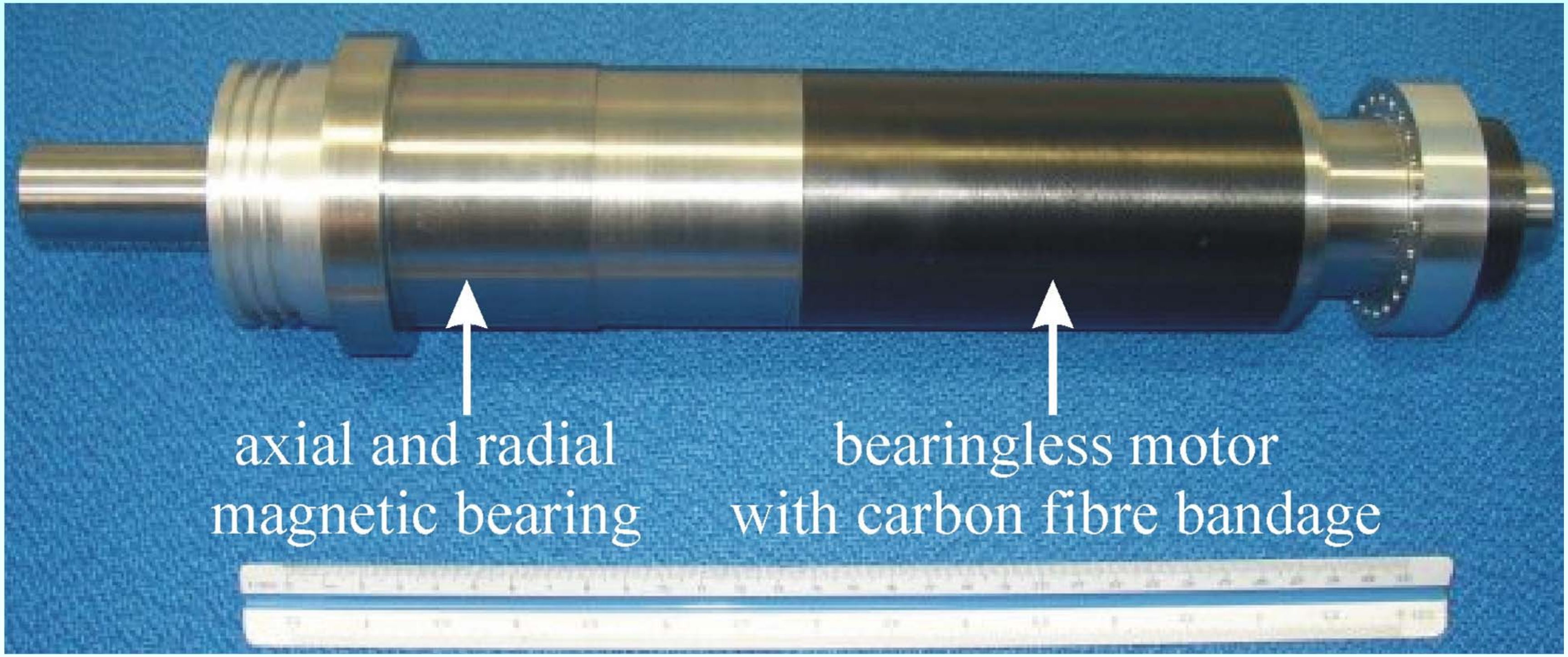
f = 1000 Hz

100 m/s

Source: TU Darmstadt,

In co-operation with Levitec Company, Lahnau, Germany

Bearingless high-speed PM motor 40 kW, 40000/min



40000/min

40 kW

Four-pole

f = 1333 Hz

167 m/s

BEARINGLESS concept:

Two 3- phase stator windings:

- **Four-pole drive winding $2p_1 = 4$**
- **Six-pole levitation winding $2p_2 = 6$**

Source: TU Darmstadt,

In co-operation with Levitec Company, Lahnau, Germany

- Rotor successfully tested at 44000/min (185 m/s)

- Electrical tests completed

Losses at high speed

No-load losses:

a) Iron losses: *x* = ca. 1.8, considering eddy-current and hysteresis losses

$$P_{Fe} \sim B^2 \cdot f^x \sim B^2 \cdot n^x$$

b) Friction losses at the rotor surface (and in the bearings)

y = 2 ... 3, depending on rotor and stator surface condition

$$P_{fr} \sim d_r \cdot l_{Fe} \cdot n^y$$

Load losses:

c) Armature winding losses (copper losses):

$$P_{Cu} = 3 \cdot R_s \cdot I_s^2$$

d) Additional losses

- in stator winding due to eddy currents of high frequency current
- in rotor magnets and rotor iron, *z* = ca. 1.5 ... 2, depending on rotor geometry and stator current shape.

$$P_{M+Fe,r} = f(n,\ \text{current shape}) \sim I^2 \cdot n^z$$

With high speed motors the loss groups a), b) and d) dominate, and therefore special care must be taken for motor design. Low loss iron sheets with only 1 W/kg losses at 1 T, 50 Hz may reduce iron losses.

Electromagnetic utilization

- Size of motor active parts: Stator bore diameter d_{si}
  Active rotor length l_{Fe}

- *Esson´s* apparent air gap power equation.

$$S_\delta = (\pi^2/\sqrt{2}) \cdot k_w \cdot A \cdot B \cdot d_{si}^2 l_{Fe} \cdot n$$

- *Esson´s* electromagnetic utilization factor: $C = (\pi^2/\sqrt{2}) \cdot k_w \cdot A \cdot B$

  Air gap flux density *B*, current loading *A*, winding factor $k_W \cong 0.91$

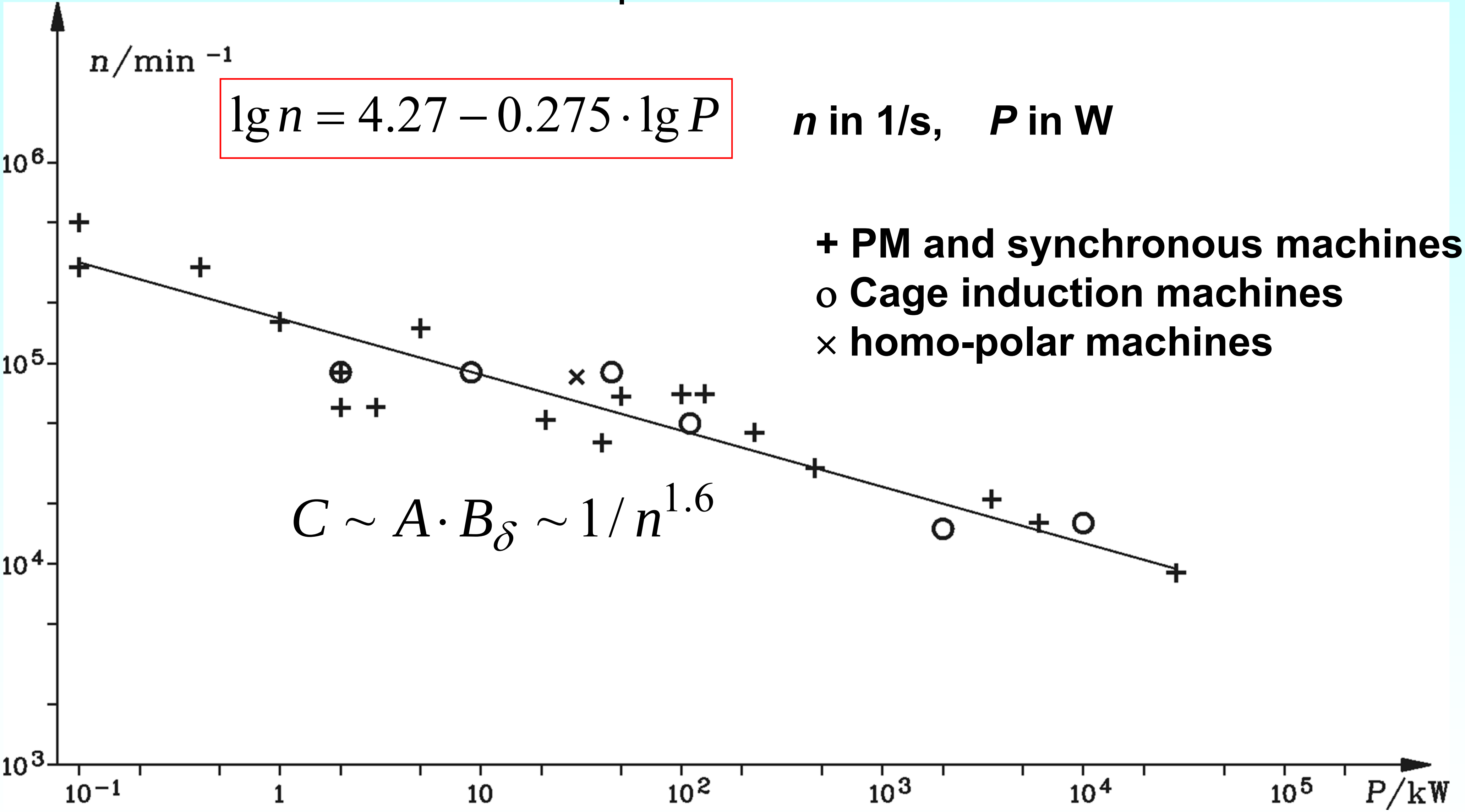
- Increased iron, friction and additional losses at high speed:

  Reduction of *A* and *B* necessary, leads to reduction of *C* !

$$\boxed{C \sim A \cdot B \sim 1/n^{1.6}}$$

Rated speed vs. rated power

from published data



$$\boxed{\lg n = 4.27 - 0.275 \cdot \lg P}$$

n in 1/s, *P* in W

+ PM and synchronous machines

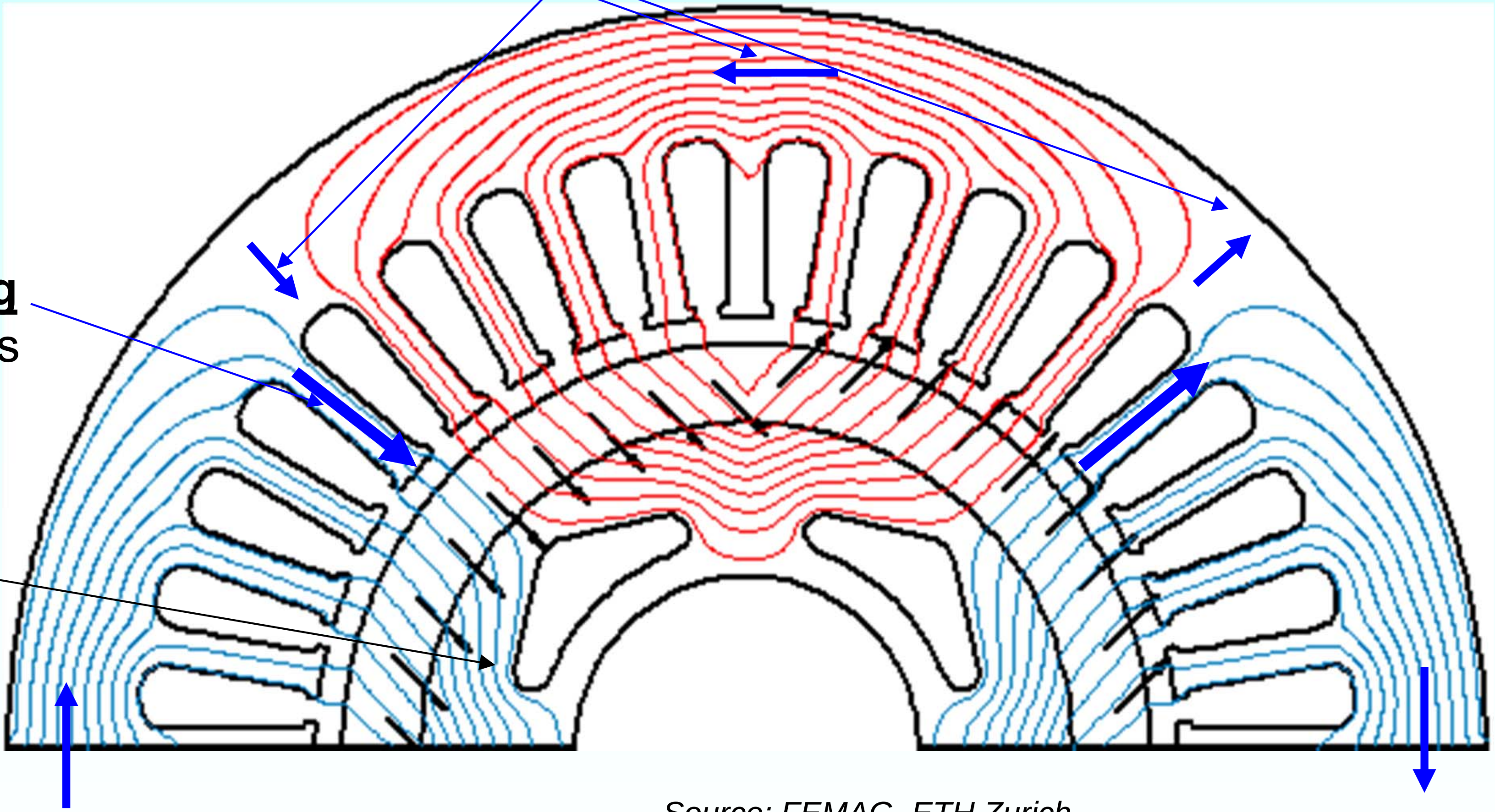
o Cage induction machines

× homo-polar machines

$$C \sim A \cdot B_\delta \sim 1/n^{1.6}$$

Pulsating and rotating hysteresis losses

Example: 4 pole PM synchronous machine, no-load



Yoke:
Mainly **rotating** hysteresis losses

Teeth:
Mainly
pulsating
hysteresis
losses

Rotor:
No time-varying
field with
fundamental
frequency = no iron
losses

Source: FEMAG, ETH Zurich

Air friction losses at high speed

- **Air friction losses P_{fr} of a rotating rotor cylinder in a smooth stator bore:** **Turbulent flow (*Reynolds*-number *Re* > 1000) at the cylinder surface:**

$$P_{fr} = 1.7 \cdot \rho_{air} \cdot n^3 \cdot (2R)^4 \cdot L \cdot \frac{1}{\mathrm{Re}^{0.15}} \qquad \mathrm{Re} = \frac{(2R) \cdot \pi \cdot n \cdot \delta}{\nu_{air}}$$

Air gap between rotor cylinder and stator bore: δ

- *Example:* C-Fiber sleeve:
  140°C: air mass density: $\rho_{air} = 0.826\,\mathrm{kg/m^3}$, kinematic viscosity $\nu_{air} = 26.5 \cdot 10^{-6}\,\mathrm{m^2/s}$
  cylinder radius & length $R = 50$ mm, $L = 100$ mm, $v_u = 200$ m/s, $\delta = 10$ mm

$$n = v_u / (2\pi R) = 636 / s = 38200 / \min$$

$$\mathrm{Re} = \frac{(2 \cdot 0.05) \cdot \pi \cdot 636 \cdot 0.01}{26.5 \cdot 10^{-6}} = 75400 > 1000$$

Air friction losses:

$$P_{fr} = 1.7 \cdot 0.826 \cdot 636^3 \cdot 0.1^4 \cdot 0.1 \cdot (75400)^{-0.15} = \underline{\underline{670}}\ \mathrm{W}$$

Losses at inverter operation

- Voltage: $U_N \sim n \cdot N_s \cdot k_w \cdot d \cdot l_{Fe} \cdot B$: high $n \sim f$, low $N_s \sim 1/f$
- Motor inductance rather small: $L \sim \mu_0 \cdot N_s^2 \cdot l_{Fe} \sim 1/f^2$
- High fundamental frequency: e. g. n = 200000 /min, two-pole motor: f = 3.3 kHz, high switching frequency $f_T \approx 5f$
- Current ripple amplitude rather big : $I_T \sim U_T/(f_T \cdot L) \sim f$ (U_T: PWM voltage ripple amplitude at f_T)

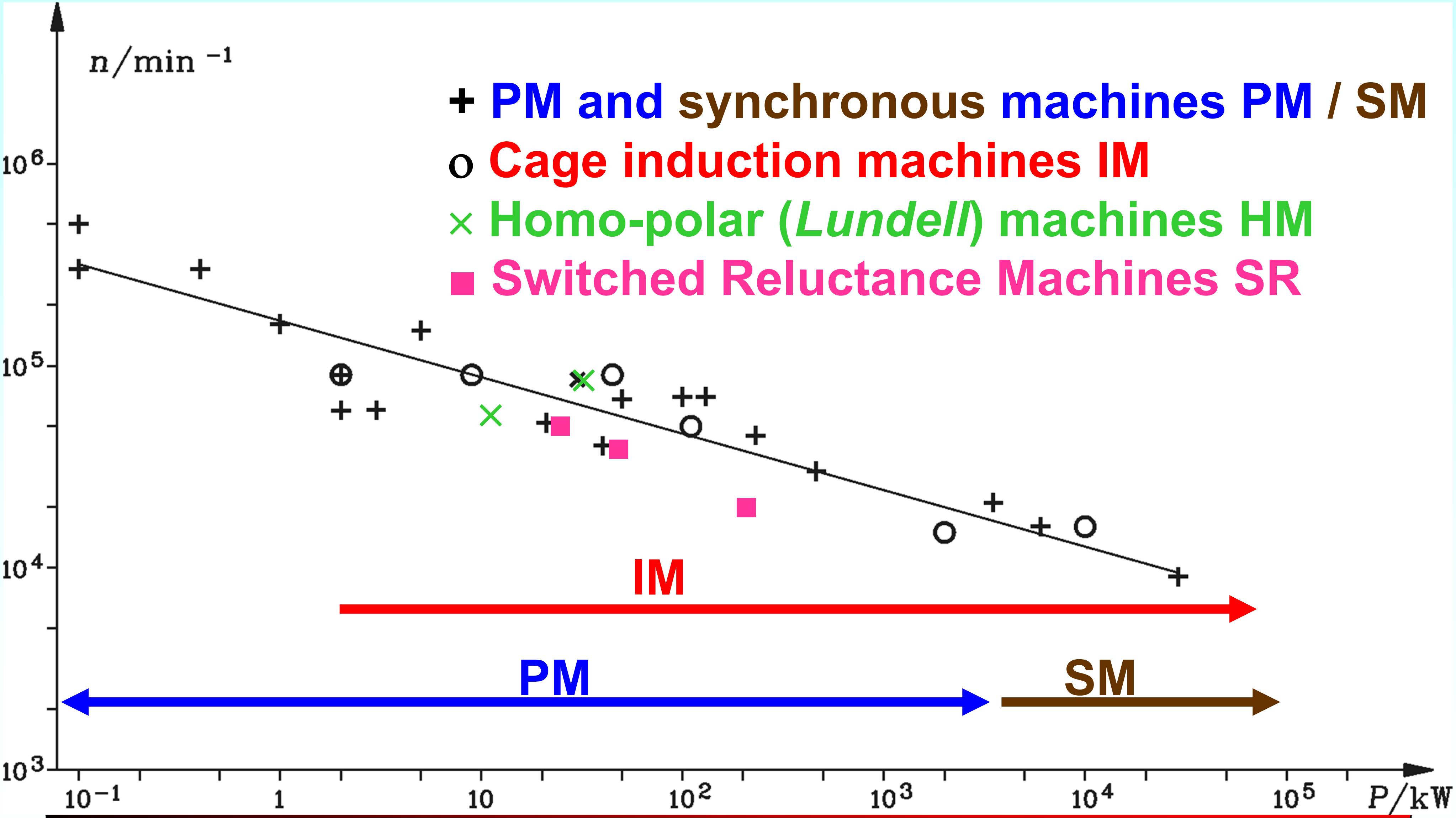
Need: Low current harmonics to reduce additional losses

a) Very high switching frequency f_T

b) 3-level-inverter

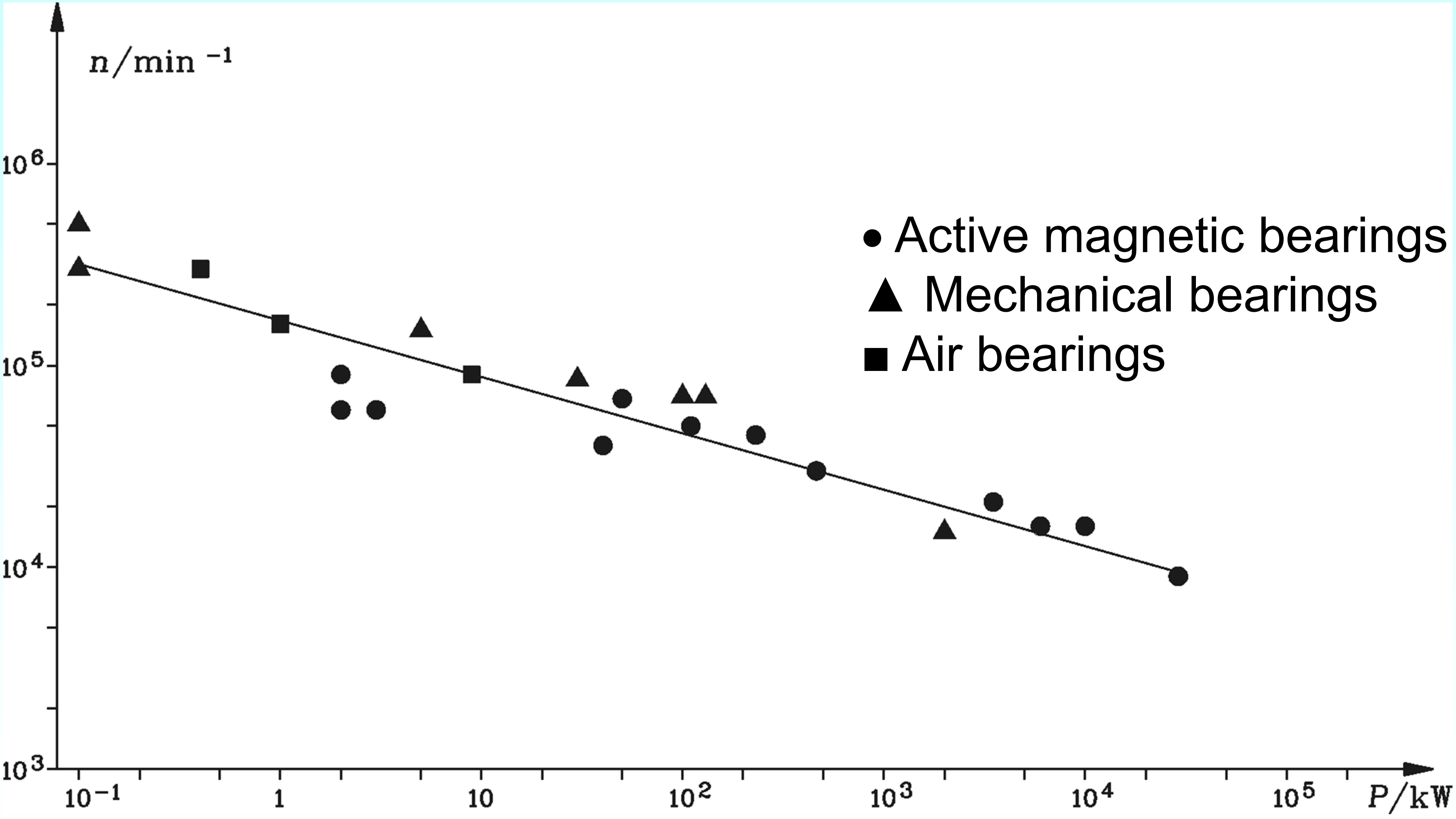
c) (Active) output sine wave filter

Types of High Speed Motors



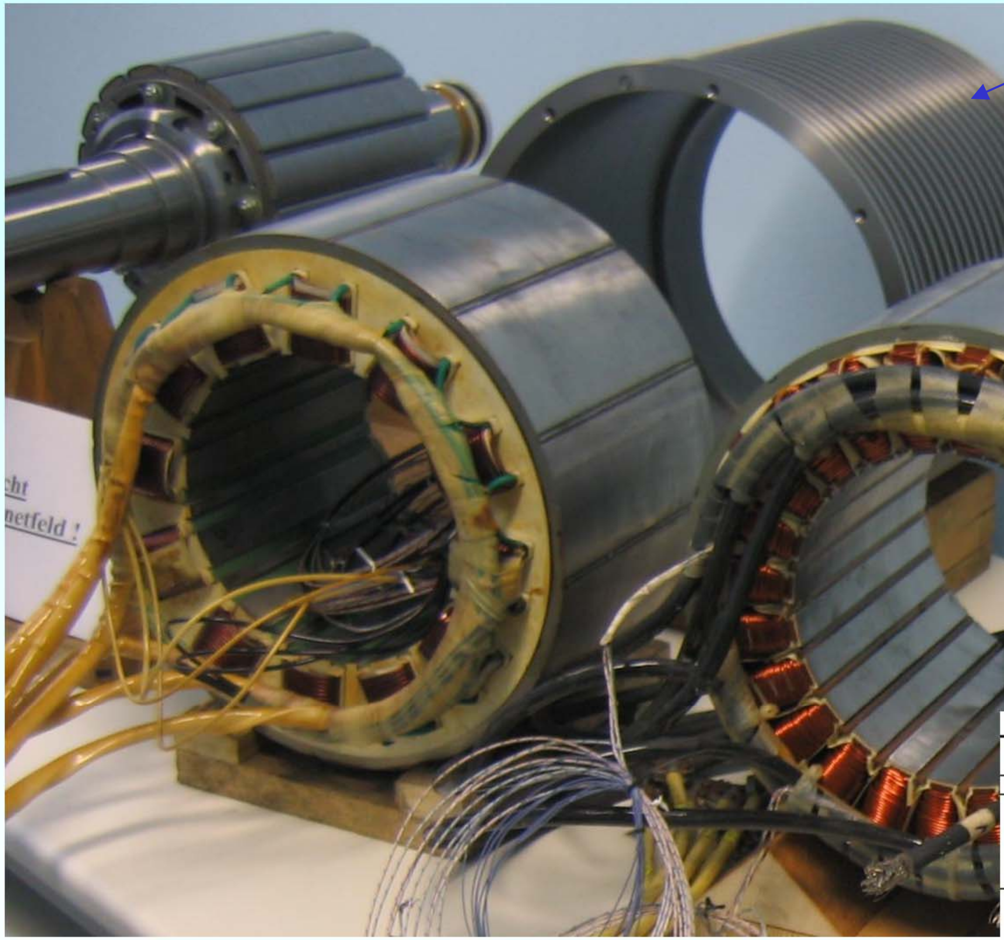
Types of bearings for high speed drives

taken from published data



- ● Active magnetic bearings
- ▲ Mechanical bearings
- ■ Air bearings

Water jacket cooling

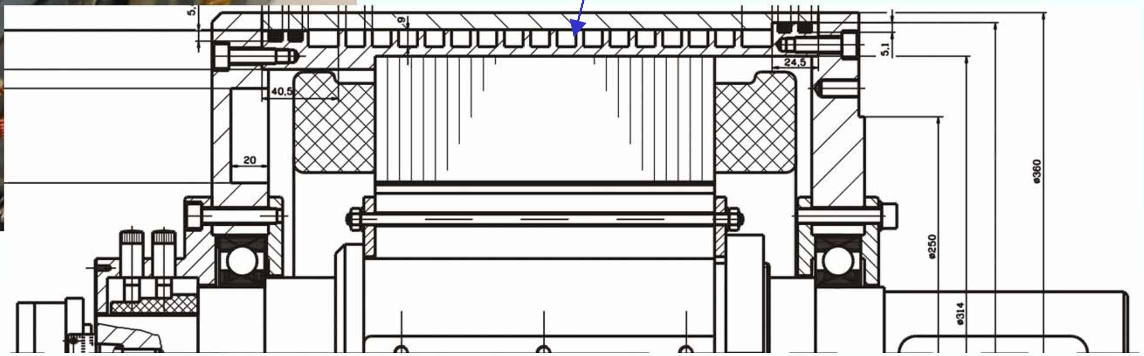


spiral water channel

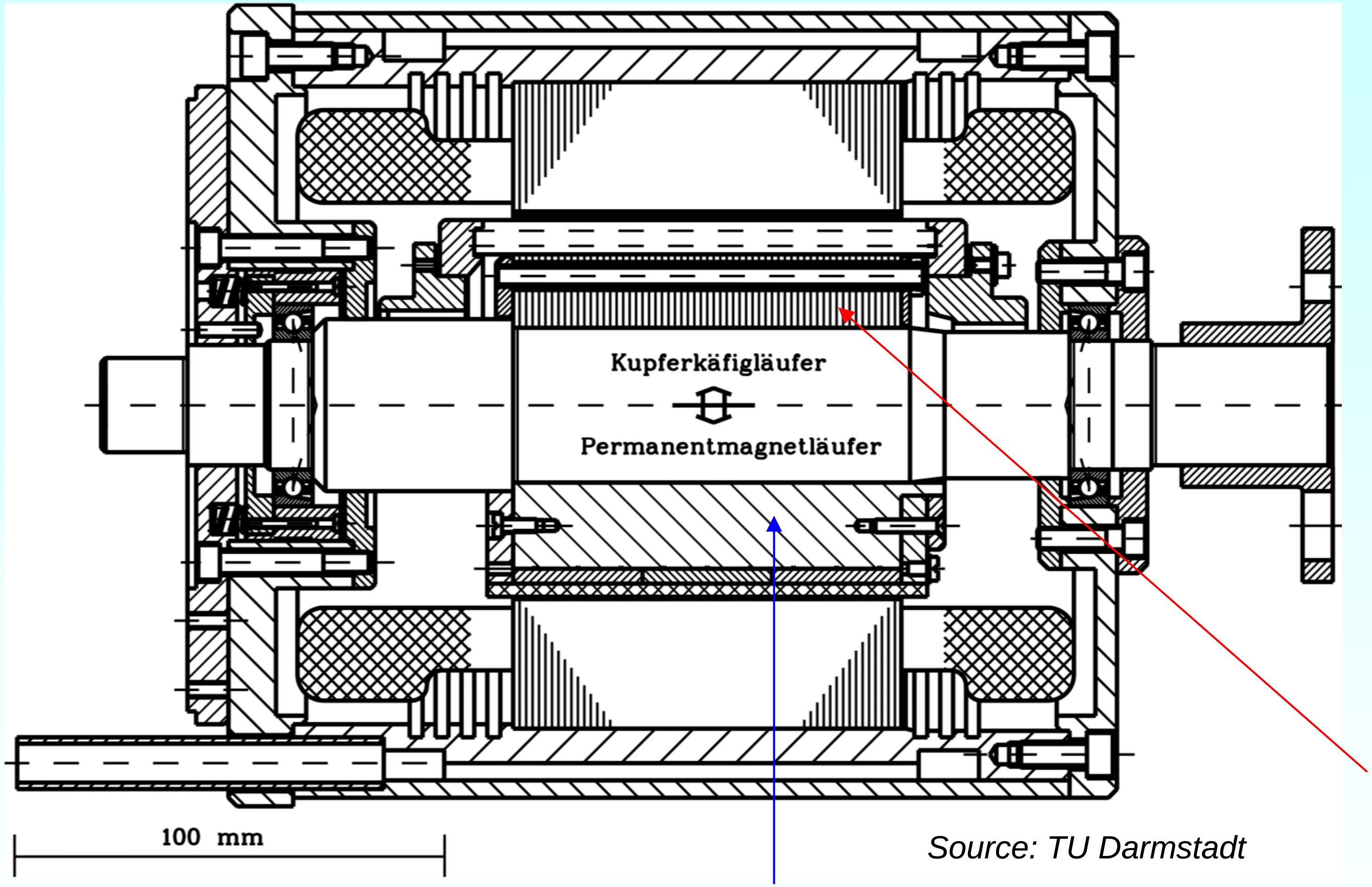
Source: TU Darmstadt

-Spiral water channel in the stator housing
- Closed machine, but intensive stator cooling
- Good for PM machines, where rotor losses are low

spiral water channel



Example: High speed motors 30 kW, 24 000 /min



Source: TU Darmstadt

- Cross section of four pole high speed AC motor 30 kW, 24 000 /min
- AC induction and PM synchronous alternatives
- Identical stator with 36 slots, two-layer winding and water jacket cooling.
- Upper half: Induction machine with copper squirrel cage rotor
- Lower half: Synchronous machine with surface-mounted Sm_2Co_{17}-Magnets and glass fibre bandage, d_r = 90 mm, 113 m/s

Hi-Speed Rotor: PM-Synchronous vs. Induction Principle

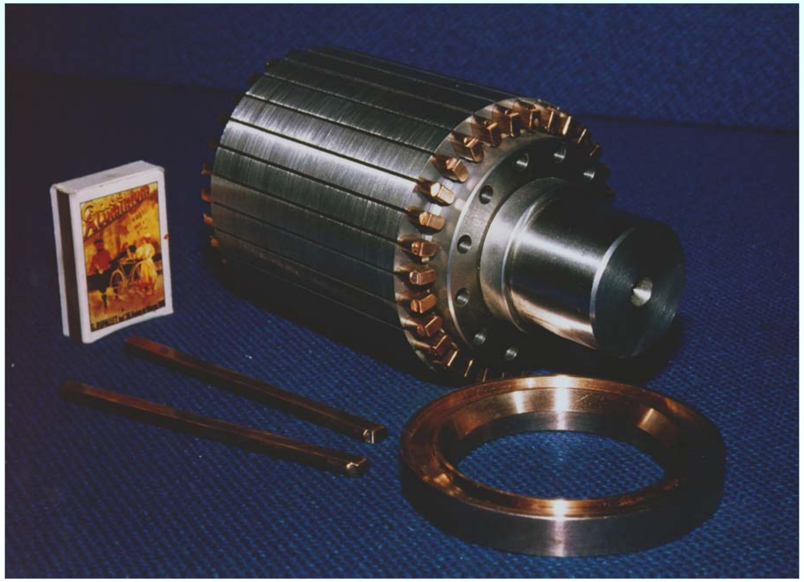
AC-rotor: 24 000/min

30 kW, $d_{si} = l_{Fe}$ = 90 mm

$$P_N /(d_{si}^2 l_{Fe}) = 41.15\mathrm{kW/dm}^3$$

4-poles PM-rotor, laminated yoke, BEFORE the pressing on of the fibre-glass sleeve

Source: TU Darmstadt



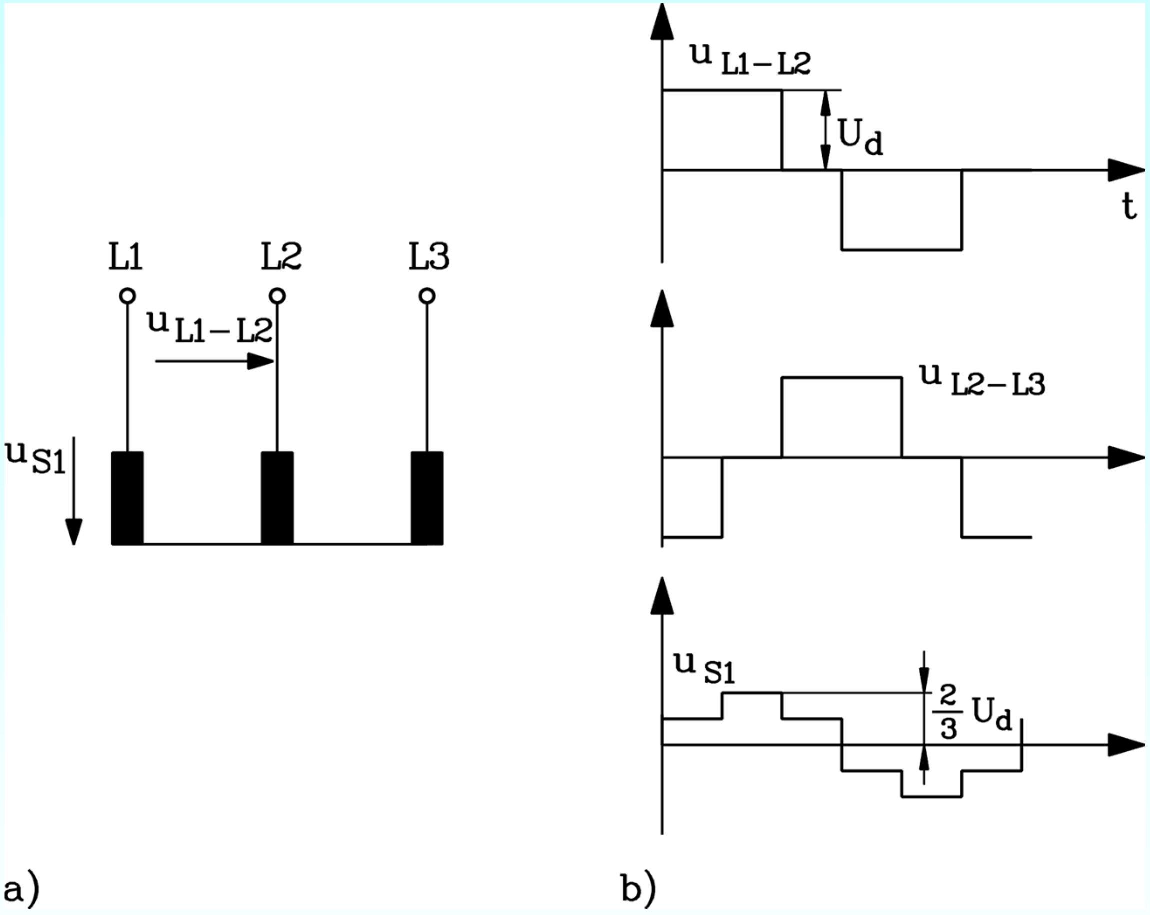
4-poles induction copper cage-rotor with oval bars:

Mass/bar: 23 grams

Centrifugal force/bar: 0.6 tons

Voltage harmonics at six-step operation

- **Inverter output phase voltage:** $u_{S1} - u_{S2} = u_{L1-L2}; \quad u_{S2} - u_{S3} = u_{L2-L3}; \quad u_{S1} + u_{S2} + u_{S3} = 0;$



we get: $u_{S1} = \dfrac{2u_{L1-L2} + u_{L2-L3}}{3}$

- Block shaped line-to-line voltage, expanded as *FOURIER*-series:

$$u_{LL}(t) = \sum_{k=1,-5,7,..}^{\infty} \hat{U}_{LL,k} \cdot \cos(k \cdot \omega_s t)$$

$$k = 1 + 6g, \qquad g = 0, \pm 1, \pm 2, ...$$

$\Rightarrow$ ***k* = 1, -5, 7, -11, 13, ...**

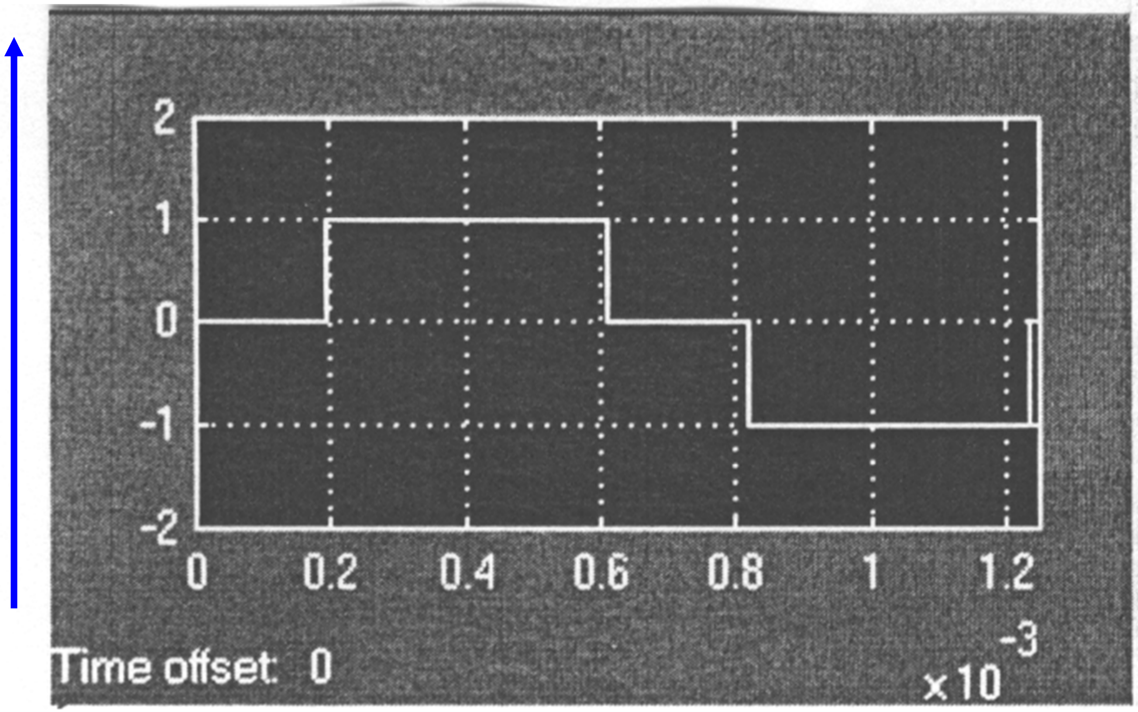
$$\hat{U}_{LL,k} = \frac{2}{\pi}\sqrt{3}\frac{U_d}{k}\left(= \frac{4}{\pi}\frac{U_d}{k}\sin(k\pi/3)\right)$$

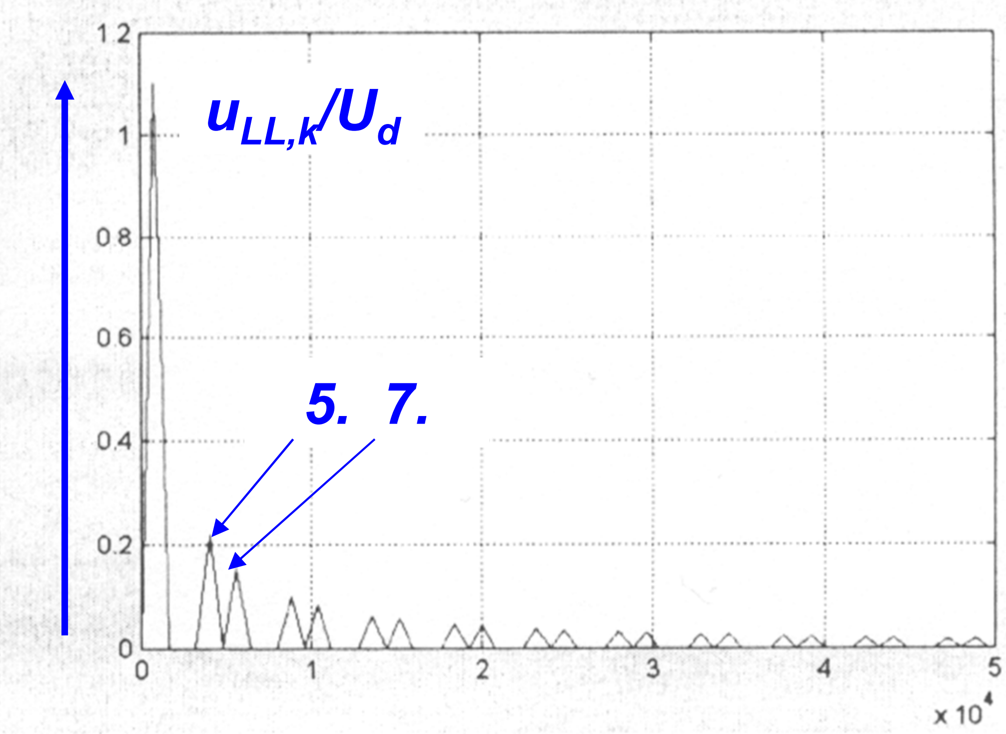
Electrical machine is fed with a blend of harmonic voltages of different amplitude, frequency and phase angle. Only fundamental (ordinal number k = 1) is desired. Voltage harmonics (|k| > 1) cause harmonic currents in electric machine with additional losses, torque pulsation, vibrations and acoustic noise.

FOURIER-Spectrum of voltage harmonics: Six-step operation (= Block voltage)

k	1	-5	7	-11	13
$\hat{U}_{LL,k}/\hat{U}_{LL,1}$	1	-0.2	0.14	-0.1	0.08

u_{LL}/U_d





$u_{LL,k}/U_d$

5. 7.

f_s = 800 Hz

t / s

f / Hz

Harmonic voltage systems

k = 1:

$$u_{U1}(t)=\hat{U}_1\cdot\cos(\omega t)$$

$$u_{V1}(t)=\hat{U}_1\cdot\cos(\omega t-2\pi/3)$$

$$u_{W1}(t)=\hat{U}_1\cdot\cos(\omega t-4\pi/3)$$

U – V – W
Positive sequence system

k = 5 ⇒ k = -5:

$$u_{U5}(t)=\hat{U}_5\cdot\cos(5\omega t)$$

$$u_{V5}(t)=\hat{U}_5\cdot\cos(5(\omega t-2\pi/3))=\hat{U}_5\cdot\cos(5\omega t+2\pi/3)$$

$$u_{W5}(t)=\hat{U}_5\cdot\cos(5(\omega t-4\pi/3))=\hat{U}_5\cdot\cos(5\omega t+4\pi/3)$$

U – W – V
Negative sequence system

- **General rule:** Positive and negative systems occur alternatively: Ordinal number *k* has a positive or negative sign: *k* =+1, -5, +7, -11, +13, …

$$\boxed{\begin{aligned}u_{Uk}(t)&=\hat{U}_k\cdot\cos(k\omega t)\\u_{Vk}(t)&=\hat{U}_k\cdot\cos(k\omega t-2\pi/3)\\u_{Wk}(t)&=\hat{U}_k\cdot\cos(k\omega t-4\pi/3)\end{aligned}}$$

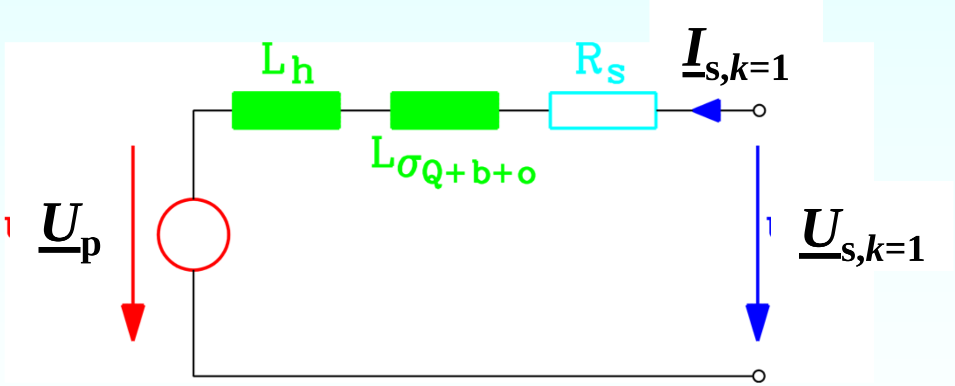
$$\boxed{k=1+6g}\qquad\boxed{g=0,\pm1,\pm2,...}$$

Current harmonics in PM synchronous machines at inverter operation

- *k^{th} voltage harmonic* $\underline{U}_{s,k}$ (frequency: $k \cdot f_s$) causes current harmonic per phase $I_{s,k}$ in the stator winding.
- These current harmonics cause fast rotating magnetic fundamental field waves ($\nu = 1$, pole count $2p$) in the air gap with clockwise or counter-clockwise rotation:

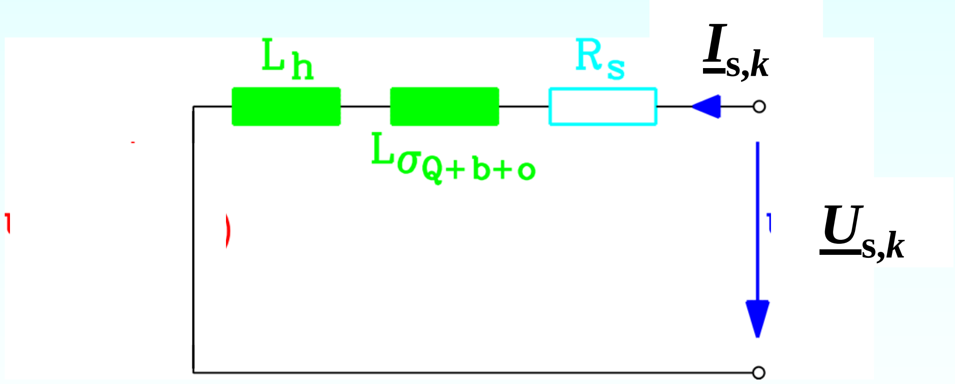
$$\boxed{n_{syn,k} = k \cdot f_s / p}$$

Equivalent circuit of the PM synchronous machine for the fundamental frequency f_s



Iron losses neglected in the equivalent circuit

Equivalent circuit of the PM synchronous machine for a harmonic frequency $k.f_s$



$$\boxed{I_{s,k} \approx \frac{U_{s,k}}{\sqrt{R_s^2 + (k\omega_s)^2 \cdot (L_{s\sigma} + L_h)^2}}}$$

Example: Harmonic currents at six step operation

Data: 8-pole motor, n_N = 1500/min, U_{pN} = 119 V, X_{dN} = 4.35 Ω, DC-link voltage of inverter: U_d = 540 V, Operation at elevated speed n = 3000/min, f = 200 Hz

Phase voltage, r.m.s.: $U_k = \hat{U}_{k,LL}/\sqrt{6} = \frac{4}{\pi}U_d\,\frac{1}{k}\sin(\frac{k\pi}{3})/\sqrt{6}$

Current harmonics: $|k|>1: I_k = \frac{U_k}{\sqrt{R^2+(k\omega L_d)^2}} \approx \frac{U_k}{|k|2\pi f L_d} \qquad k = 1+6g, \quad g = 0,\pm1,\pm2,...$

k	U_k / V	I_k / A
1	243	5.64
-5	42	0.97
7	30	0.49
-11	19.1	0.20
13	16.2	0.14

- For k = 1 the fundamental equivalent circuit has to be taken for calculating the current.

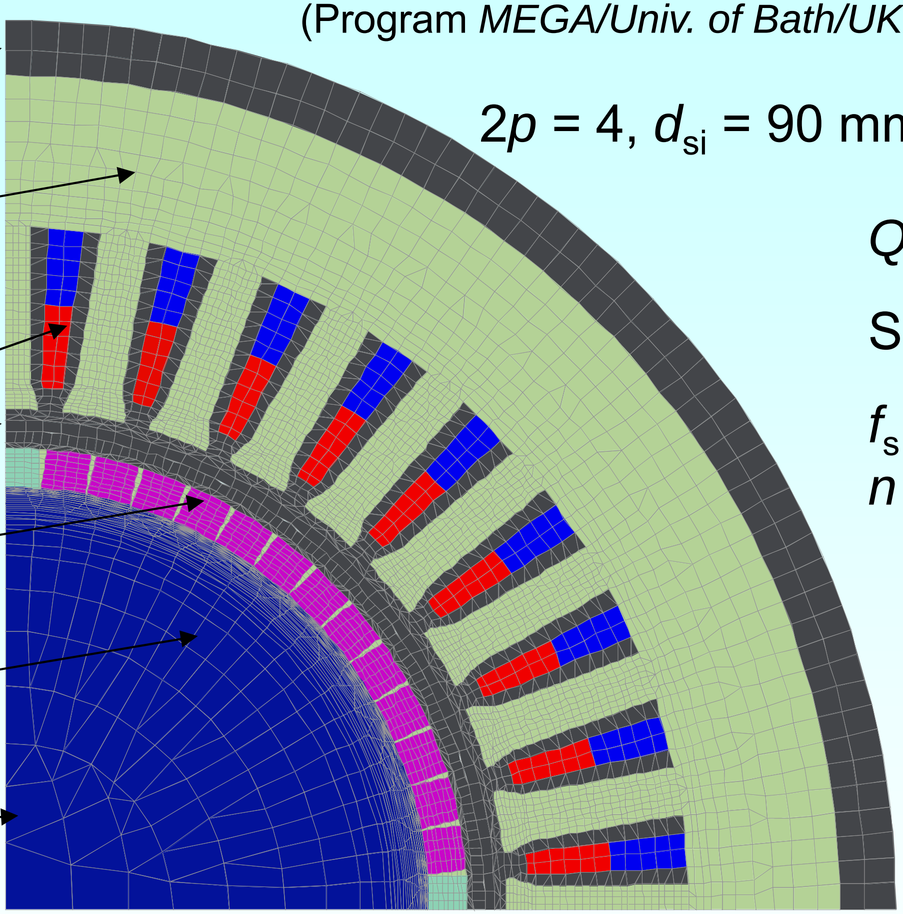
$$\hat{U}_{k,LL} = \frac{4}{\pi}U_d\,\frac{1}{k}\sin(\frac{k\pi}{3})\bigg|_{k=1} = \frac{4}{\pi}540\cdot\sin(\frac{\pi}{3}) = 595V$$

$$U_{k=1} = 595/\sqrt{6} = 243V = U_{\max}$$

$$I_{s,k=1} = \frac{\sqrt{U_{\max}^2 - U_{pN}^2\cdot\left(\frac{n}{n_N}\right)^2}}{\frac{n}{n_N}X_{dN}} = \frac{\sqrt{243^2 - 119^2\cdot\left(\frac{3000}{1500}\right)^2}}{\frac{3000}{1500}\cdot 4.35} = 5.64A$$

Numerical Calculation of Additional Losses in Rotor

(Program *MEGA/Univ. of Bath/UK*)



Water jacket cooling

Stator core stack

2-layer, 3-phase winding

Air gap and GF bandage

Rotor magnets

Rotor yoke

Rotor shaft

$2p = 4$, $d_{si} = 90$ mm, $l_{Fe} = 90$ mm,

$Q = 36$ stator slots,

Sm_2Co_{17}-magnets

$f_s = 800$ Hz,
$n = f_s/p = 24000$/min

Results: **solid** rotor yoke: too high losses with block-voltage supply

Remedy: laminated yoke or sine wave filter necessary

Example: PM rotor iron losses - six step voltage operation

- Loss calculation in massive rotor iron for k^{th} current harmonic, neglecting influence of losses in the segmented SmCo-magnets:

- Analytical calculation of rotor iron losses due to k-th current harmonic: $P_{\mathrm{Fe,r},k}$

- Numerical calculation with finite element method (program *MEGA/Univ. of Bath/UK*)

k	n_{rel} 1/min	current loading A/m	$P_{Fe,r,k}$ / W analytical	$P_{Fe,r,k}$ / W MEGA numerical
-5	144000	4610	120	114
7	144000	2338	31	29
-11	288000	971	12	11
13	288000	633	5	5
		Total:	168	159

$$n_{syn} = 24000/\min$$

$$n_{rel} = |k-1| \cdot n_{syn}$$

Losses in solid PM rotor iron back and in massive magnet rings

Six step voltage inverter operation, 24000/min, full load

- Losses in for k^{th} current harmonic in rotor iron: $P_{Fe,r,k}$
  in solid SmCo-magnet rings: $P_{M,k}$

k	n_{rel} 1/min	Harmonic current loading A/m	$P_{M,k}$ / $P_{Fe,r,k}$ / W analytical	P_{Mk} / $P_{Fe,r,k}$ / W numerical MEGA
-5	144000	4610	92.6 / 4.1	64 / 13
7	144000	2338	24.1 / 1.1	16.7 / 3.3
-11	288000	971	4.2 / 0.13	3.0 / 0.5
13	288000	633	2.2 / 0.07	1.3 / 0.2
		Total	123.1 + 5.4 = 128.5	85 + 17 = 102

Result:

The rotor iron back is shielded by the self-field of the eddy currents in the magnet shell

Losses in laminated rotor iron back & in massive magnet rings

- Losses in laminated rotor iron ($\kappa_{Fe} = 0$) : $P_{Fe,r,k}$, in magnet rings: $P_{M,k}$

  Six step voltage inverter operation, 24000/min, full load

- *Magnet rings are shielding the rotor, so not much difference between solid and laminated rotor iron back!*

k	n_{rel} 1/min	Harmonic current loading A/m	P_{Mk} / $P_{Fe,r,k}$ / W analytical	P_{Mk} / $P_{Fe,r,k}$ / W numerical MEGA
-5	144000	4610	101.8 / 0	81 / 0
7	144000	2338	26.0 / 0	21 / 0
-11	288000	971	4.5 / 0	3.5 / 0
13	288000	633	2.3 / 0	1.5 / 0
		Total	134.6	107

Example: Losses in segmented magnets

- Only losses in magnets P_{Mk} considered,
  rotor iron back is assumed laminated: $\kappa_{Fe} = 0$
- Six step voltage inverter operation, 24000/min, full load

k	n_{rel} 1/min	Harmonic current loading A/m	Harmonic flux density mT	P_{Mk} / W analytical	P_{Mk} / W numerical MEGA
-5	144000	4610	19.9	13.8	14.2
7	144000	2338	10.2	3.6	3.6
-11	288000	971	4.1	2.4	2.5
13	288000	633	2.9	1.2	1.1
				Total: 21	22

- *Magnet segments have only small eddy current losses. They cannot shield the rotor iron. There is a big difference in losses between solid rotor (big iron losses) and laminated rotor iron back (small losses)!*

Example: **Additional losses of high speed PM synchronous motor**

Result:

Segmented surface magnets and laminated rotor iron back yield lowest additional rotor losses.

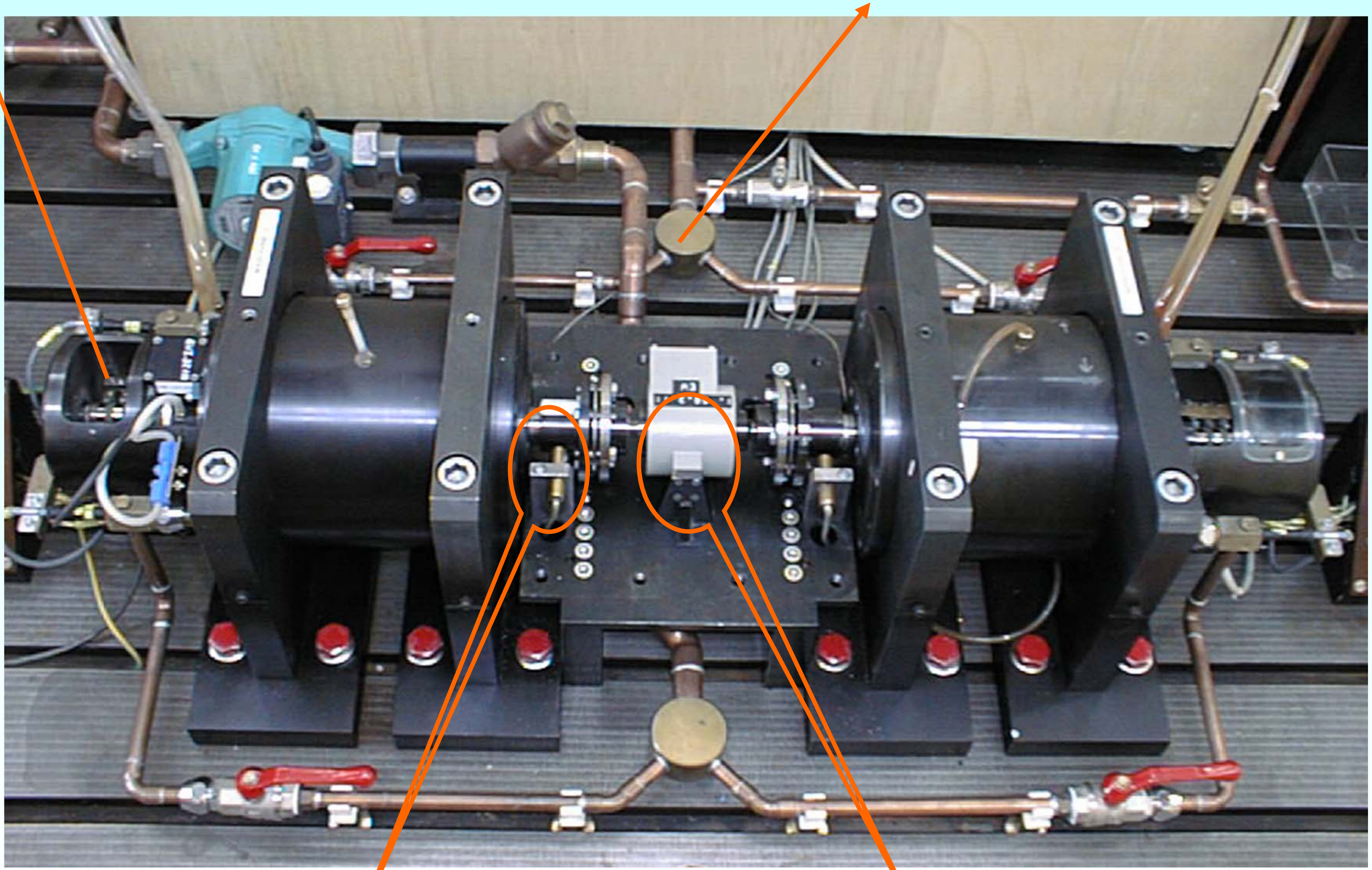
- Built prototypes: 30 kW, 24000/min, 4 poles, segmented magnets

(Source: PhD thesis Lu Tong: TU Darmstadt)

Calculated additional rotor losses at six-step voltage inverter operation:

Solid rotor iron, magnetic shell:	128.5 W:	η = 95.66%
Laminated rotor, magnetic shell:	134.6 W:	η = 95.65%
Laminated rotor, segmented magnets:	21 W:	η = 96.06%

Motor Test Bench for 24 000/min, 30 kW



rotor temperature measurement

water cooling circulation

PM-Synchronous
Test-Motor

- 30 kW
- 24 000 RPM
- 12 Nm
- 800 Hz

IM
Induction
machine
Load

speed measurement

torque transducer

Source: TU Darmstadt

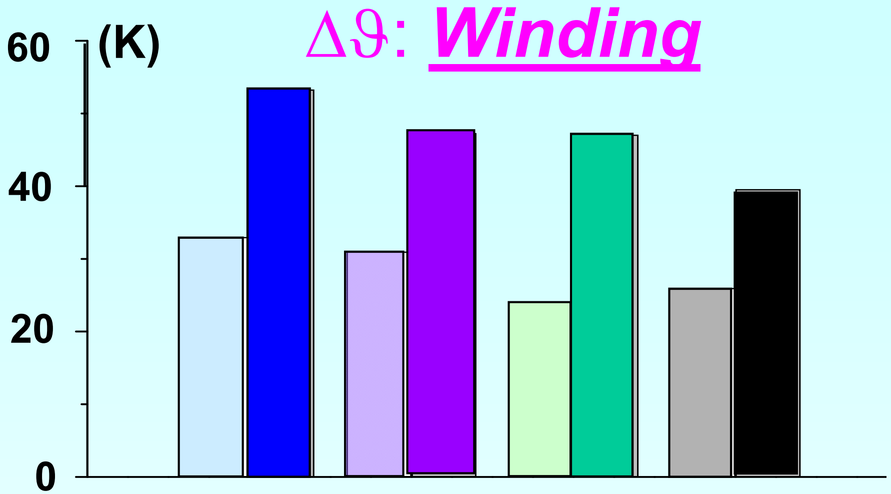
Additional losses at high speed

	current harmonics	additional losses
Sine wave voltage operation:	**few**	**small**
Block current operation:	**rather big**	**rather big**
Block voltage operation:	**considerable**	**considerable**

Permanent magnet synchronous motor: magnets $h_M = 3.5$ mm, $d_B = 2.8$ mm, $\delta = 0.7$ mm
Massive rotor iron, segmented magnets (so no shielding effect for rotor iron)

Fundamental voltage, current, power factor	*Ideal voltage sine wave operation*	*Voltage six step inverter operation*
$U_{s,(1)}$ (line to line), I_s, $\cos\varphi_{(1)}$	301 V, 67.4 A, 0.89	309 V, 71.9 A, 0.84
Motor output power P_{out}	30 144 W	30 159 W
P_{Fe}	560 W	560 W
P_{fr}	440 W	440 W
$P_{Cu,s}$	430 W	522 W
$\boldsymbol{P_{M+Fe,r}}$	**50 W**	**520 W**
Efficiency	95.3 %	93.65 %

PM-Synchronous Motor: Measured Heating in Stator Windings and Rotor Magnets

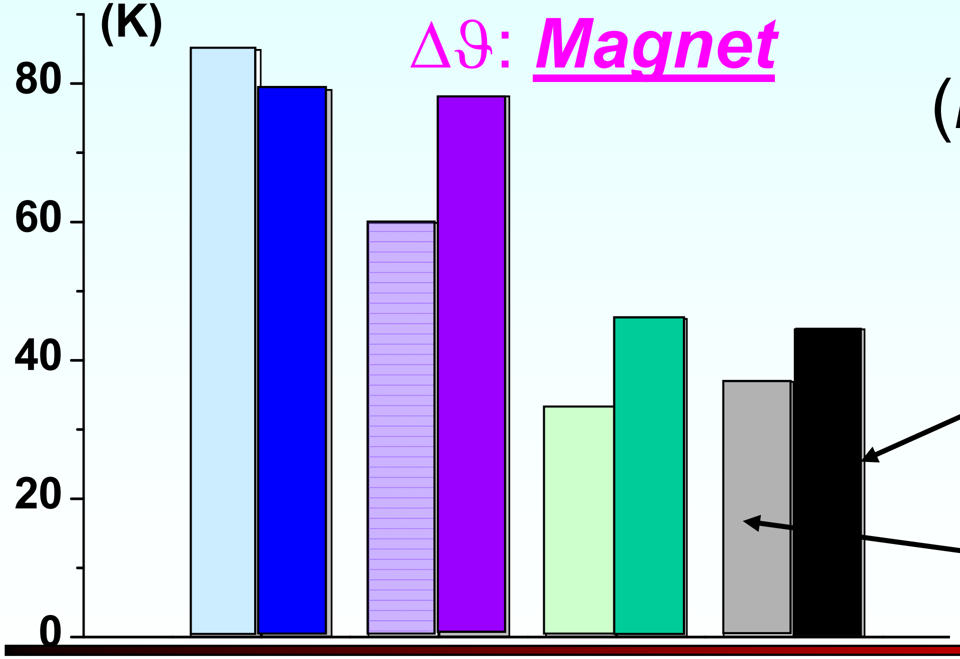

$\Delta\vartheta$: ***Winding***

PWM with output choke **+**

Block voltage **++**

PWM with sinusoidal filter **+++**

Sinusoidal converter **+++**


$\Delta\vartheta$: ***Magnet***

(*PWM* without filter: $\Delta\vartheta$ too high)

at 30 kW, 24 000 /min

under no-load, 24 000 /min