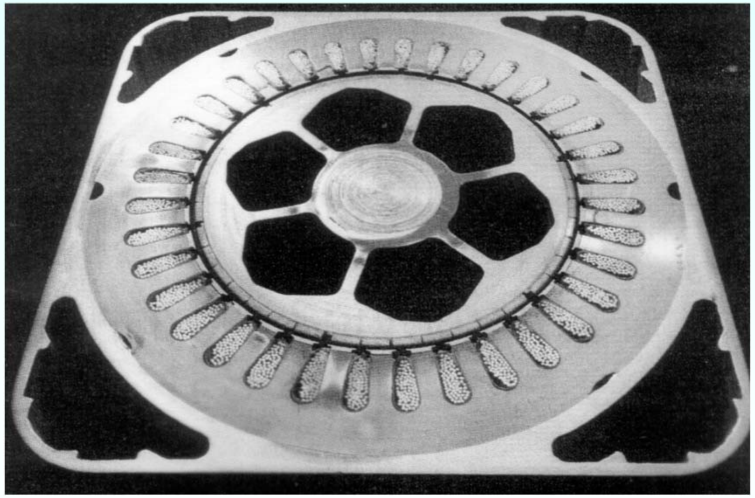
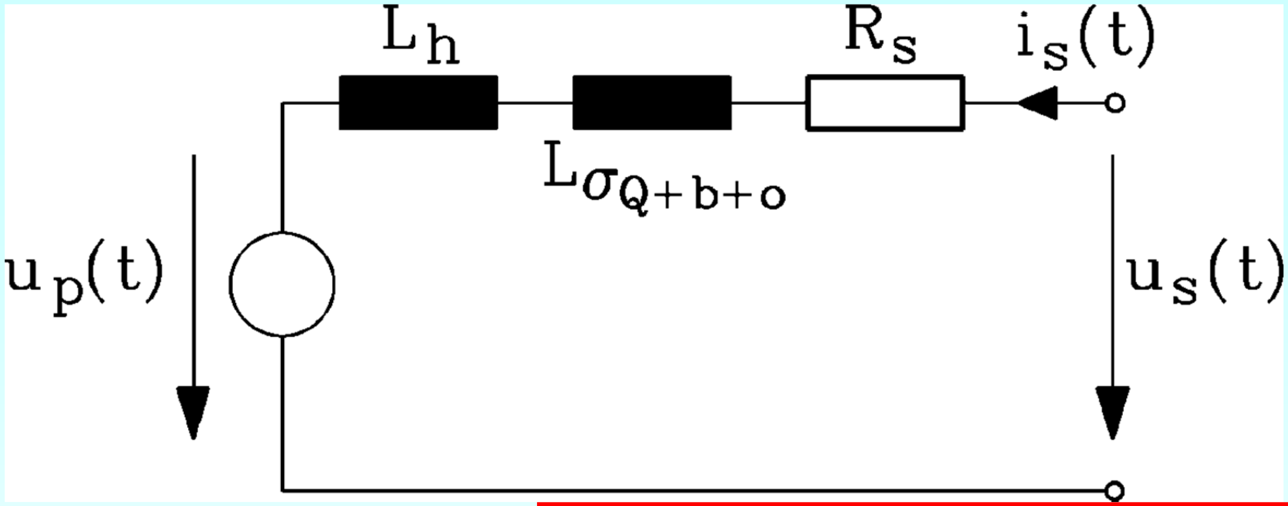
1. Permanent magnet synchronous machines as “brushless DC drives”

1.1.5 Equivalent circuit of PM synchronous machine



Source: Siemens AG, Germany

Equivalent circuit of PM synchronous machines



Voltage equation per phase:

$$u_s(t) = R_s \cdot i_s(t) + L_d \frac{di_s(t)}{dt} + u_p(t)$$

- back EMF $u_p(t)$
- self-induced voltage ~ di_s/dt
- resistive voltage drop
- voltage from feeding inverter: $u_s(t)$

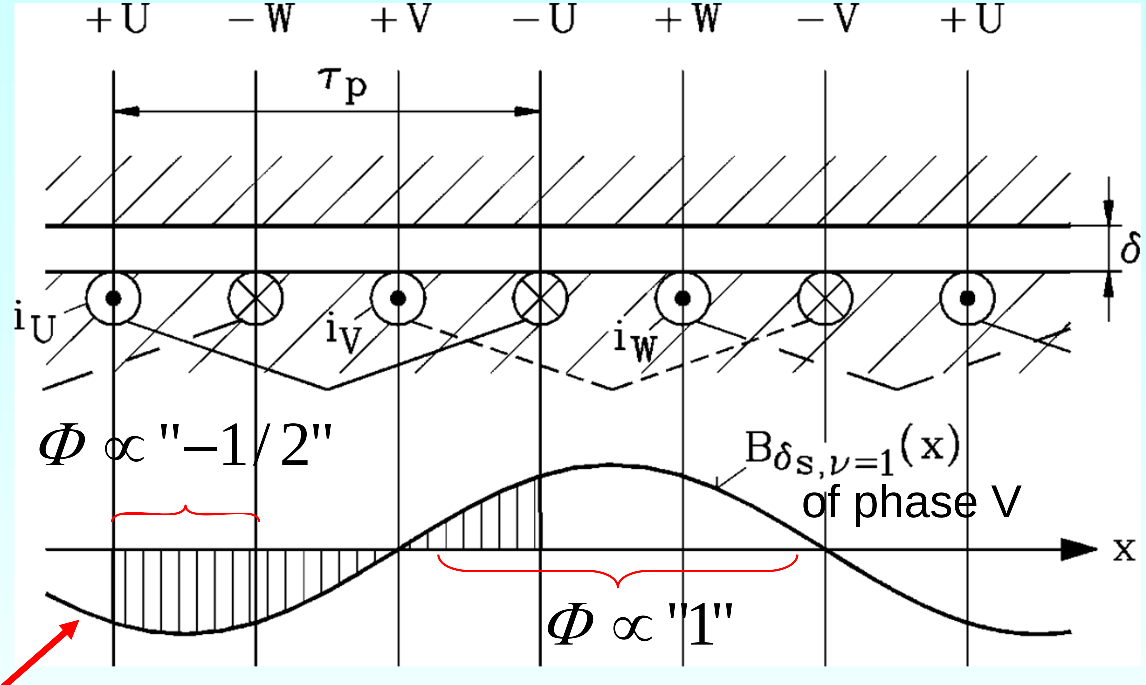
Synchronous inductance: $L_d = L_h + L_\sigma$

main and leakage inductance:

Leakage:

Q: slot; b: overhang, o: harmonic

Air gap flux density excited by the distributed stator winding



Resulting air-gap:

$$\delta_{res} = \delta + h_B + h_M$$

mechanic air gap

bandage thickness

magnet height

Fundamental of this flux density induces back into stator winding, thus linking phases U, V, W, generating a self-inductance (main inductance) L_h, which is equal for all three phases (here is shown flux excited by phase V, linked with coil of phase U)

$$L_h = \frac{U_{s,s}}{\omega_s \cdot I_s} = \mu_0 \cdot (N_s \cdot k_{ws})^2 \cdot \frac{2m_s}{\pi^2 \cdot p} \cdot \frac{\tau_p l_{Fe}}{\delta_{res}}$$

Fundamental air gap flux linkage between the three phases – Consideration for phase U

$$\Psi_U = \Psi_{UU} + \Psi_{UV} + \Psi_{UW} = L_{Uph} i_U + M_{UV} i_V + M_{UW} i_W$$

$$\Psi_{self} = L_{ph} i \sim \Phi \qquad \Psi_{mutual} = M \cdot i \sim -\Phi / 2 \qquad M = -L_{ph} / 2$$

$$\Phi \propto "1" \qquad \Phi \propto "-1/2"$$

$$i_U + i_V + i_W = 0 \Rightarrow i_U = -i_V - i_W$$

$$\Psi_U = L_{Uph} i_U - L_{ph} i_V / 2 - L_{ph} i_W / 2 = (3/2) \cdot L_{ph} i_U = L_h i_U$$

$$\boxed{L_h = (3/2) \cdot L_{ph}}$$

Alternative derivation: Self inductance of the stator fundamental air gap field wave

Self induced voltage:

$$u_{s,s,\nu=1}(t) = \omega_s \cdot N_s \cdot k_{w1} \cdot \frac{2}{\pi} \cdot \tau_p l_{Fe} B_{\delta,\nu=1} \cdot \sin(\omega_s t) = \sqrt{2} \cdot U_{s,s} \cdot \sin(\omega_s t)$$

Fundamental stator field amplitude, excited by the stator sine current I_s:

$$B_{\delta,\nu=1} = \frac{\mu_0}{\delta_{res}} \cdot \frac{\sqrt{2}}{\pi} \cdot \frac{m_s}{p} \cdot N_s \cdot k_{w1} \cdot I_s$$

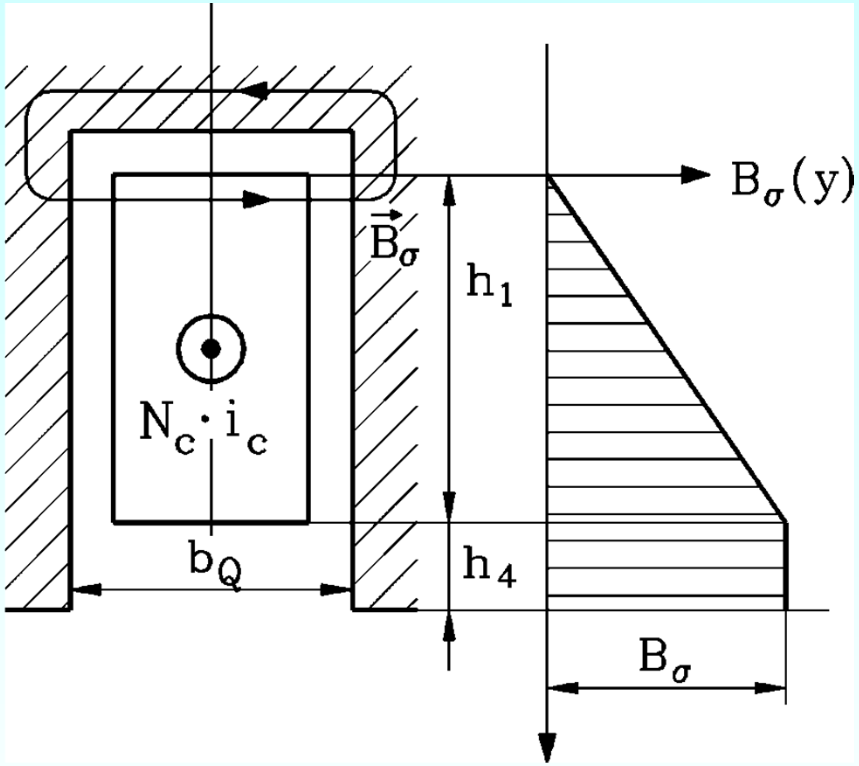
Fundamental stator field winding factor: $k_{w1} = k_{ws}$

Self inductance L_h of the fundamental stator field:

$$L_h = \frac{U_{s,s}}{\omega_s \cdot I_s} = \mu_0 \cdot (N_s \cdot k_{ws})^2 \cdot \frac{2 m_s}{\pi^2 \cdot p} \cdot \frac{\tau_p l_{Fe}}{\delta_{res}}$$

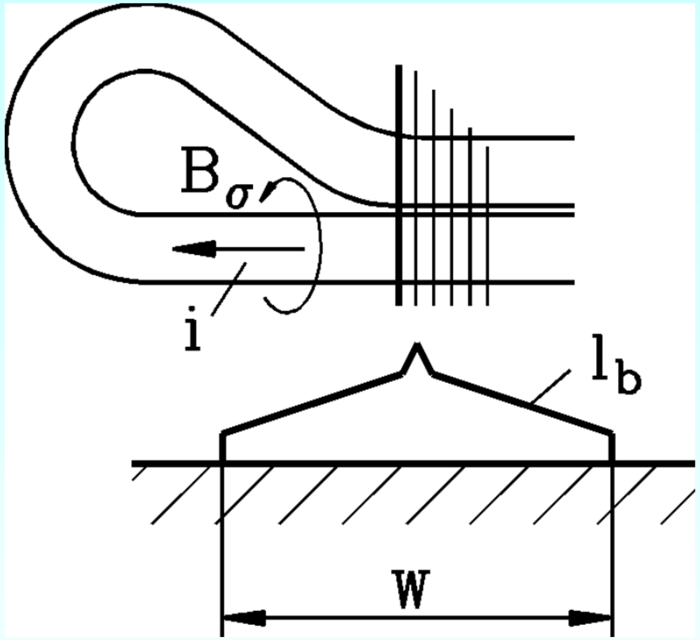
Schematic drawing of stray flux lines

Slot leakage flux



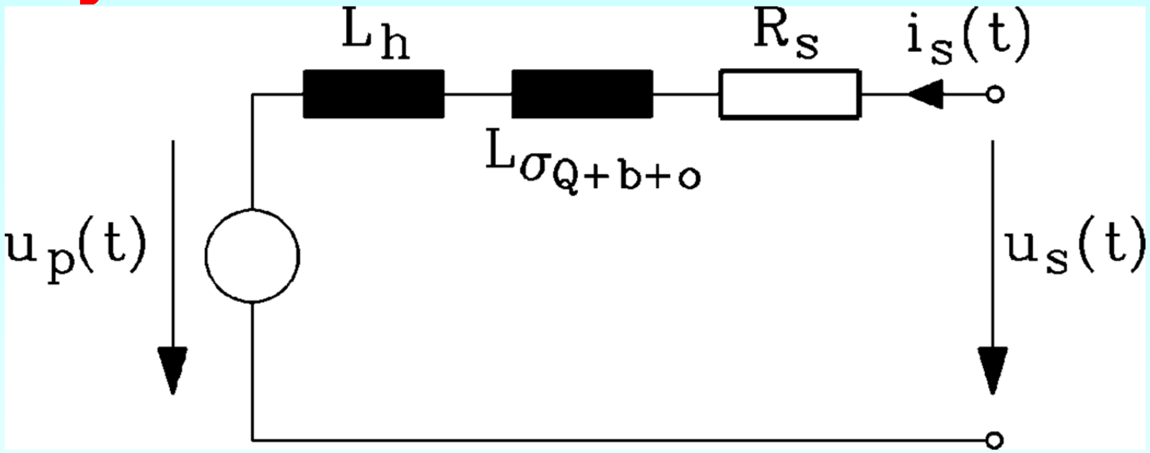
Slot leakage flux, rising linear from bottom to top of slot according to *Ampere´s* law

Winding overhang leakage flux



Leakage flux density of winding overhangs

Equivalent circuit per phase of synchronous PM machine

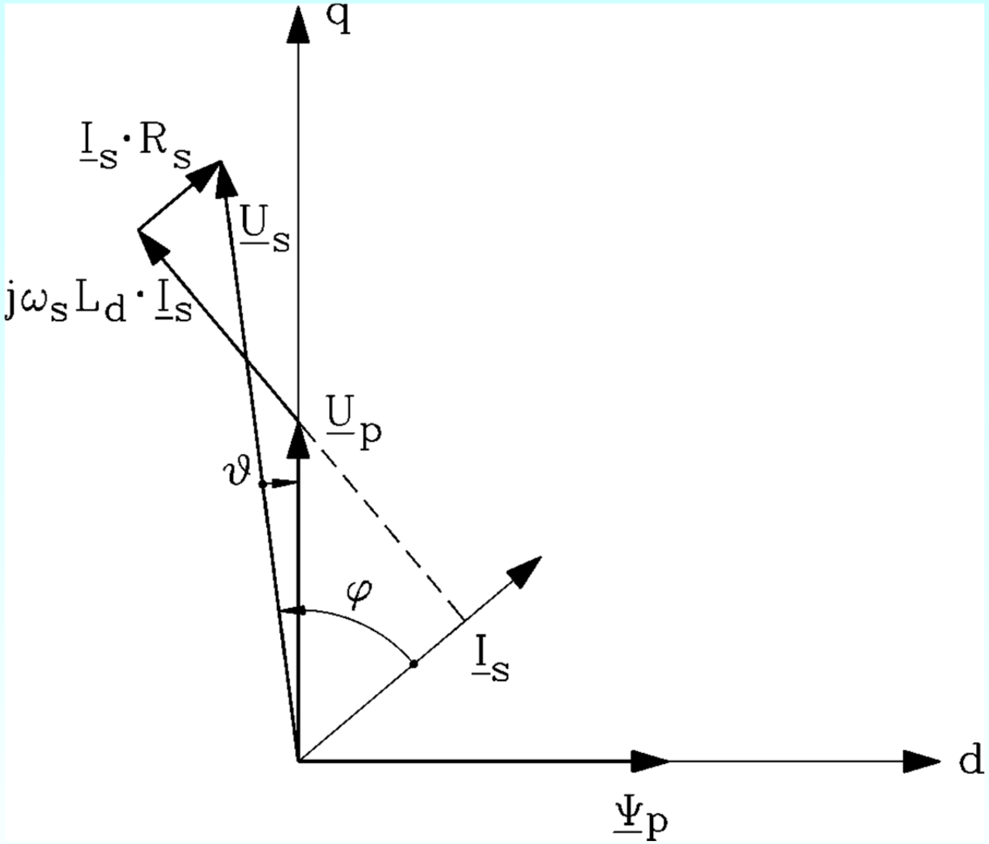


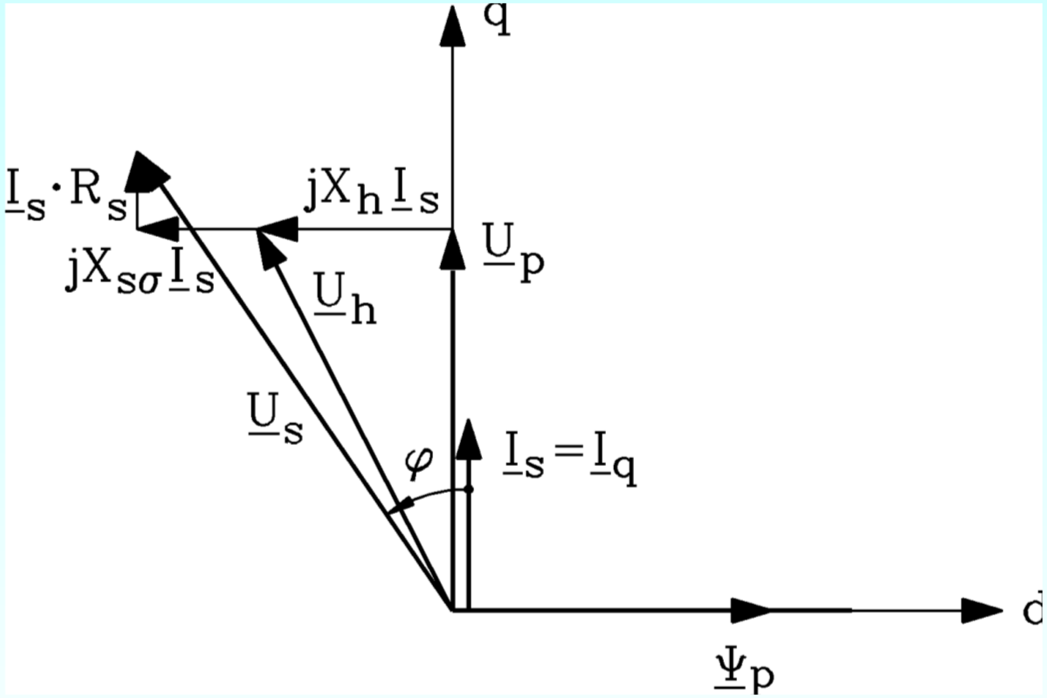
$$u_s(t) = R_s \cdot i_s(t) + L_d \frac{di_s(t)}{dt} + u_p(t)$$

- Considering only time fundamentals = use complex phasors $\underline{U}_s$, $\underline{U}_p$ and $\underline{I}_s$!
- Field oriented operation = current in phase with back EMF: ***q*-axis current I_q.**
- Surface magnets: inductivity for *d*- and *q*-axis identical ($L_d = L_q = L_s$)

$$\underline{U}_s = R_s \underline{I}_s + j\omega L_d \underline{I}_s + \underline{U}_p$$

Phasor diagram per phase of synchronous PM machine at operation with sinusoidal voltage and current



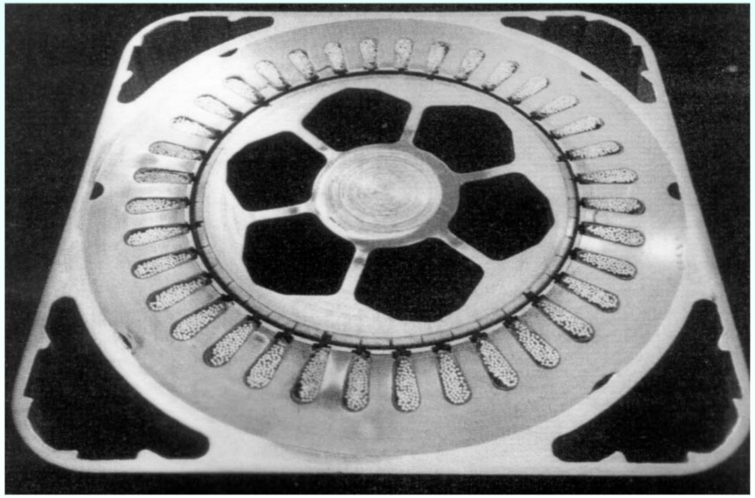


arbitrary current phase shift between back EMF U_p and phase current

field-oriented control = current **in phase** with back EMF U_p **("brushless DC drive")**

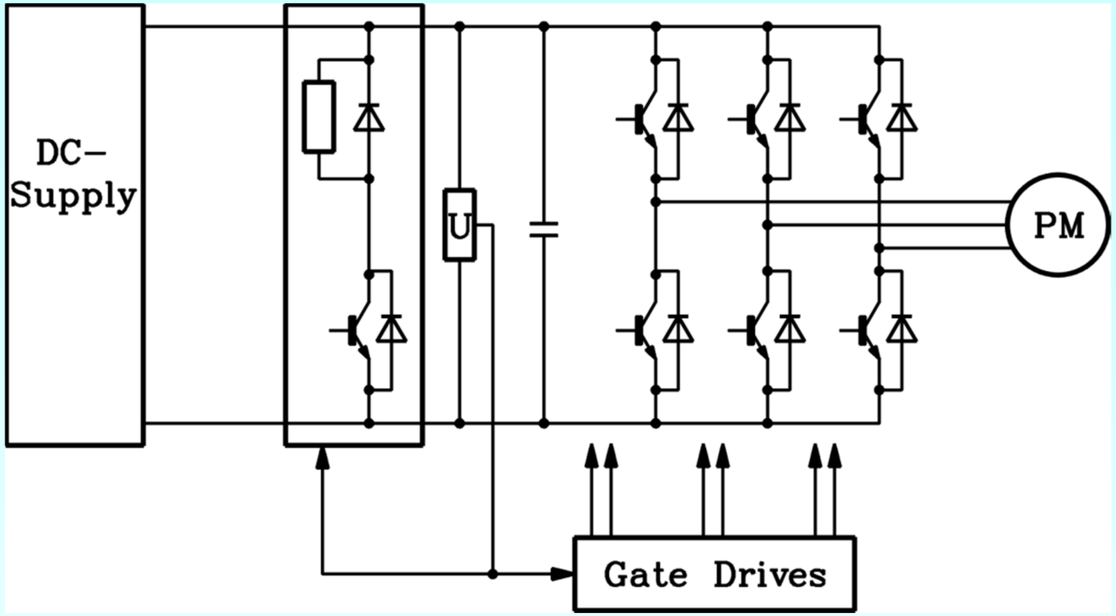
1. Permanent magnet synchronous machines as “brushless DC drives”

1.1.6 Stator current generation



Source: Siemens AG, Germany

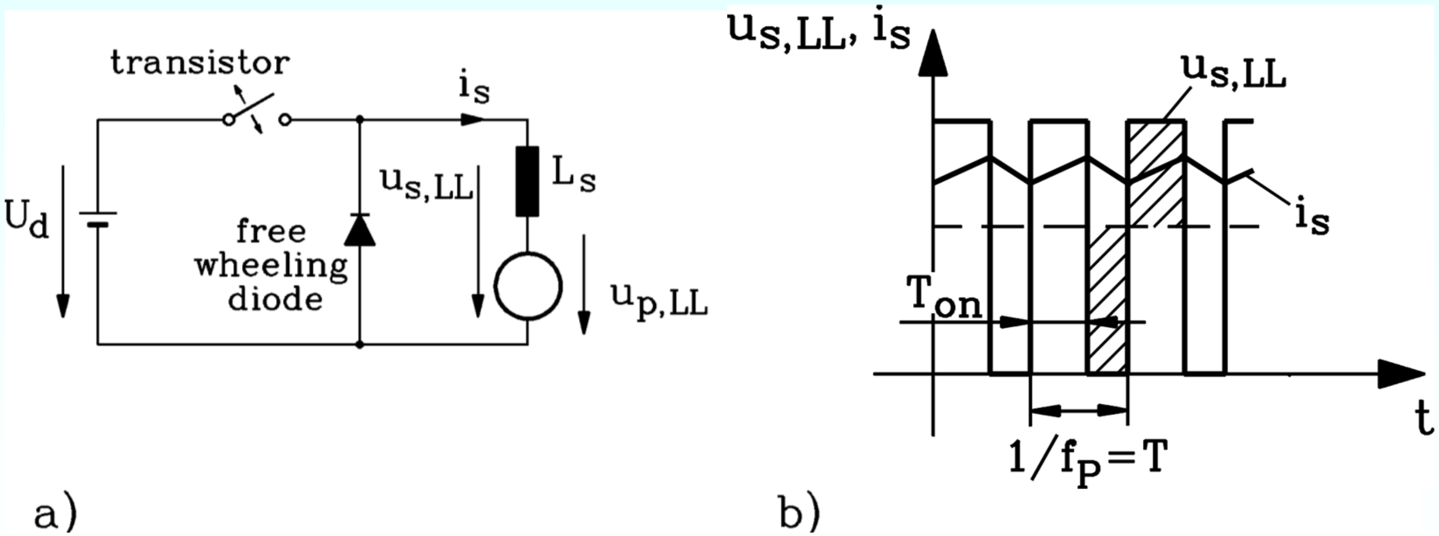
Stator current generation



DC link voltage source inverter with switching transistors and free-wheeling diodes

R_s neglected:

$$U_d - U_{p,LL} \approx L_s \cdot di_s / dt$$

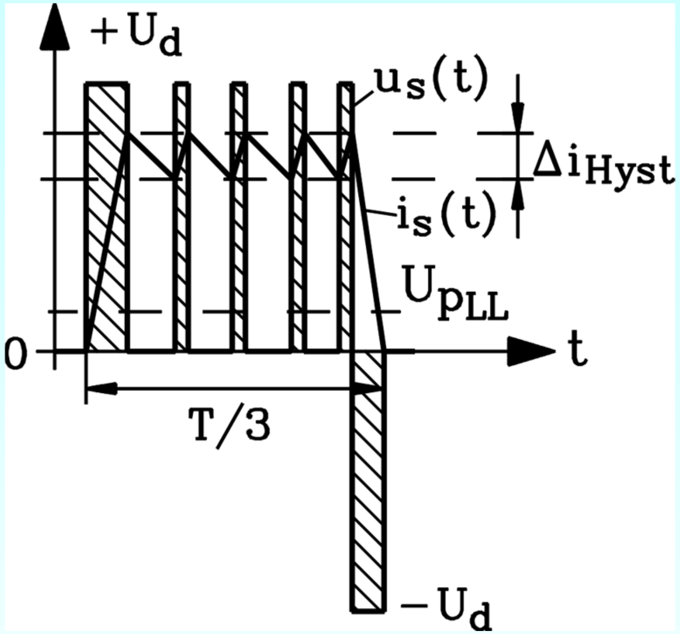


a) Equivalent switching scheme of DC link voltage source inverter, connected to the two phases with switching transistor and free-wheeling diode,

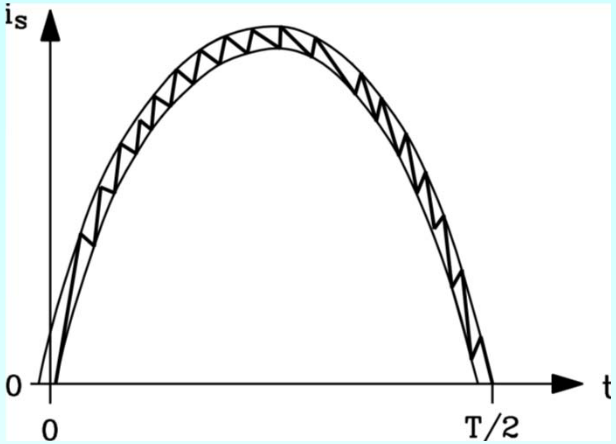
b) Current ripple and chopped inverter voltage

Hysteresis band current control

block commutation



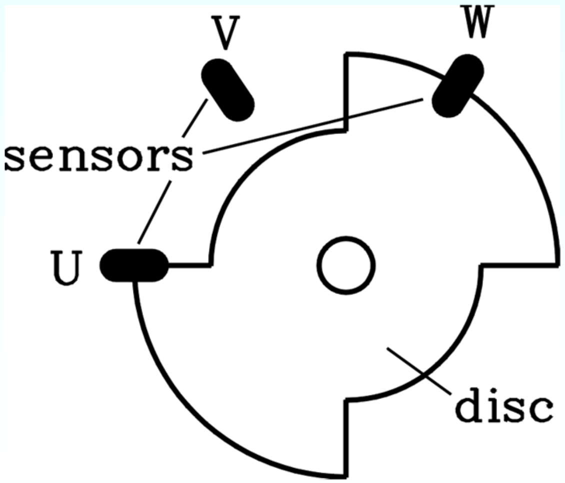
sine wave commutation



Shaping of current with hysteresis band

Current commutation from phase U to V etc.:

Determination of current phase shift (= firing angle) by encoder to get rotor position. Current shall be in phase with back EMF !



Block commutation: Six step encoder:

A rotor disc and three stator-fixed sensors U, V, W, spaced by 120°/p (p: number of pole pairs), are sufficient for rotor position sensing for block commutation (here: $2p = 4$)

Measuring rotor position for sine wave commutated synchronous PM machine

Source: Heidenhain, Traunreut,Germany

Optical incremental encoder,

to be mounted on motor non-drive shaft end

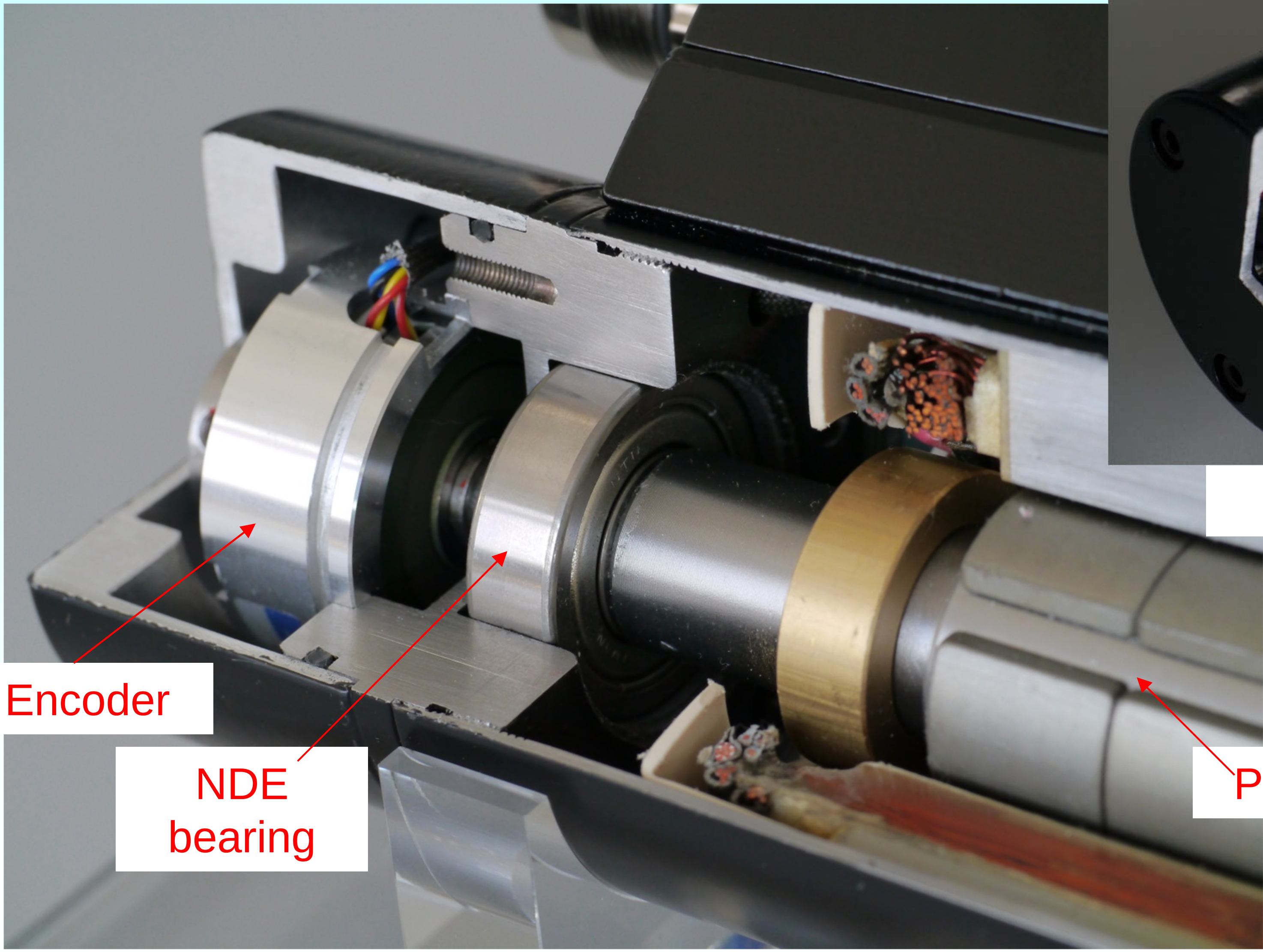
Rotor position must be known at every moment, as frequency might change at every moment, hence changing sine wave shape !

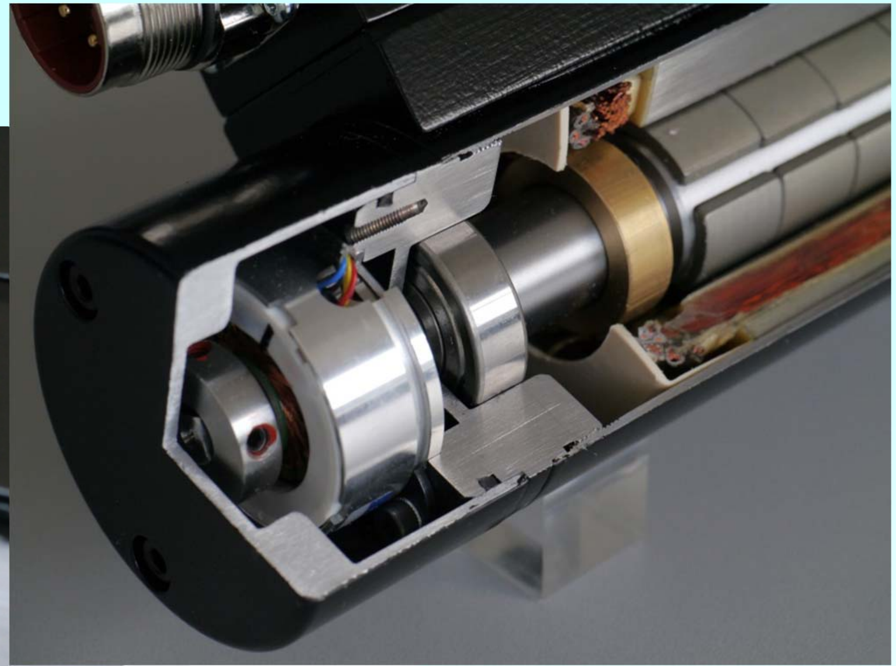
Position measurement:

a) Resolver: Continuous measurement of position (analogue electromagnetic device)

b) Incremental encoder: High resolution necessary (e.g. 1024 x 4 counts per revolution), hence optical sensors !

Rotor position encoder at NDE motor side





Source: Engel Elektroantriebe GmbH, Germany

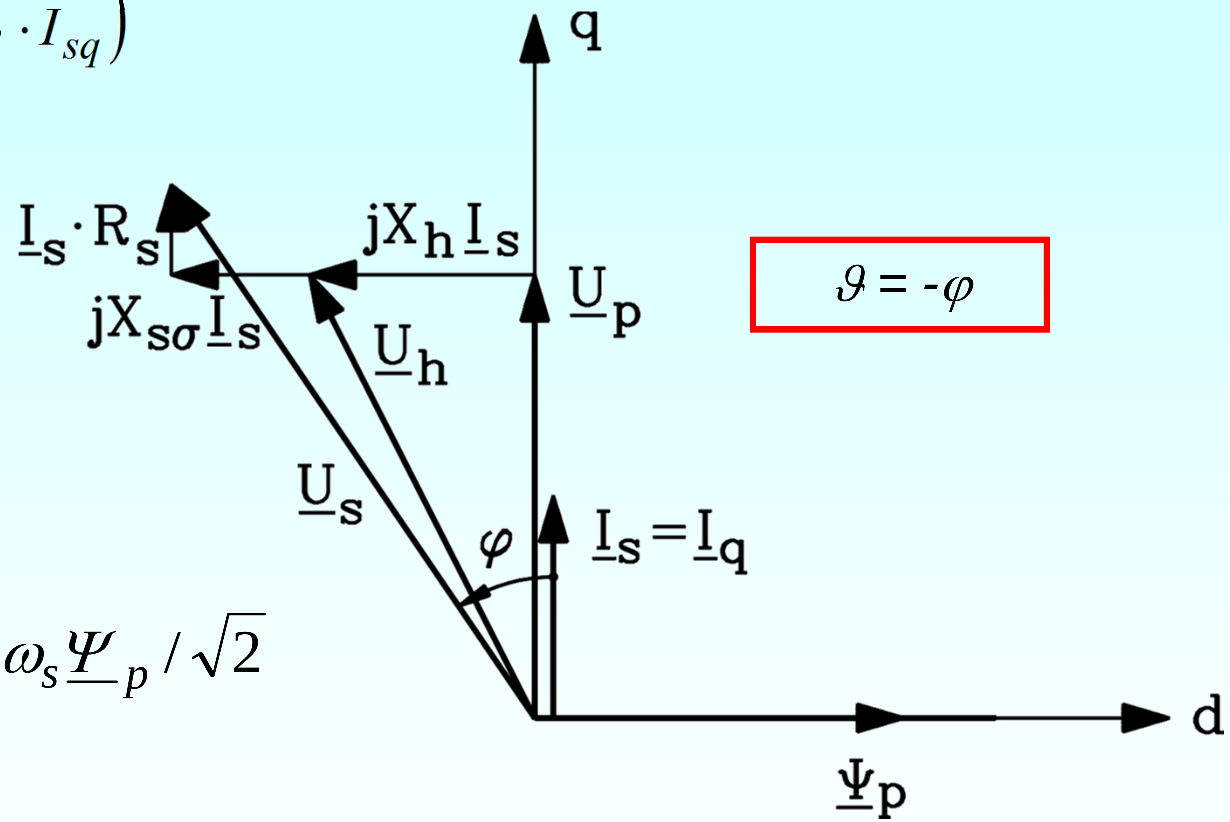
Inverter operation with *q*-current

At a given current we get the maximum possible torque at $I_s = I_{sq}, I_{sd} = 0$, if we do not have a reluctance torque component!

$$M_e = \frac{m_s}{\Omega_{syn}} \cdot \left(U_p \cdot I_{sq} + (X_d - X_q) \cdot I_{sd} \cdot I_{sq} \right)$$

No reluctance difference between *d*- and *q*-axis: $X_d = X_q$:

$$M_e = m_s \cdot U_p \cdot I_{sq} / \Omega_{syn}$$



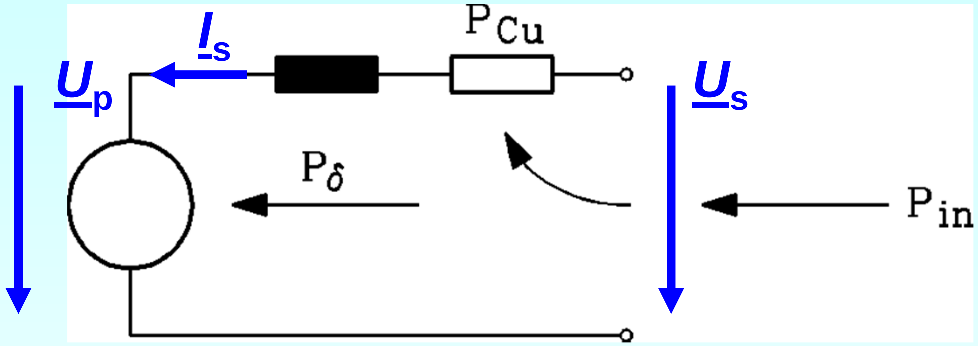
$\vartheta = -\varphi$

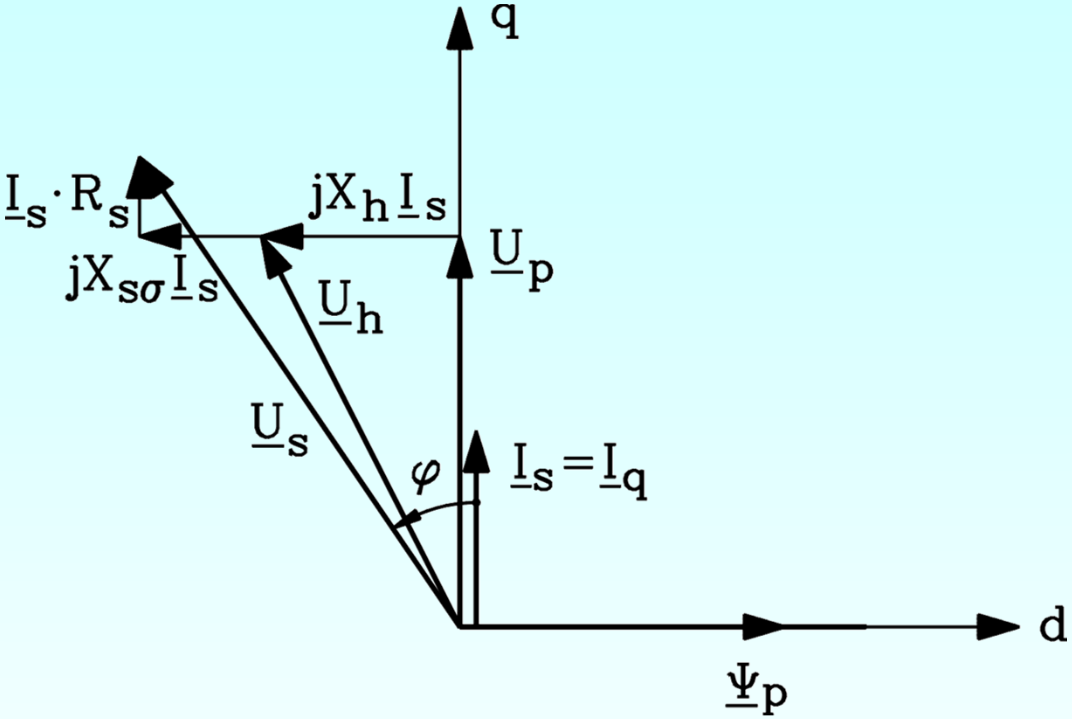
The torque is at $I_s = I_{sq}$ at maximum, because due to $L_d = L_q$ only the *q*-current I_{sq} produces a torque.

Back EMF of PM machine: $\underline{U}_p = j\omega_s \underline{\Psi}_p / \sqrt{2}$

$$M_e = p \cdot m_s \cdot \Psi_p \cdot I_{sq} / \sqrt{2}$$

Air gap power at *q*-current operation

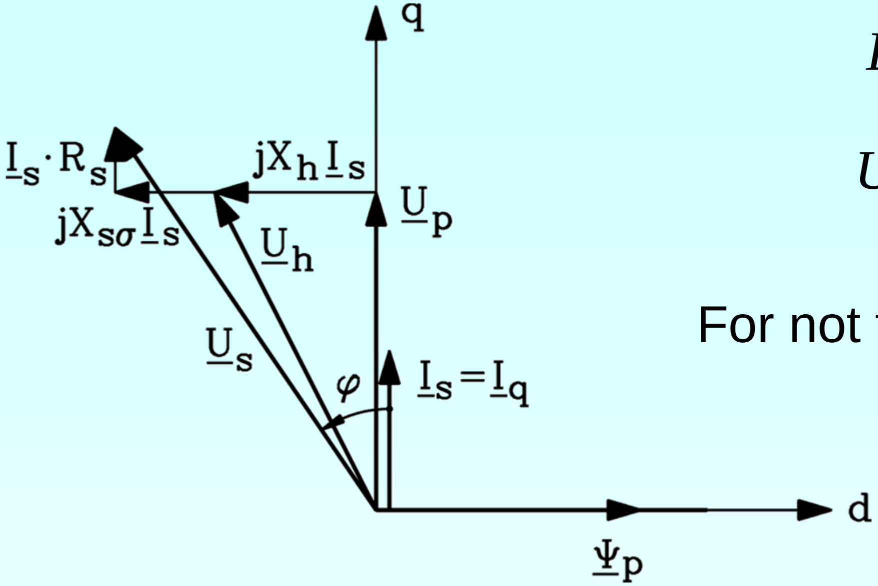




$$P_\delta = m_s \cdot U_p \cdot I_{sq} = M_e \cdot \Omega_{syn}$$

- At $\underline{I}_{sq}$-current operation current $\underline{I}_s$ and back EMF $\underline{U}_p$ are in phase.
- The air gap power is therefore at maximum.
- Hence also the torque is for a given current at maximum: Maximum Torque per Ampere MTPA

Inverter-operated PM synchronous machine



$$L_d = L_q :$$

$$U_s = \sqrt{(\omega_s L_q I_{sq})^2 + (R_s I_{sq} + \omega_s \Psi_p / \sqrt{2})^2}$$

For not too small stator angular frequency ω_s we get:

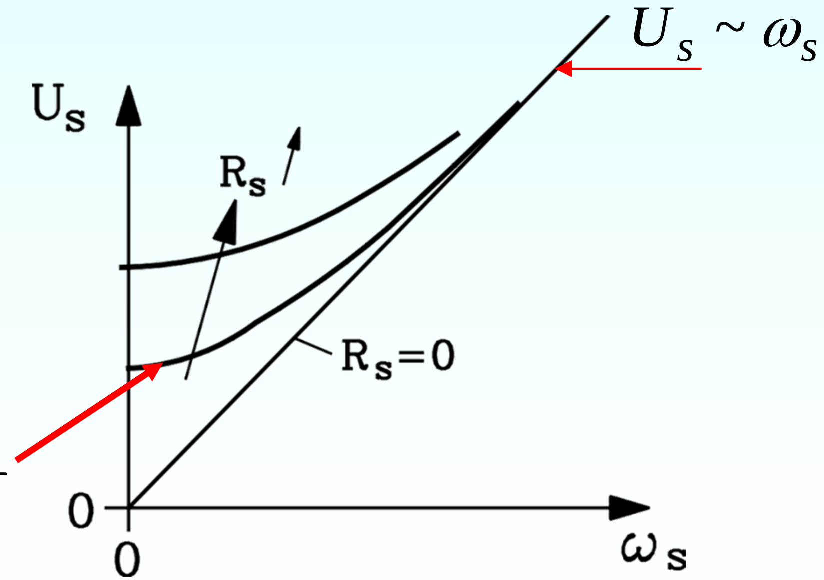
$$U_s = \omega_s \sqrt{L_q^2 I_{sq}^2 + (\Psi_p / \sqrt{2})^2}$$

- **Control law for the inverter**
  (similar to induction machine operation):

$$U_s \sim \omega_s$$

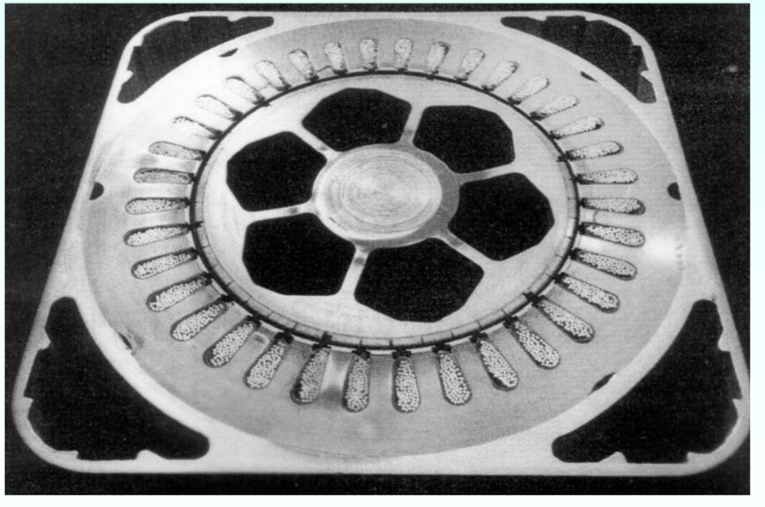
- **Influence of stator resistance R_s at low stator frequency:**

$$U_s = \sqrt{(\omega_s L_d I_{sq})^2 + (R_s I_{sq} + \omega_s \Psi_p / \sqrt{2})^2}$$



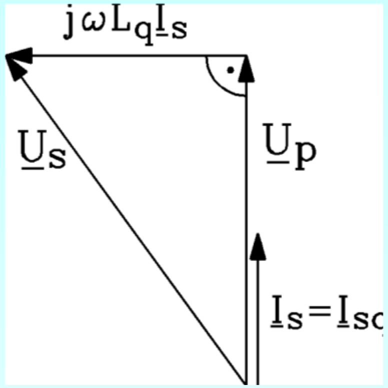
1. Permanent magnet synchronous machines as "brushless DC drives"

1.1.7 Operating limits of brushless DC drives

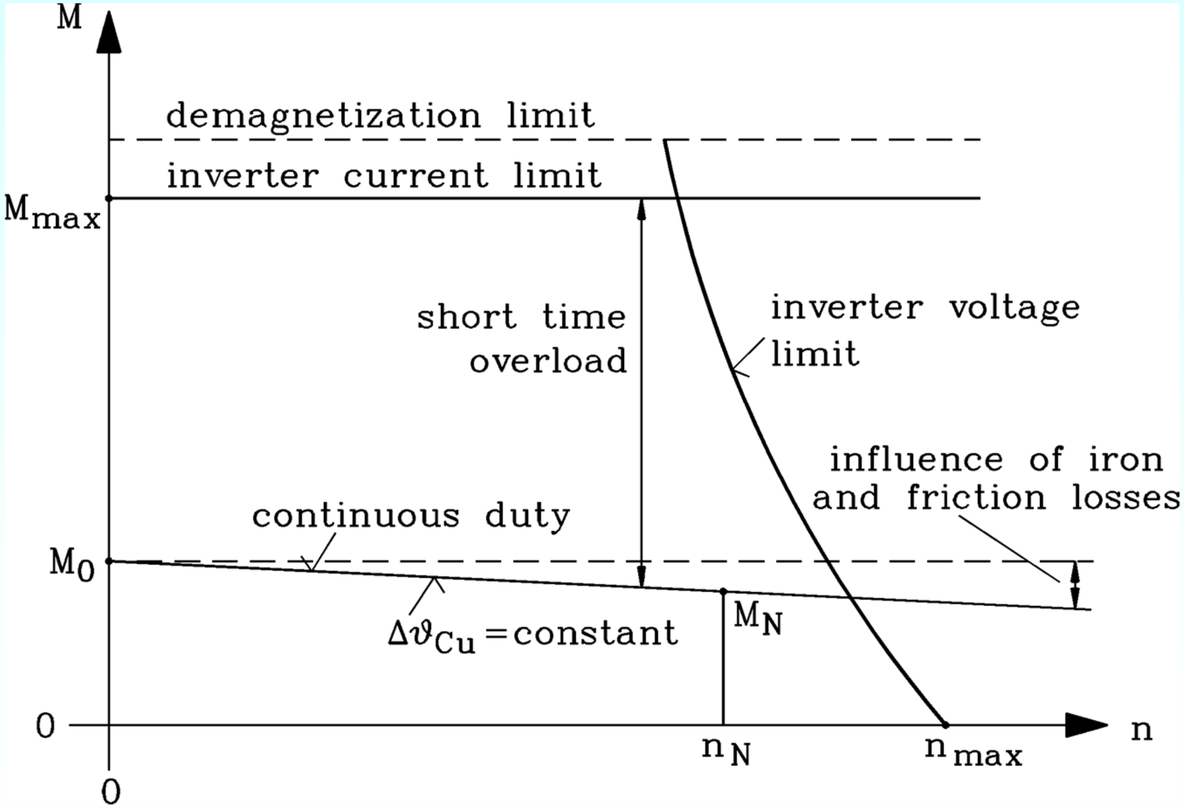


Source: Siemens AG, Germany

Operating limits of brushless DC drive



Phasor diagram per phase of synchronous PM machine at high speed with neglected stator resistance; field-oriented control with current in phase with back EMF, no saliency assumed $L_d = L_q$



Speed-torque curve limit for synchronous PM machine with field-oriented control (current in phase with back EMF)

Operation limits for brushless DC drive

- **Steady state torque:** temperature limit of insulation material of stator winding.

  E.g.: Temperature limit (IEC 34-1): 105 K for insulation class F (ambient temperature 40°C).

  Stand-still torque: M_0 at $n = 0$: only resistive losses

  Rated torque an rates speed: M_N at n_N: Resistive losses, friction and iron losses, additional losses.

  For constant temperature and self-cooled machine: Total losses must be constant $\Rightarrow$ Resistive losses must decrease at n_N, hence current decreases: $M_N < M_0$.

- **Dynamic torque (Overload** up to about $4M_0$ **): A**ccelerating and braking: short time operation (several seconds). Temperature rise according to thermal time constant T_{th} of the motor winding stays below temperature limit. So **dynamic torque** overload up to about $4M_0$ is only possible for.

- **Maximum torque: inverter current limit**.

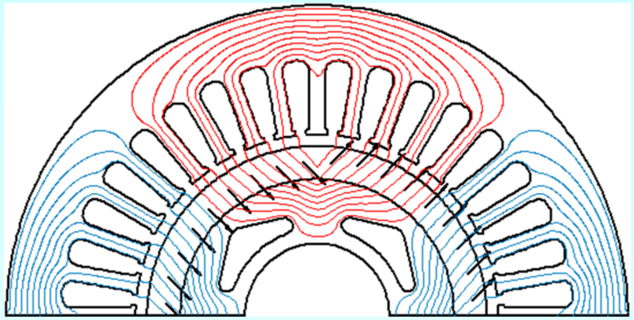
- **Demagnetization limit:** Inverter current limit must be below the critical motor current which would cause irreversible demagnetization of the hot rotor magnets (at 150°C).

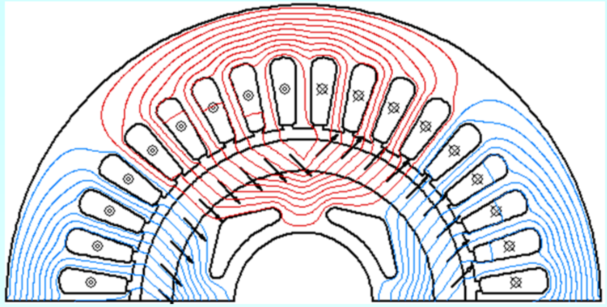
- **Mechanical maximum speed limit** n_{max} > rated speed n_N !

- **Inverter voltage limit:** Internal motor voltage reaches inverter voltage limit, hence current input and torque decreases

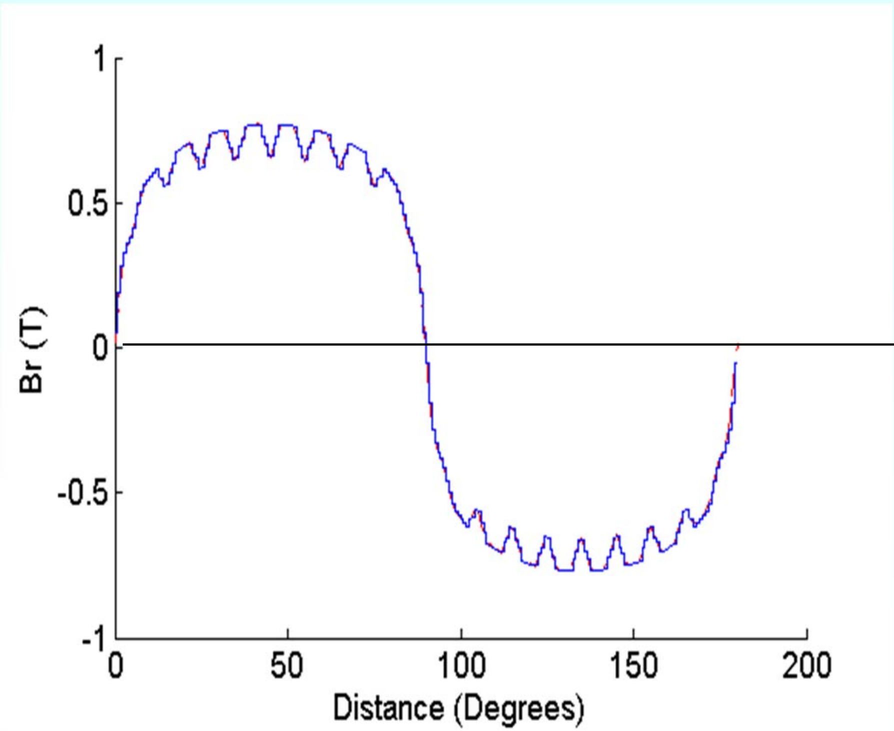
No-load and load radial air gap field component

- Stator slot opening influence INCLUDED, constant air gap

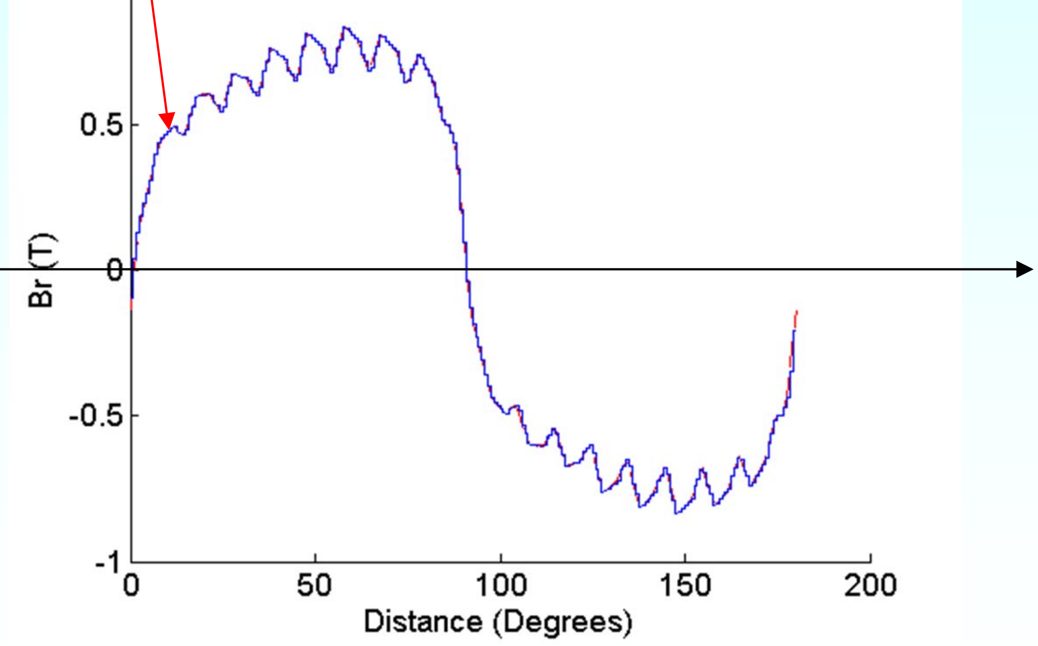




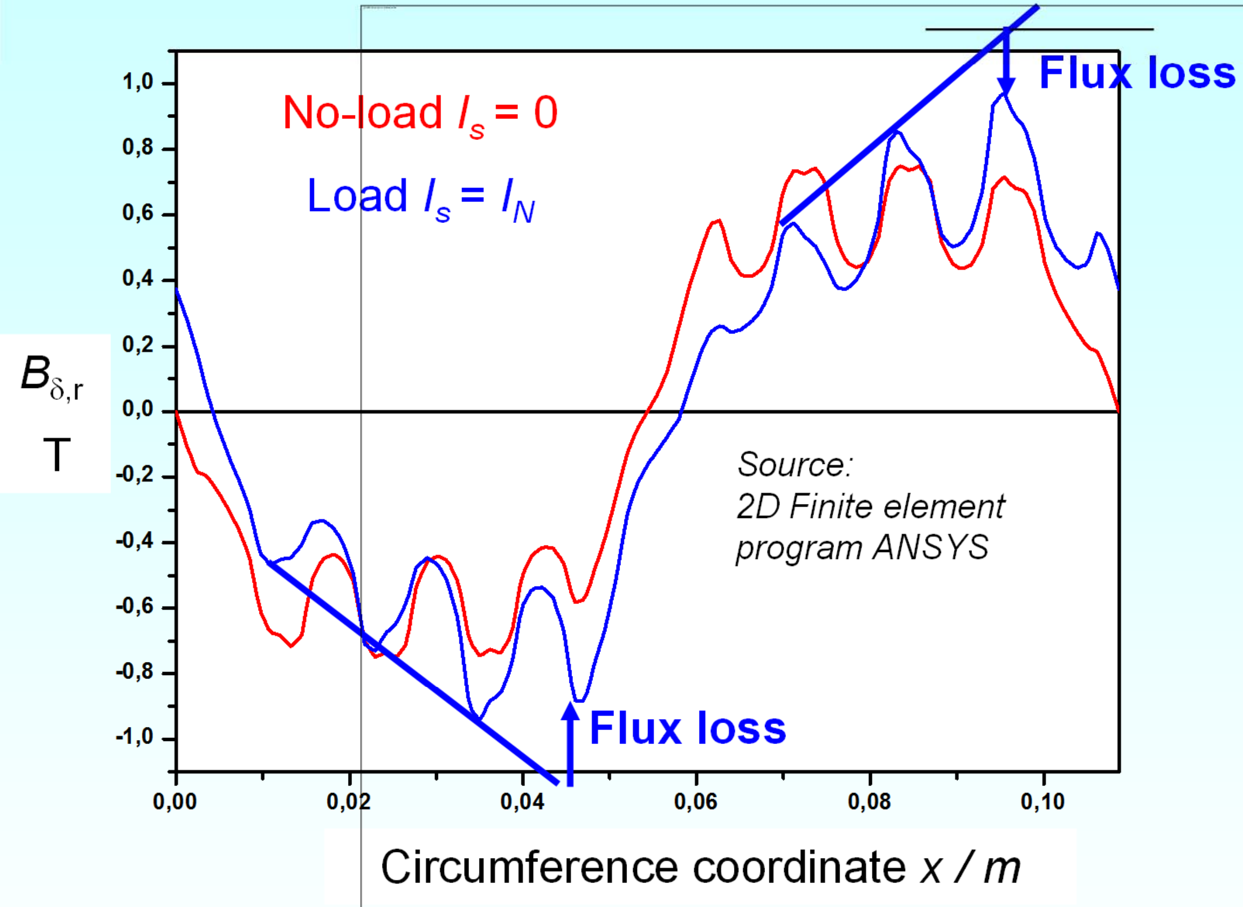
Critical magnet edge

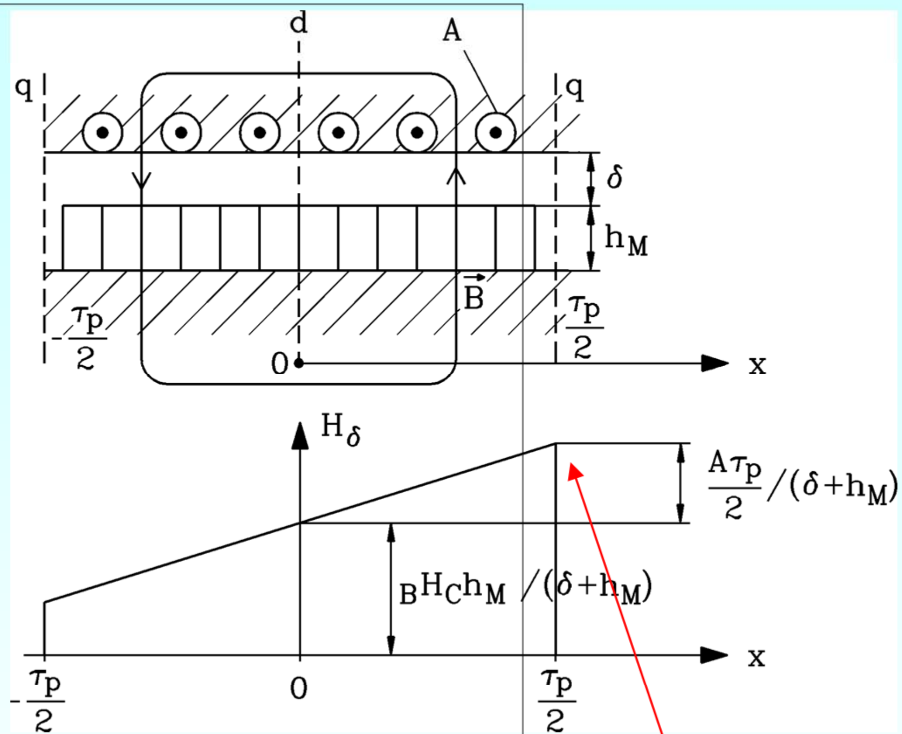


Source: 2D Finite element program FEMAG, ETH Zurich



Surface mounted PM synchronous machines: Load dependent saturation





Increase of field at leading magnet edge may cause load-dependent saturation of stator teeth.

This causes a loss in flux there.

Flux linkage is dependent on load current:

$$\Psi(I_s) < \Psi(I_s = 0)$$

Example: Load dependent saturation = Loss in torque

Source: TU Darmstadt, Germany

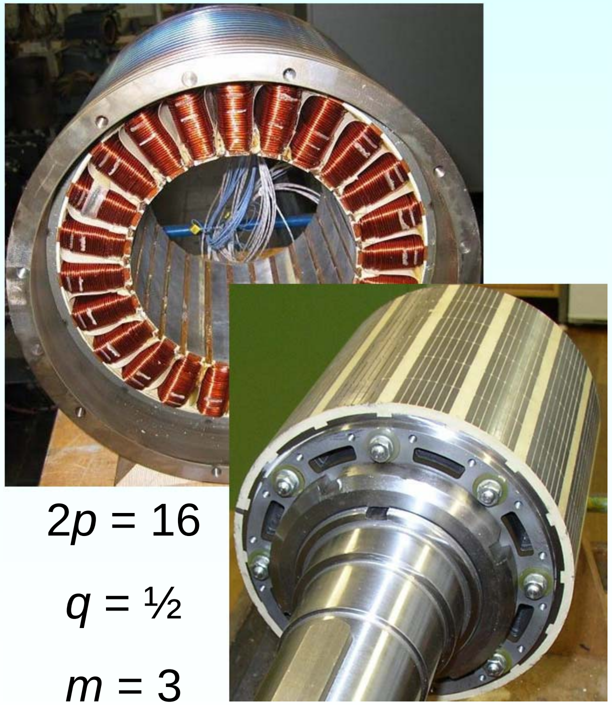
I_N = 94.6 A

M_{IN} = 477.4 Nm

$2I_N$ = 189.2 A

M_{2IN} = 695.1 Nm

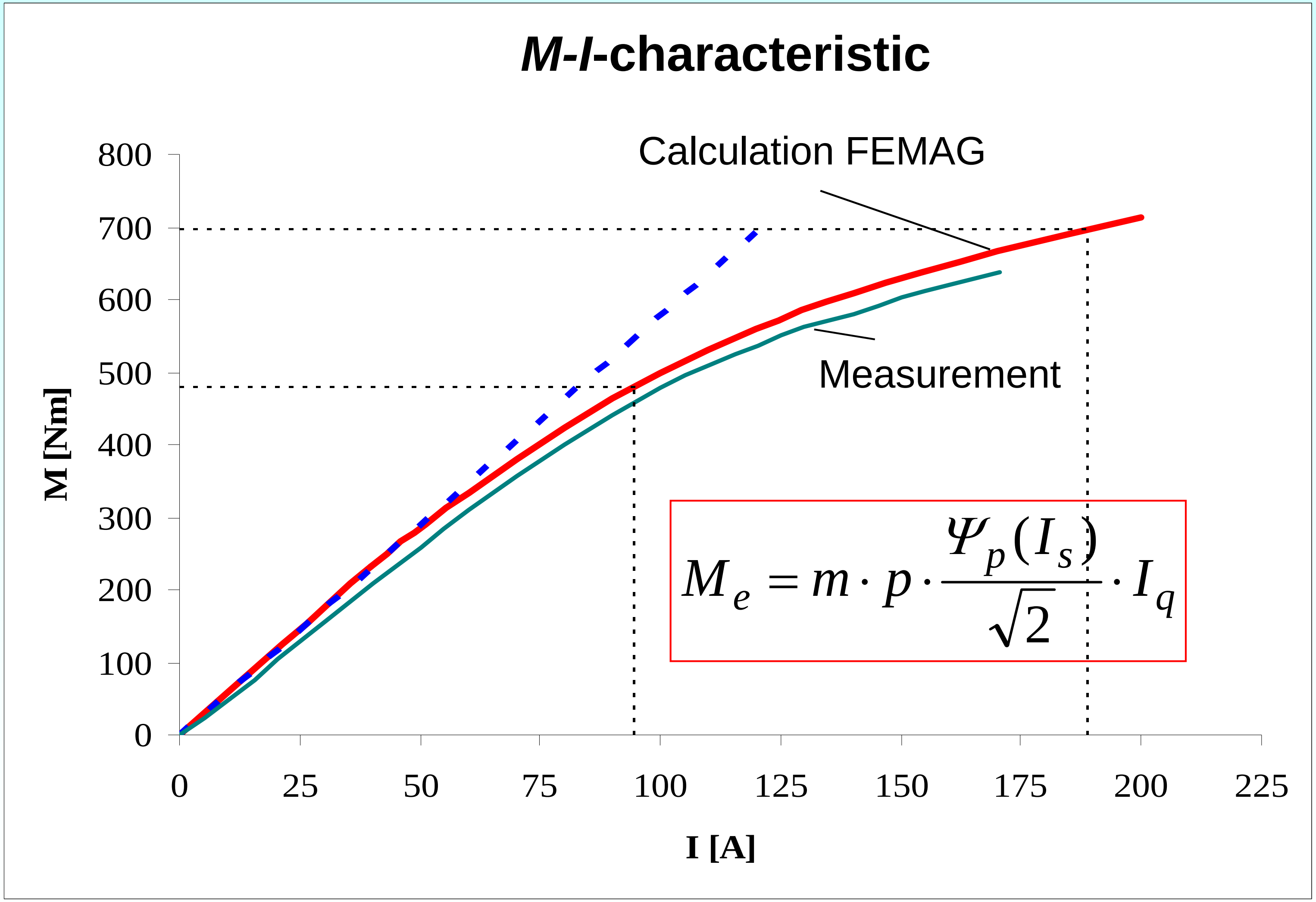
M_{2IN} / M_{IN} = **1.46**



$2p = 16$

$q = ½$

$m = 3$

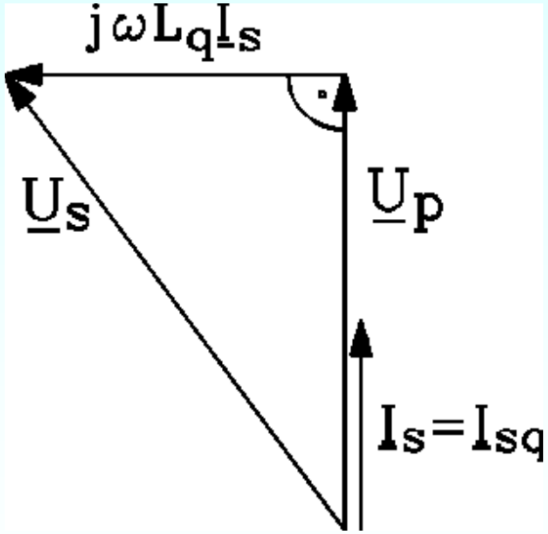


Torque formula from air gap power at q-current operation

$$M_e = \frac{P_\delta}{2\pi n} = \frac{P_\delta}{\omega_s / p} = \frac{m \cdot U_p I_{sq}}{\omega_s / p}$$

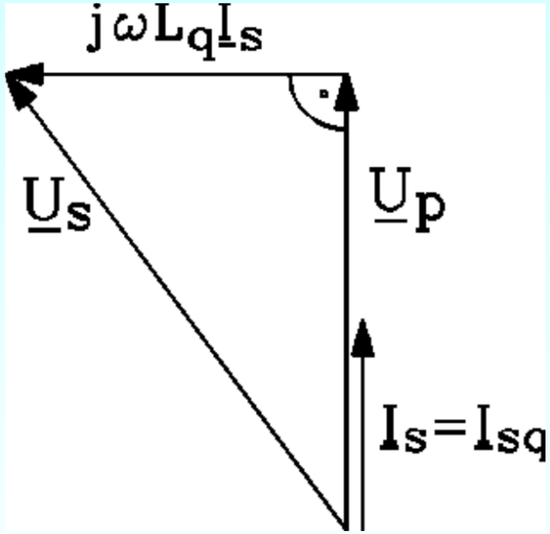
$$M_e = \frac{m \cdot (\omega_s \Psi_p(I_s) / \sqrt{2}) \cdot I_{sq}}{\omega_s / p} = m \cdot p \cdot \frac{\Psi_p(I_s)}{\sqrt{2}} \cdot I_q$$

$$\boxed{M_e = m \cdot p \cdot \frac{\Psi_p(I_s)}{\sqrt{2}} \cdot I_q}$$



Voltage limit for brushless DC drive

- Inverter-output voltage U_{max} defines maximum possible motor speed n_{max}.



At high speed neglect $R_s << X_q$:

$$I_{s,\lim} = \frac{1}{\omega_s L_d} \cdot \sqrt{U_{s,\max}^2 - (\omega_s \Psi_p / \sqrt{2})^2}$$

$$M_{\lim} = m \cdot p \cdot \frac{\Psi_p}{\sqrt{2}} I_{\lim}(n)$$

- **Maximum possible motor speed** = No-load speed at voltage limit:

$$U_{s,\max} = \omega_{s,\max} \cdot \Psi_p / \sqrt{2}$$

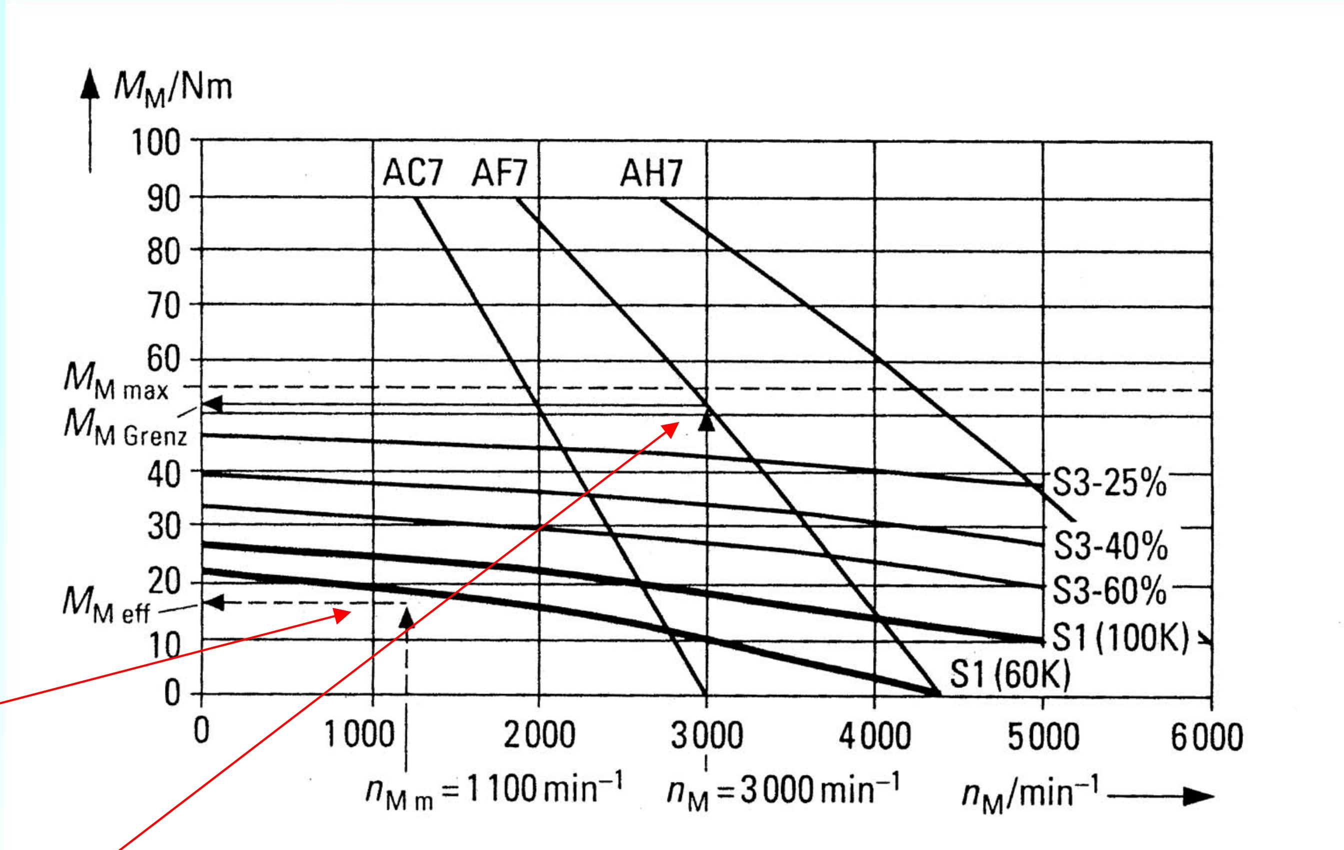
$$n_{\max} = n_N \frac{U_{\max}}{U_{pN}}$$

Typical *M(n)* characteristic of a PM servo drive

Different number of turns per phase:
variants AC7, AF7, AH7

Admissible torque:
- S1 steady state operation,
- S3 short term operation

Continuous duty torque must stay below S1-line

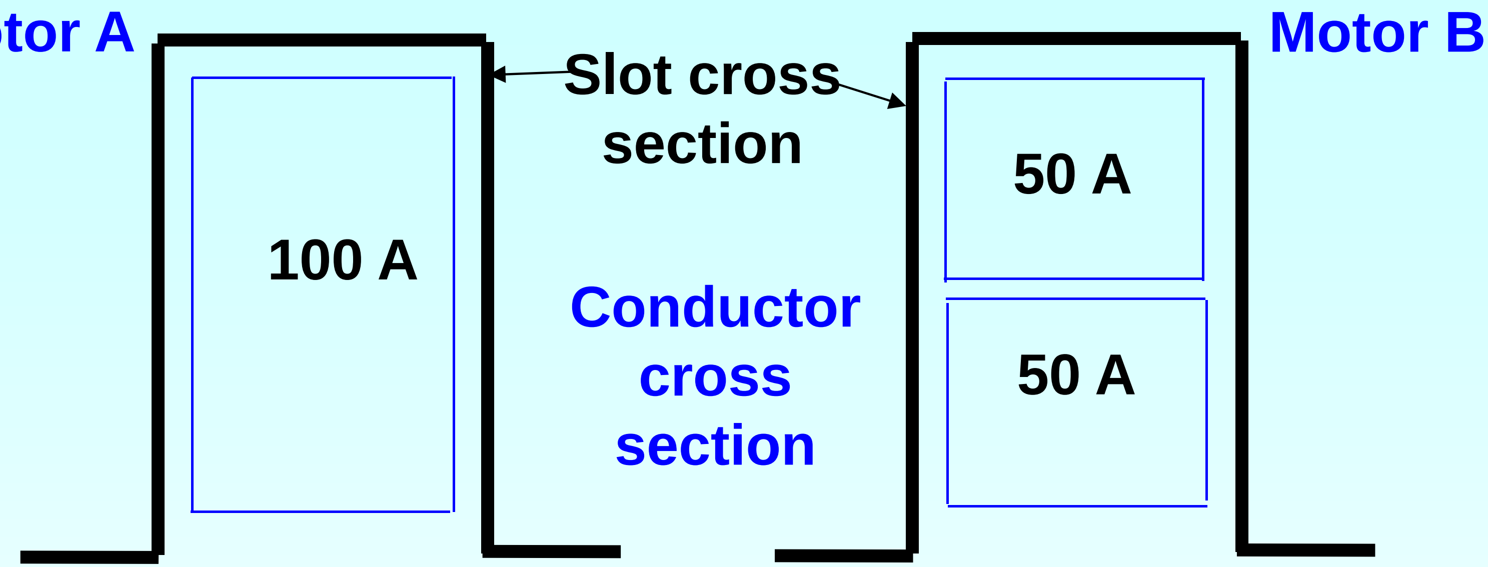


For acceleration maximum possible torque is used, but only for short time !

Source: Groß/Hamann/Wiegärtner: Elektrische Vorschubantriebe in der Automatisierungstechnik, Publicis Verlag, Munich, Germany

Different „Number of turns“ for one motor

- **Identical** are:
  - Copper mass
  - Iron mass
  - Motor size
  - Torque
  - Current density
  - I^2R-losses
  - Magnet flux



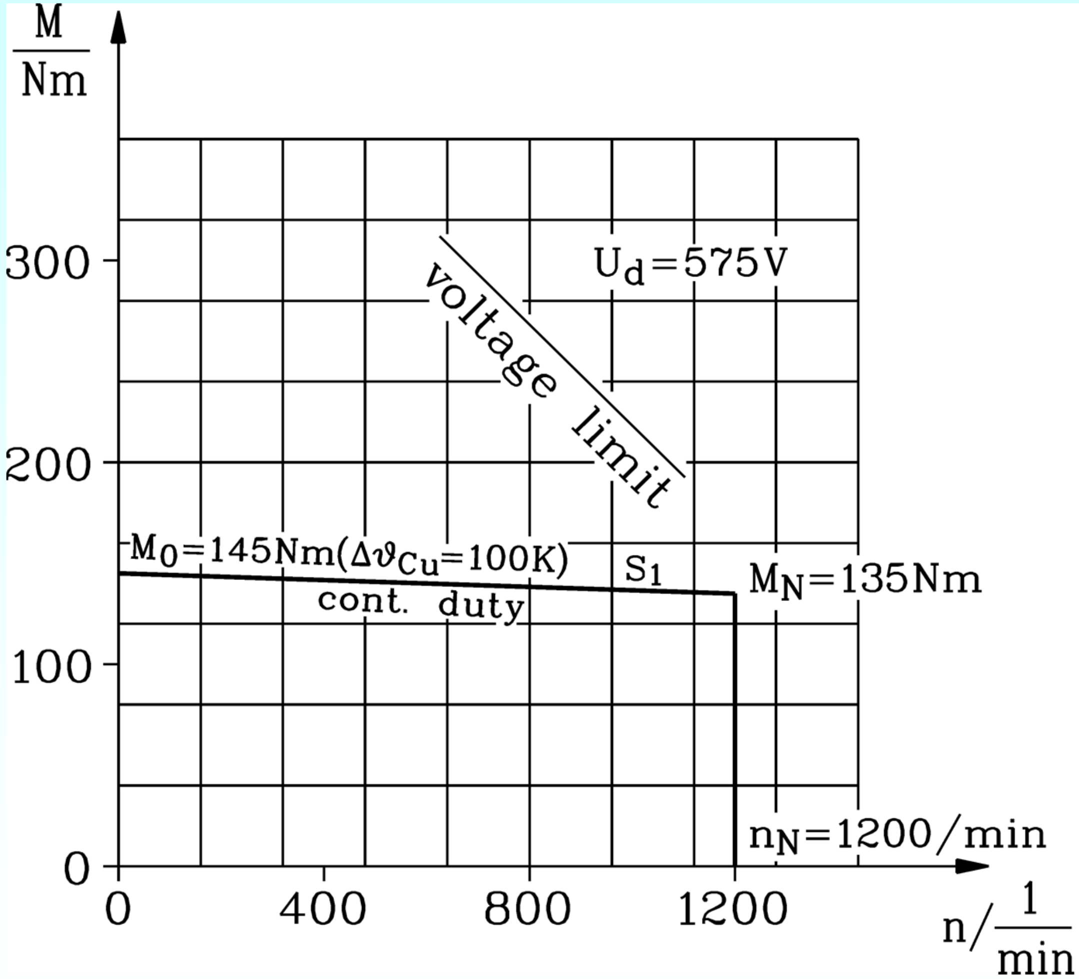
	Motor A	Motor B
Number of turns per phase:	N	$2N$
Conductor cross section:	q_{Cu}	$q_{Cu}/2$
Current:	I_s	$I_s/2$
Back EMF at speed *n*:	$U_i \sim N \cdot n \cdot \Phi$	$2U_i \sim 2N \cdot n \cdot \Phi$
Maximum speed:	$n_{max} \sim U_{max}/(N \cdot \Phi)$	$n_{max}/2 \sim U_{max}/(2N \cdot \Phi)$
Inverter power at n_{max}:	$S = 3U_{max}I_s$	$S/2 = 3U_{max}I_s/2$

Example: M(n) characteristic of a PM servo drive

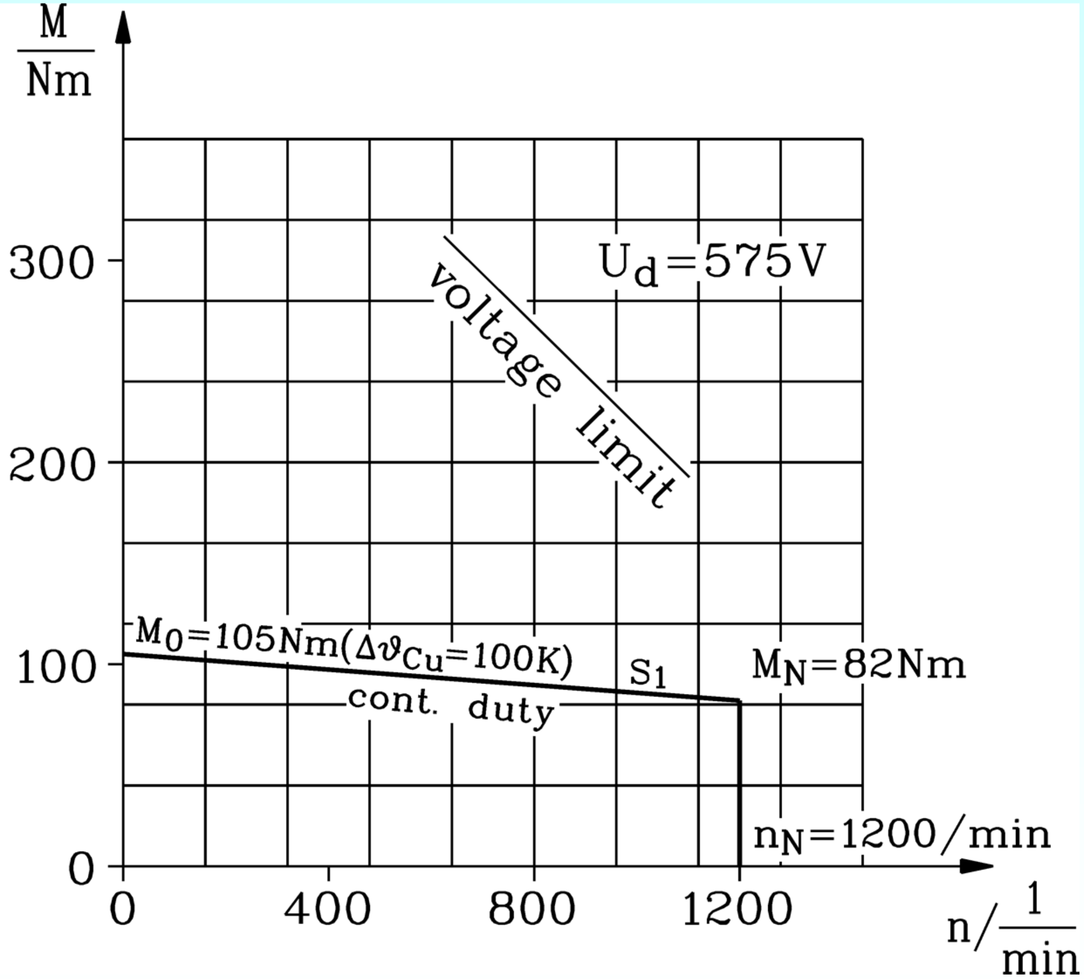
Source: Siemens motor catalogue SIMODRIVE, Erlangen, Germany

The same closed motor, but

a) External fan for forced cooling

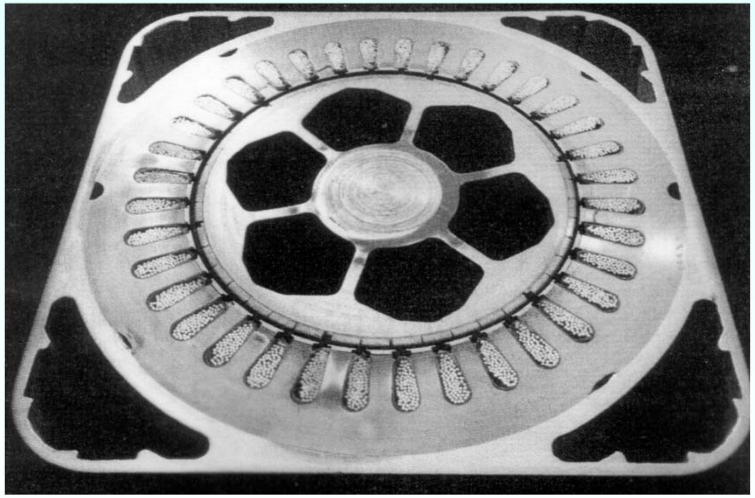


b) Self-cooling = no fan



1. Permanent magnet synchronous machines as "brushless DC drives"

1.1.8 Use of reluctance torque to increase the torque

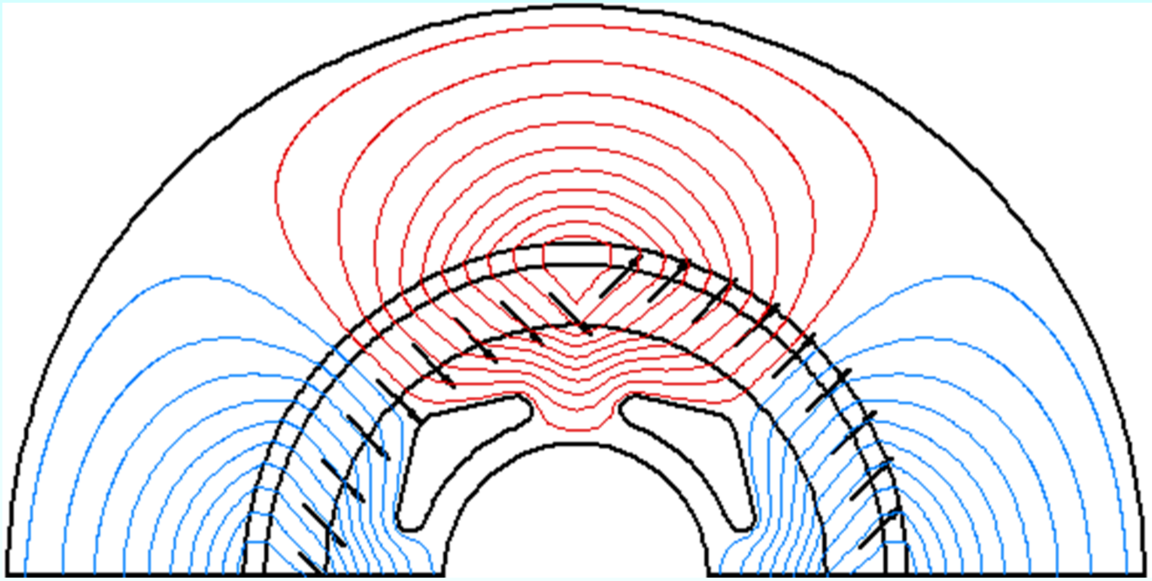


Source: Siemens AG, Germany

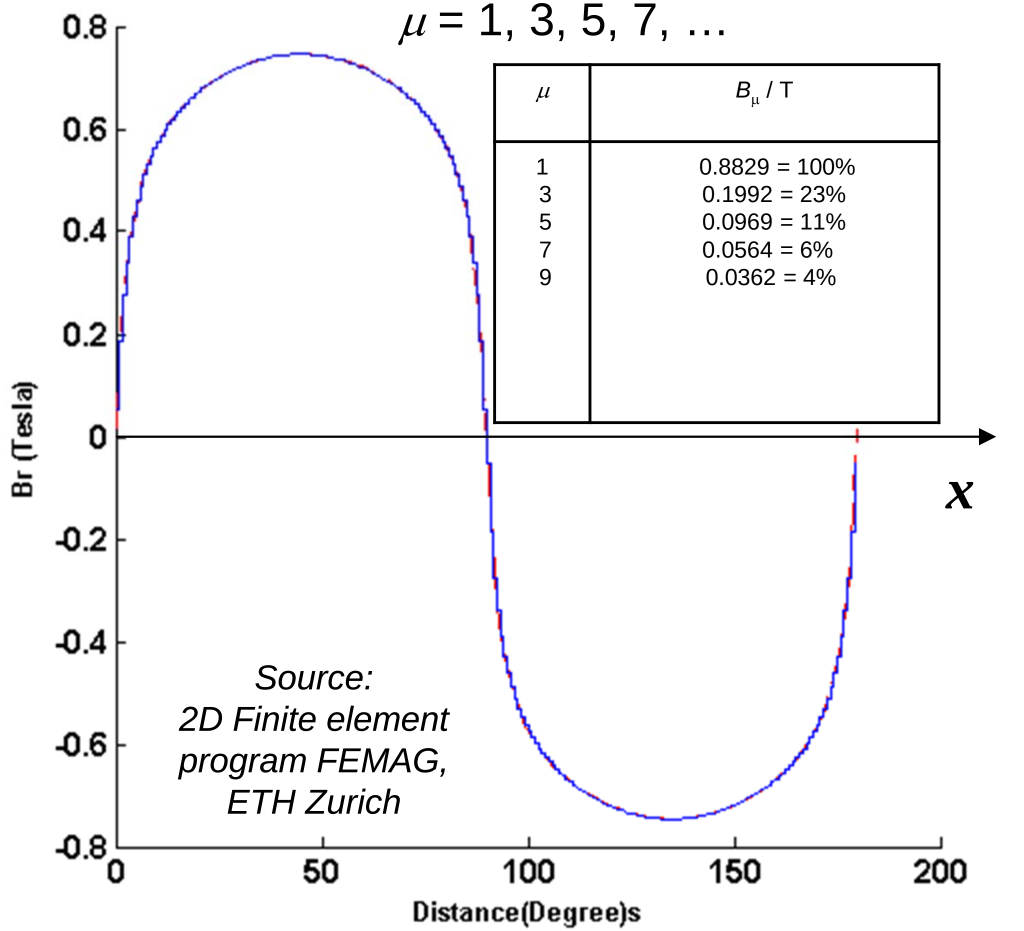
No-load rotor air gap field with <u>parallel</u> PM magnetization - No reluctance variation

- Surface mounted magnets, stator slot opening influence neglected

Pole coverage ratio $\alpha_e = 1$



Rotor air gap field amplitudes:
$\mu = 1, 3, 5, 7, \ldots$



- Parallel magnetization yields at low pole count considerable difference to radial magnetization

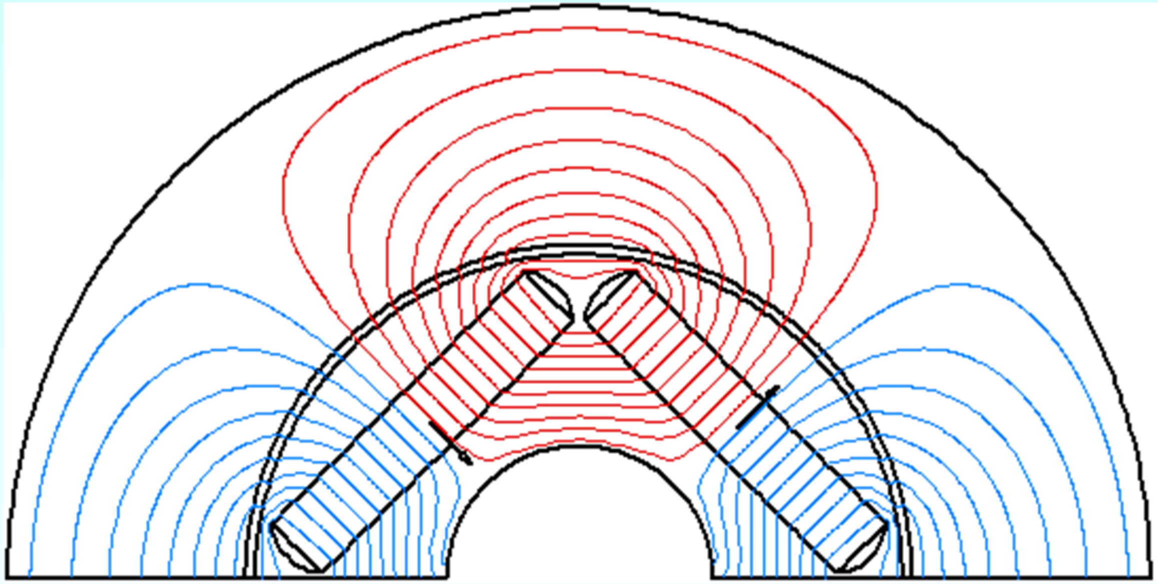
- A better approximation of sinusoidal field is achieved, higher harmonics B_μ are reduced

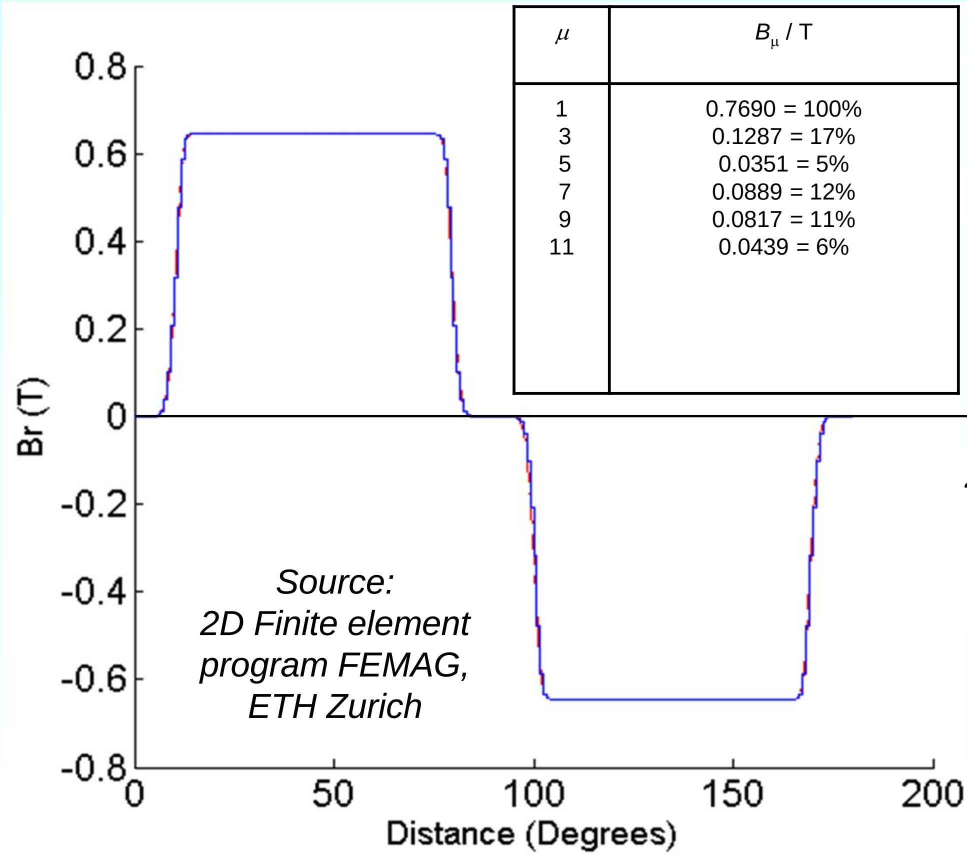
No-load rotor air gap field with buried magnets – Rotor reluctance variation occurs

- Stator slot opening influence neglected, constant air gap

Pole coverage ratio $\alpha_e < 1$

Rotor air gap field amplitudes:
$\mu = 1, 3, 5, 7, \ldots$





- Depending
  - a) on the shape of air gap
  - b) on the saturation of iron bridge

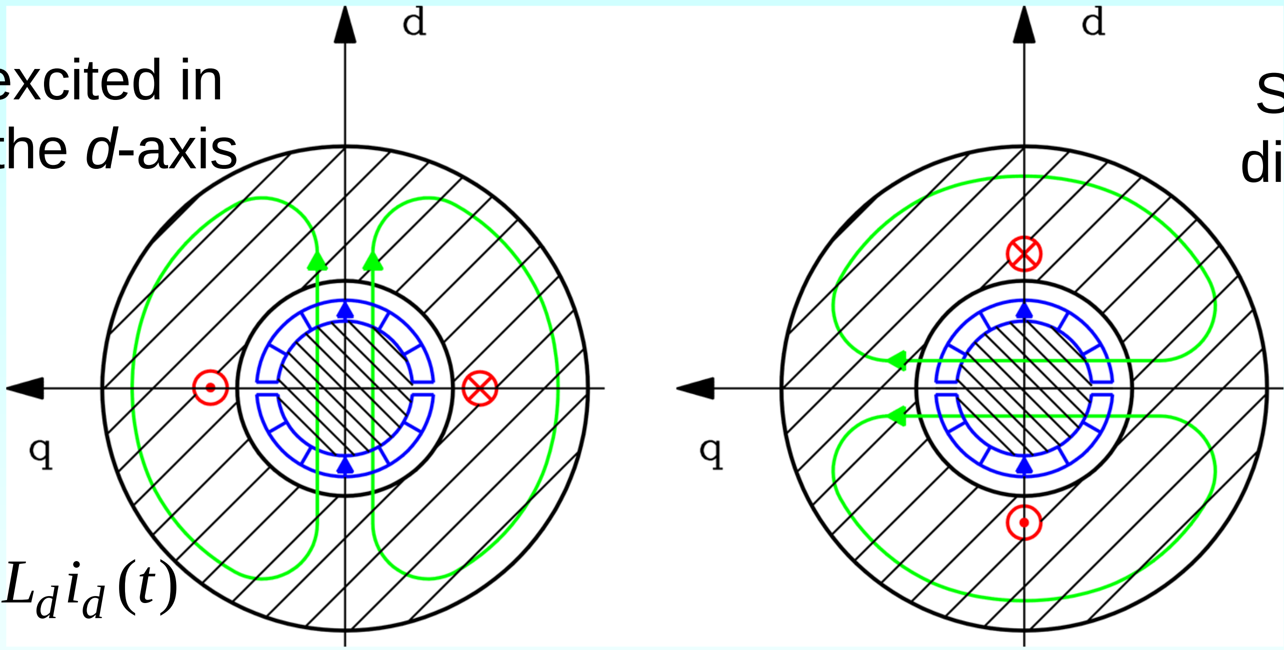
the air gap flux density may vary.

- Numerical calculation is recommended.

Torque from power balance at $L_d \neq L_q$

Stator flux excited in direction of the *d*-axis

Stator flux excited in direction of the *q*-axis



$$\psi_d(t) = \Psi_p + L_d i_d(t)$$

$$\psi_q(t) = L_q i_q(t)$$

Electromagnetic torque at $L_d \neq L_q$: *m* = 3 phases

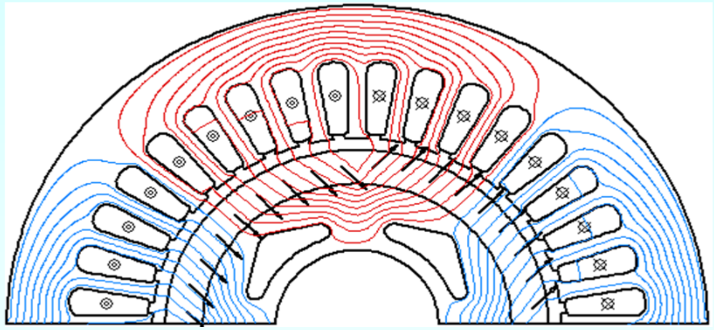
$$M_e(t) = p \cdot m \cdot (\psi_d(t) i_q(t) - \psi_q(t) i_d(t))/2$$

$$M_e(t) = p \cdot m \cdot (\underbrace{\Psi_p i_q}_{\text{PM torque}} - \underbrace{(L_q - L_d) i_d i_q}_{\text{reluctance torque}})/2$$

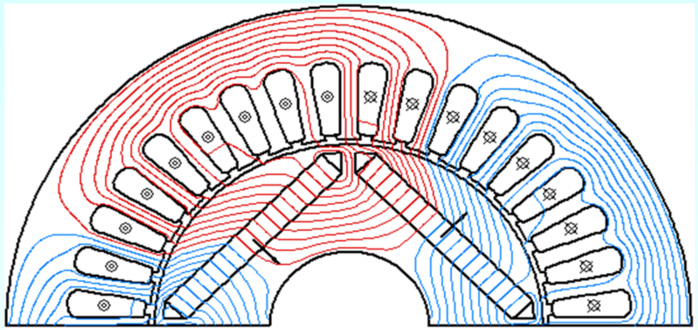
PM torque **reluctance torque**

Example: Rotor geometries A, B, C for a stator with distributed integer slot winding

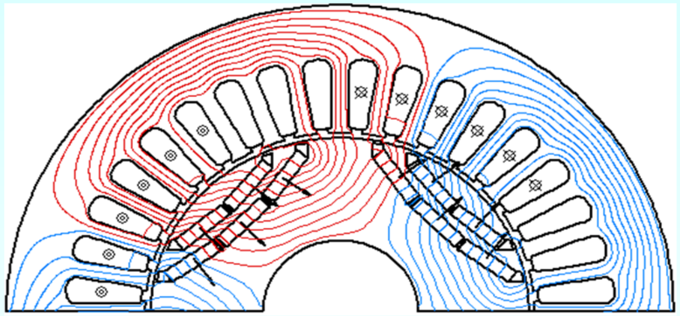
Rotor A



Rotor B



Rotor C



Motor A:
Surface magnet motor A (three phases m = 3, four poles 2p = 4, 15000/min, over-speed: 18000/min)

$$M_e = p \cdot m \cdot (\Psi_p I_q + (L_q - L_d) I_d I_q) / 2$$

Source:
2D Finite element program FEMAG, ETH Zurich

Numerically calculated torque for motors A, B, C

Motor	Average torque M_e (Nm)	PM torque (Nm) / (percent %)	Reluctance torque $M_e - M_p$ (Nm) / (percent %)
A	7.95	7.95 / 100%	0 / 0%
B	7.88	6.14/ 78%	1.74 / 22%
C	7.76	5.42 / 70%	2.34 / 30%

Comparison of

- the average total torque,

- the PM torque and

- the **reluctance torque** of motors A, B, C for the same stator geometry, winding, sinusoidal current 44.5 A rms per phase, calculated with FEMAG.

Reduction of PM material due to reluctance torque

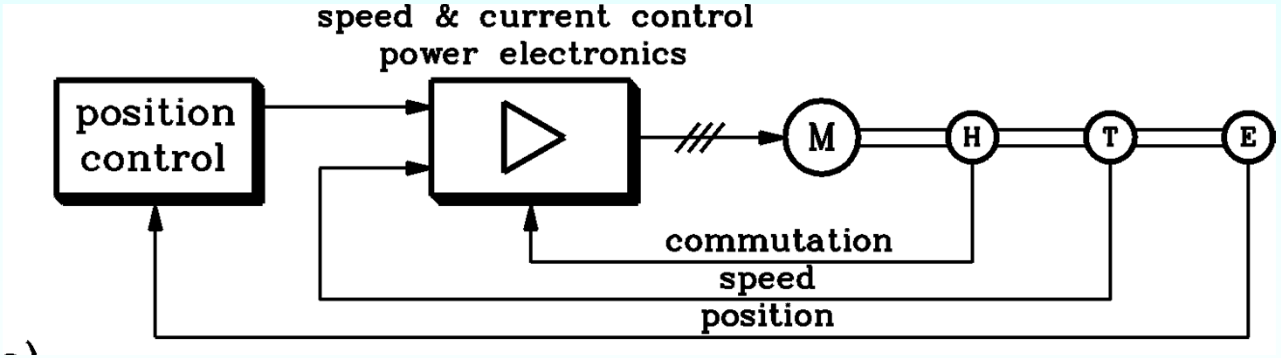
Motor	Voltage ph. U_s / V rms	Angle β	Inductances L_d / L_q (μH)	App. Power S (kVA / %)	Less magnet volume
A	95.17	0	127.5 / 136.0	12.7	-
B	99.7	-26°	198.0 / 575.0	13.31 / +5.0%	-34%
C	99.92	-30°	200.0 / 650.0	13.39/+5.5%	-53.1%

- Comparison of inverter and motor data of motors A, B, C for the same stator geometry, winding

- Calculated with FEMAG.

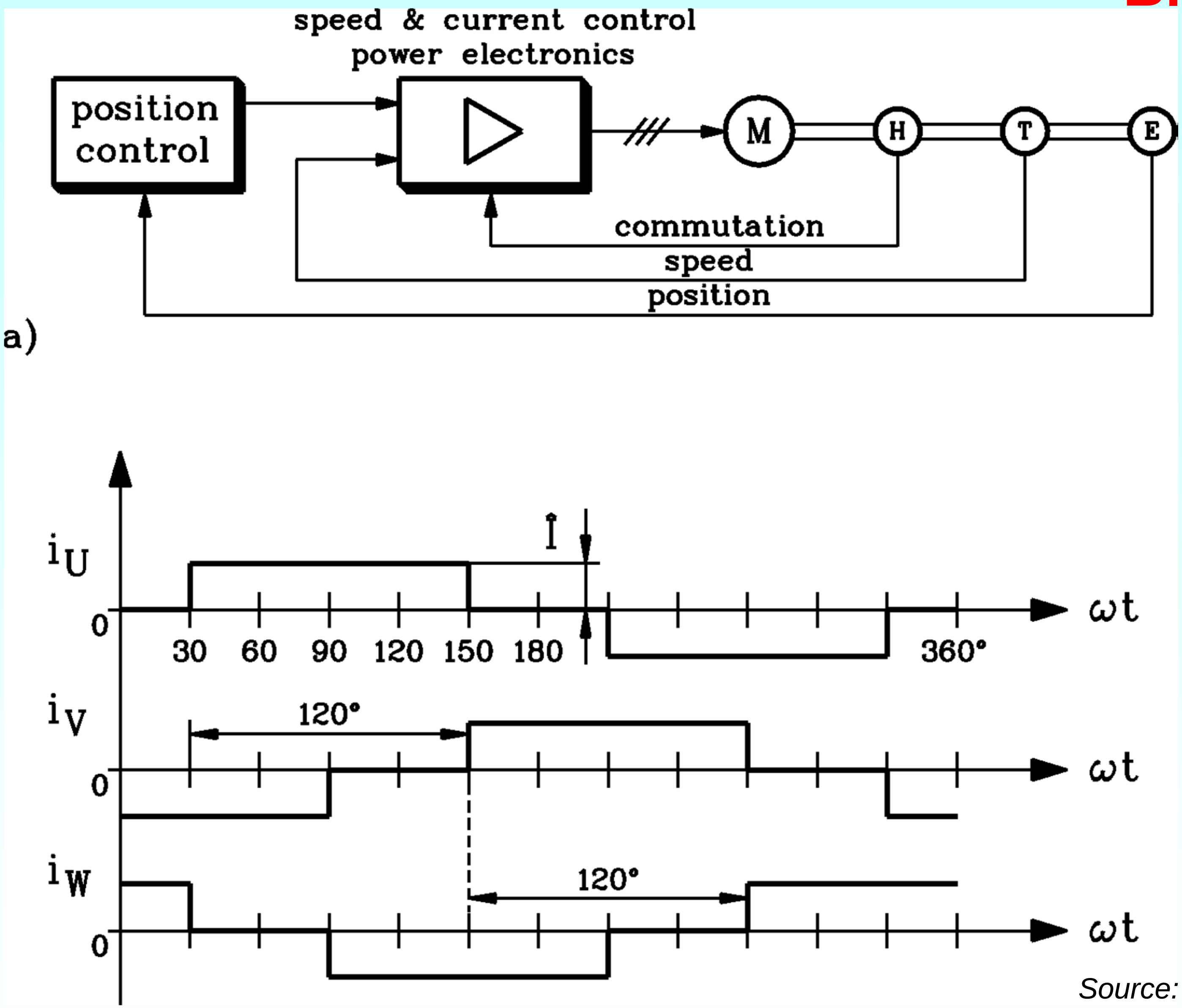
- Motors B, C have for the same torque 7.5 Nm less winding losses and temperature rise than motor A.

1.2 Brushless DC drive systems



Source:
Lehmann R.: Technik bürstenloser Servoantriebe, Elektrotechnik 21: 96-101, 1989

Block current commutated drive system



- Inner loop current control as in DC machines (“brushless DC drive”)

- Outer loop speed control

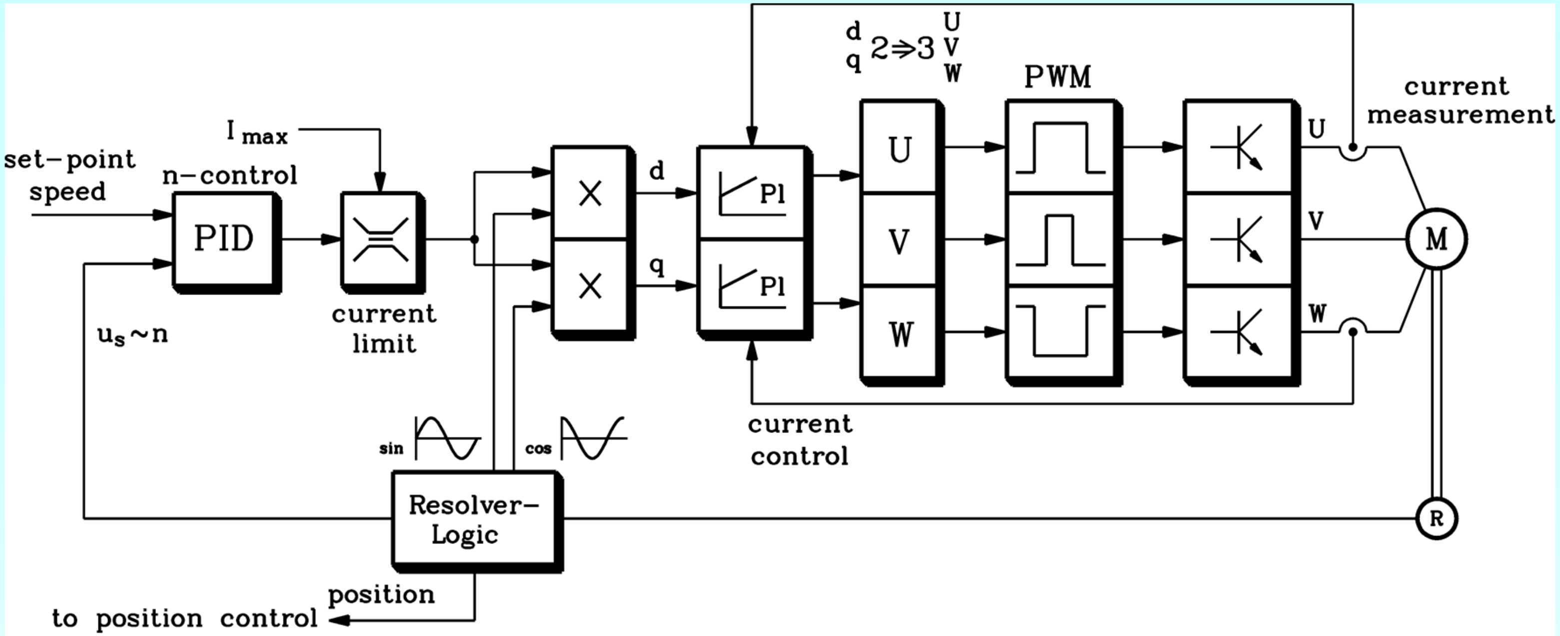
- Master position control

- PWM inverter with block phase currents

- Simple rotor position measurement (H : Hall sensors), additional speed sensor (T: Tacho) and high resolution position sensor (E: encoder) for position control

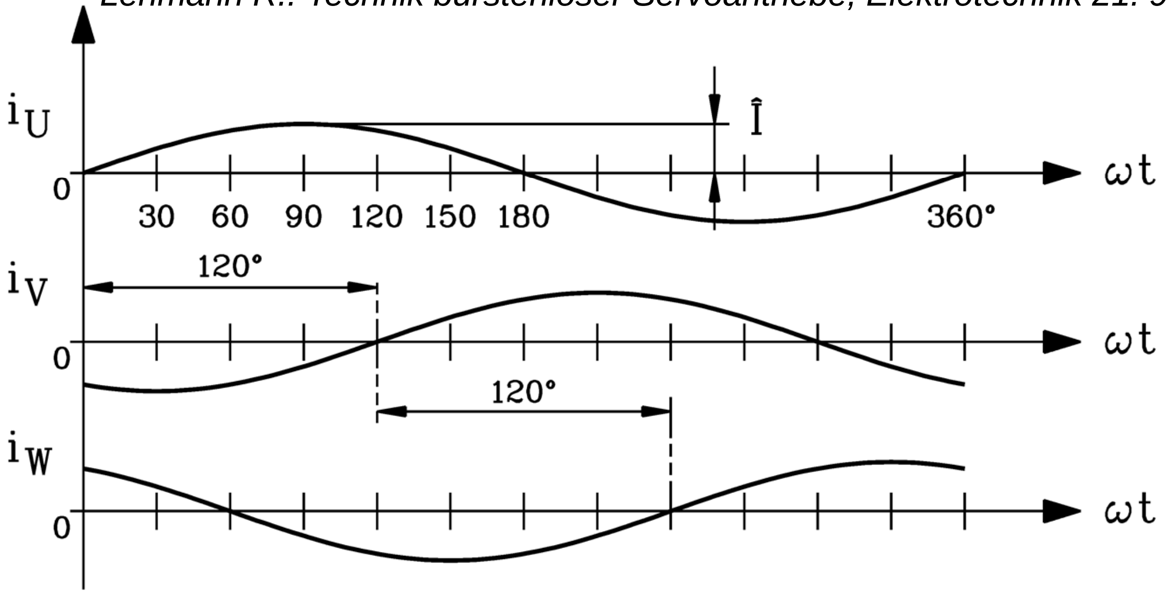
Source:
Lehmann R.: Technik bürstenloser Servoantriebe, Elektrotechnik 21: 96-101, 1989

Sine wave commutated drive system



a)

Source:
Lehmann R.: Technik bürstenloser Servoantriebe, Elektrotechnik 21: 96-101, 1989



b)

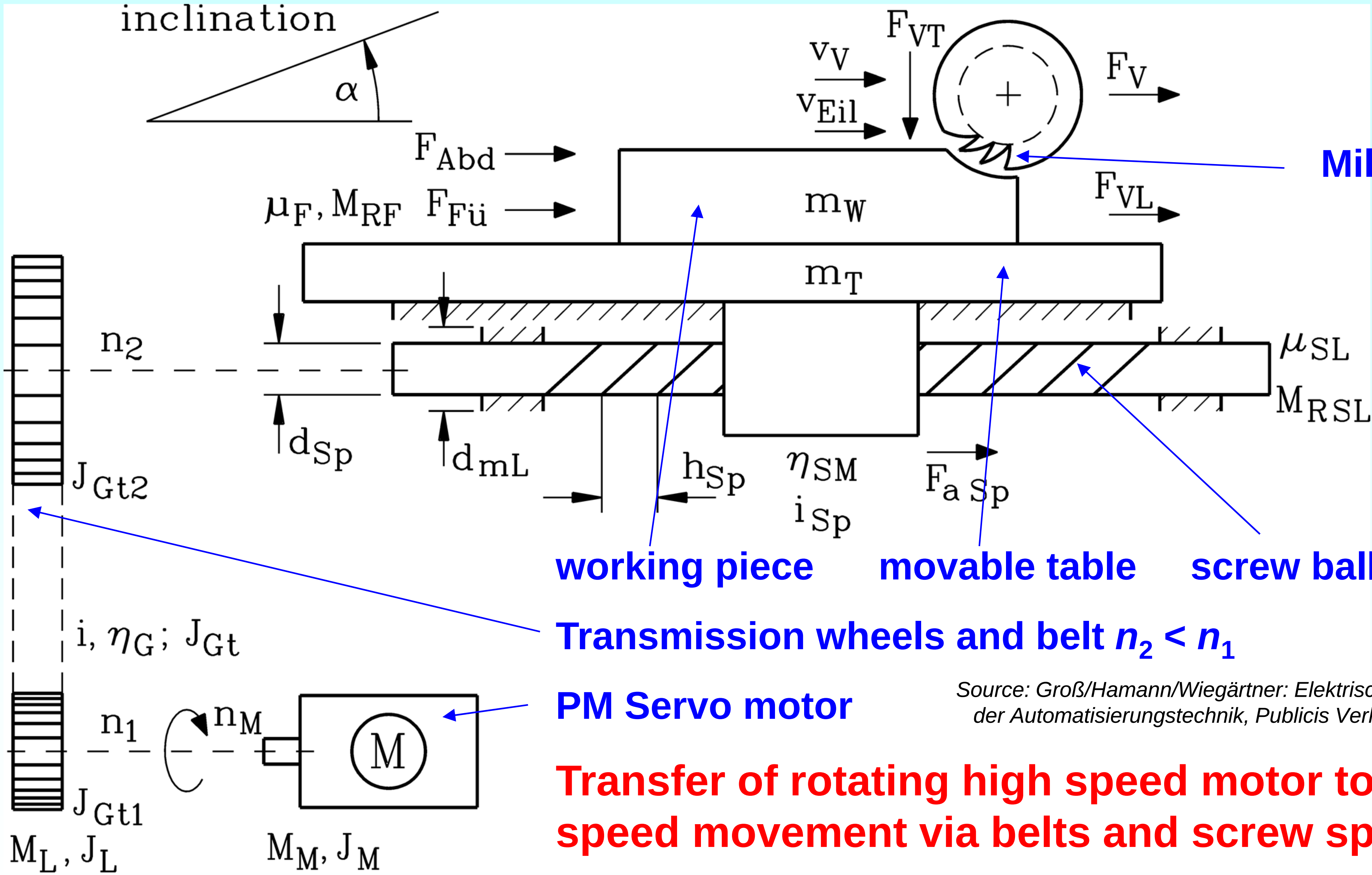
- Inner loop space vector current control in rotor reference frame (d-q-system)

- Outer loop speed control

- PWM inverter with sinusoidal phase currents

- Resolver (R) or encoder for high resolution rotor position measurement

Application of PM servo drive in milling machine



Milling

working piece **movable table** **screw ball spindle**

Transmission wheels and belt $n_2 < n_1$

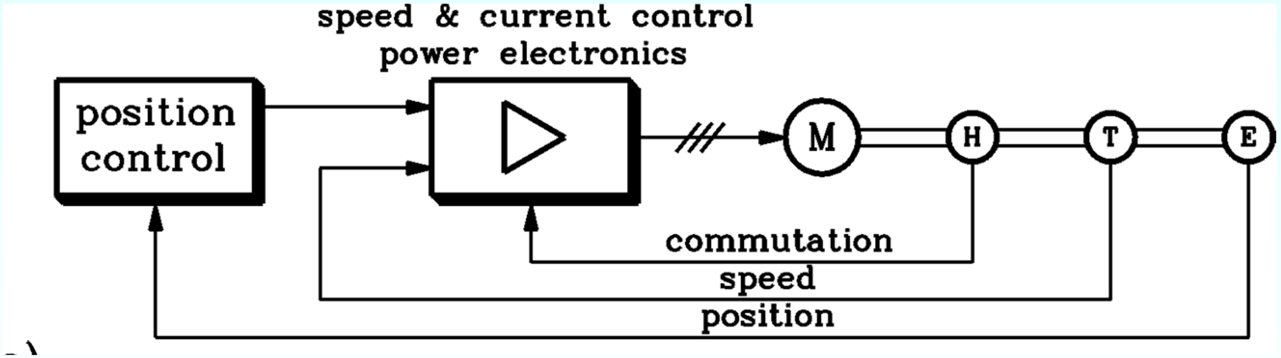
PM Servo motor

Source: Groß/Hamann/Wiegärtner: Elektrische Vorschubantriebe in der Automatisierungstechnik, Publicis Verlag, Munich, Germany

Transfer of rotating high speed motor to linear low speed movement via belts and screw spindle

1. Permanent magnet synchronous machines as "brushless DC drives"

1.2.1 Comparison of block and sine wave commutated motors



Source:
Lehmann R.: Technik bürstenloser Servoantriebe, Elektrotechnik 21: 96-101, 1989

Steady state torque of block and sine commutated motors

- For same **thermal limit**, only copper losses, same stator geometry, identical winding, identical magnet material and magnet height:

- Sine wave and block commutated motors give for the same copper losses the same output power.

PM machine	*Block commutation (B)*	*Sine wave commutation (S)*
Air gap flux density amplitude	$B_\delta = B_p$	$B_{\delta,1} = \frac{4B_p}{\pi} \cdot \sin\left(\frac{\alpha_e \pi}{2}\right)$
Back EMF	$\hat{U}_{pB} = 2N_s \cdot 2f\tau_p \cdot B_p \cdot l_{Fe}$	$\hat{U}_{pS} = 2\pi f \cdot N_s \cdot k_w \cdot \frac{2}{\pi} \tau_p l_{Fe} B_{\delta,1}$
Stator copper losses	$P_{Cu} = 2 \cdot R_s \cdot \hat{I}_{sB}^2$	$P_{Cu} = (3/2) \cdot R_s \cdot \hat{I}_{sS}^2$
Air gap power	$P_{\delta B} = 2 \cdot \hat{U}_{pB} \cdot \hat{I}_{sB}$	$P_{\delta S} = (3/2) \cdot \hat{U}_{pS} \cdot \hat{I}_{sS}$

Equal copper losses:

$$\hat{I}_{sS} / \hat{I}_{sB} = 2/\sqrt{3}$$

$$\frac{P_{\delta S}}{P_{\delta B}} = \frac{(3/2) \cdot \hat{U}_{pS} \cdot \hat{I}_{sS}}{2 \cdot \hat{U}_{pB} \cdot \hat{I}_{sB}} = \frac{\sqrt{3} \cdot 2}{\pi} \cdot k_w \cdot \sin\left(\frac{\alpha_e \pi}{2}\right)$$

Comparison of block and sine wave commutated motors

Example A: **Operation at same copper losses :**

Winding factor k_w = 0.933, pole coverage ratio α_e = 0.85:

$$\frac{P_{\delta S}}{P_{\delta B}} = \frac{\sqrt{3}\cdot 2}{\pi}\cdot 0.933\cdot \sin\left(\frac{0.85\cdot\pi}{2}\right) = 1.00035$$

Example B:

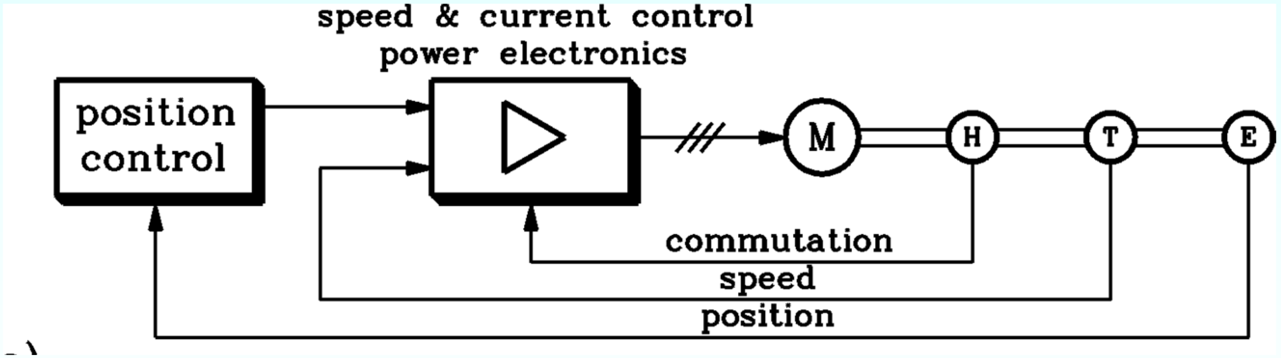
Operation at inverter current limit $I_{s,max}$: Block or sine wave commutated motor delivers the higher short term torque ?

$$\frac{M_{e,S}}{M_{e,B}} = \frac{P_{\delta S}}{P_{\delta B}} = \frac{(3/2)\cdot \hat{U}_{pS}\cdot \hat{I}_{s,\max}}{2\cdot \hat{U}_{pB}\cdot \hat{I}_{s,\max}} = \frac{\sqrt{3}\cdot 2}{\pi}\cdot k_w\cdot \sin\left(\frac{\alpha_e \pi}{2}\right)\cdot \frac{\sqrt{3}}{2} = \frac{1}{1.15}$$

The block commutated motor is able to deliver at the SAME current amplitude 15% higher maximum torque, whereas for the same copper losses the thermal steady state torque for block and sine wave commutated PM machine is equal.

1. Permanent magnet synchronous machines as “brushless DC drives”

1.2.2 Torque ripple of brushless DC motors

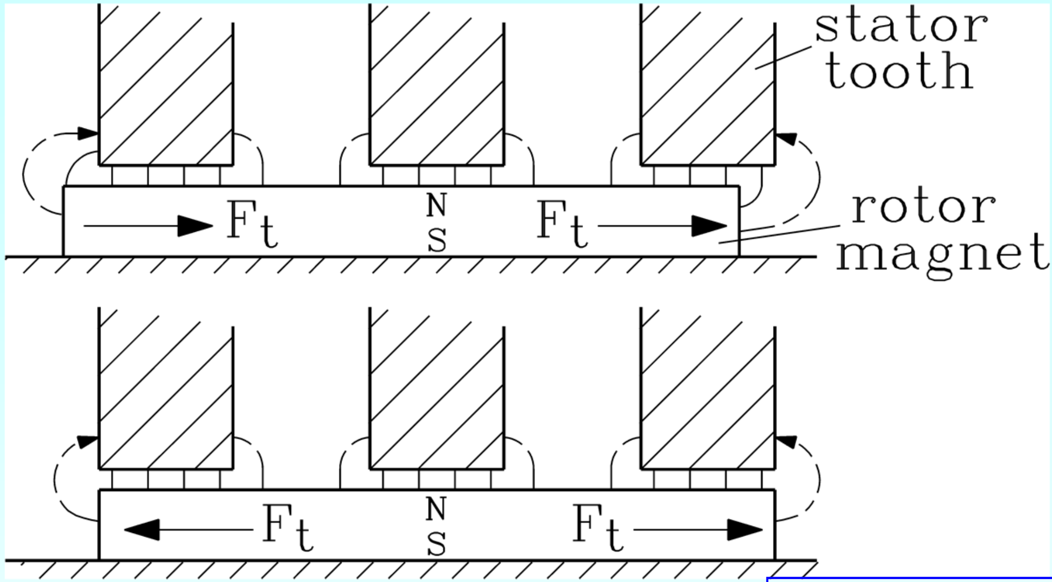


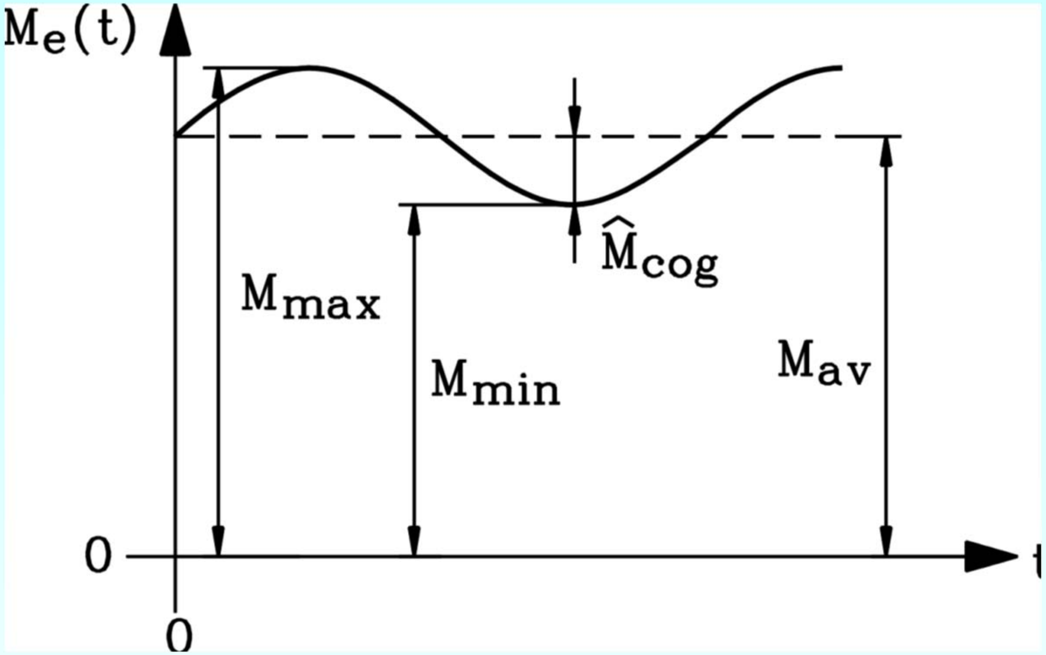
Source:
Lehmann R.: Technik bürstenloser Servoantriebe, Elektrotechnik 21: 96-101, 1989

Torque ripple of brushless DC motors

- **Electromagnetic air gap torque $m_e(t)$:** May exhibit a considerable ripple, which might be also visible at the shaft as a torque ripple of the shaft torque $m_s(t)$
- **Cogging torque:** No-load torque ripple due to rotor magnets and stator slot openings. The stator current is zero.
- **Pulsating torque at ideal sine wave current:** Torque variation at load due to interaction between stator and rotor field. Step-like stator m.m.f. distribution due to distributed stator winding may be regarded as FOURIER sum of space harmonics (ordinal number ν), causing pulsating torque components with the rotor magnet field space harmonics.
- **Pulsating torque due to current ripple:** Inverter switching causes current ripple = current time harmonics. Each current harmonic causes a stator fundamental field, which interacts with rotor PM fundamental field $\mu = 1$.

Cogging torque M_{cog} and pulsating load torque





Cogging torque frequency: $f_Q = n \cdot Q_s$

Cogging effect at no-load (i_s = 0):
Unaligned position: rotor tangential magnetic pull F_t on stator tooth sides generates torque,
Aligned position: sum F_t = 0, no torque

Typical good values: $\hat{w}_{M0} \sim 0.5\%...1\%$

Pulsating load torque:
Quantification of torque ripple from measured torque time function, e.g. measured with strain gauge torque-meter:

$$\hat{w}_M = \frac{\hat{M}_{cog}}{M_{av}} = \frac{(M_{max} - M_{min})/2}{(M_{max} + M_{min})/2}$$

Load torque of fundamental fields, calculated via internal power

- Internal power varies with time, leading to torque and speed variation
- Speed variation much smaller than torque variation due to rotor inertia, hence we assume CONSTANT speed
- Internal power gives electromagnetic torque:

$$m_e(t) = \left(u_{p,U}(t)\cdot i_U(t) + u_{p,V}(t)\cdot i_V(t) + u_{p,W}(t)\cdot i_W(t)\right)/(2\pi n)$$

- **Ideal sine wave current feeding:** NO rotor field space harmonics = Back EMF is ideally sinusoidal; NO inverter current ripple:

$$p_\delta(t) = \hat{U}_p\cos(\omega t)\cdot\hat{I}\cos(\omega t) + \hat{U}_p\cos(\omega t - 2\pi/3)\cdot\hat{I}\cos(\omega t - 2\pi/3) + \hat{U}_p\cos(\omega t - 4\pi/3)\cdot\hat{I}\cos(\omega t - 4\pi/3)$$

$$p_\delta(t) = \frac{\hat{U}_p\hat{I}}{2}\cdot\left[\cos(2\omega t)+1\right] + \frac{\hat{U}_p\hat{I}}{2}\cdot\left[\cos(2\omega t - \frac{4\pi}{3})+1\right] + \frac{\hat{U}_p\hat{I}}{2}\cdot\left[\cos(2\omega t - \frac{8\pi}{3})+1\right]$$

$$p_\delta(t) = m\frac{\hat{U}_p\hat{I}}{2} = const. \qquad \boxed{M_e = \frac{(3/2)\cdot\hat{U}_p\cdot\hat{I}}{2\cdot\pi\cdot n} = const.}$$

No load torque ripple occurs due to ideally sinusoidal back EMF & current time function !

Load torque of fundamental fields, calculated via air gap fields

Air gap torque: $M_e(t) = z \cdot F_c(t) \cdot d_{si}/2 = \sum_{j=1}^{z} i_c(x_j,t) \cdot B_r(x_j,t) \cdot l \cdot d_{si}/2$

Air gap torque via current loading: $M_e(t) \sim l \cdot \int_0^{2p\tau_p} A_s(x_s,t) \cdot B_r(x_s,t) \cdot dx_s$

Determination of stator current loading from stator air gap field:

$$B_{\delta s}(x_s,t) = \mu_0 H_{\delta s}(x_s,t) = \mu_0 V_s(x_s,t)/\delta = \mu_0 \cdot \int A_s(x_s,t) \cdot dx_s/\delta$$

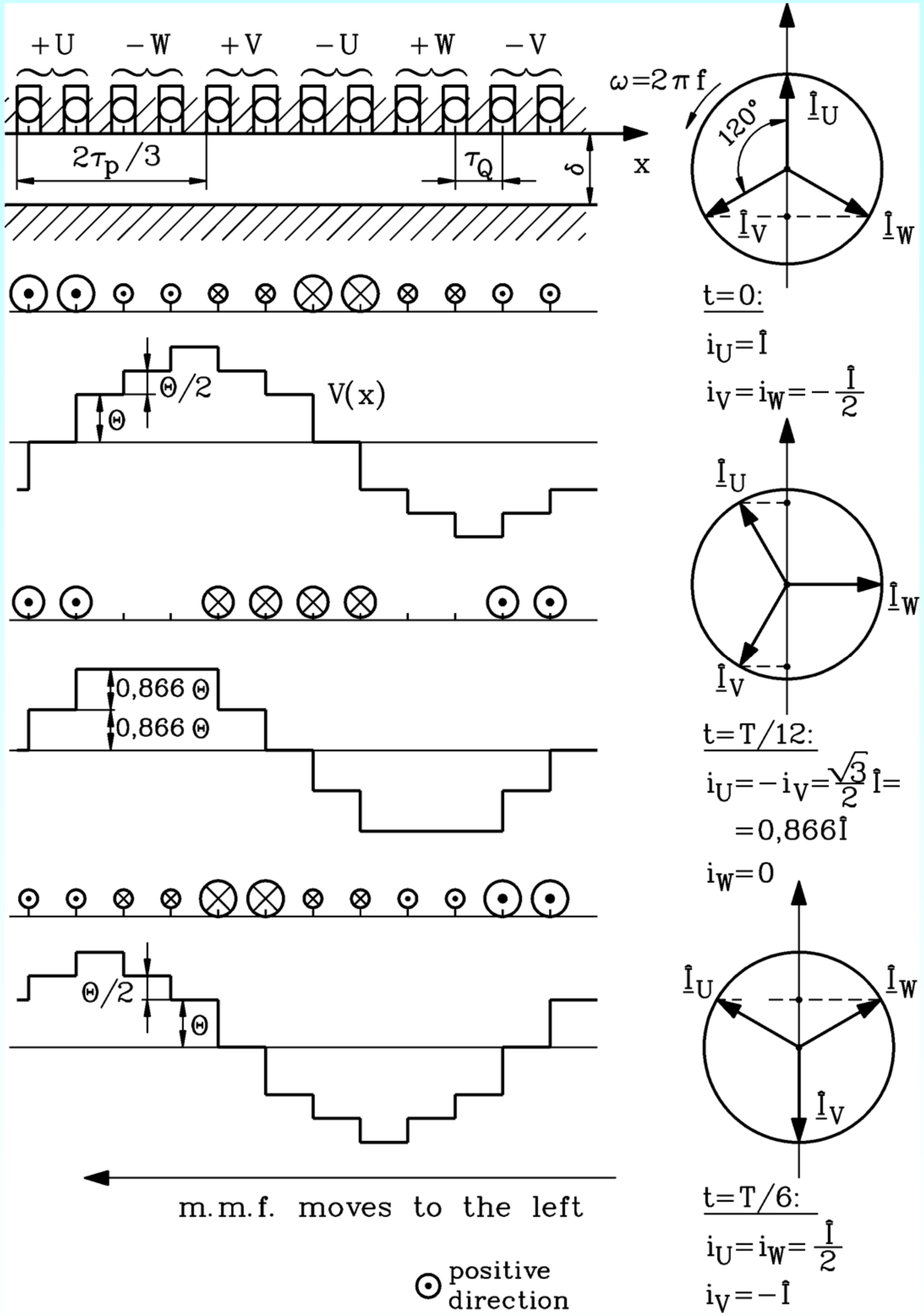
$$A_s(x_s,t) = (\delta/\mu_0) \cdot dB_{\delta s}(x_s,t)/dx_s$$

Air gap torque of stator and rotor fundamental field wave $\nu = 1$ and $\mu = 1$ for $I_s = I_q$:

$$M_e(t) \sim \int_0^{2p\tau_p} A_{s1}\cos(x_s\pi/\tau_p - \omega t) \cdot B_{r1}\cos(x_s\pi/\tau_p - \omega t) \cdot dx_s$$

Air gap torque is constant: $M_e(t) \sim A_{s1}B_{r1} \cdot p\tau_p = const. = M_e$

Torque ripple due to space harmonics (1)

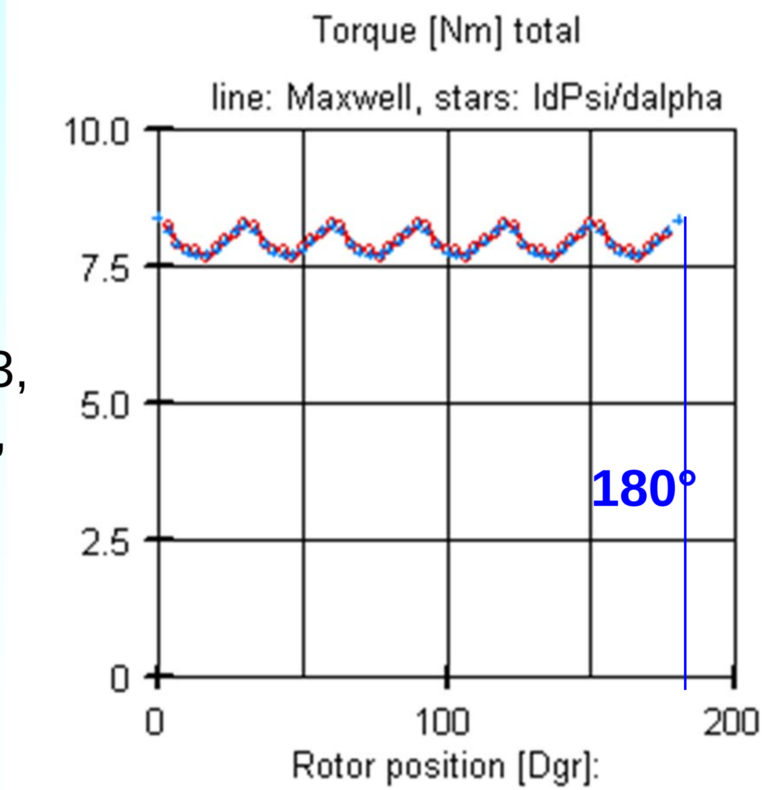


- Current supply is purely sinusoidal
- Field wave changes shape six times per period $T = 1/f_s$
- Torque pulsates with $6f_s$

<u>*Example:*</u>

4-pole motor, q =3, semi-closed slots, surface magnets

Source: 2D Finite element program FEMAG, ETH Zurich



Torque ripple due to space harmonics (2)

Air gap torque of stator and rotor 5-th field wave $\nu = -5$ and $\mu = 5$ for $I_s = I_q$:

$$m_{e,5}(t) \sim \int_0^{2p\tau_p} A_{s5}\cos(5x_s\pi/\tau_p + \omega t)\cdot B_{r5}\cos(5x_s\pi/\tau_p - 5\omega t)\cdot dx_s$$

$$m_{e,5}(t) \sim (A_{s5}B_{r5}/2)\cdot \int_0^{2p\tau_p}\left[\cos(10x_s\pi/\tau_p - 4\omega t) + \cos(6\omega t)\right]\cdot dx_s$$

$$\boxed{m_{e,5}(t) \sim A_{s5}B_{r5}\cdot p\tau_p\cdot\cos(6\omega t)}$$

Air gap torque of stator and rotor 7-th field wave $\nu = 7$ and $\mu = 7$ for $I_s = I_q$:

$$m_{e,7}(t) \sim \int_0^{2p\tau_p} A_{s7}\cos(7x_s\pi/\tau_p - \omega t)\cdot B_{r7}\cos(7x_s\pi/\tau_p - 7\omega t)\cdot dx_s$$

$$m_{e,7}(t) \sim (A_{s7}B_{r7}/2)\cdot \int_0^{2p\tau_p}\left[\cos(14x_s\pi/\tau_p - 8\omega t) + \cos(6\omega t)\right]\cdot dx_s$$

$$\boxed{m_{e,7}(t) \sim A_{s7}B_{r7}\cdot p\tau_p\cdot\cos(6\omega t)}$$

Air gap torque of 5th and 7th space harmonics add to a pulsating torque with $6f_s$.

Alternative torque ripple calculation via air gap power

Rotor 5-th field wave μ = 5 induces stator back EMF with $5f_s$: $u_{p,5}(t) = \hat{U}_{p5} \cdot \cos(5\omega t)$

$$p_{\delta 5}(t) = \hat{U}_{p5}\cos(5\omega t)\cdot \hat{I}\cos(\omega t) + \hat{U}_{p5}\cos\left(5\omega t - 5\cdot\frac{2\pi}{3}\right)\cdot \hat{I}\cos\left(\omega t - \frac{2\pi}{3}\right) + \hat{U}_{p5}\cos\left(5\omega t - 5\cdot\frac{4\pi}{3}\right)\cdot \hat{I}\cos\left(\omega t - \frac{4\pi}{3}\right)$$

$$p_{\delta 5}(t) = \hat{U}_{p5}\cos(5\omega t)\cdot \hat{I}\cos(\omega t) + \hat{U}_{p5}\cos\left(5\omega t - \frac{4\pi}{3}\right)\cdot \hat{I}\cos\left(\omega t - \frac{2\pi}{3}\right) + \hat{U}_{p5}\cos\left(5\omega t - \frac{2\pi}{3}\right)\cdot \hat{I}\cos\left(\omega t - \frac{4\pi}{3}\right)$$

$$p_{\delta 5}(t) = \frac{\hat{U}_{p5}\cdot\hat{I}}{2}\cdot\left[\cos(6\omega t) + \cos(4\omega t) + \cos(6\omega t) + \cos\left(4\omega t - \frac{2\pi}{3}\right) + \cos(6\omega t) + \cos\left(4\omega t - \frac{4\pi}{3}\right)\right]$$

$$p_{\delta 5}(t) = \frac{3\hat{U}_{p5}\cdot\hat{I}}{2}\cdot\cos(6\omega t) = m_{e5}(t)\cdot 2\pi n$$

Rotor 7-th field wave μ = 7 induces stator back EMF with $7f_s$: $u_{p,7}(t) = \hat{U}_{p7} \cdot \cos(7\omega t)$

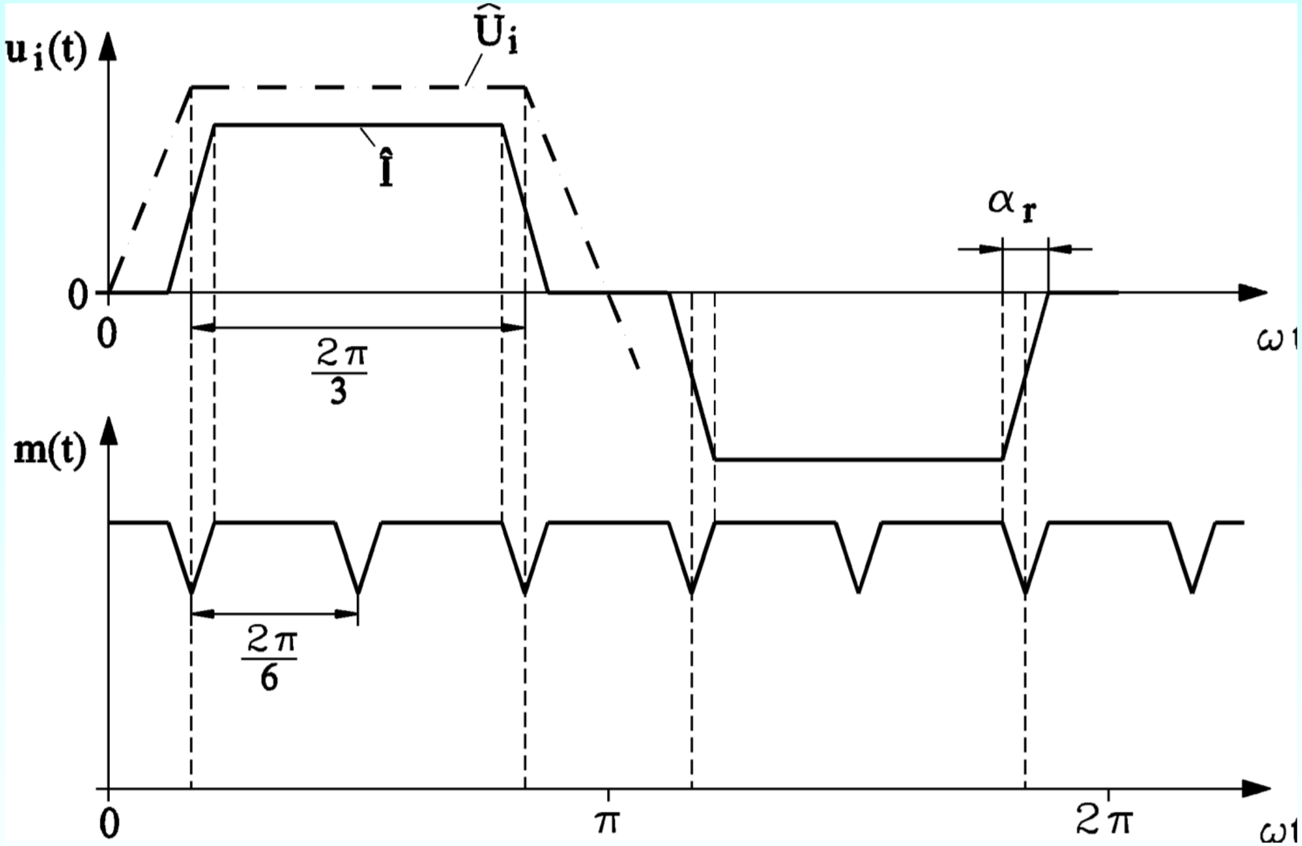
$$p_{\delta 7}(t) = \hat{U}_{p7}\cos(7\omega t)\cdot \hat{I}\cos(\omega t) + \hat{U}_{p7}\cos\left(7\omega t - 7\cdot\frac{2\pi}{3}\right)\cdot \hat{I}\cos\left(\omega t - \frac{2\pi}{3}\right) + \hat{U}_{p5}\cos\left(7\omega t - 7\cdot\frac{4\pi}{3}\right)\cdot \hat{I}\cos\left(\omega t - \frac{4\pi}{3}\right)$$

$$p_{\delta 7}(t) = \frac{\hat{U}_{p7}\cdot\hat{I}}{2}\cdot\left[\cos(8\omega t) + \cos(6\omega t) + \cos\left(8\omega t - \frac{4\pi}{3}\right) + \cos(6\omega t) + \cos\left(8\omega t - \frac{2\pi}{3}\right) + \cos(6\omega t)\right]$$

$$p_{\delta 7}(t) = \frac{3\hat{U}_{p7}\cdot\hat{I}}{2}\cdot\cos(6\omega t) = m_{e7}(t)\cdot 2\pi n$$

Air gap torque of 5th and 7th space harmonics add to a pulsating torque with $6f_s$.

Load torque ripple in block commutated brushless DC machines



Generation of load torque ripple due to block current commutation with finite current rise time t_r (corresponding angle α_r)

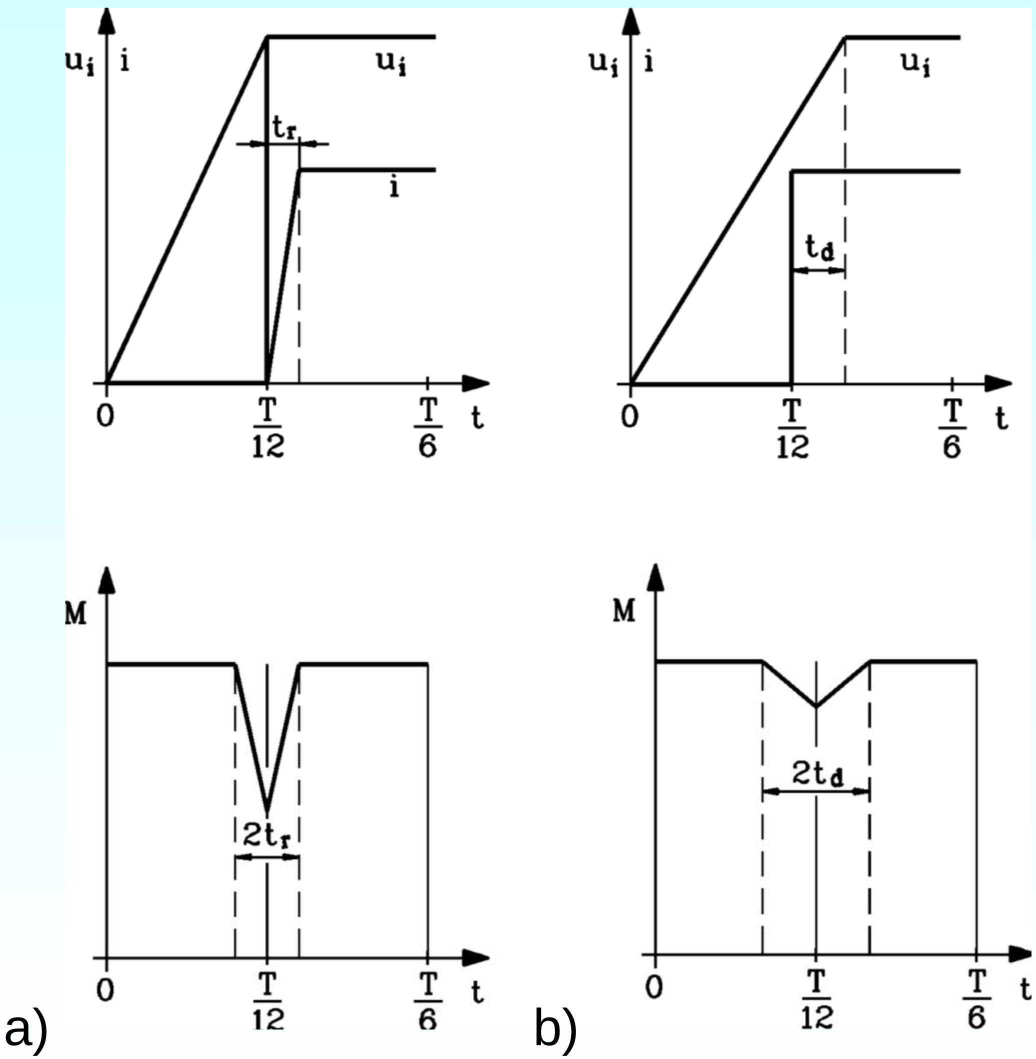
Typical block commutation torque ripple values:

$$\hat{w}_M \sim 3\%...5\%$$

Facit:

The generated load torque ripple is with <u>six times fundamental frequency</u>.

Two typical reasons for load torque ripple with block commutated brushless DC motors



a) deviation of block current from ideal rectangular shape (finite rise time t_r),

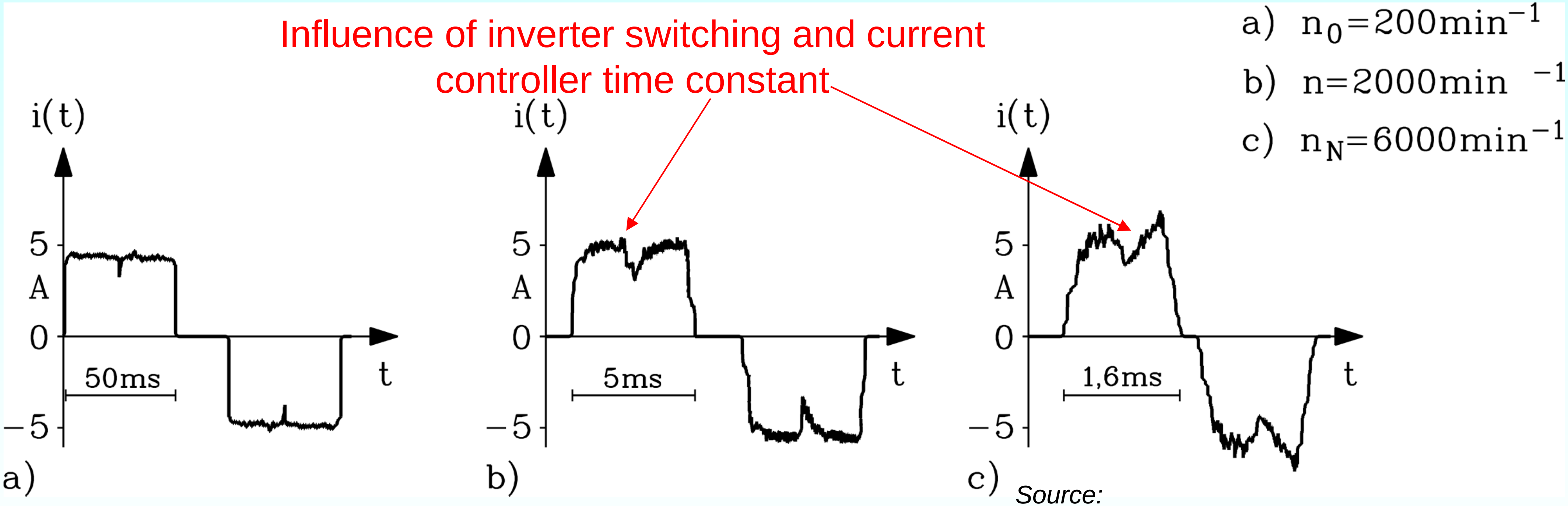
b) deviation of trapezoidal back EMF from ideal shape (slope increased by t_d)

Facit:

The sine wave commutated motor has a lower load dependent torque ripple (~ 1%) than the block commutated brushless DC drive (ca. 4 ... 5%).

Pulsating torque due to current ripple

- Inverter switching causes current ripple = current time harmonics. Each current harmonic causes a stator fundamental field, which interacts with rotor fundamental PM field $\mu = 1$.



Source:
Henneberger G, Schleuter W (1989) Elektrotechn. Z. 110: 274-279

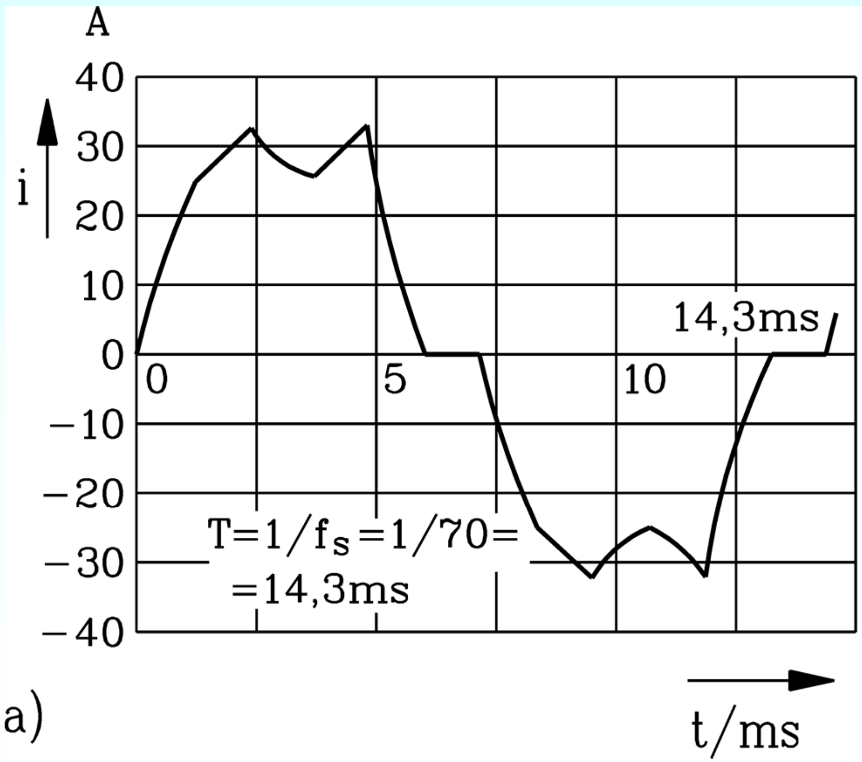
Measured block wave commutated stator phase current at low, medium and rated speed.

Block commutated PM servo: Torque ripple at voltage limit

- At very high speed and frequency f_s, when voltage limit is reached, only six-step operation is possible, if inverter switching frequency is too low with respect to maximum f_s.

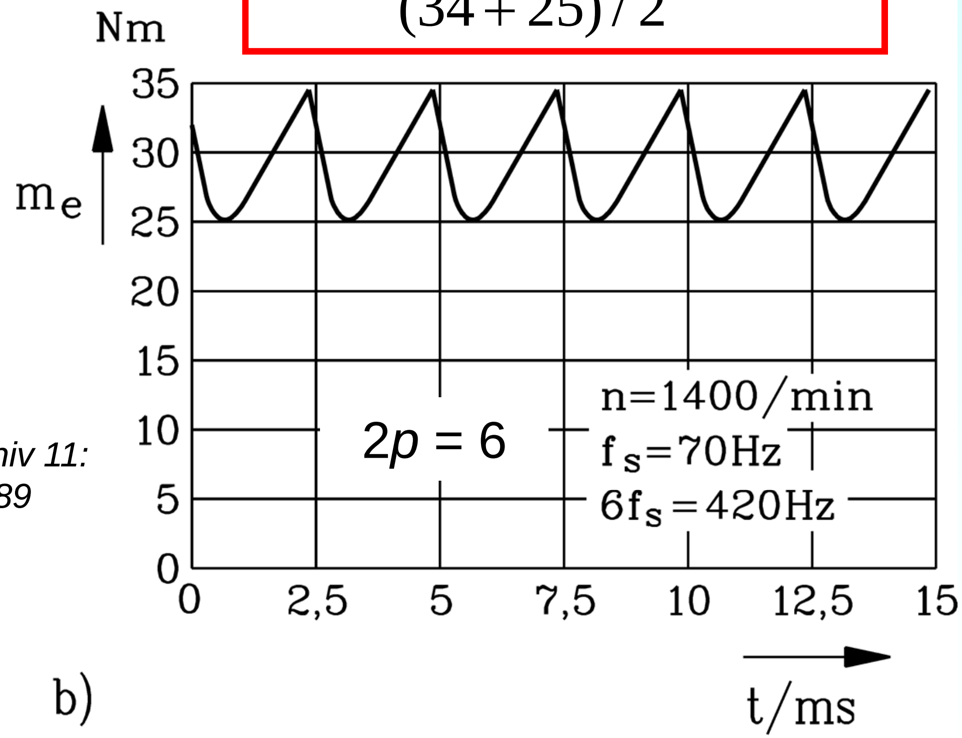
- Phase current a) loses block shape, and torque b) shows MAXIMUM ripple with $6f_s$!

$$\hat{w}_M = \frac{(34-25)/2}{(34+25)/2} = 15.3\%$$



a)

Source:
Huth G, etz-Archiv 11: 401-408, 1989

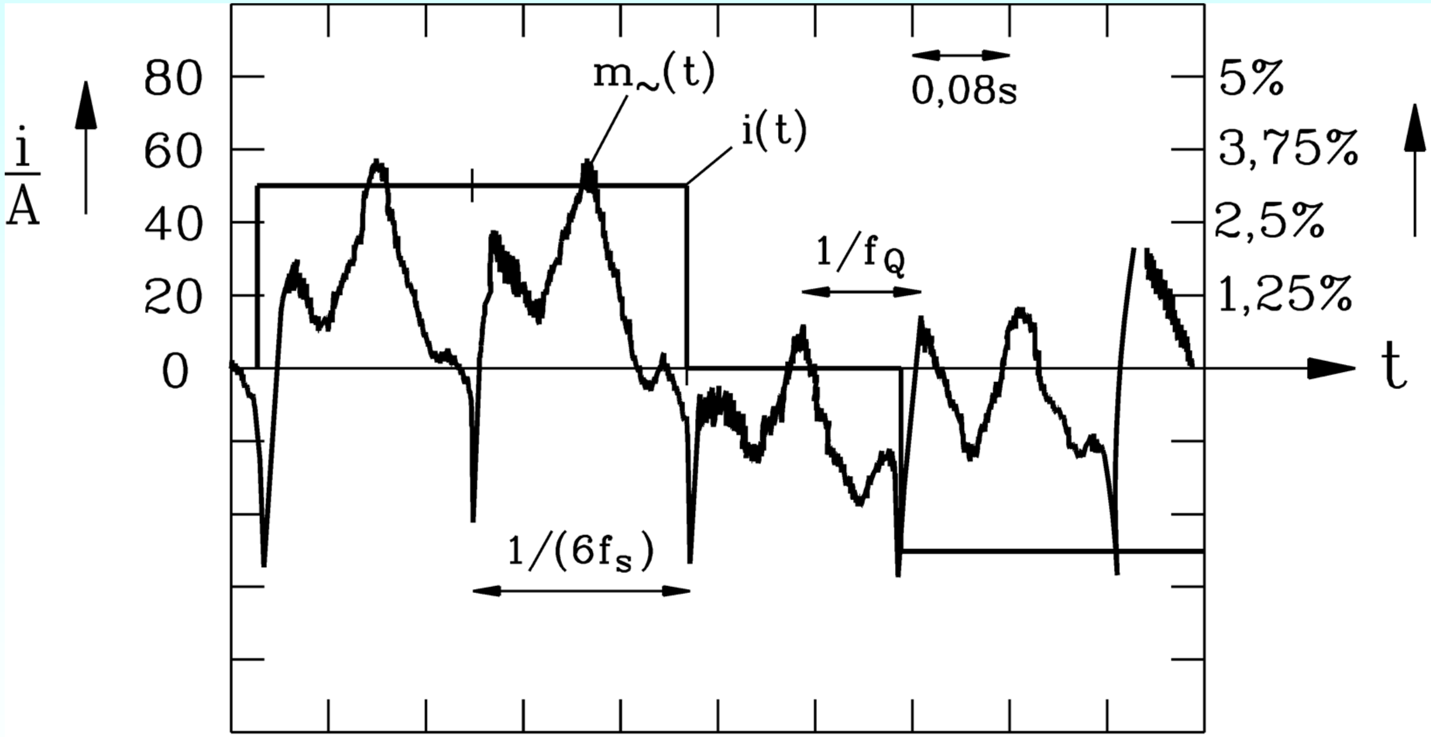


b)

Block commutated PM servo: Torque ripple at very low speed

- At very low speed and frequency f_s, due to fast switching with respect to f_s, phase current looks perfectly block-like.

- Torque ripple is MINIMUM, being determined by (1) cogging with slot frequency, (2) current commutation with $6f_s$!



$$\hat{w}_M = 3.6\%$$

no-load: slot frequency

$$f_Q = n \cdot Q = \frac{20}{60} \cdot 36 = 12\,\mathrm{Hz}$$

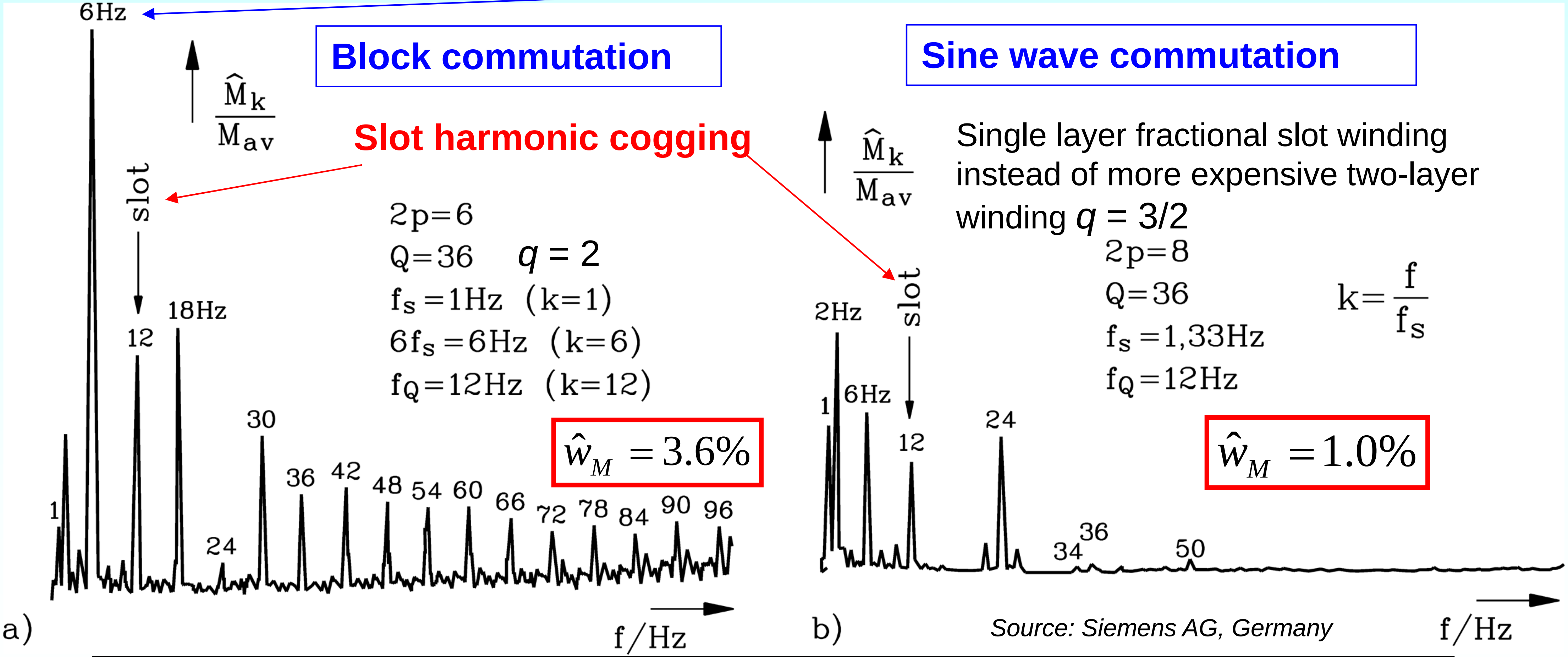
load: commutation frequency: $f = 6f_s = 6\,\mathrm{Hz}$

$\hat{I} = 51{,}5\,\mathrm{A}$; $f_s = 1\,\mathrm{Hz}$; $n = f_s/p = 20/\mathrm{min}$; $2p = 6$

Source: Siemens AG, Germany

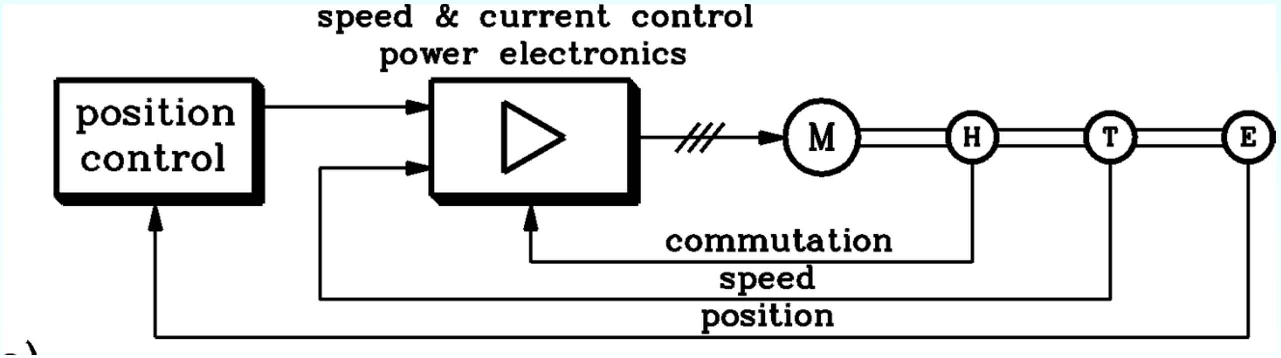
Comparison of torque ripple of block and sine wave commutated PM motors at low speed

- Measured FFT-analysis of torque signal $m_e(t)$ at 20/min
- Block commutated motor has higher torque ripple **due to 6f_s component**



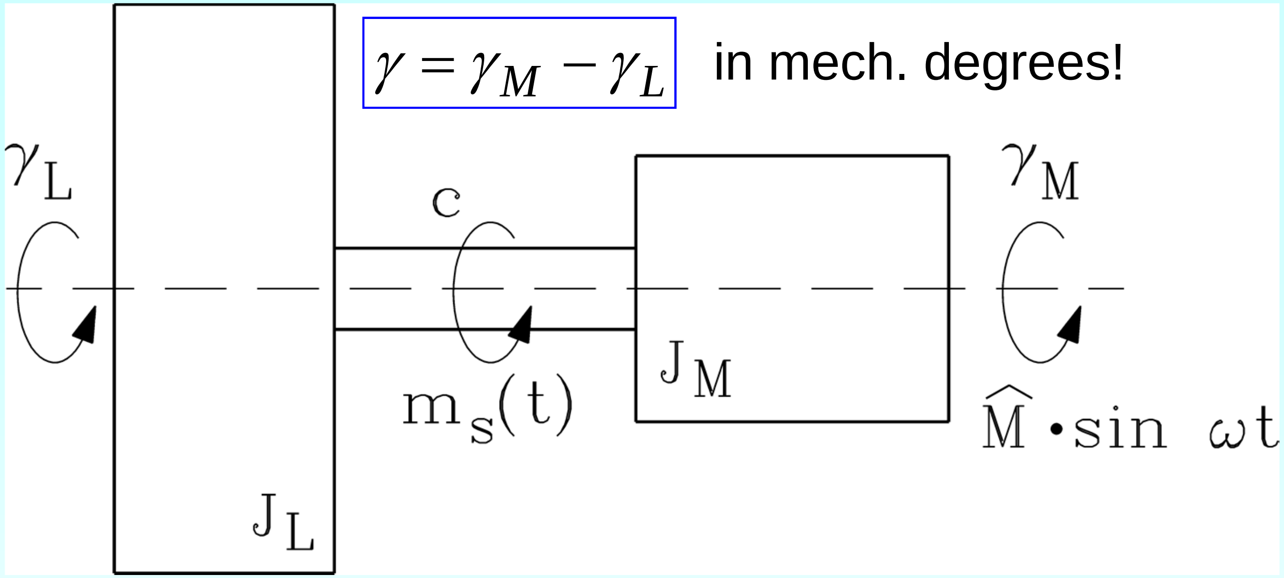
1. Permanent magnet synchronous machines as “brushless DC drives”

1.2.3 Significance of torque ripple for motor and drive performance



Source:
Lehmann R.: Technik bürstenloser Servoantriebe, Elektrotechnik 21: 96-101, 1989

Torsion resonance



- Rotor of motor coupled to rotating load via an elastic coupling
- Coupling stiffness c
- Inertia of motor and load J_M, J_L
- Exciting k-th harmonic air gap torque $m_{e,k}$

$$J_L \ddot{\gamma}_L - m_s = 0 \quad , \quad J_M \ddot{\gamma}_M + m_s = m_{e,k} \quad , \quad m_s = c \cdot (\gamma_M - \gamma_L)$$

$$\ddot{\gamma}_L = m_s / J_L \quad , \quad \ddot{\gamma}_M = (m_{e,k} - m_s)/J_M \quad \Rightarrow \quad \ddot{\gamma}_M - \ddot{\gamma}_L = -\left(\frac{1}{J_M} + \frac{1}{J_L}\right) \cdot m_s + \frac{m_{e,k}}{J_M}$$

Differential equation:

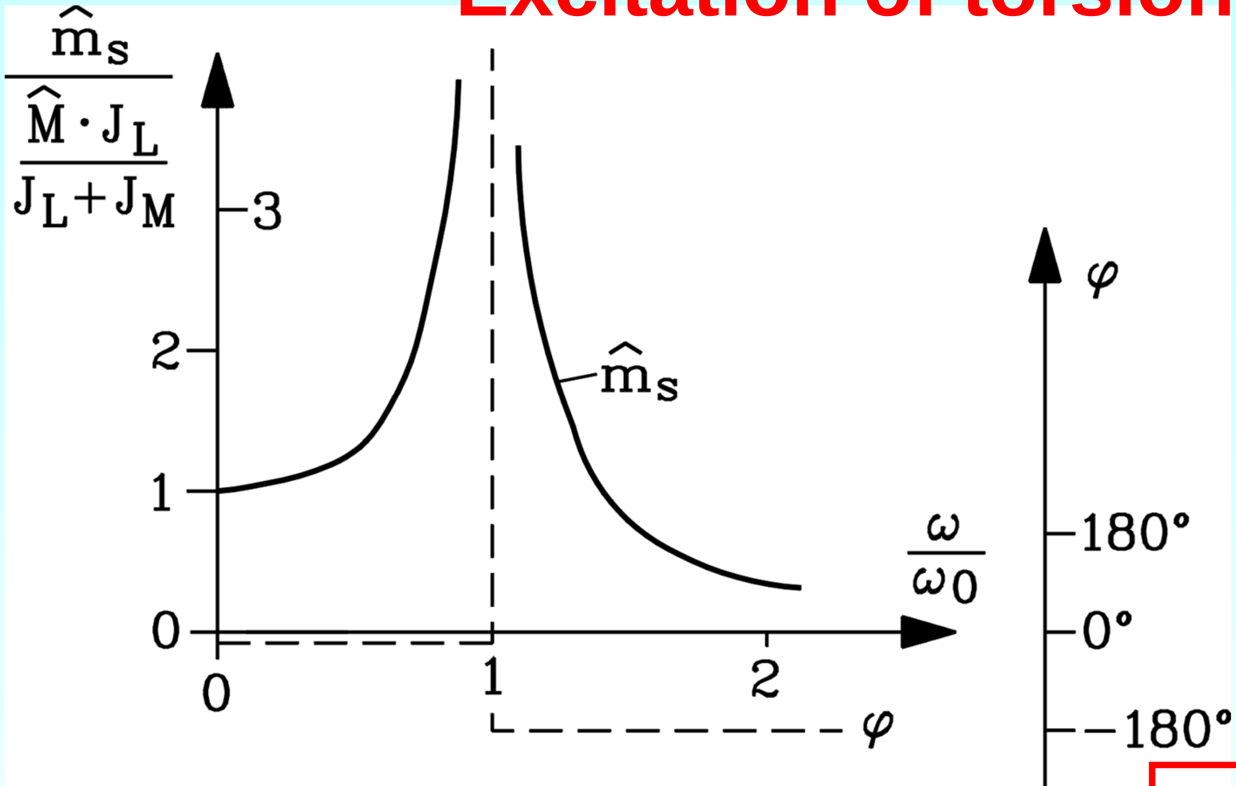
$$\ddot{\gamma} + \frac{c \cdot (J_M + J_L)}{J_M \cdot J_L} \cdot \gamma = \frac{m_{e,k}}{J_M}$$

Homogeneous solution leads to

torsion resonance frequency:

$$f_0 = \omega_0 / (2\pi) = \frac{1}{2\pi} \cdot \sqrt{c \cdot \frac{J_M + J_L}{J_M \cdot J_L}}$$

Excitation of torsion vibrations



Pulsating torque excites torsion vibrations:

$$m_{e,k}(t) = \hat{M} \cdot \sin(\omega t)$$

$$\omega = k \cdot (2\pi \cdot n \cdot p)$$

Solution of differential equation with exciting torque ripple yields vibration angle γ and oscillating shaft torque m_s :

$$\gamma(t) = \frac{\hat{M}}{J_M} \cdot \frac{1}{\omega_o^2 - \omega^2} \cdot \sin(\omega t)$$

$$m_s(t) = c \cdot \gamma(t) = \frac{\hat{M}}{J_M} \cdot \frac{c}{\omega_o^2 - \omega^2} \cdot \sin(\omega t)$$

1. It must be avoided that the dominant pulsating torque frequency excites the torsion resonance of the drive system. This can be achieved by designing the drive with a stiff coupling (c: high value) to stay with the pulsating torque frequency below the resonance.

2. The high frequency pulsating air gap torque ($\omega > \omega_0$) is filtered and not visible in the shaft torque ripple.

Speed ripple due to torque pulsation

Speed ripple definition: $n(t) = n + \Delta n(t)$

From solution of torsion oscillation we know oscillation angle:

Angular acceleration is: $\ddot{\gamma}_M(t) = \dfrac{m_e(t) - m_s(t)}{J_M} = \dfrac{\hat{M}}{J_M} \cdot \left(1 - \dfrac{c/J_M}{\omega_o^2 - \omega^2}\right) \cdot \sin(\omega t)$

Speed ripple:

$$\Delta n(t) = \dot{\gamma}_M(t)/(2\pi) \qquad \Delta n(t) = -\frac{\hat{M}}{2\pi \cdot \omega \cdot J_M} \cdot \left(1 - \frac{c/J_M}{\omega_o^2 - \omega^2}\right) \cdot \cos(\omega t)$$

Staying below the resonance $\omega << \omega_0$, we observe that especially at low speed the speed ripple amplitude, expressed as percentage of actual speed, increases with DECREASING speed:

$$\left|\frac{\Delta n}{n}\right| = \left|\frac{\hat{M}}{(2\pi)^2 \cdot k \cdot p \cdot n^2 \cdot J_M} \cdot \left(1 - \frac{c/J_M}{\omega_o^2 - \omega^2}\right)\right| \approx \frac{\hat{M}}{(2\pi)^2 \cdot k \cdot p \cdot n^2 \cdot (J_L + J_M)} \sim \frac{1}{n^2}$$

At low speed (servo operation !) the speed ripple is big and of importance!

Block commutated brushless DC drive systems

- permanent magnet motor with three phase, single layer winding, skewed slots and 100% pole coverage ratio for the rotor magnets

- simple rotor position sensor with rotor disc according to pole number,

- brushless DC tachometer for speed measurement,

- encoder with high resolution for precise positioning,

- voltage source inverter system (power transistors), speed and current control, implemented in a micro-computer system; motor current measurement devices such as shunts or DC transformers with *Hall*-sensors,

- motion control system, implemented in a second microcomputer. It allows position control, but also different motion control such as special velocity profiles according to the needs of the driven load.

Sine commutated brushless DC drive systems

- permanent magnet motor with three phase, double layer winding (or special single layer winding with non-integer q), skewed slots and 80%...85% pole coverage ratio for the rotor magnets,

- high resolution rotor position sensor (optical encoder or magnetic resolver), which acts also as speed sensor and sometimes as position sensor for precise positioning,

- voltage source inverter system (power transistors), speed and current control, implemented in a micro-computer system and motor current measurement devices such as shunts or DC transformers with *Hall*-sensors,

- motion control system, which is implemented in a second microcomputer and allows position control, but also different motion control such as special velocity profiles according to the needs of the driven load.

Block and sine commutated brushless DC drives

Block commutation / Sine commutation

Advantages

Block commutation:

- cheap rotor position and speed sensor,
- cheap motor winding
- 15% higher overload at inverter current limit

Sine commutation:

- 10% ... 15% reduced amount of magnets,
- very low torque ripple below 1%,
- reduced additional losses at high speed,
- reduced torque ripple sensitivity due to misalignment of rotor position sensor.

Disadvantages

Block commutation:

- extra encoder for accurate drive positioning,
- higher minimum torque ripple (2% ... 4% at low speed),
- increased additional losses in the motor due to extra eddy currents especially in the rotor due to the rapid chance of stator flux at commutation
- Strongly increased load torque ripple at mis-aligned rotor position sensor.

Sine commutation:

- expensive encoder for current com-mutation and speed measurement,
- expensive stator winding, if two layer winding is used,
- 15% lower overload capability at inverter current limit,
- more complex mathematical model for motor control.