Motor Development for Electrical Drive Systems

Lectures SS 2+1

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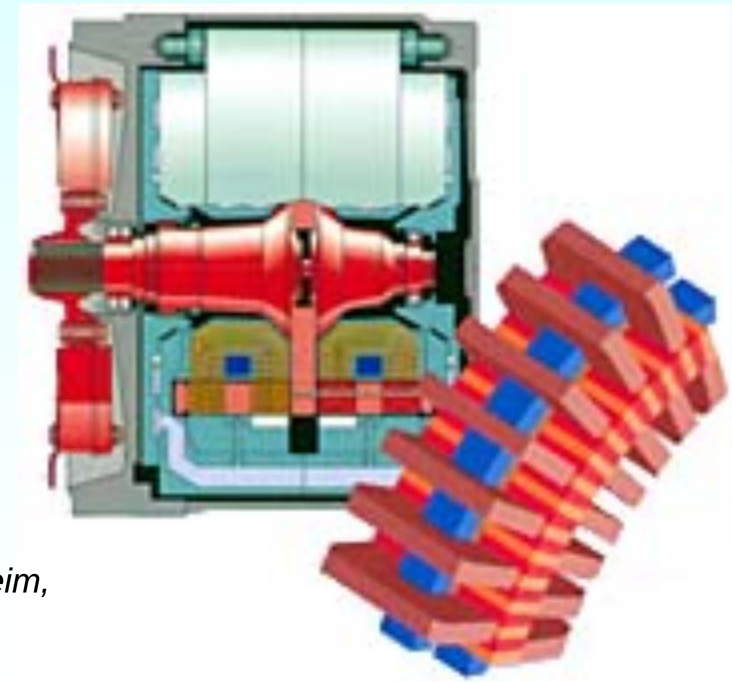


Source: Voith, Heidenheim, Germany

Transversal flux machine

Contents of lectures

- Permanent magnet synchronous machines as “brushless DC drives”
- Basic principles of brushless DC drives
- PM Linear machines
- High Torque & High Speed Machines

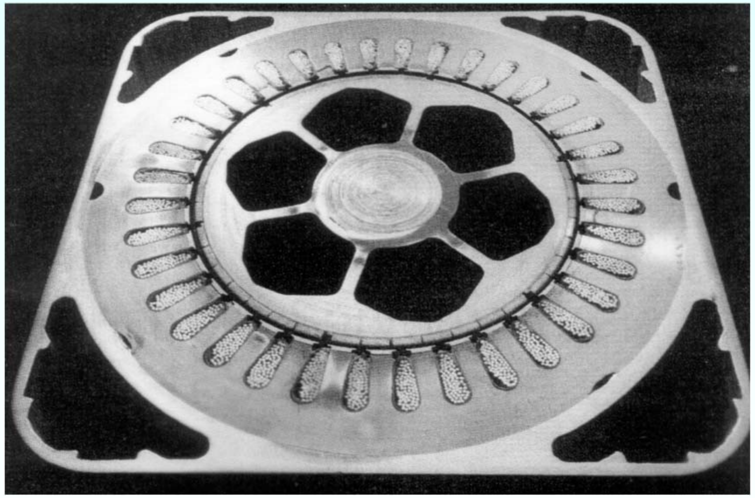
- Reluctance motors
- Switched Reluctance Drives
- Synchronous Reluctance Drives

Add-on offers to the lectures:
Tutorials, excursion to industry
Power point presentation (down load), CD ROM
full text book, collection of calculation examples

- PM Synchronous Machines with Cage Rotor
- Induction machines - Harmonic effects
- Inverter-fed Induction Machines
- Mechanical Rotor Design

1. Permanent magnet synchronous machines as “brushless DC drives”

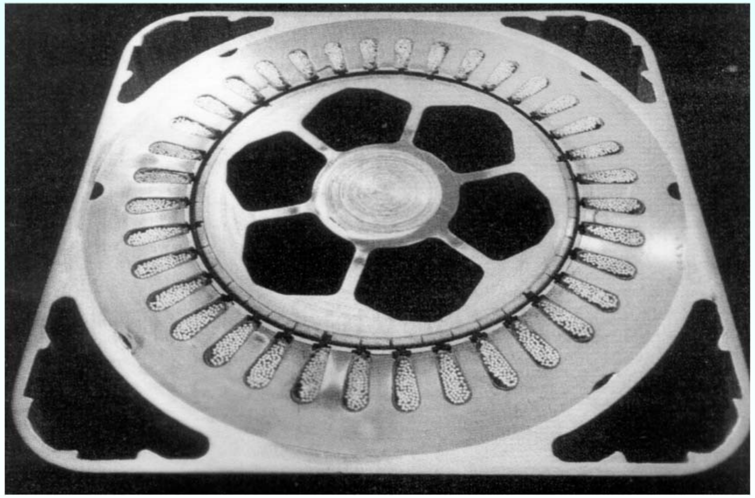
1.1 Basic principles of brushless DC drives



Source: Siemens AG, Germany

1. Permanent magnet synchronous machines as “brushless DC drives”

1.1.1 Basic function of PM synchronous machines



Source: Siemens AG, Germany

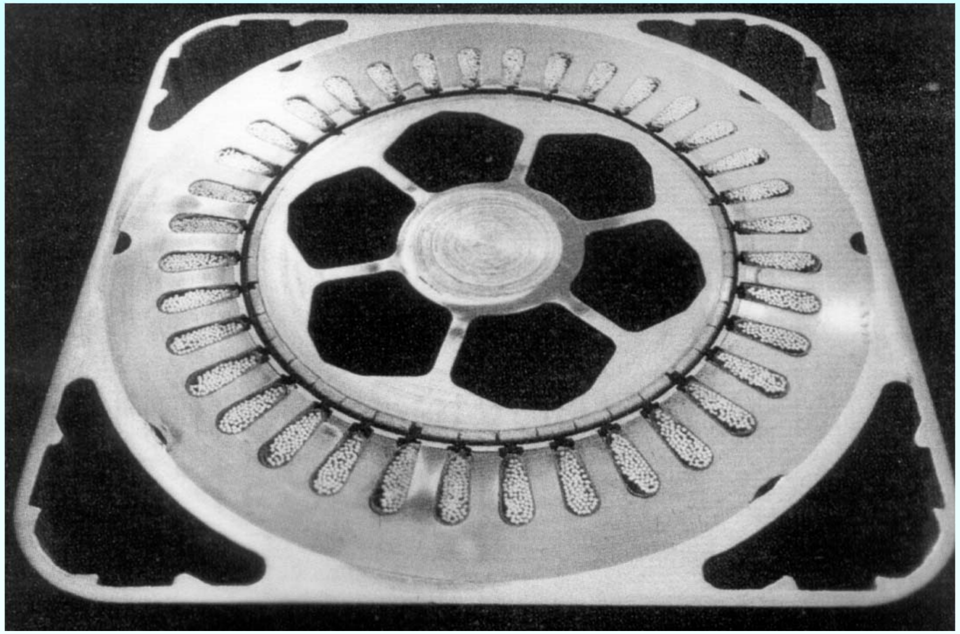
Applications of modern PM synchronous machines

Robot drives

Tooling machines

Source: Kuka, Germany

Industrial Robot with PM Drives



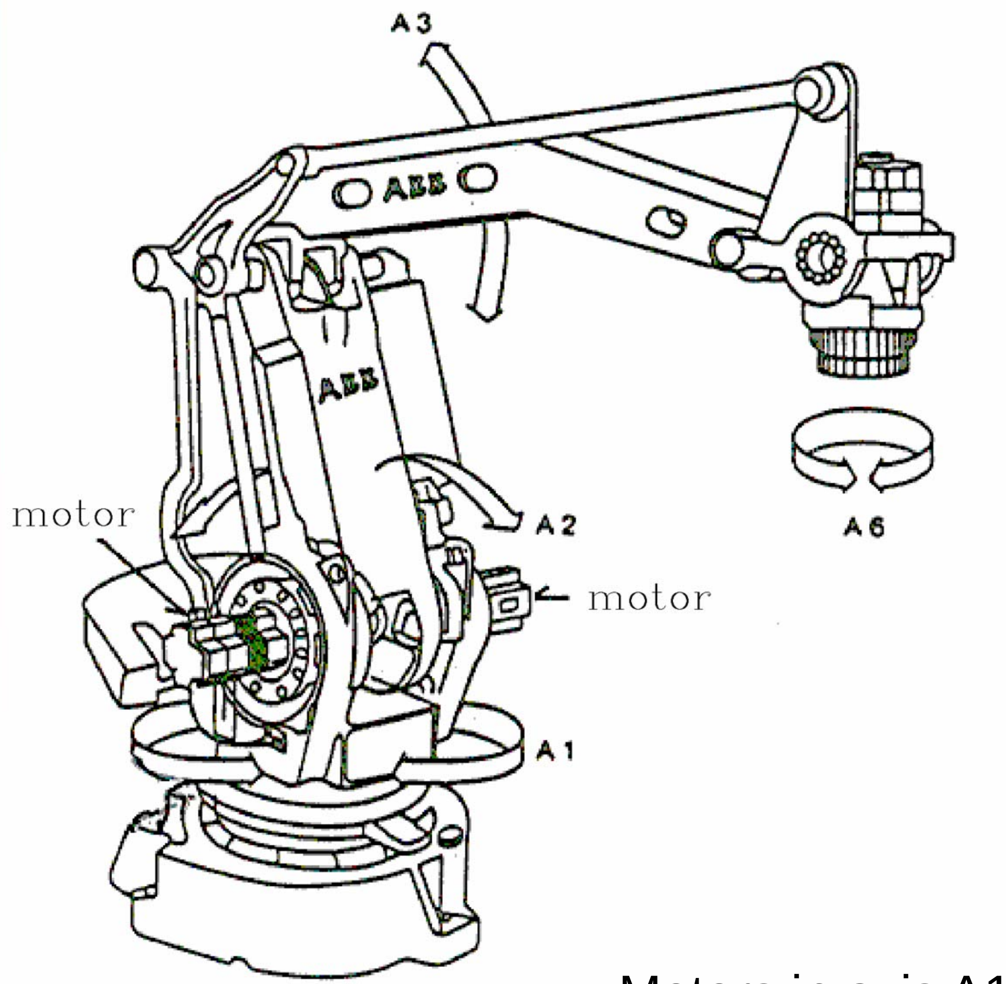
Source: Siemens AG, Germany

Cross section of Six-pole PM Motor

- High precision positioning
- Robust (no cooling)

- High acceleration /deceleration (“dynamic” !)
- Smooth torque

PM synchronous machines ("brushless DC") as robot drives

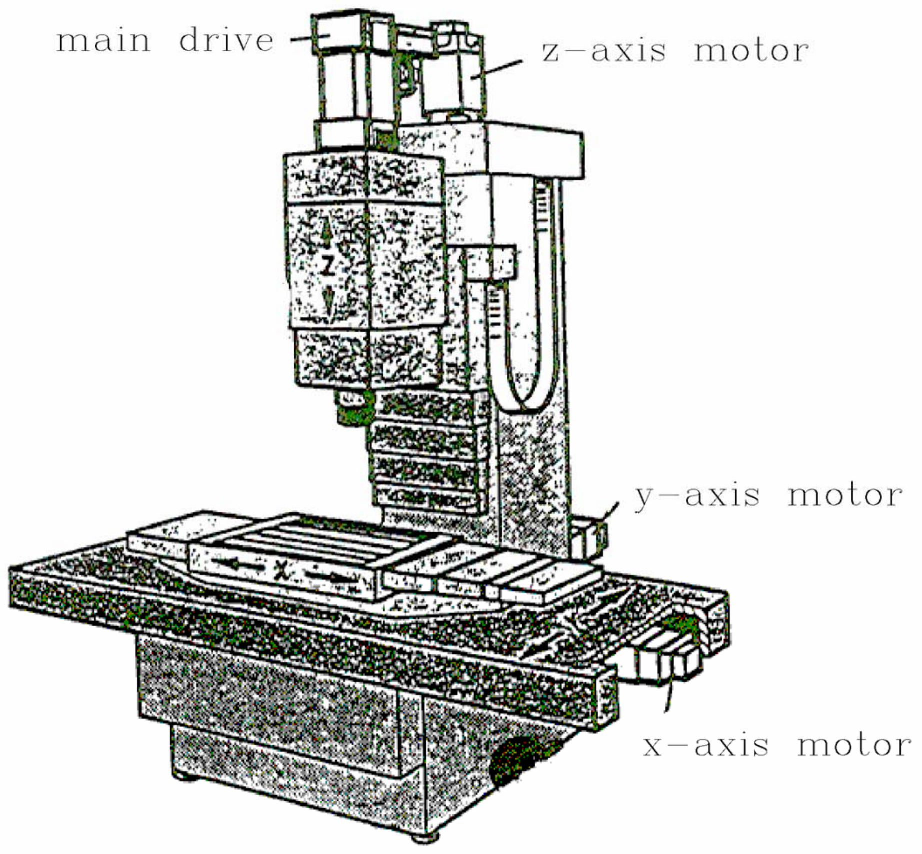


Source: Kuka, Germany

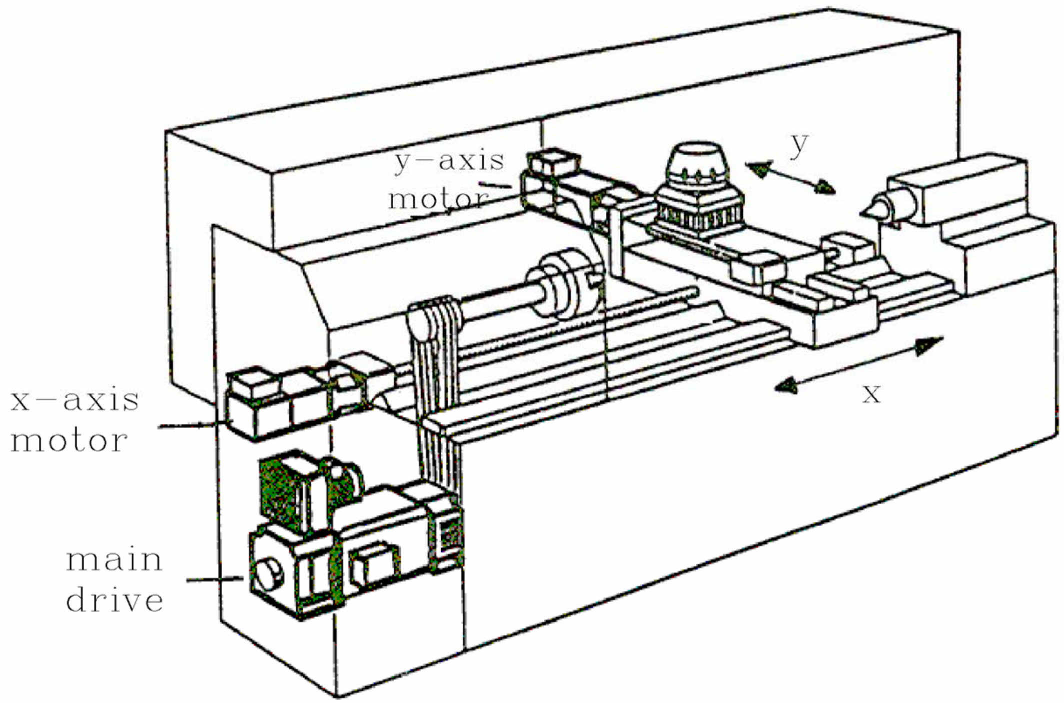
Motors in axis A1, A2, A3 and A6

Source: ABB, Sweden

"Brushless DC" servo drives in tooling machines



Main drives are of 10 … 30 times higher power, either induction or PM synchronous machines



Milling machine: PM motors for servo application in x-, y- and z-axis

Cutting machine: PM motors for servo application in x- and y-axis

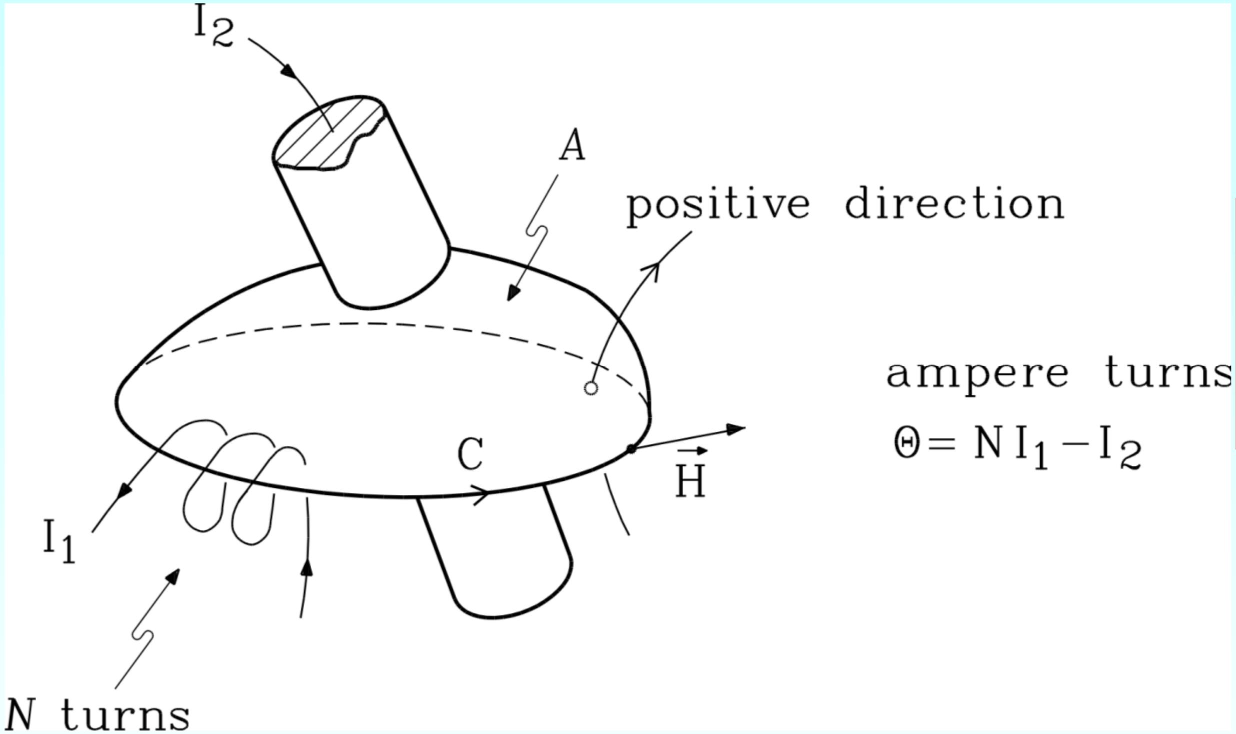
AMPERE´s law: Excitation of magnetic field by electric current

<u>Example:</u>

Two different currents I_1, I_2 with two different numbers of turns 1 and N and two different flow directions:

<u>Ampere turns Θ:</u>

$\Theta = N I_1 - I_2$

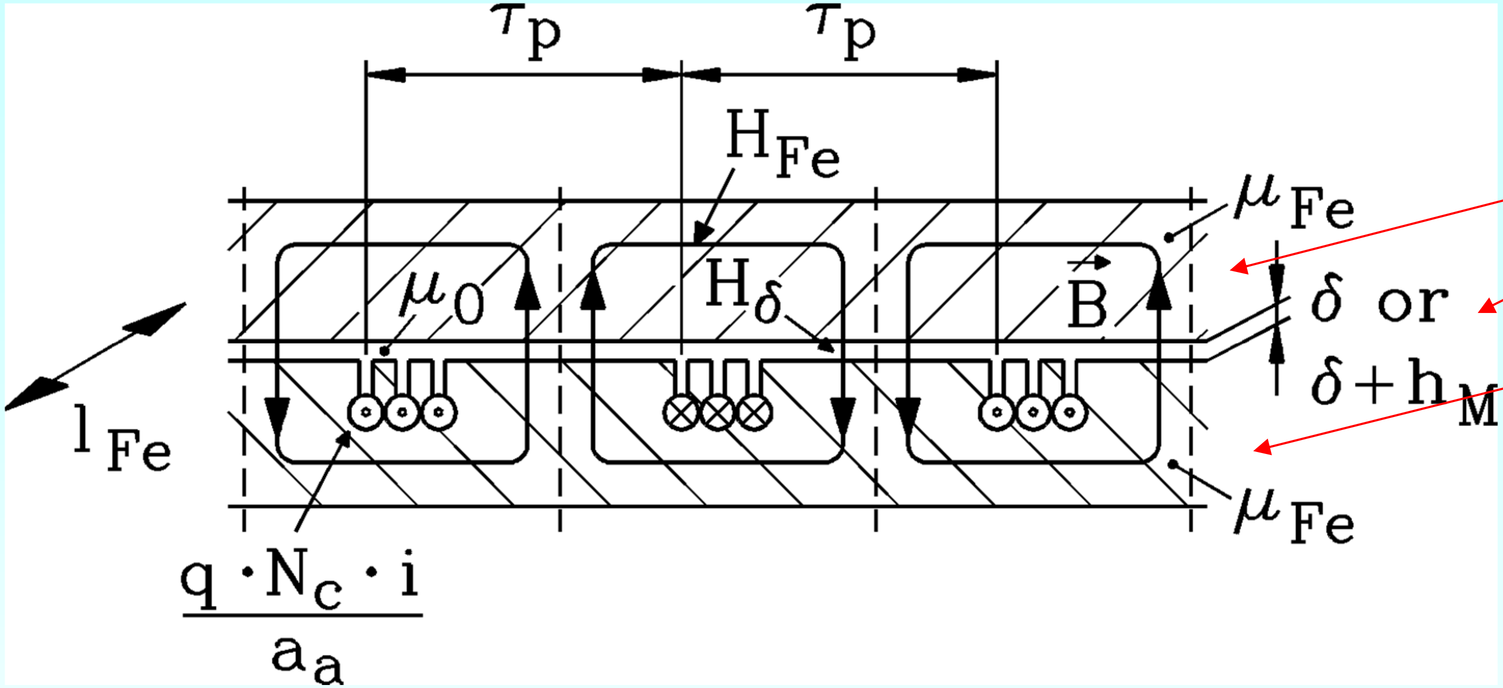


Source: A. Prechtl, TU Wien, Austria

$$\oint_C \vec{H} \cdot d\vec{s} = \Theta$$

- **The integration of <u>magnetic field strength *H*</u> along closed loop (curve *C*), which spans the area *A*, is equal to the resulting current flow (<u>Ampere turns Θ</u>) penetrating through the area *A*.**
- **Positive field direction is connected to positive current flow direction by RIGHT HAND RULE.**

Basic function of PM synchronous machine



Electric machine:

Unslotted rotor

Air gap

Slotted stator:

Inserted coils with two coil sides each, where coil current *i* flows

Ampere´s law for air gap field: $\oint_C \vec{H} \bullet d\vec{s} = H_{Fe,s} \cdot s_{Fe,s} + H_{Fe,r} \cdot s_{Fe,r} + H_{\delta,right} \cdot \delta + H_{\delta,left} \cdot \delta = \Theta_Q$

Infinite iron permeability μ_{Fe} assumed: $\oint_C \vec{H} \bullet d\vec{s} = H_{\delta,right} \cdot \delta - H_{\delta,left} \cdot \delta = \Delta V_\delta(x) = \Theta_Q(x)$

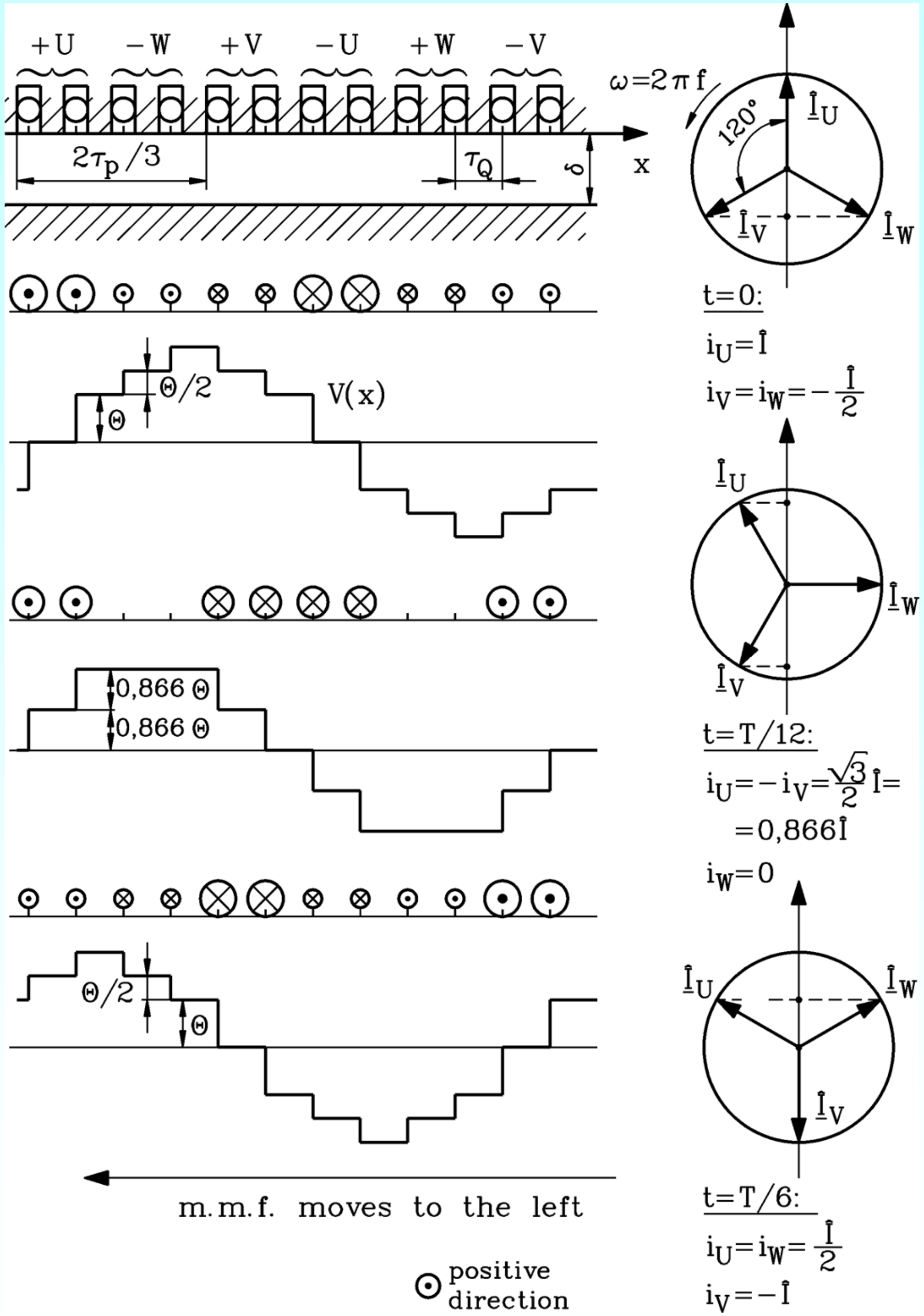
Magnetomotive force distribution in air gap: $V_\delta(x)$

Magnetic field strength distribution in air gap: $H_\delta(x)$

Magnetic flux density distribution in air gap: $B_\delta(x)$

$$V_\delta = H_\delta \cdot \delta$$

$$B_\delta = \mu_0 V_\delta / \delta$$



Magnetic moving field

- Field curve moves with increasing time *t* to the left !
- After time *T* the field curve has passed the distance $2\tau_p$
- Velocity of <u>linear</u> movement is called

$$v_{syn} = \frac{2\tau_p}{T} = 2f\tau_p$$

synchronous velocity !

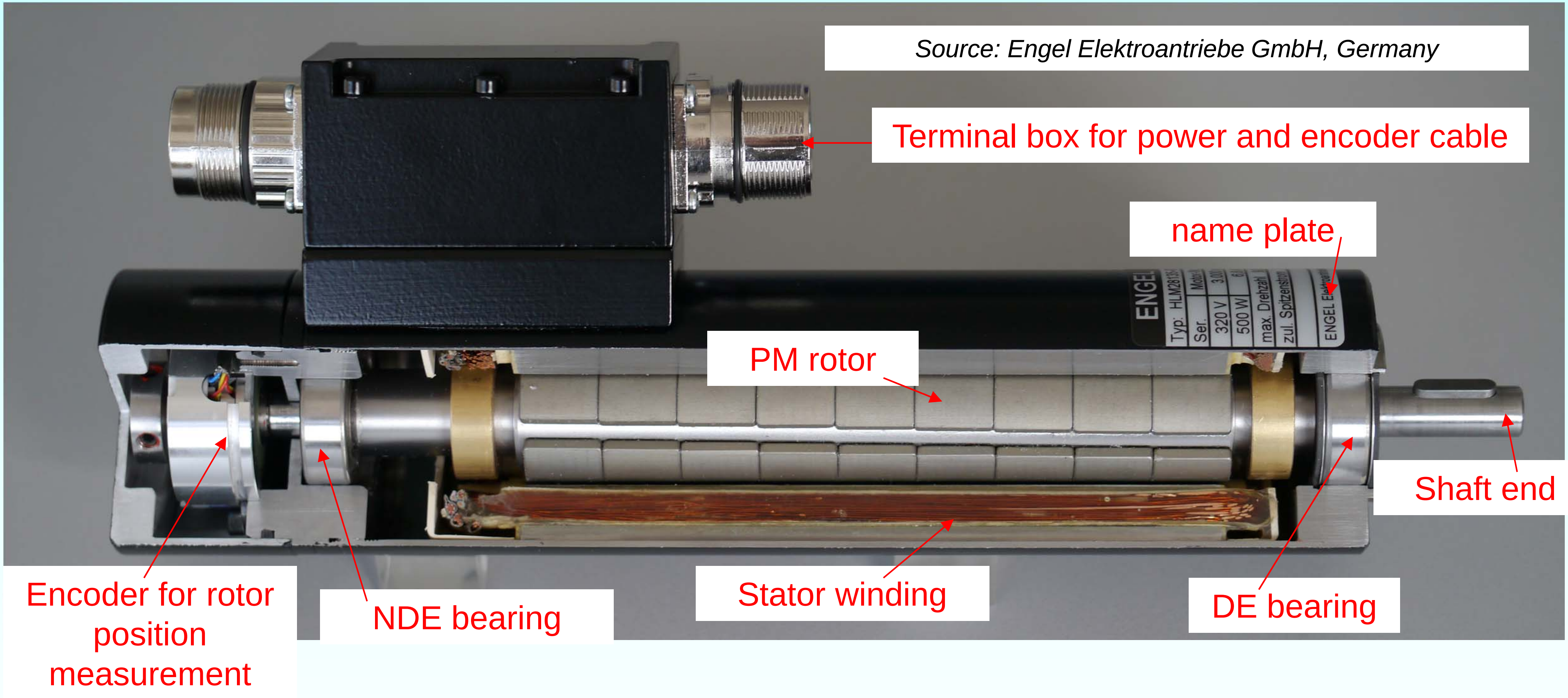
Synchronous rotational speed n_{syn}

in case of <u>rotating</u> field arrangement:

$$\omega_{syn} = 2\pi n_{syn} = \frac{v_{syn}}{d_{si}/2} = \frac{v_{syn}}{p\tau_p/\pi} = \frac{2\pi f}{p}$$

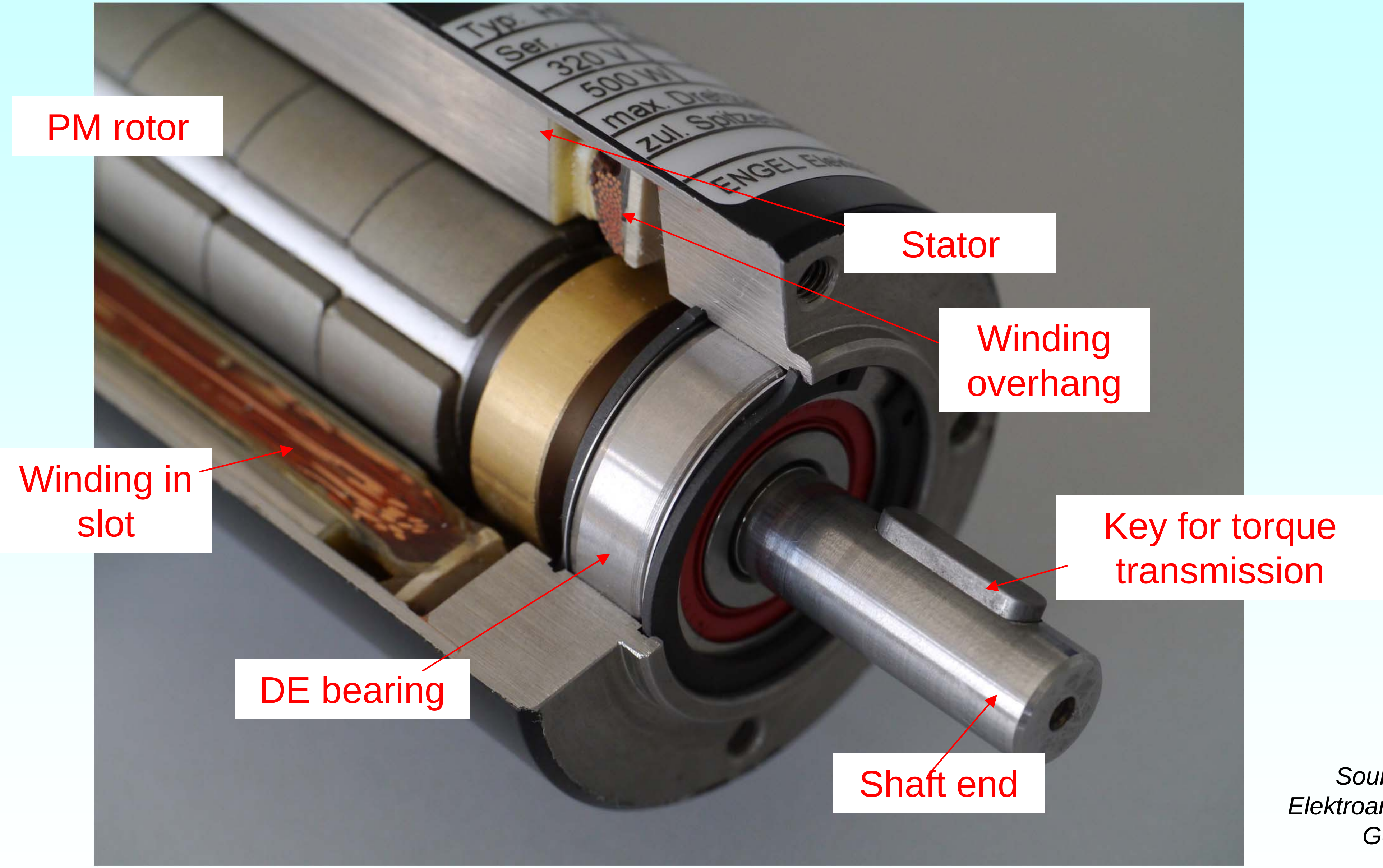
$$n_{syn} = \frac{f}{p}$$

PM synchronous servo motor overview



Small 500 W, 320 V, self-cooled 4-pole three-phase PM synchronous servo motor

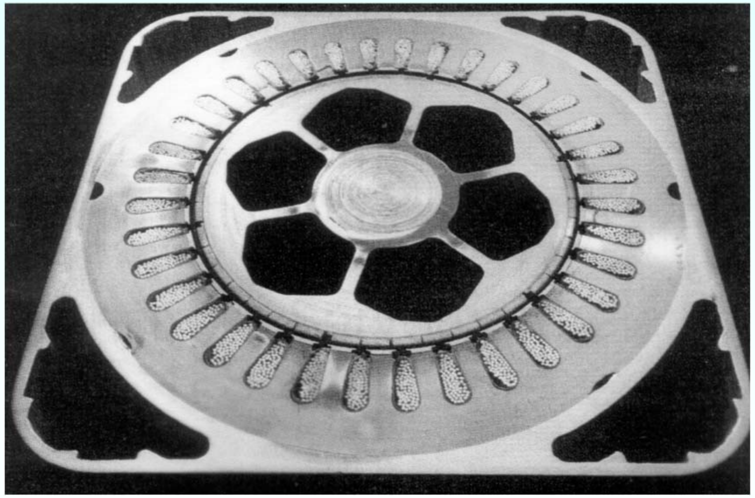
Single-layer three-phase round-wire stator winding



Source: Engel Elektroantriebe GmbH, Germany

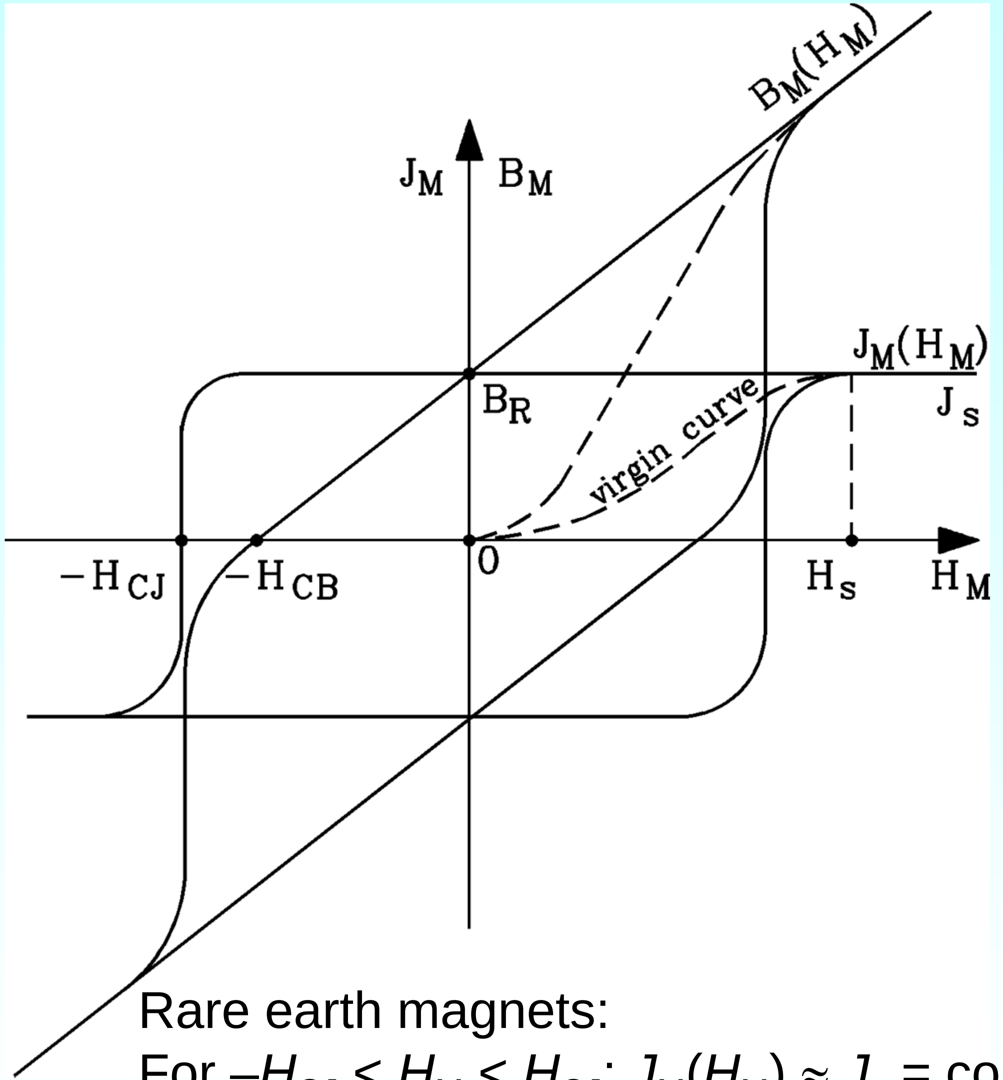
1. Permanent magnet synchronous machines as “brushless DC drives”

1.1.2 Permanent magnet technology



Source: Siemens AG, Germany

Permanent magnet technology



Permanent magnets:

- AlNiCo,
- Ba-Ferrite and Sr-Ferrite,
- Rare earth magnets SmCo and NdFeB

Magnetic field inside permanent magnet:

$$B_M = \mu_0 H_M + J_M$$

J_M: magnetic polarization

Saturated values: Subscript *s*

Remanence flux density: $B_R = J_R$

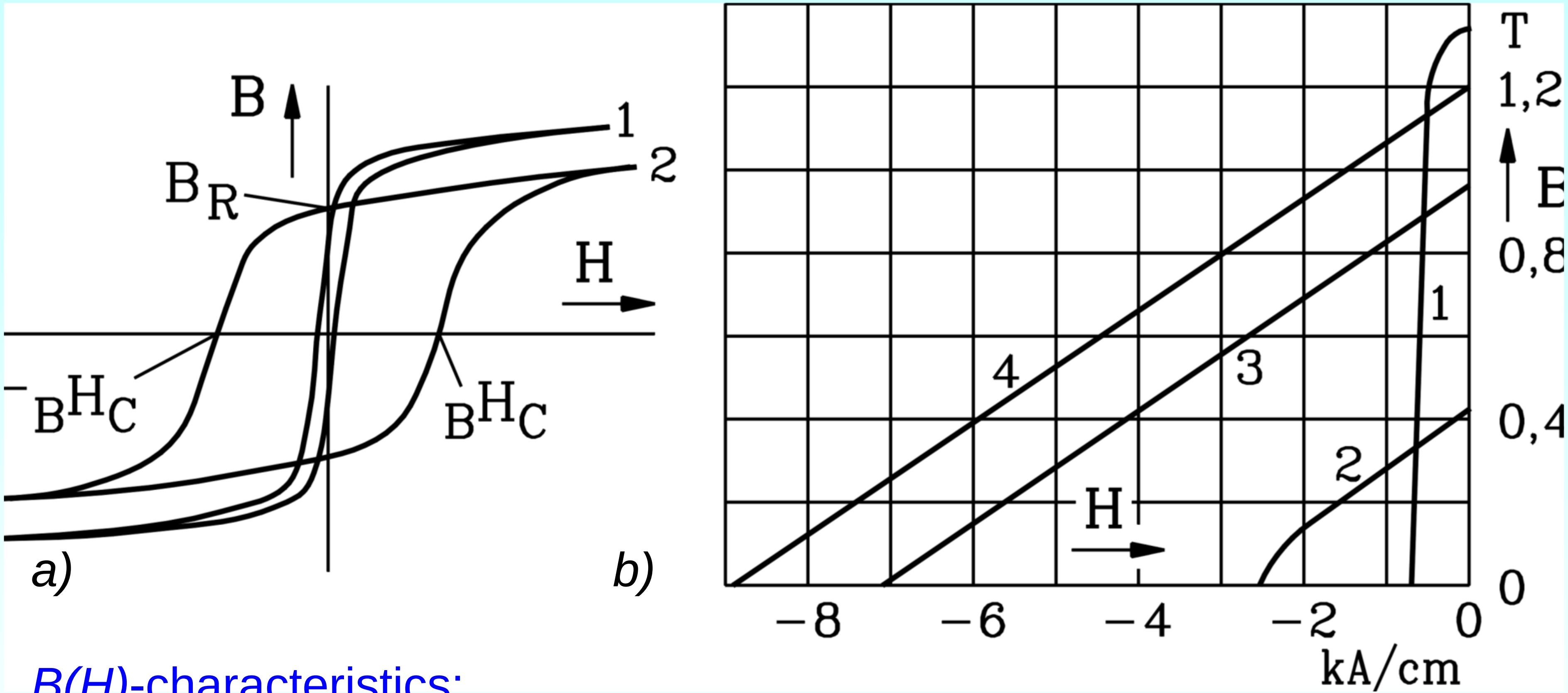
Coercive field strength: H_{CJ} and H_{CB}

Rare earth magnets:

For $-H_{CJ} < H_M < H_{CJ}$: $J_M(H_M) \approx J_s$ = const.: $B_M \approx \mu_0 H_M + J_s$

$$B_M = \mu_M H_M + B_R = \mu_M H_M + J_s \qquad \mu_M \approx \mu_0 \text{ typical}: \mu_M \approx 1.05\mu_0$$

Permanent magnet properties



B(H)-characteristics:

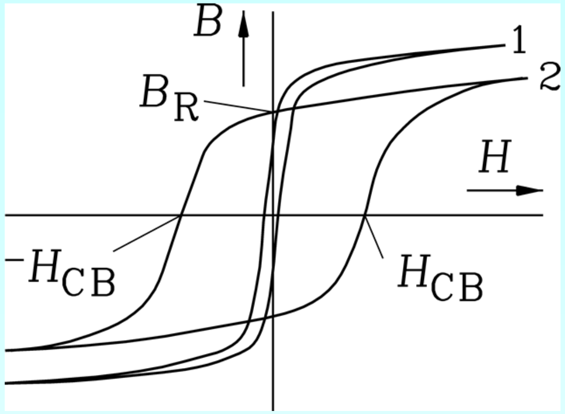
a) (1) Soft magnetic material, (2) PM magnet,

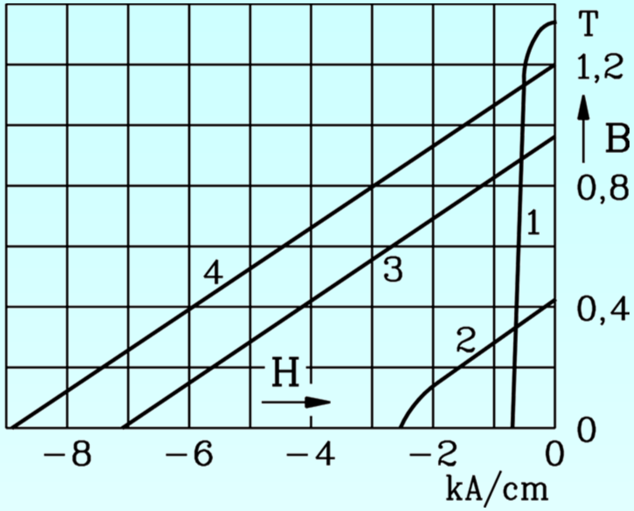
b) PM magnets: Second quadrant at 20°C;

(1): Al-Ni-Co, (2): Ba-Ferrite, Sr-Ferrite, (3): Sm_2Co_{17} (ϑ_{max} = 350°C)

(4): NdFeB (ϑ_{max} = 180°C)

Permanent magnet materials





Source: R. Fischer, hanser-Verlag

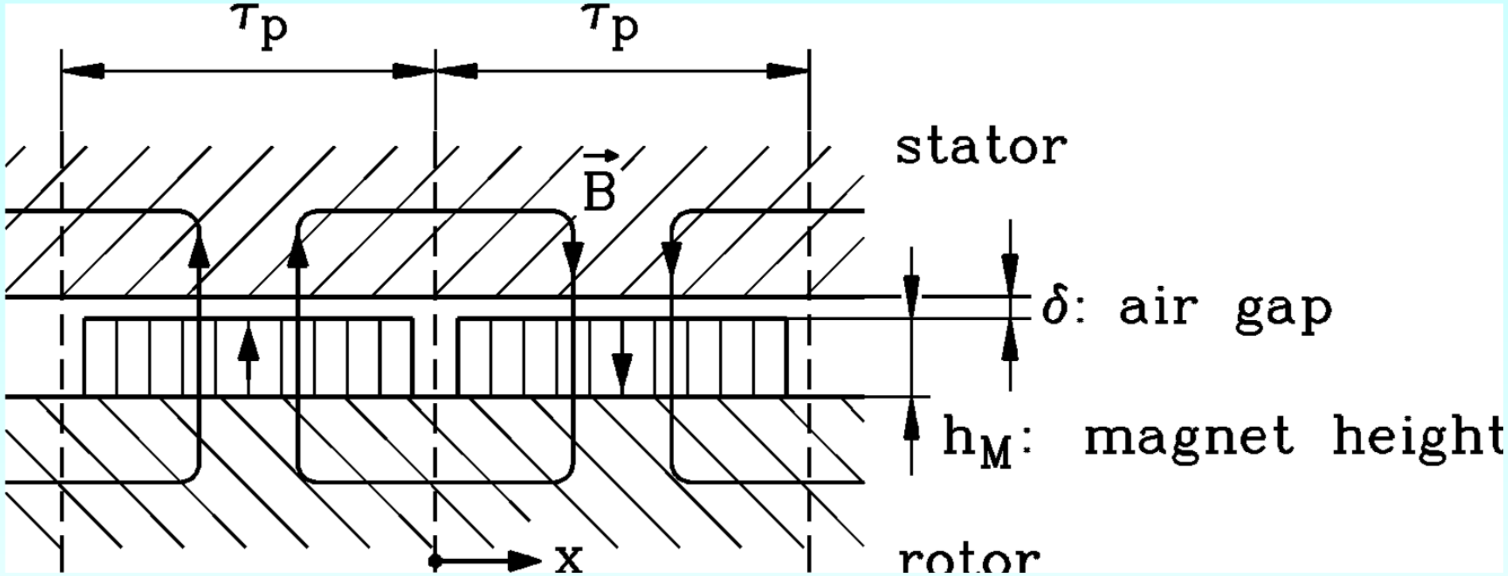
- **B_R: Remanence flux density**
- **H_{CB}: Coercive field strength of the *B(H)*-loop**
- **Hysteresis loop *B(H)*:** statically measured (here: values at 20°C)

- **Soft magnetic materials ("ferromagnetics") (1):** Iron, nickel, cobalt: B_R and H_{CB} are small: well suited for AC field operation due to small hysteresis losses
- **Hard magnetic materials (2):** = permanent magnets: B_R and H_{CB} are big: well suited for the excitation of magnetic DC fields, but not suited for AC fields

1. **Aluminum-Nickel-Cobalt-Magnets** (AlNiCo) with high B_R, but low H_{CB}
2. **Ferrites** (e.g. Barium-Ferrite) with lower B_R, but higher H_{CB}
3. **Rare-Earth-Magnets** Samarium-Cobalt: high B_R & H_{CB}, small influence of temperature
4. **Rare-Earth-Magnets** Neodymium-Iron-Boron: very high B_R & H_{CB}, decreasing with increasing magnet temperature

- Magnetic point of operation is in the **2nd quadrant of the *B(H)*-loop**

Design of magnet dimensions



Magnet field at no-load: No electric current in stator winding !

Ampere´s law: $\oint_C \vec{H} \bullet d\vec{s} = 2(H_\delta \delta + H_M h_M) = \Theta = 0 \qquad \mu_{Fe} \to \infty$

Flux continuity: $\Phi = B_M A_M = B_\delta A_\delta$

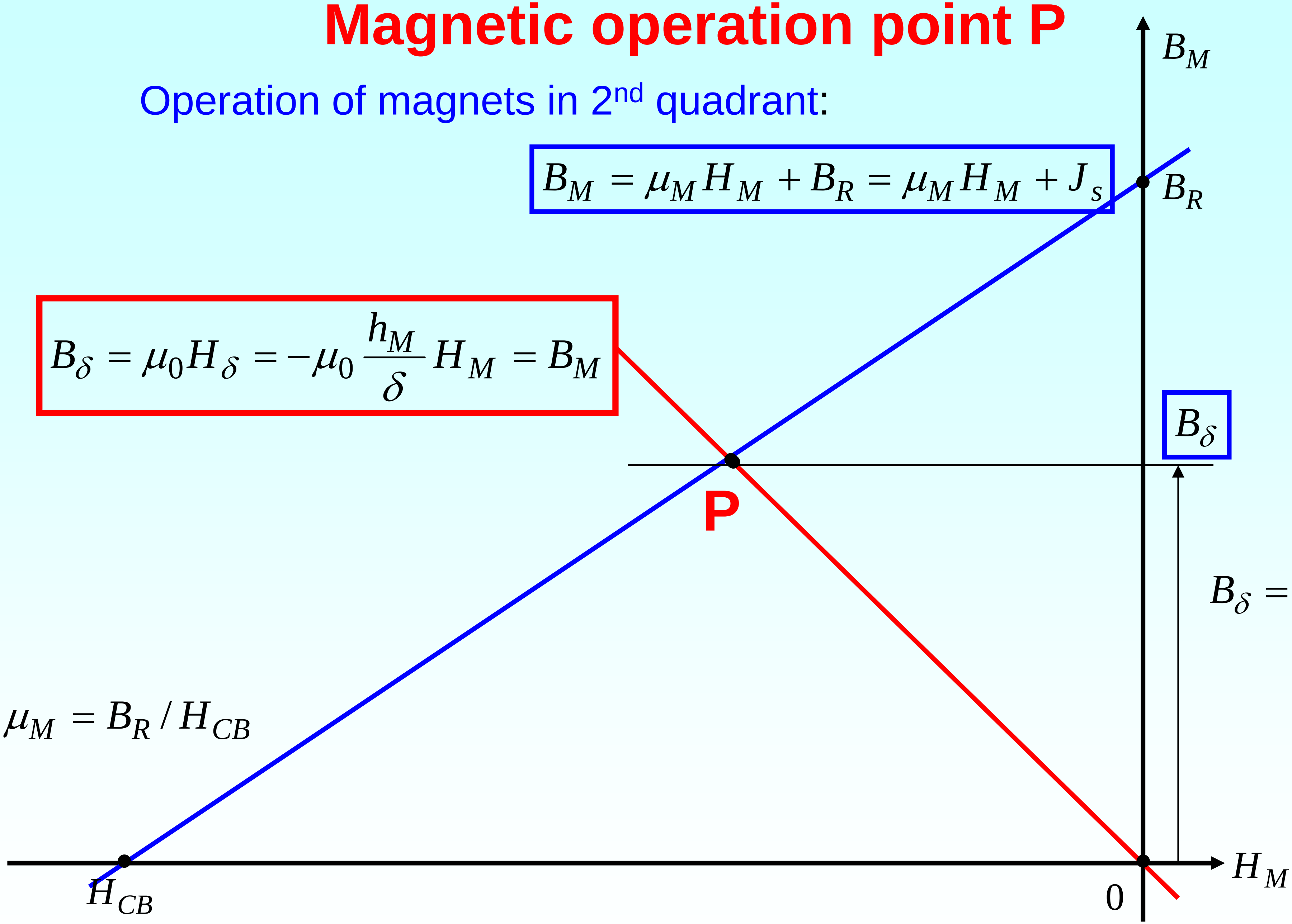
Surface mounted magnets: $A_M = A_\delta$, hence: $B_M = B_\delta$

Operation of magnets in 2nd quadrant:

$$B_\delta = \mu_0 H_\delta = -\mu_0 \frac{h_M}{\delta} H_M = B_M$$

Magnetic operation point P

Operation of magnets in 2nd quadrant:



$$B_M = \mu_M H_M + B_R = \mu_M H_M + J_s$$

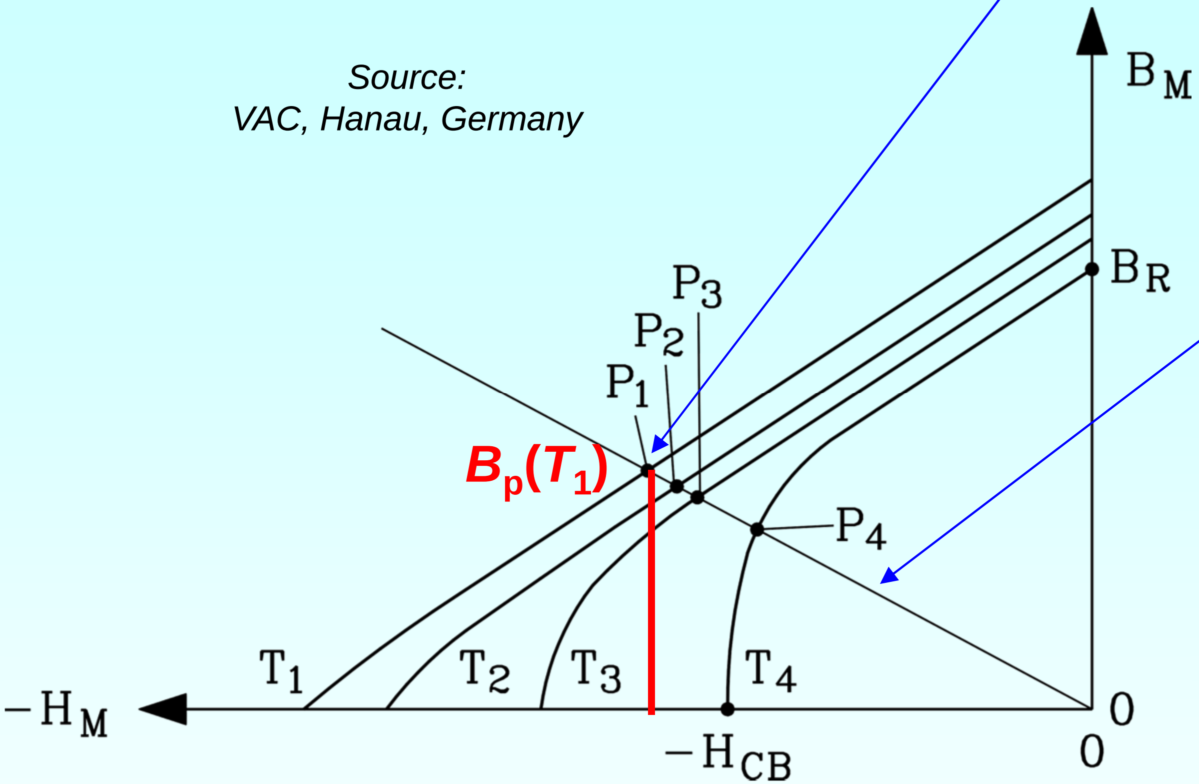
$$B_\delta = \mu_0 H_\delta = -\mu_0 \frac{h_M}{\delta} H_M = B_M$$

$$B_\delta = \frac{B_R}{1 + \frac{\mu_M}{\mu_0} \cdot \frac{\delta}{h_M}}$$

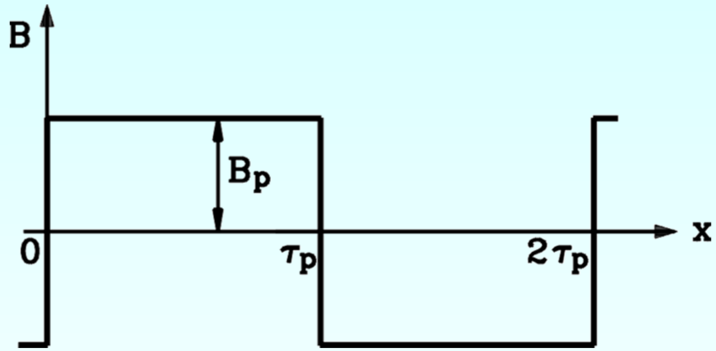
$$\mu_M = B_R / H_{CB}$$

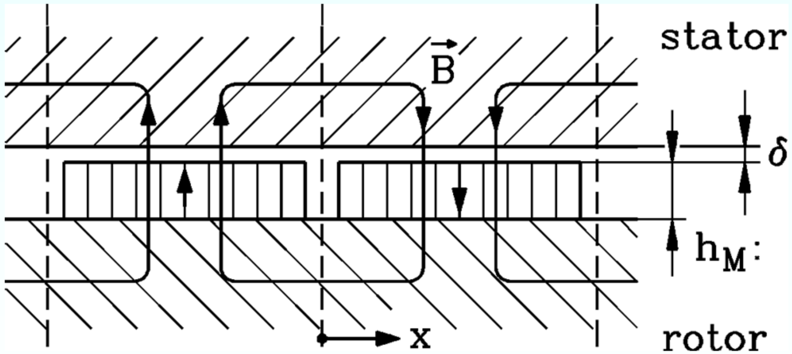
No-load magnetic point of operation *P*

Source:
VAC, Hanau, Germany

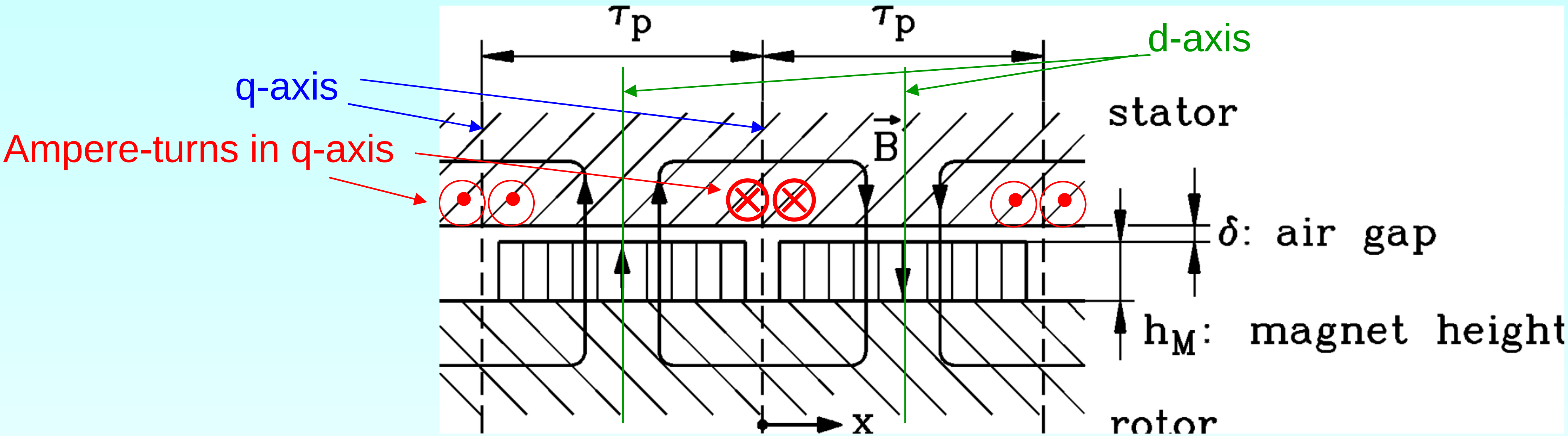


- **Temperature influence *T*:**
  $B_M(H_M)$-loop shrinks with increasing temperature T. Hence the PM flux density decreases with increasing T: $T_1 < T_2 < T_3 < T_4$.





Stator ampere-turns in the q-axis = d-current operation



Magnet field supported in d-axis, when electric current flows in stator winding in the q-axis ("d-axis current operation")!

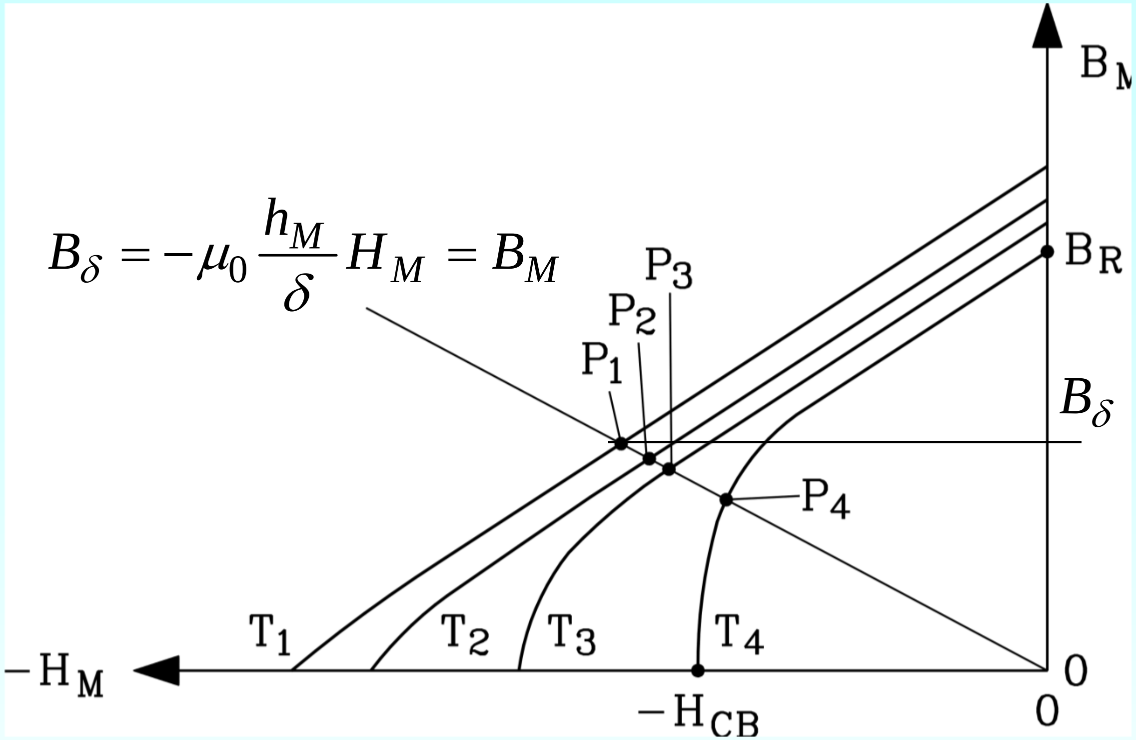
Ampere´s law: $\oint_C \vec{H} \bullet d\vec{s} = 2(H_\delta \delta + H_M h_M) = 2\Theta$

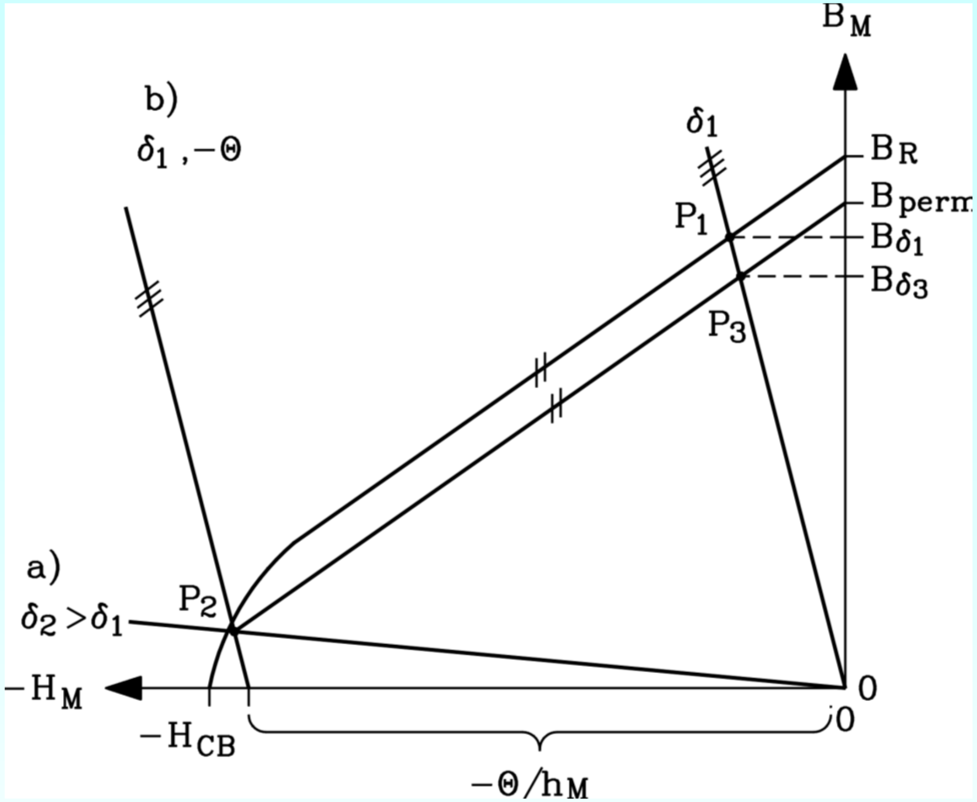
Flux continuity: $\Phi = B_M A_M = B_\delta A_\delta$

Surface mounted magnets: $A_M = A_\delta$, hence: $B_M = B_\delta$

Operation of magnets in 2nd quadrant: $B_\delta = \mu_0 H_\delta = -\mu_0 \frac{h_M}{\delta} \cdot (H_M - \Theta / h_M) = B_M$

Operation of magnets in 2nd quadrant of *B(H)*-plane

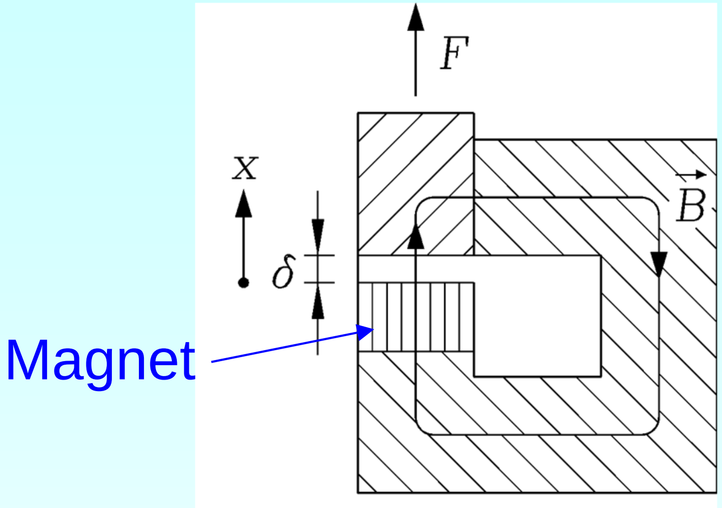




Reversible demagnetization of permanent magnet by air gap. The operating region of the magnet is the second quadrant. With increased temperature the remanence and coercive field is decreasing, yielding reduced air gap flux density according to operation points P_1 to P_4.

Irreversible demagnetization of permanent magnet a) by increased air gap $\delta_1 < \delta_2$ b) by external opposite field $-\Theta/h_M$, reaching an operating point P_2 below the "knee" of the hysteresis loop.

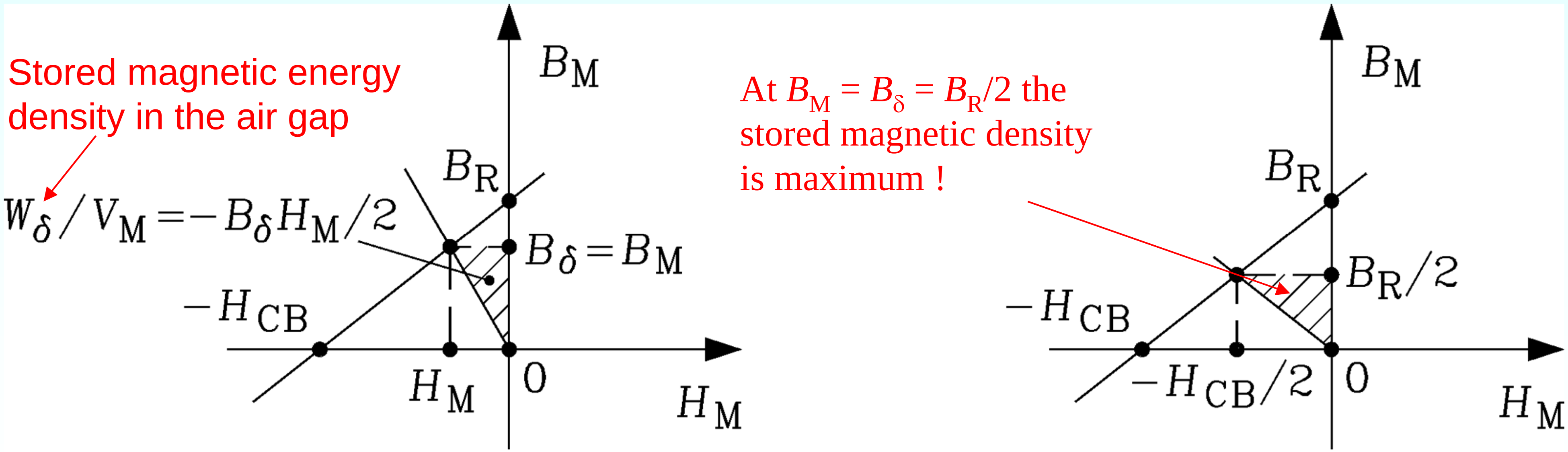
Stored magnetic energy density in the air gap (1)



- At $B_M = B_\delta = B_R/2$ the stored magnetic density is maximum !

- But this is NOT a motor design rule!

- Usually one tries to obtain a high B_δ close to the remanence B_R to get a strong motor.



Stored magnetic energy density in the air gap (2)

$$W_m = \int_V w_m dV \qquad w_m = \int_0^B \vec{H} \cdot d\vec{B}$$

$$w_m = \int_0^B \vec{H} \cdot d\vec{B} = \int_0^{B_\delta} H_\delta \cdot dB_\delta = \int_0^{B_\delta} (B_\delta / \mu_0) \cdot dB_\delta = \frac{B_\delta^2}{2\mu_0} \qquad W_m = \frac{B_\delta^2}{2\mu_0} \cdot A_\delta \cdot \delta$$

$$W_m = \frac{B_\delta \cdot A_\delta}{2\mu_0} \cdot B_\delta \cdot \delta = \frac{B_\delta \cdot A_\delta}{2} \cdot H_\delta \cdot \delta = -\frac{B_M \cdot A_M}{2} \cdot H_M \cdot h_M$$

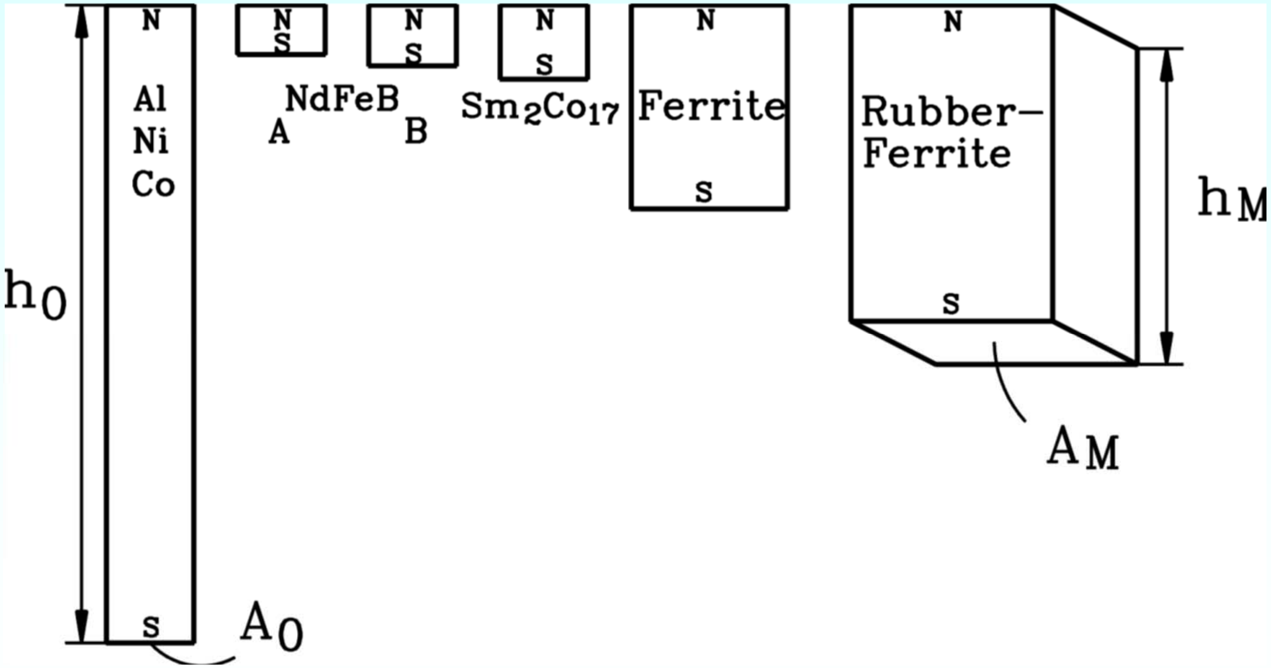
$$W_m = -\frac{B_M \cdot H_M}{2} \cdot (A_M \cdot h_M) = -\frac{B_M \cdot H_M}{2} \cdot V_M$$

$$W_{m,\max} = -\frac{(B_R / 2) \cdot (H_{CB} / 2)}{2} \cdot V_M$$

The stored magnetic energy in the air gap is proportional to the „energy product“ and to the magnet volume!

Comparison of different magnetic material for the same flux and demagnetization limit

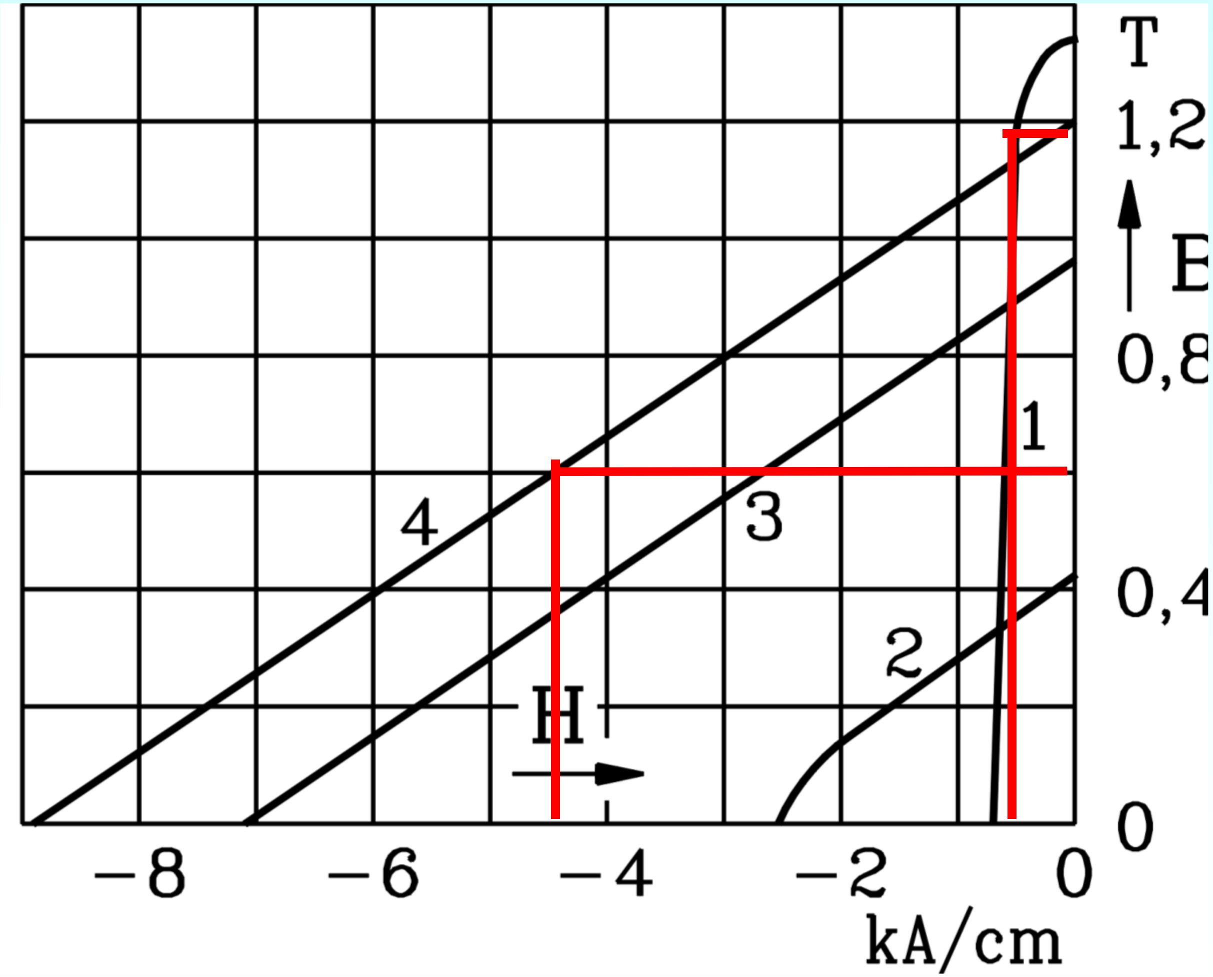
at 20°C	AlNiCo	NdFeB, A	NdFeB, B	Sm_2Co_{17}	Ba-ferrite	rubber ferrite
B_R / T	1.3	1.4	1.2	0.95	0.4	0.24
H_{CB} / kA/m	90	1100	900	710	270	175
A_M/A_0	*1*	*0.93*	*1.08*	*1.36*	*3.25*	*5.4*
h_M/h_0	*1*	*0.08*	*0.1*	*0.13*	*0.33*	*0.51*
V_M/V_0	*1*	*0.076*	*0.11*	*0.18*	*1.08*	*2.8*



Rare earth magnets allow for the same flux and the same demagnetization limit a much smaller magnetic volume of only about 10%, which yields compact PM motors, but it is expensive.

Energy product of permanent magnets

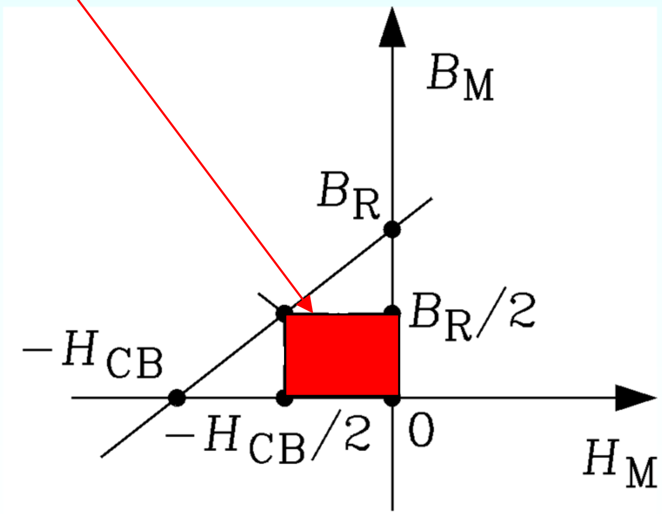
"Energy product": $(B_M H_M)_{\max}$ *It characterizes the strength of a magnet!*
Maximum rectangular area under $B_M(H_M)$-curve in the 2^{nd} quadrant = TWICE the maximum stored energy $w_{m,max}$ per volume in the air gap



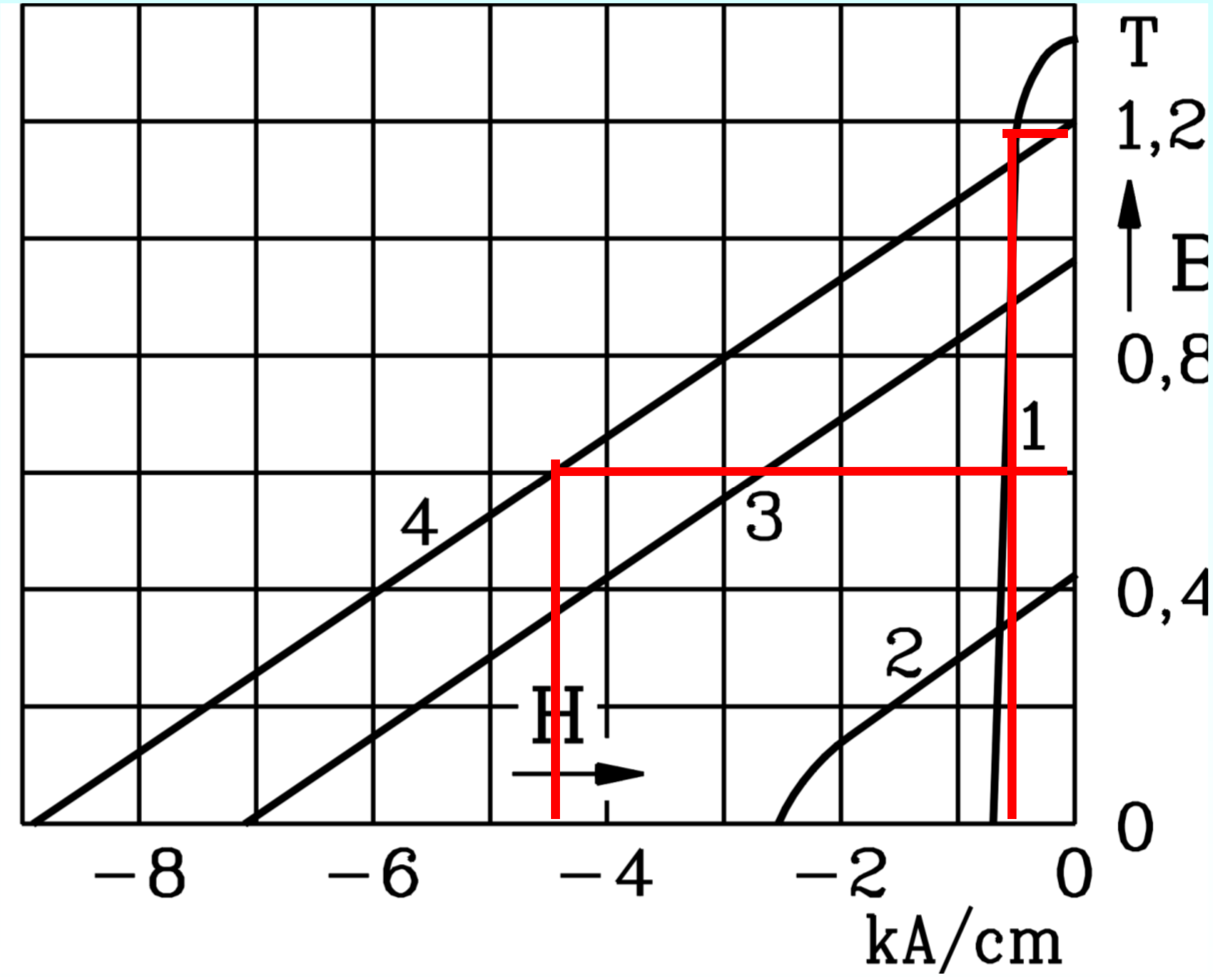
In case of rare earth magnets with straight $B_M(H_M)$-curve in the 2^{nd} quadrant:

$$w_{m,\max} = \frac{1}{2} \cdot \frac{B_R}{2} \cdot \frac{H_{CB}}{2}$$

"Energy product": $\frac{B_R}{2} \cdot \frac{H_{CB}}{2}$



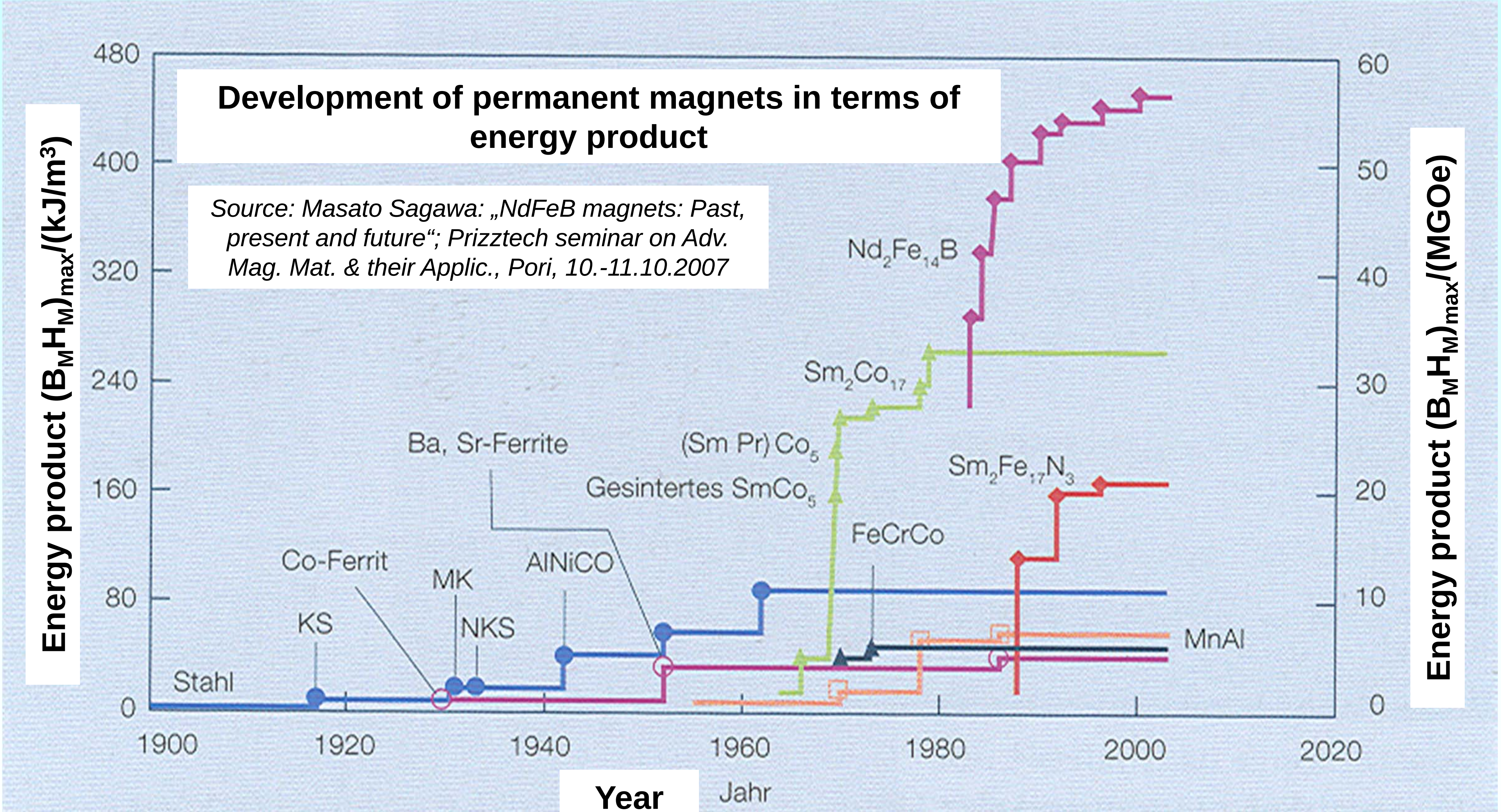
Example: Energy product of permanent magnets



"Energy product": $(B_M H_M)_{\max}$

Al-Ni-Co:	60 kJ/m^3
Ba-Ferrit:	30 kJ/m^3
Sm_2Co_{17}:	170 kJ/m^3
NdFeB:	270 kJ/m^3

Development of permanent magnets

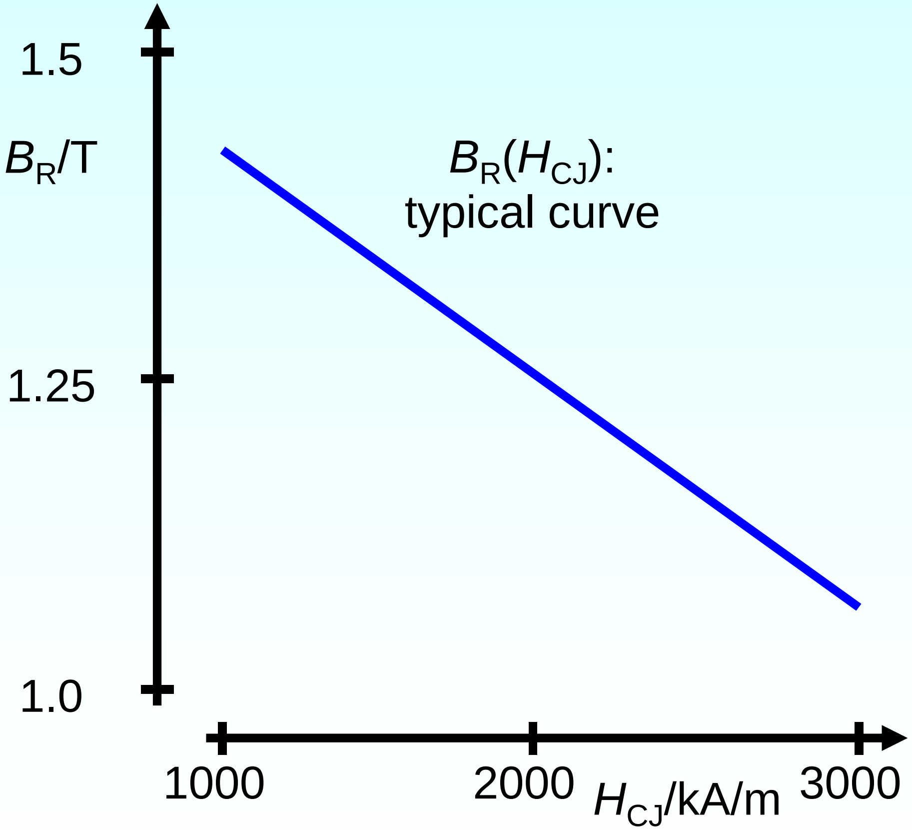


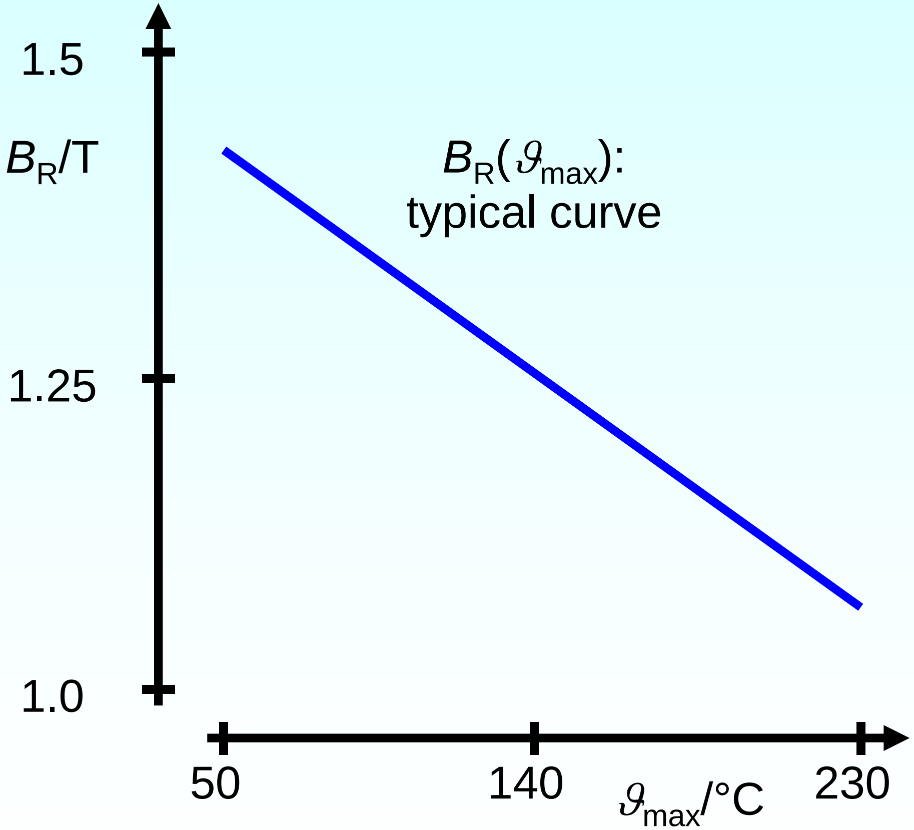
NdFeB-magnets: Temperature limit and coercive field strength

With increasing remanence flux density B_R:

a) the operation temperature limit ϑ_{max} decreases.

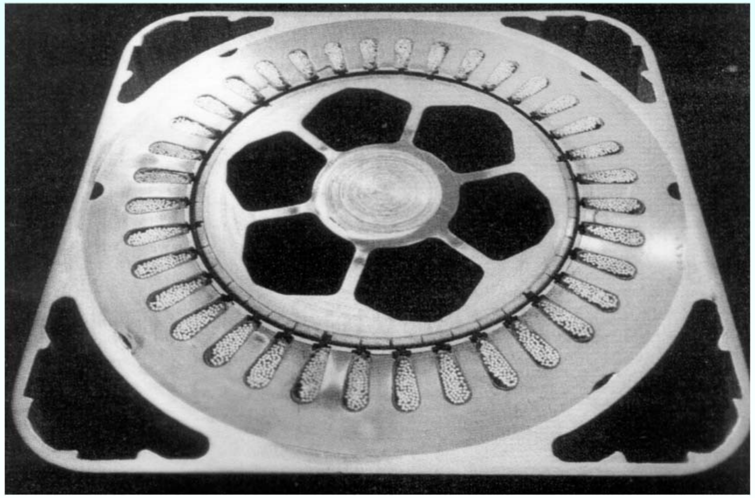
b) The coercive field strength H_{CJ} decreases.





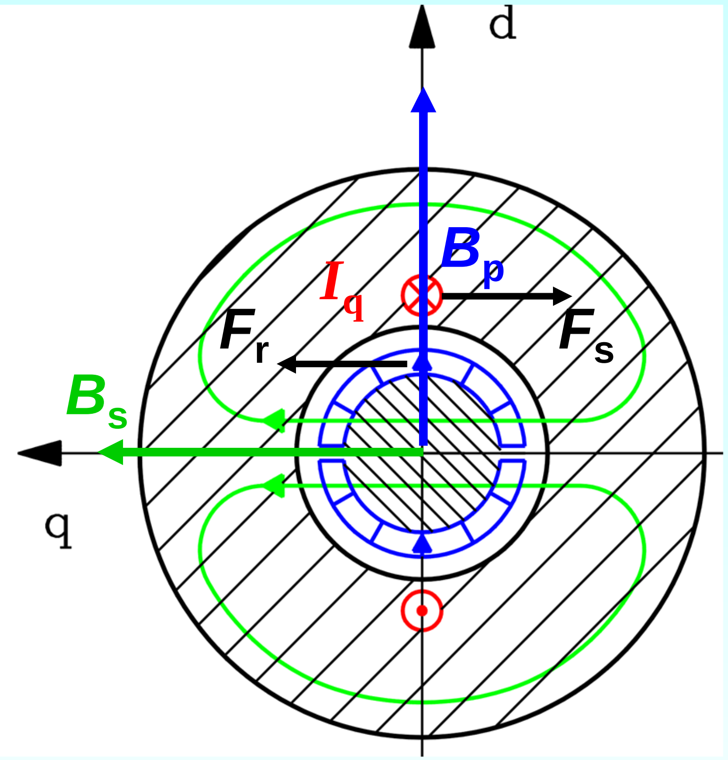
1. Permanent magnet synchronous machines as “brushless DC drives”

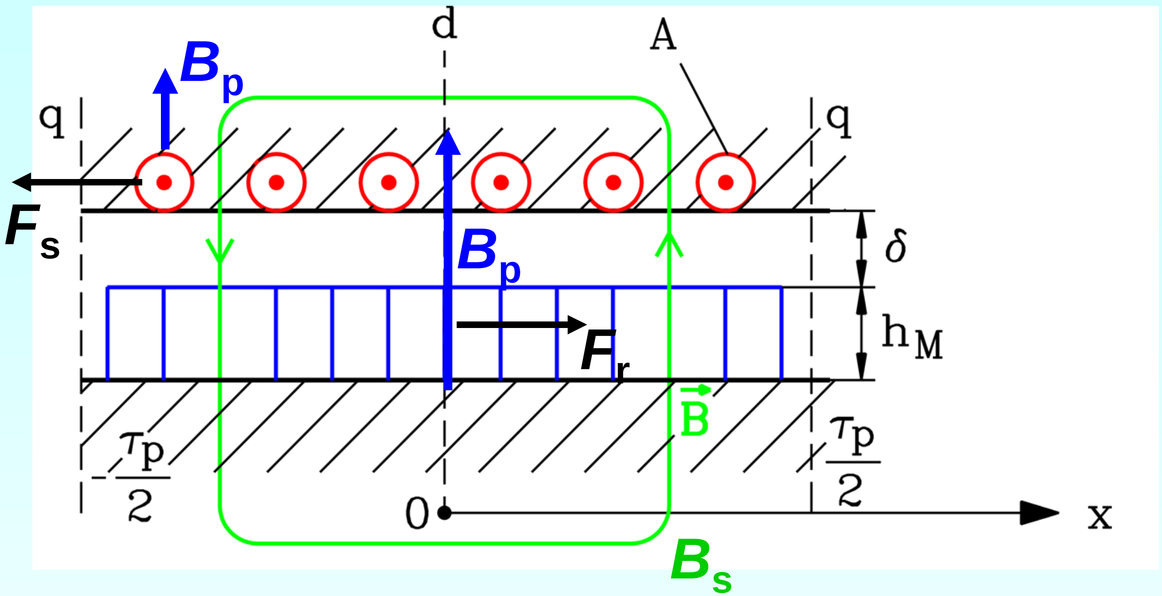
1.1.3 Torque generation in PM machines



Source: Siemens AG, Germany

PM-Motor – Torque generation at *q*-current operation





Torque due to LORENTZ-force:

$$M_e \sim F_r \cdot r \sim I_q \cdot B_p \cdot l_{Fe} \cdot r$$

The torque is directly proportional to the *q*-component of the stator current!

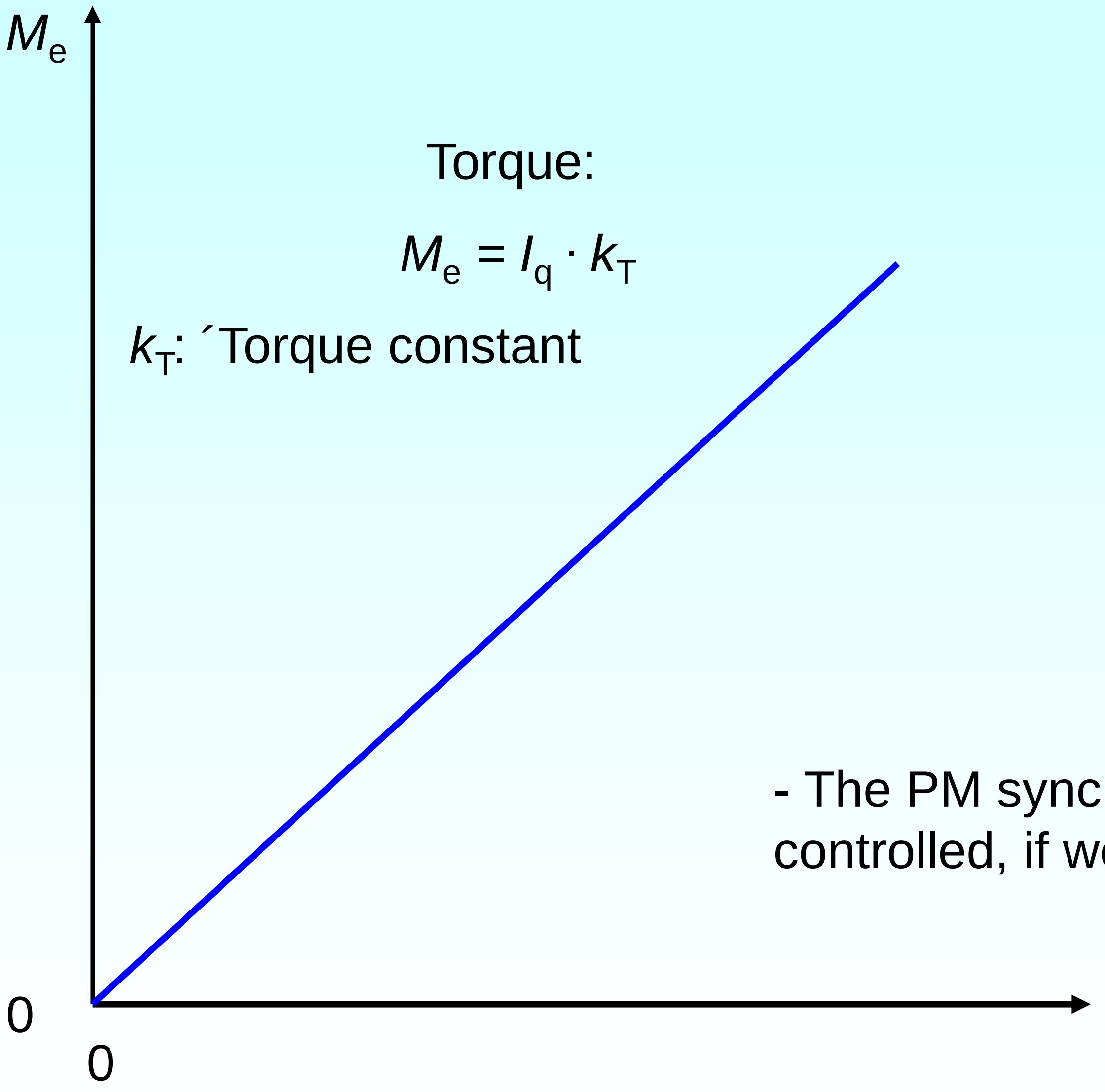
- **$\underline{I}_{sq}$-Current operation:** All conductors with the same direction of current flow are opposite of the rotor magnets with identical polarity.
- Hence all tangential *LORENTZ*-forces point into the same direction, yielding maximum torque per current.

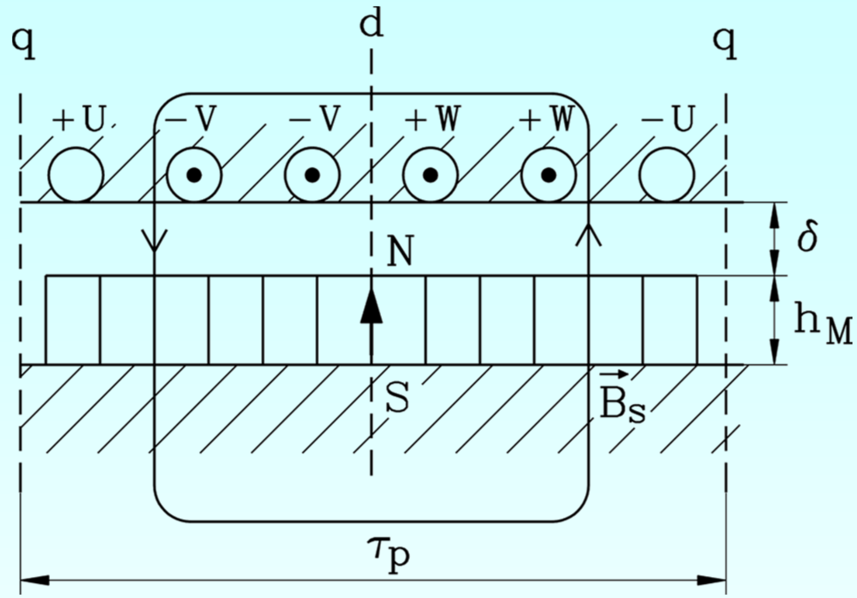
The torque is proportional to the *q*-current component!

Torque:

$$M_e = I_q \cdot k_T$$

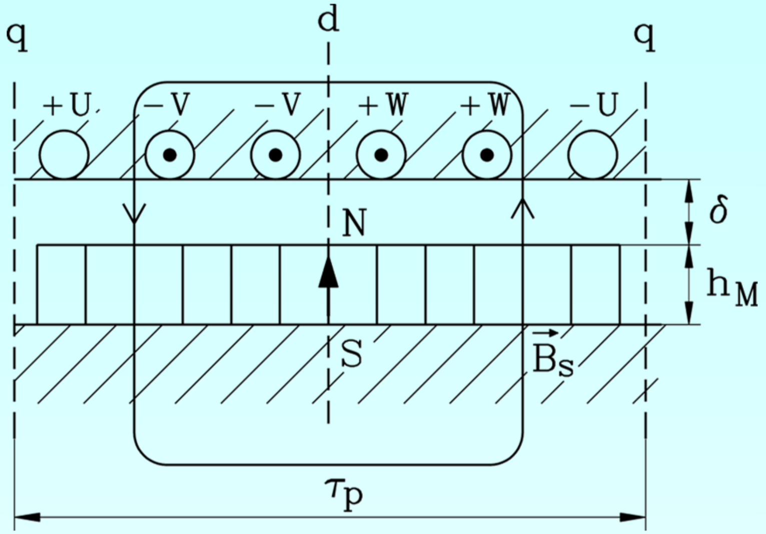
k_T: ´Torque constant





- The PM synchronous machine can be easily controlled, if we have „current ~ torque!“

PM synchronous machine as “Brushless-DC”drive

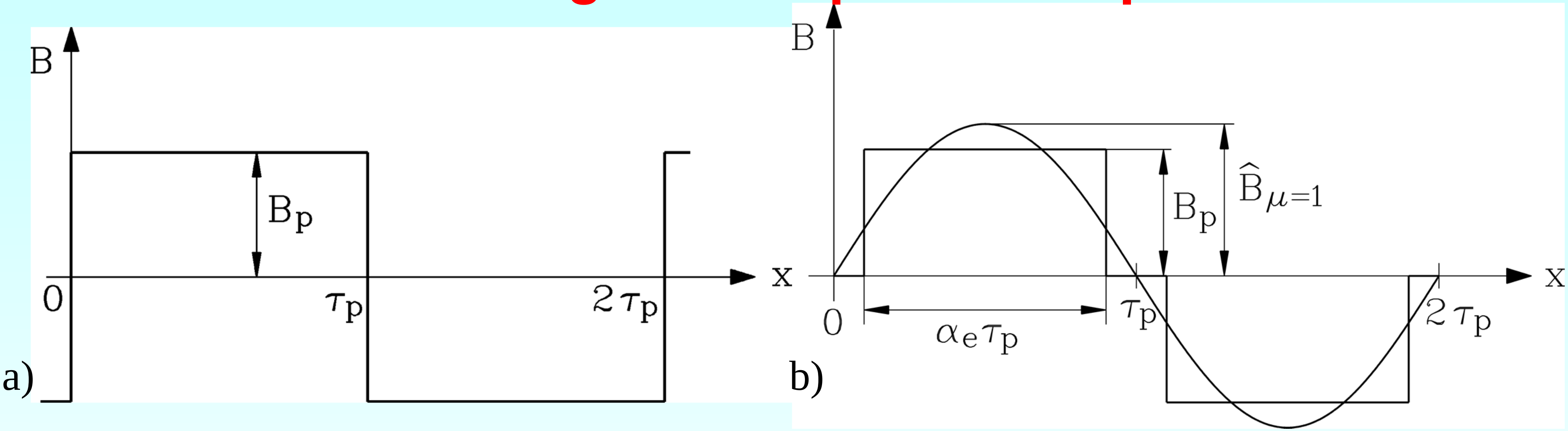


- Torque: $M_e \sim \Phi_p \cdot I_{sq}$
- like in a DC machine: $M_e \sim \Phi \cdot I_a$

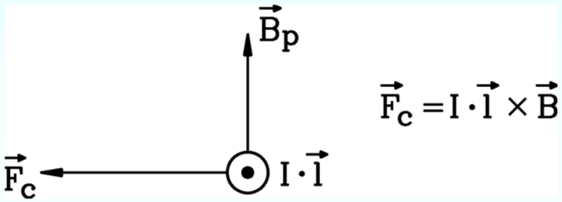
DC machine:	PM synchronous machine with I_q-current operation: (“brushless DC”-drive)
Commutator + brushes	*Inverter + Rotor position measurement*
Rotor armature winding	*Stator three-phase winding*
Stator field winding	*Rotor magnetic poles*

ADVANTAGE of brushless DC drive: NO brushes = reduced maintenance
Less rotor mass = lower rotor inertia
More robust motor system with less losses

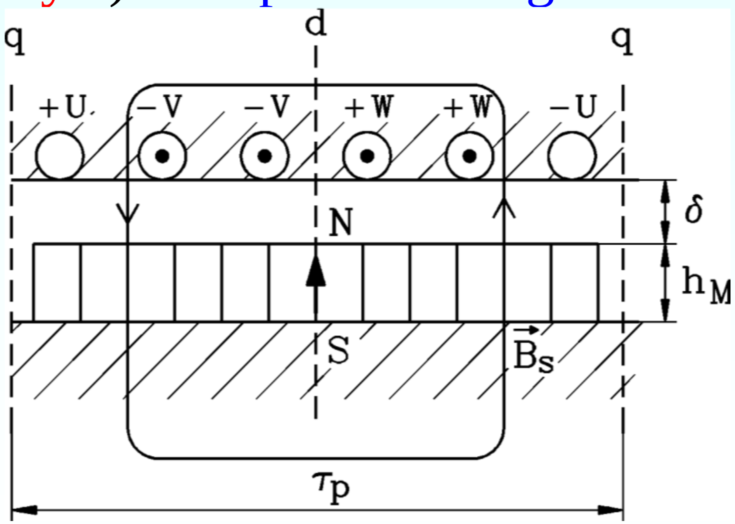
Torque generation in PM machines with surface mounted magnets at q-current operation



No-load air gap magnetic flux density a) with pole coverage ratio $\alpha_e = 1$, and b) with $\alpha_e < 1$



Tangential *Lorentz*-force per conductor

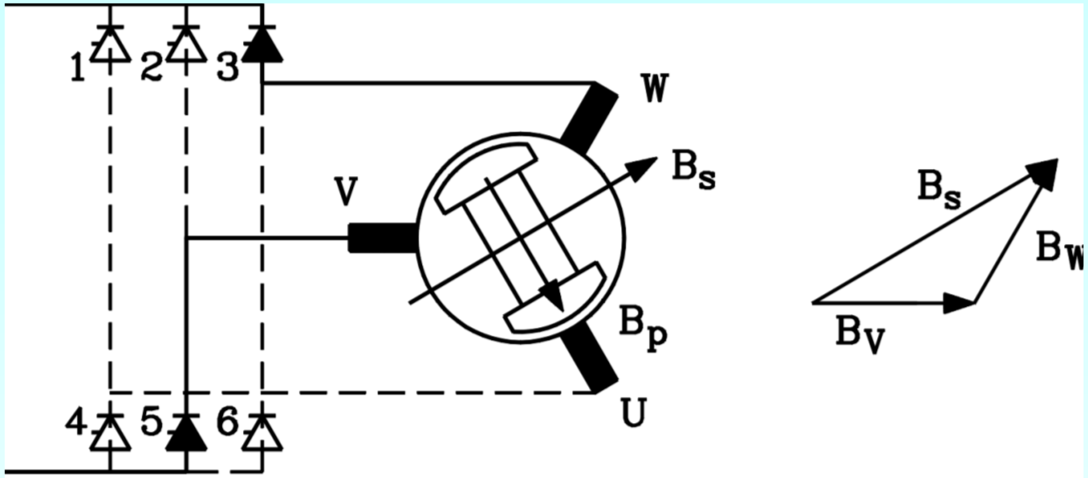


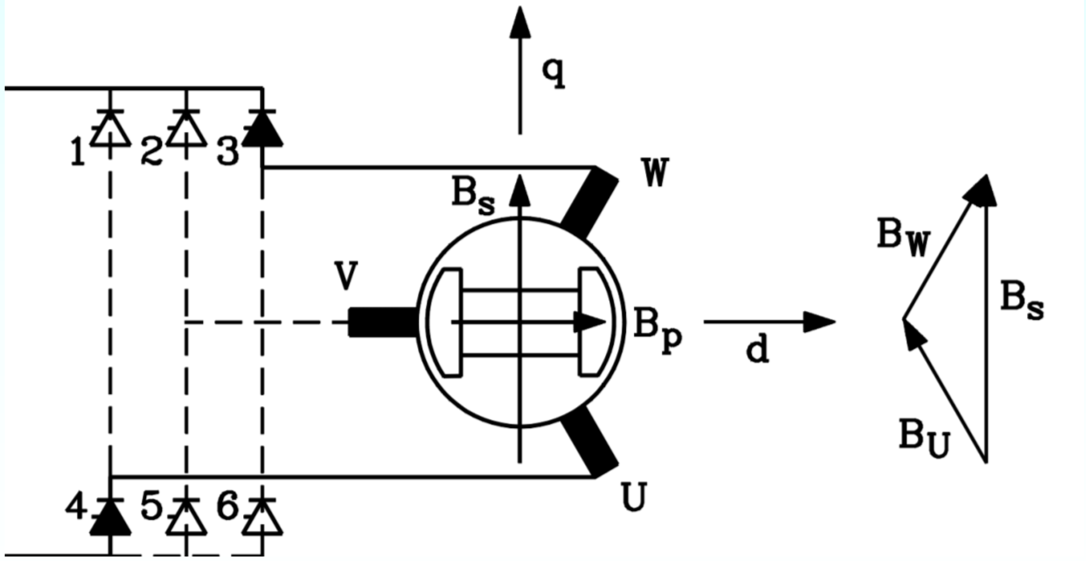
Current flow in stator coils produces with rotor PM air gap field a tangential force on rotor.

Force gives torque !

$$M_e = F \cdot d_{si} / 2$$

How to get maximum force resp. torque for given current amplitude ?





Stator air gap field B_s must be perpendicular to rotor PM field B_p to get maximum torque. Then ALL phase currents have the same polarity under one pole and give there fore the **SAME force direction. This is like in DC machines = brushless DC Drive system !**

Stator field is directed into gaps between rotor magnetic poles (rotor q-axis) = q-axis current operation !

By rotor position sensor the stator currents are switched with inverter to get the right phase shift for q-current operation !

An inverter is needed !

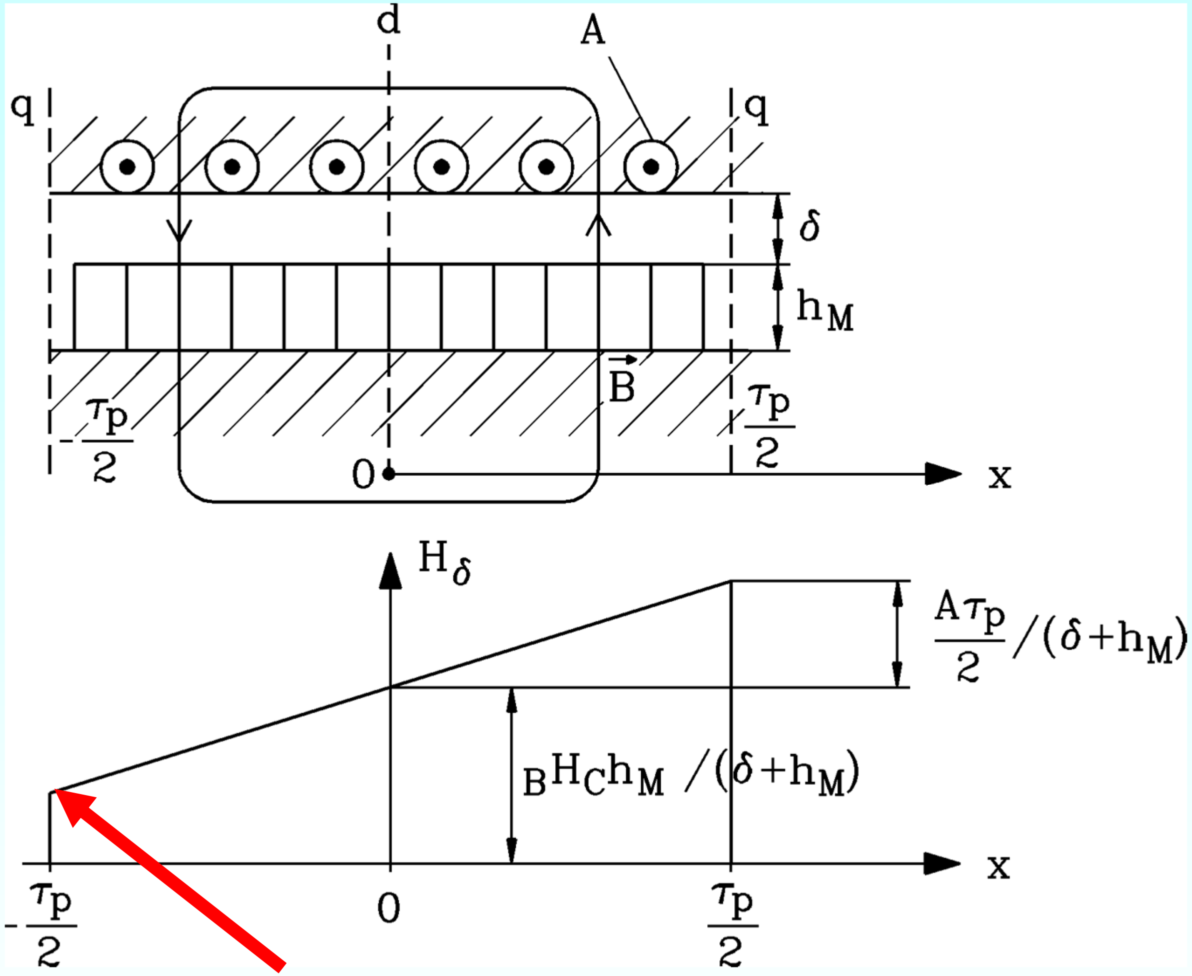
No-load air gap field expressed by B_R or H_{CB}

$$B_\delta = \frac{B_R}{1 + \frac{\mu_M}{\mu_0} \cdot \frac{\delta}{h_M}}$$

$$B_\delta = \frac{\mu_M H_{CB}}{1 + \frac{\mu_M}{\mu_0} \cdot \frac{\delta}{h_M}} \qquad H_\delta = \frac{\frac{\mu_M}{\mu_0} \cdot H_{CB}}{1 + \frac{\mu_M}{\mu_0} \cdot \frac{\delta}{h_M}}$$

$$\mu_M \approx \mu_0 \qquad H_\delta \cong \frac{H_{CB}}{1 + \frac{\delta}{h_M}} = \frac{h_M \cdot H_{CB}}{h_M + \delta}$$

Air gap magnetic flux density for one pole under load



„Current layer“ (loading):

$$A = \Theta / \tau_p = \frac{2 \cdot m \cdot N_s \cdot I_s}{2p \cdot \tau_p}$$

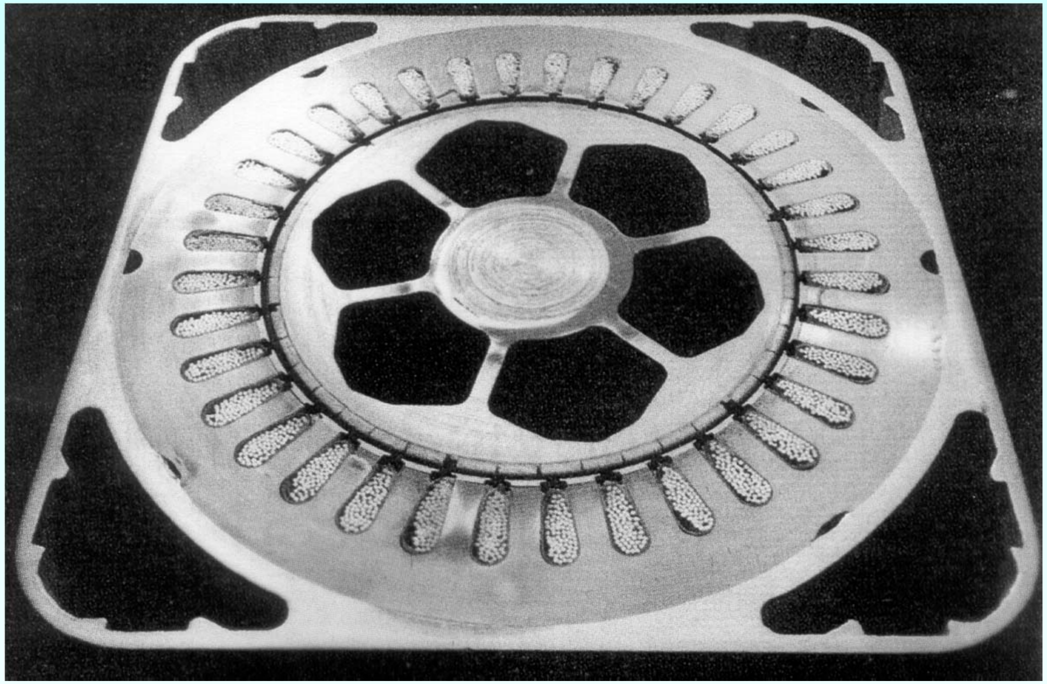
Ampere´s law gives stator field:

$$\oint_C \vec{H} \bullet d\vec{s} = 2 \cdot H_{\delta,s} \cdot (\delta + h_M) = 2 \cdot A \cdot x$$

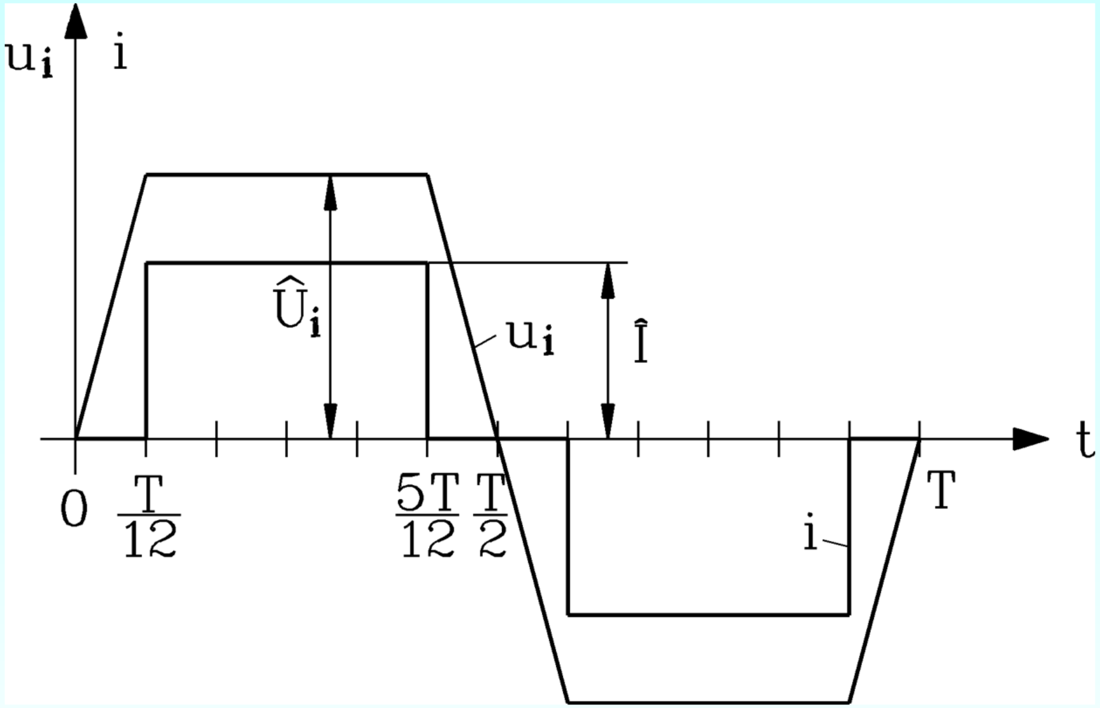
$$\Rightarrow \quad H_{\delta,s} = \frac{A}{\delta + h_M} \cdot x$$

***Under load** surface mounted rotor magnets experience danger of demagnetization at the trailing pole edge, especially when magnet is hot (typically 150 °C).*

Block current feeding



a) *Source: Siemens AG, Germany*

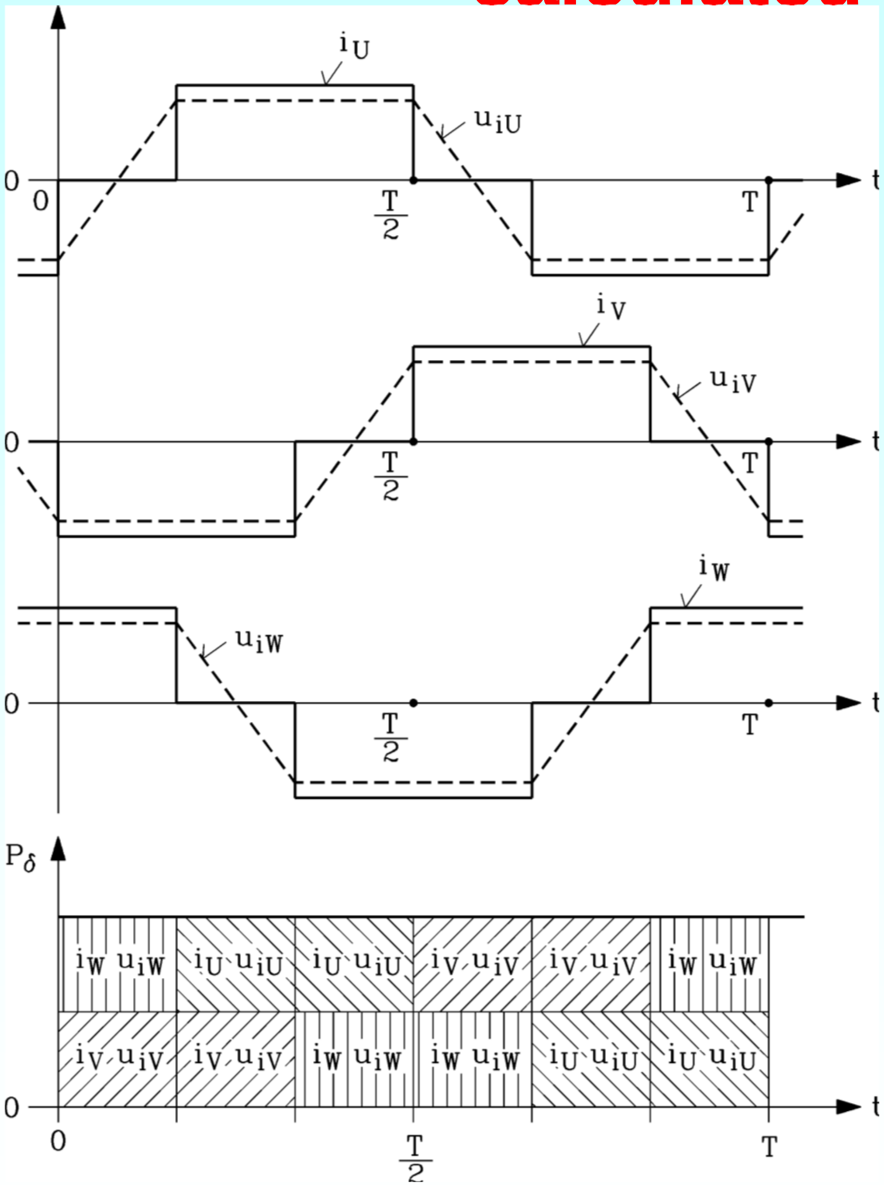


b)

a) Cross section of PM synchronous machine with 100% pole coverage ratio

b) Trapezoidal no-load stator phase voltage (back EMF); block shaped current impressed in phase with back EMF

Torque generation with block current feeding, calculated via internal power



Torque generation with block current feeding, calculated via internal power.

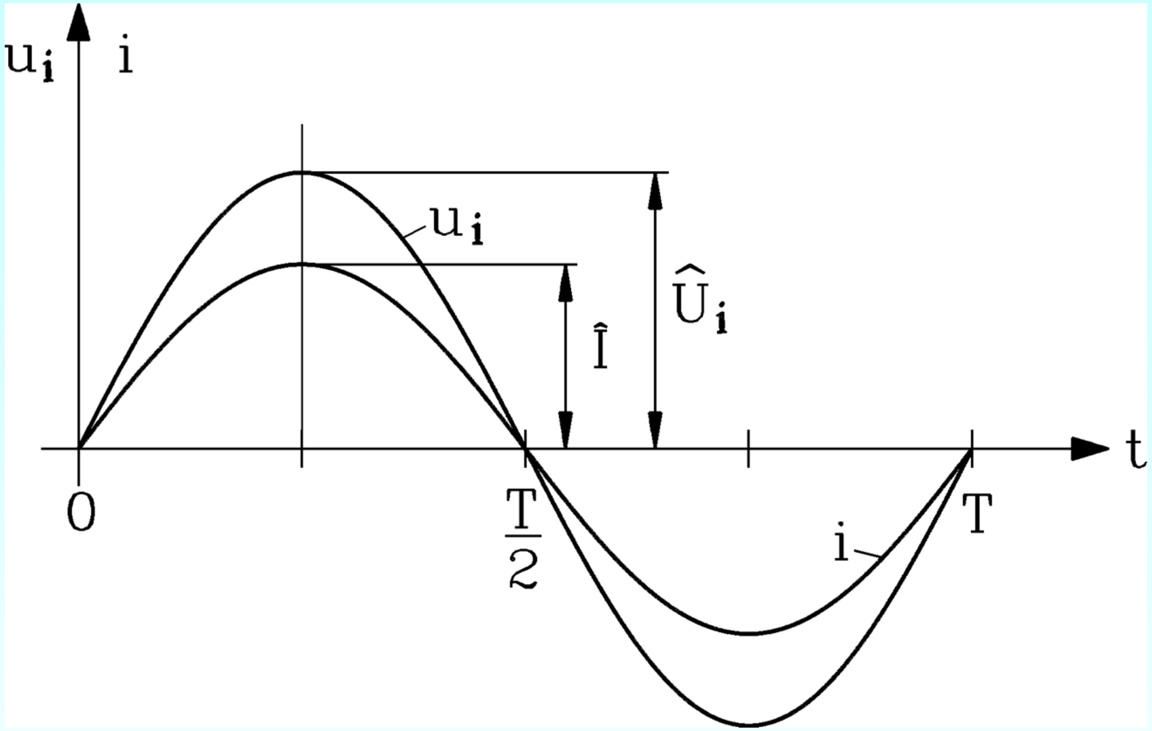
Air gap power: $P_\delta = 2\pi \cdot n_{syn} \cdot M_e$

$$p_\delta(t) = u_{pU}(t) \cdot i_U(t) + u_{pV}(t) \cdot i_V(t) + u_{pW}(t) \cdot i_W(t)$$

Electromagnetic torque: $$M_e = \frac{2 \cdot \hat{U}_p \cdot \hat{I}}{2 \cdot \pi \cdot n}$$

A smooth torque without any ripple is theoretically produced with contribution of two phases at each moment.

Torque generation with sine wave current feeding



Sinusoidal phase current is impressed by inverter in phase with sinusoidal back EMF, resulting in

a) pulsating power per phase, but

b) smooth constant power and constant torque for all three phases.

Using internal power per phase we get

constant resulting power:

$$p_\delta(t) = \hat{U}_p \cos(\omega t) \cdot \hat{I}\cos(\omega t) + \hat{U}_p \cos(\omega t - 2\pi/3) \cdot \hat{I}\cos(\omega t - 2\pi/3) + \hat{U}_p \cos(\omega t - 4\pi/3) \cdot \hat{I}\cos(\omega t - 4\pi/3)$$

$$p_\delta(t) = \frac{\hat{U}_p \hat{I}}{2} \cdot [\cos(2\omega t) + 1] + \frac{\hat{U}_p \hat{I}}{2} \cdot \left[\cos(2\omega t - \frac{4\pi}{3}) + 1\right] + \frac{\hat{U}_p \hat{I}}{2} \cdot \left[\cos(2\omega t - \frac{8\pi}{3}) + 1\right]$$

m: phase count

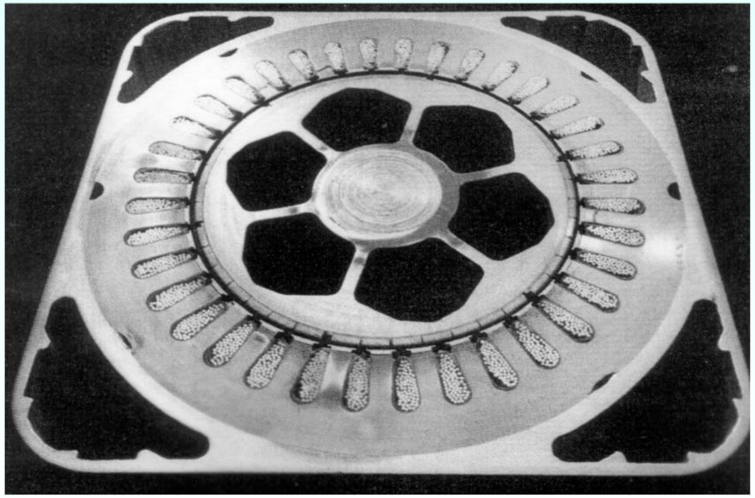
Here: $m = 3$

$$p_\delta(t) = m \frac{\hat{U}_p \hat{I}}{2} = const.$$

$$M_e = \frac{(3/2) \cdot \hat{U}_p \cdot \hat{I}}{2 \cdot \pi \cdot n}$$

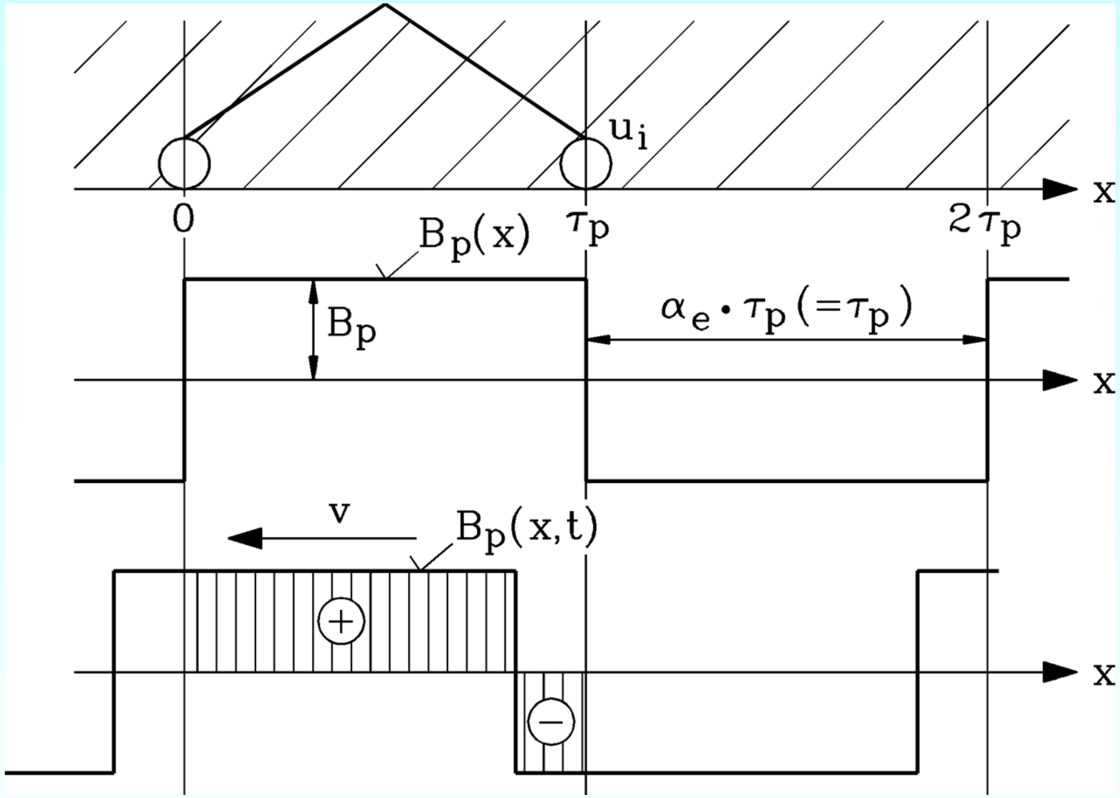
1. Permanent magnet synchronous machines as “brushless DC drives”

1.1.4 Induced no-load voltage in PM machines

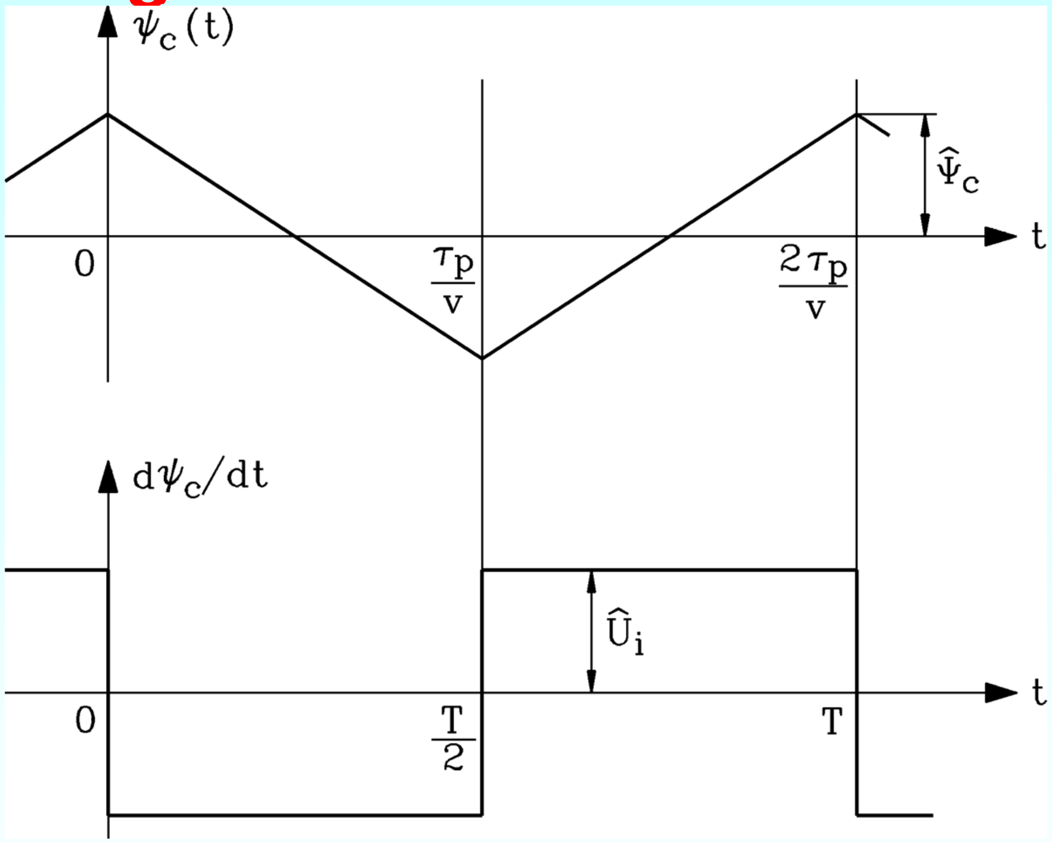


Source: Siemens AG, Germany

Induced no-load voltage ("back EMF") in one coil at 100% pole coverage ratio



a)

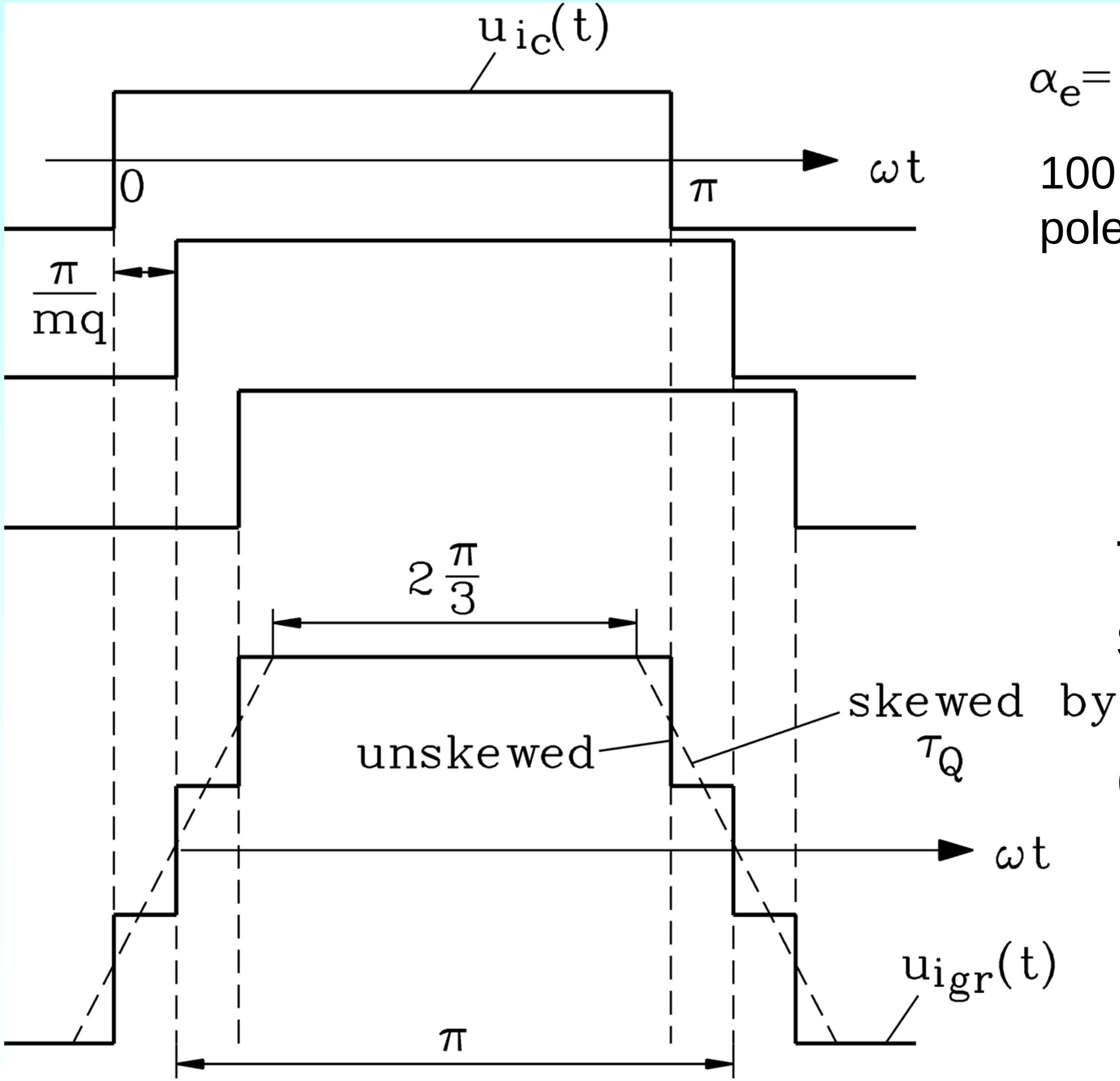


b)

a) Rectangular air gap flux density distribution = 100% pole coverage ratio leads to

b) triangular <u>coil</u> flux linkage time function, causing rectangular shaped induced <u>coil</u> voltage $u_{i,c} = -d\psi_c/dt$

Example: Coil group with *q* = 3 coils



$\alpha_e = 1, \quad q = 3$

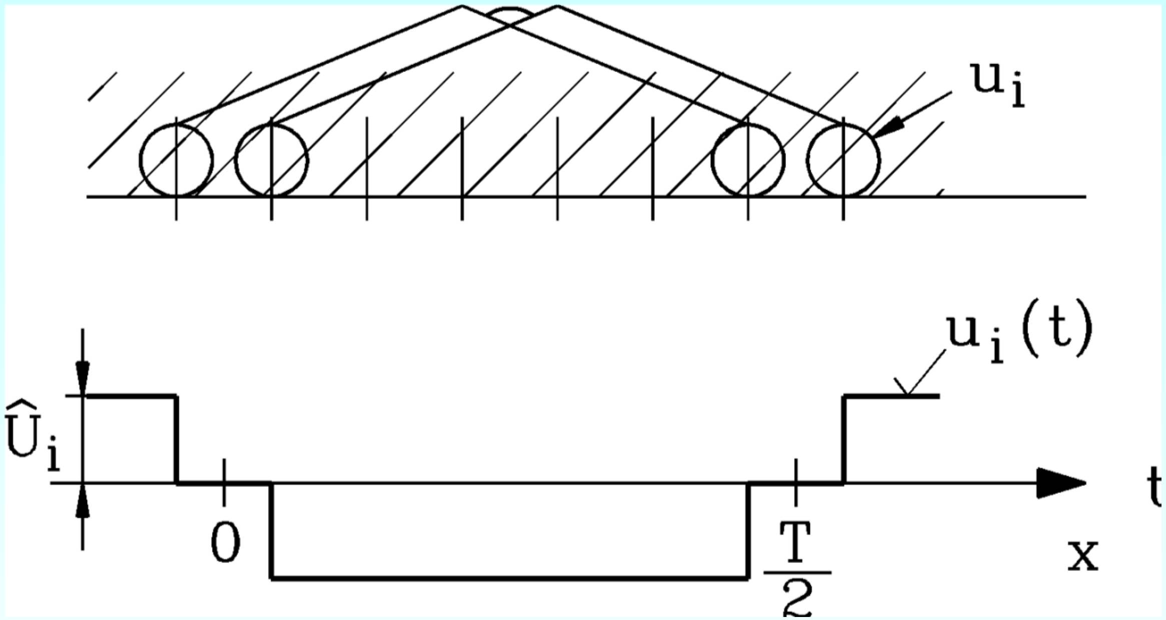
100 % pole coverage ratio, 3 slots per pole and phase

Unskewed coils:
The induced back EMF is step-like !

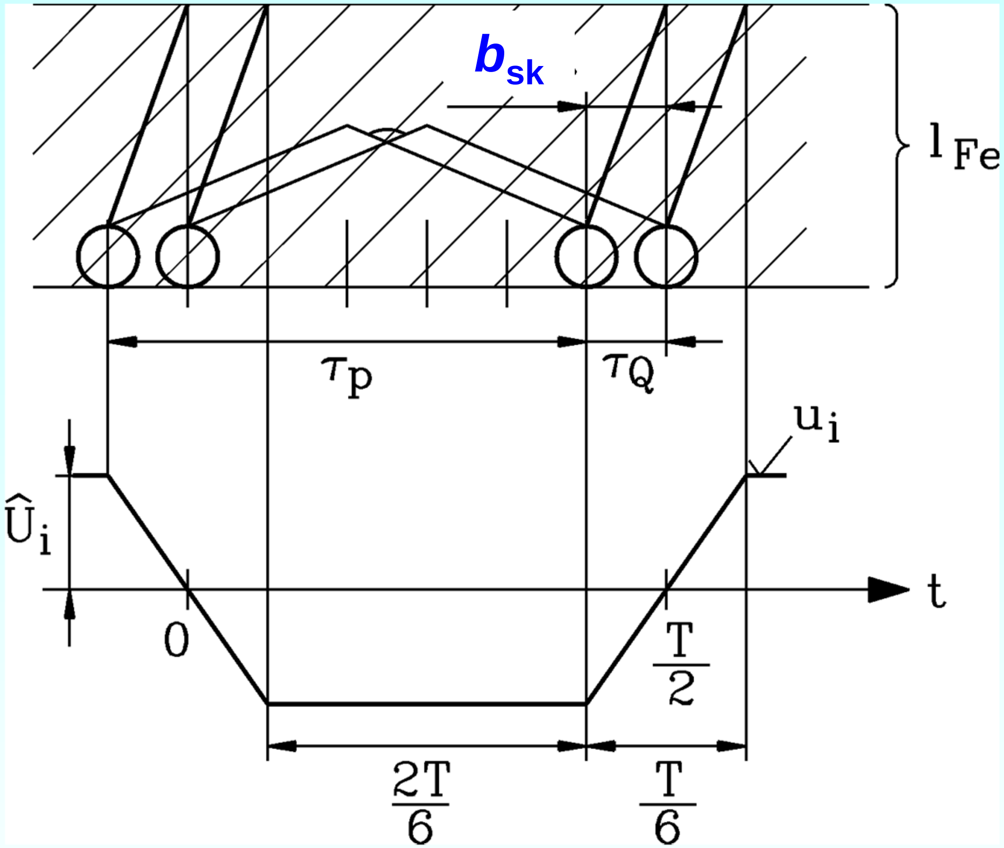
Coils - skewed by one slot pitch - yield a trapezoidal back EMF !

Back EMF in coil group with *q* >1 coils

Example: ***q* = 2**



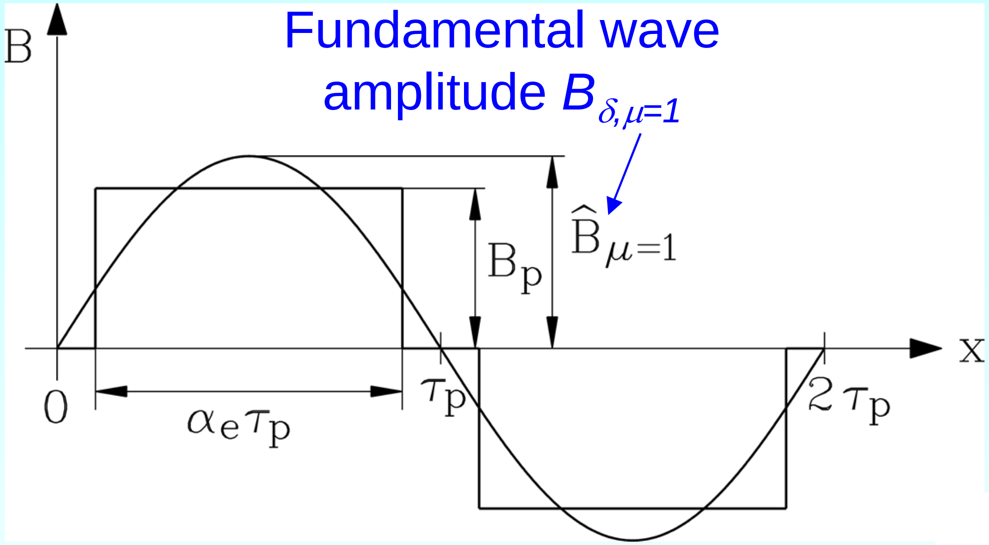
a) Unskewed coils:
The induced back EMF is step-like, when being induced by rectangular air gap flux density distribution !



b) Coils skewed by one slot pitch yield a trapezoidal back EMF

Coil skew: b_{sk}

Air gap field function developed as a FOURIER series of sine waves with shrinking wave lengths



$$B_\delta(x_r) = \sum_{\mu=1,3,5,\ldots}^{\infty} B_{\delta,\mu}\cdot\cos\left(\mu\frac{x_r\pi}{\tau_p}\right)$$

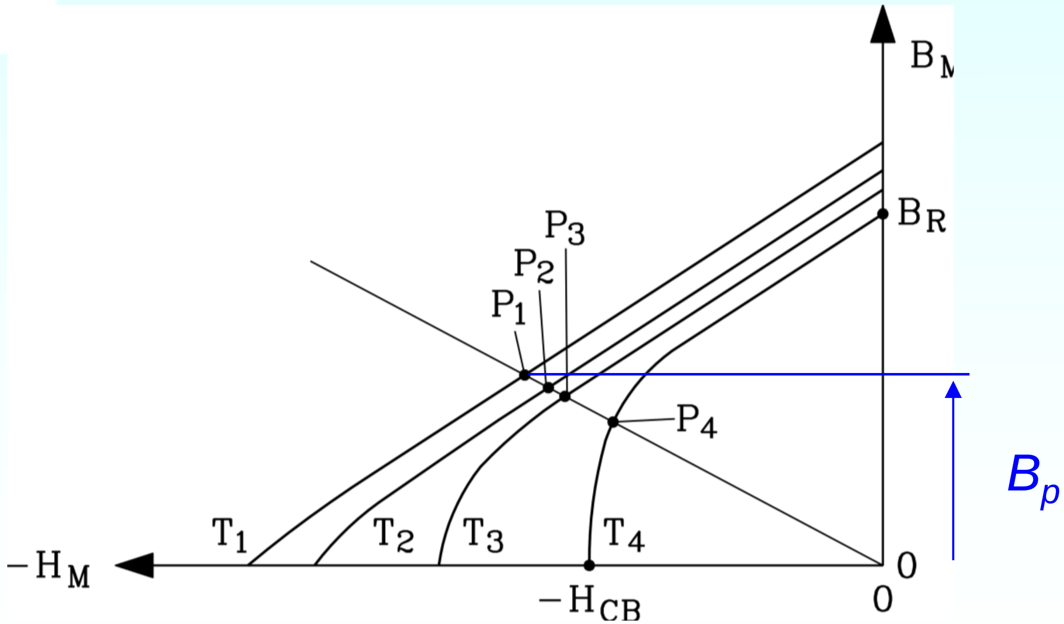
Rotor air gap field amplitudes:

$$B_{\delta,\mu} = \frac{4\cdot B_\delta}{\pi\cdot\mu}\cdot\sin(\mu\cdot\alpha_e\cdot\pi/2)$$

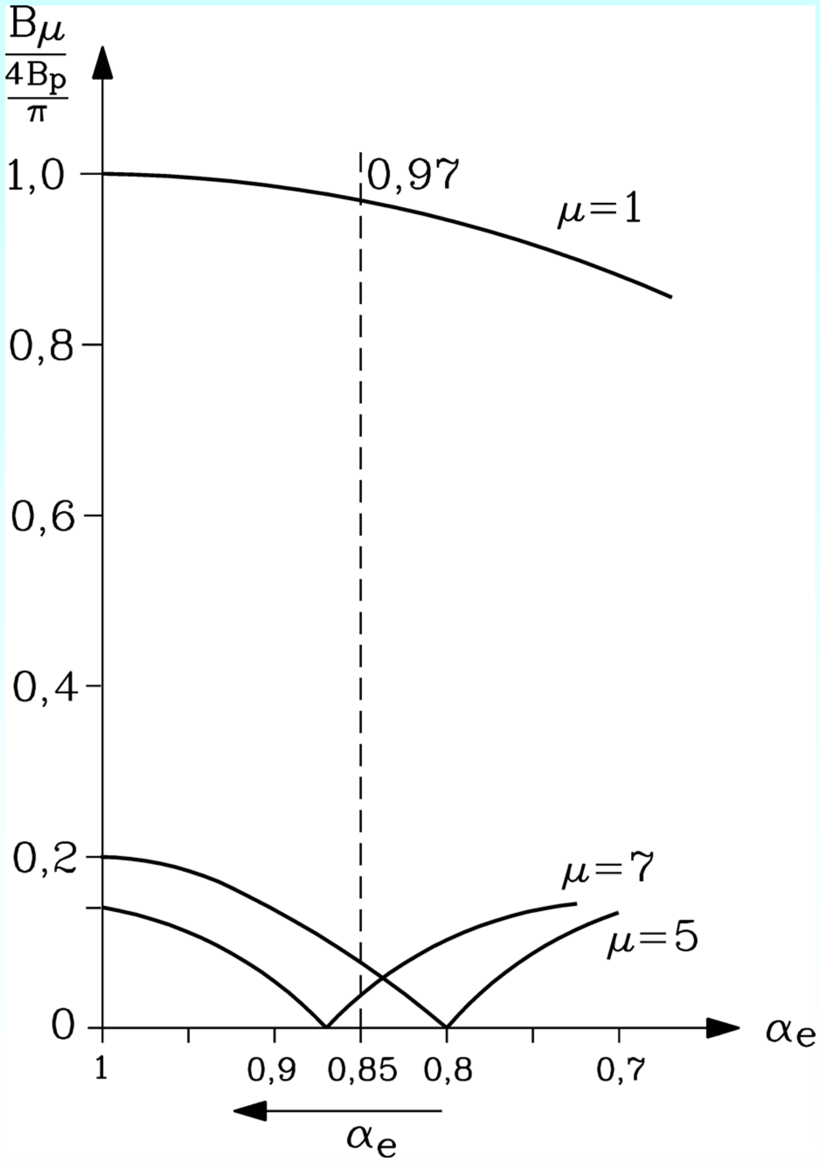
- $x = x_r$: Circumference coordinate in rotor reference frame

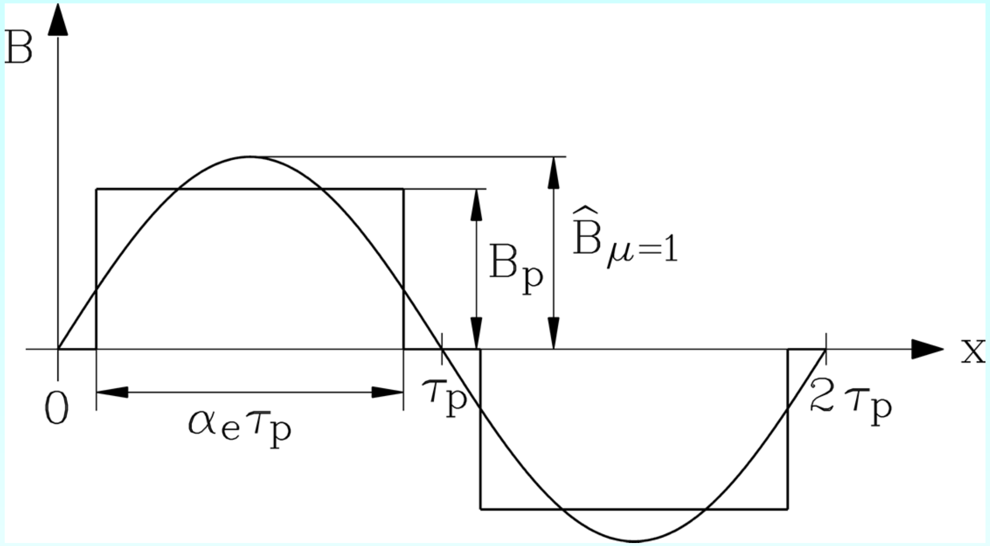
- Field function symmetry to abscissa: No even ordinal numbers μ

- Air gap flux density $B_\delta = B_p$ due to permanent magnets depends on magnet temperature T



Influence of pole coverage ratio α_e on the rotor flux density amplitudes





Rotor air gap field amplitudes:

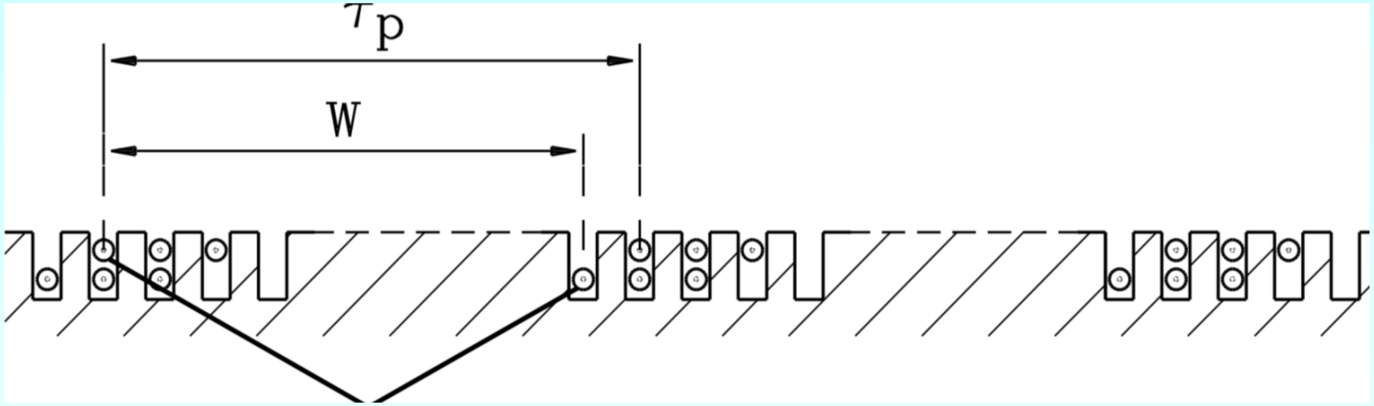
$$B_{\delta,\mu} = \frac{4 \cdot B_\delta}{\pi \cdot \mu} \cdot \sin(\mu \cdot \alpha_e \cdot \pi / 2)$$

At $\alpha_e = 0.85$ the fundamental amplitude is reduced by 3%, but 5th and 7th harmonic are reduced drastically.

How to get sinusoidal induced back EMF from NON-sinusoidal rotor air gap field ?

Use of pitched coils:

(= two-layer winding needed!)



Flux linkage of pitched coil:

$$\psi_{c\mu}(t) = N_c l_{Fe} \int_{-W/2}^{W/2} B_{\delta,\mu}(x,t) \cdot dx = N_c \cdot \frac{2}{\pi} \cdot \frac{\tau_p}{\mu} l_{Fe} B_{\delta\mu} \cdot \sin(\mu \cdot \frac{W}{\tau_p} \cdot \frac{\pi}{2}) \cdot \cos(\mu\omega t)$$

Pitch factor:

$$k_{p\mu} = \sin(\mu \cdot \frac{W}{\tau_p} \cdot \frac{\pi}{2})$$

Harmonic flux per coil of one harmonic field wave

Circumference coordinate in stator reference frame:

$$x = x_s = x_r + v_{syn} \cdot t \qquad v_{syn} = 2 \cdot n_{syn} \cdot p \cdot t \qquad n_{syn} = n$$

Permanent magnet field in stator reference frame:

$$B_\delta(x_s,t) = \sum_{\mu=1,3,5,\ldots}^{\infty} B_{\delta,\mu} \cdot \cos\left(\mu \frac{x_s \pi}{\tau_p} - \mu \cdot 2 \cdot n \cdot p \cdot \pi \cdot t\right) \qquad \omega_\mu = \mu \cdot 2\pi \cdot n \cdot p$$

Harmonic angular frequencies of harmonic field waves:

$$\omega_\mu = \mu \cdot \omega = \mu \cdot 2\pi \cdot n \cdot p \qquad \omega = 2\pi \cdot n \cdot p = 2\pi \cdot f$$

Harmonic flux linkage per coil:

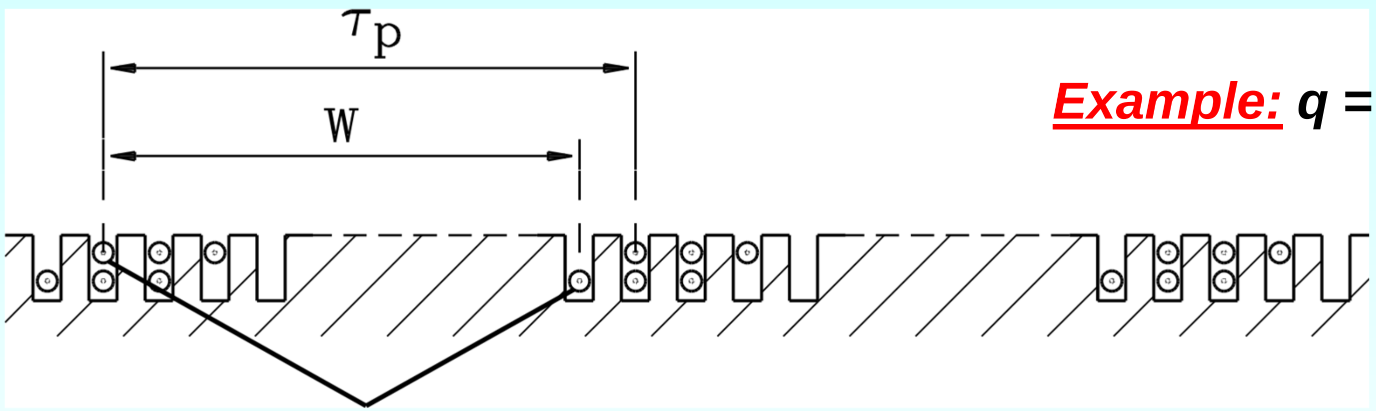
$$\psi_{c\mu}(t) = N_c l_{Fe} \int_{-W/2}^{W/2} B_{\delta,\mu}(x,t) \cdot dx = N_c l_{Fe} \int_{-W/2}^{W/2} B_{\delta,\mu} \cdot \cos(\mu \cdot \pi \cdot x / \tau_p - \omega_\mu t) \cdot dx$$

$$\psi_{c\mu}(t) = N_c \cdot \frac{2}{\pi} \cdot \frac{\tau_p}{\mu} l_{Fe} B_{\delta,\mu} \cdot \sin(\mu \cdot \frac{W}{\tau_p} \cdot \frac{\pi}{2}) \cdot \cos(\omega_\mu t)$$

Harmonic induced voltage per coil

FARADAY´s law of induction:

$$u_{i,c\mu}(t) = -d\psi_{c\mu}(t)/dt = \omega_\mu N_c \cdot \frac{2}{\pi} \cdot \frac{\tau_p}{\mu} l_{Fe} B_{\delta,\mu} \cdot \sin(\mu \cdot \frac{W}{\tau_p} \cdot \frac{\pi}{2}) \cdot \sin(\omega_\mu t)$$



Example: ***q*** **= 3,** ***m*** **= 3, pitch 8/9**

Pitch factor: $$k_{p\mu} = \sin(\mu \cdot \frac{W}{\tau_p} \cdot \frac{\pi}{2})$$

Example: ***q*** **= 2,** ***m*** **= 3, pitch 5/6:**

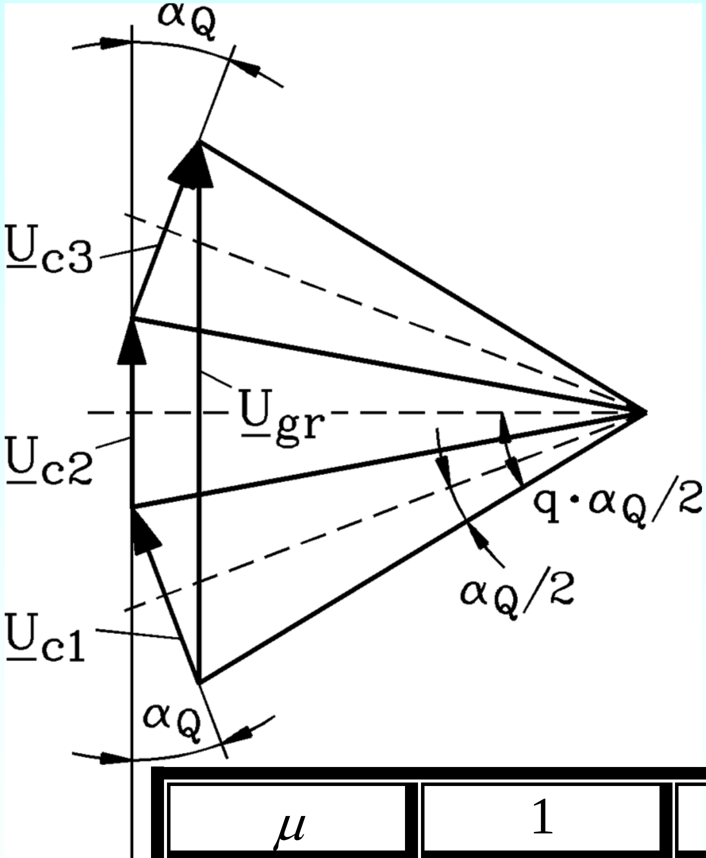
μ	1	3	5	7	9	11	13
$k_{p\mu}$	0.966	-0.707	0.259	0.259	-0.707	0.966	-0.966

Reduction of coil flux linkage due to chording $W/\tau_p = 5/6$.

Coil groups help to get sinusoidal voltage !

Example: Three-phase winding, 9 slots per pole, $q = 3$ coils per pole and phase.

$$\alpha_{Q\mu=1} = \alpha_Q = \pi \cdot (\tau_Q / \tau_p) = \pi / 9 \leftrightarrow 180° / 9 = 20°$$



Distribution factor:

$$k_{d,\mu} = \frac{\hat{U}_{i,gr,\mu}}{q\hat{U}_{i,c,\mu}} = \frac{2\sin\left(q\frac{\alpha_{Q,\mu}}{2}\right)}{q \cdot 2\sin\left(\frac{\alpha_{Q,\mu}}{2}\right)} = \frac{\sin\left(\mu\frac{\pi}{2m}\right)}{q \cdot \sin\left(\mu\frac{\pi}{2mq}\right)}$$

$$u_{i\mu}(t) = \mu\omega \cdot N_s \cdot k_{d\mu} \cdot k_{p\mu} \cdot \frac{2}{\pi} \cdot \frac{\tau_p}{\mu} l_{Fe} B_{\delta\mu} \cdot \sin(\mu\omega t)$$

μ	1	3	5	7	9	11	13
$k_{d\mu}$	0.960	0.667	0.218	-0.177	-0.333	-0.177	0.218

Reduction of coil flux linkage due to coil group arrangement $q = 3$.

Winding factor = Coil pitch + coil groups

Winding factor: $k_{w\mu} = k_{d\mu} \cdot k_{p\mu}$

Pitching factor:

$$k_{p\mu} = \sin(\mu \cdot \frac{W}{\tau_p} \cdot \frac{\pi}{2})$$

Distribution factor:

$$k_{d,\mu} = \frac{\sin\left(\mu \frac{\pi}{2m}\right)}{q \cdot \sin\left(\mu \frac{\pi}{2mq}\right)}$$

Star connected three phase winding suppresses 3rd harmonic line-to-line voltages and 3rd phase currents

Harmonic induced voltage:

$$u_{i\mu}(t) = \mu\omega \cdot N_s \cdot k_{w\mu} \cdot \frac{2}{\pi} \cdot \frac{\tau_p}{\mu} l_{Fe} B_{\delta\mu} \cdot \sin(\mu\omega t) = U_{i\mu} \cdot \sin(\mu\omega t)$$

If the stator winding is **star connected**, the third harmonic voltages in all three phases U, V, W are IN phase and IDENTICAL:

$$u_{U3}(t) = U_3 \cdot \cos(3\omega t)$$

$$u_{V3}(t) = U_3 \cdot \cos(3(\omega t - 2\pi/3)) = U_3 \cdot \cos(3\omega t) = u_{U3}(t)$$

$$u_{W3}(t) = U_3 \cdot \cos(3(\omega t - 4\pi/3)) = U_3 \cdot \cos(3\omega t) = u_{U3}(t)$$

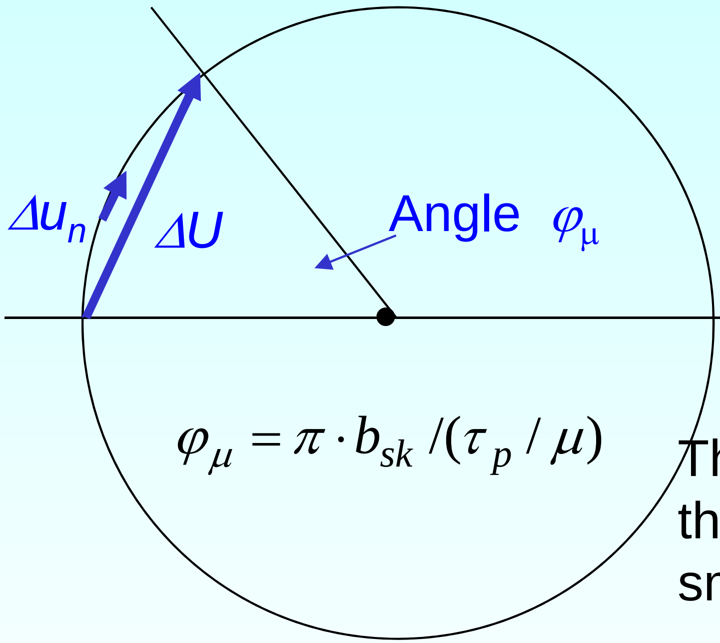
Therefore the line-to-line voltages show NO 3rd harmonic component:

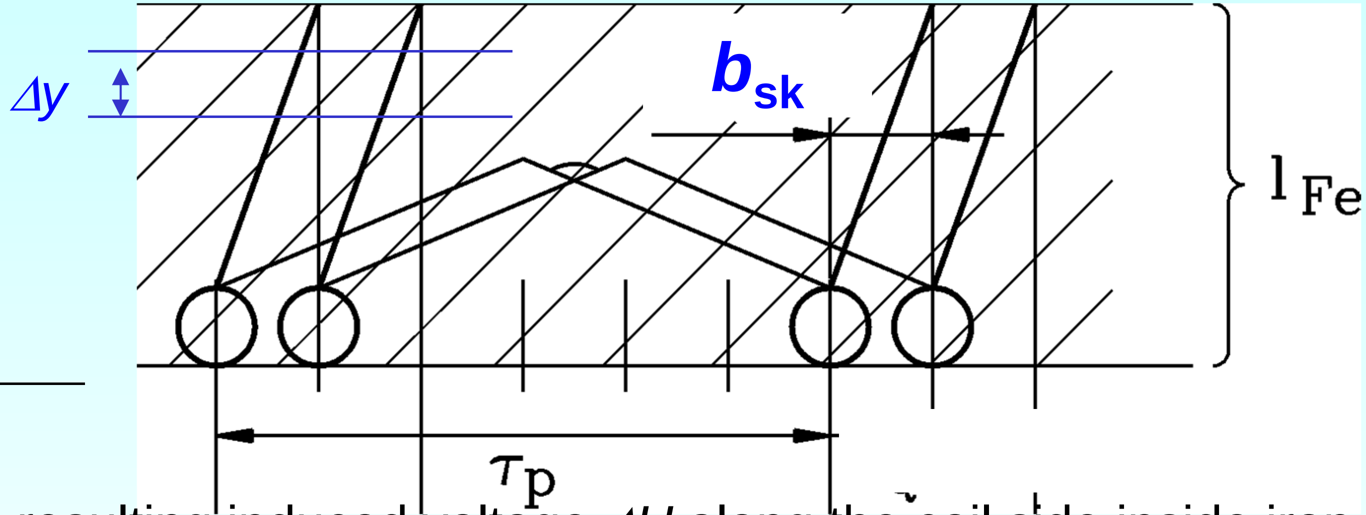
$$u_{UV3}(t) = u_{U3}(t) - u_{V3}(t) = u_{U3}(t) - u_{U3}(t) = 0$$

$$\underline{I}_3 = \underline{U}_3 / \underline{Z}_3 \quad \Rightarrow \quad \underline{I}_{U3} + \underline{I}_{V3} + \underline{I}_{W3} = 3\underline{I}_3 = 0 \quad \Rightarrow \quad \underline{I}_3 = 0$$

Skewing influence on the induced voltage

Skewing by the distance b_{sk} represents a phase shift between coil side beginning and end with respect to the μ-th harmonic field wave of $\varphi_\mu = \pi \cdot b_{sk} / (\tau_p / \mu)$





The resulting induced voltage ΔU along the coil side inside iron is the sum of differential small voltages Δu_n along the differential small coil lengths Δy.

The resulting voltage ΔU is given by:

$$\frac{\Delta U}{\sum_{n=1}^{\infty} \Delta u_n} = \frac{2 \cdot \sin(\varphi_\mu / 2)}{\varphi_\mu} = \chi_\mu = \sin(S_\mu) / S_\mu \qquad S_\mu = \frac{\mu \pi b_{sk}}{2 \tau_p}$$

Skewing helps to suppress slot harmonic back EMF

Skewing by the distance b_{sk} reduces flux linkage and therefore induced voltage further by the so-called **skewing factor**

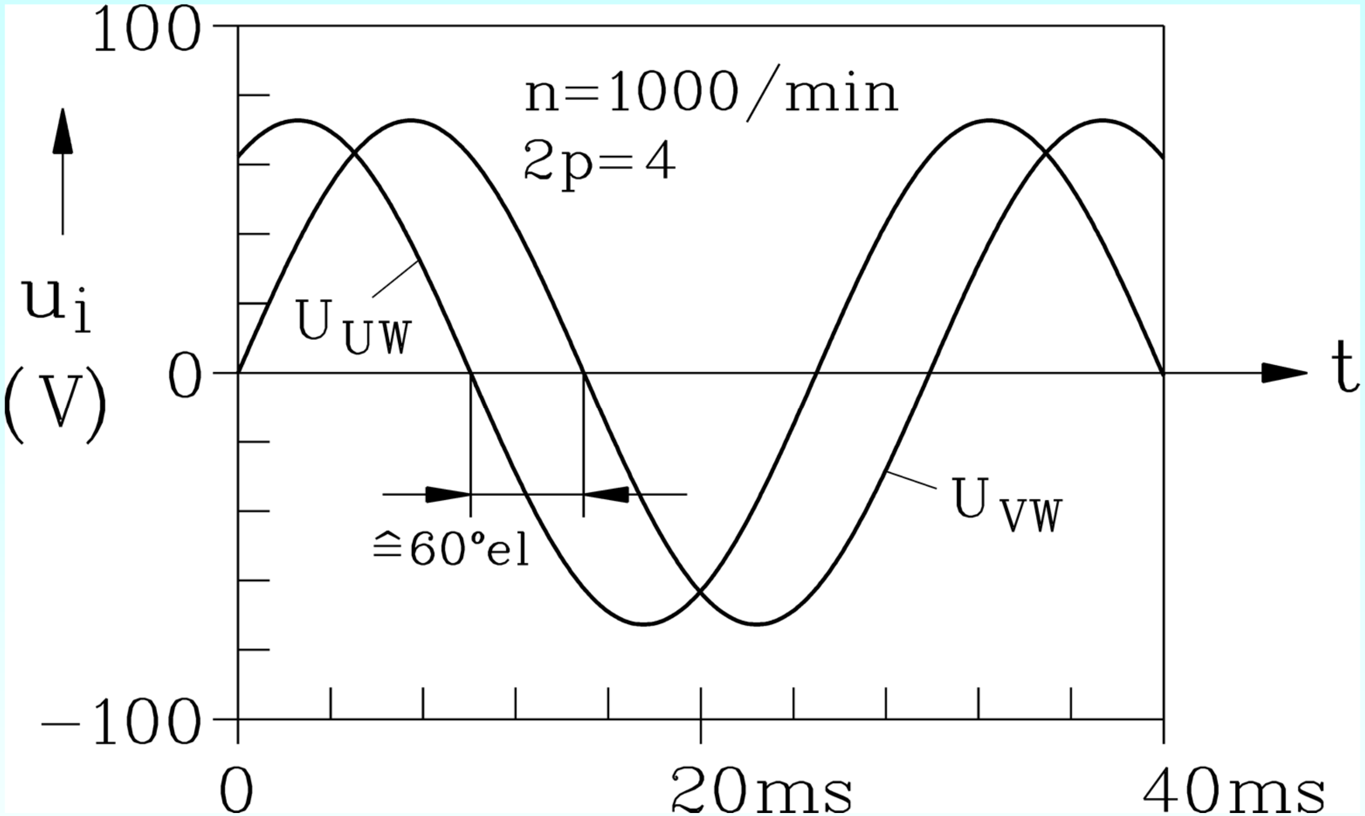
$$\chi_\mu = \sin(S_\mu)/S_\mu \qquad S_\mu = \frac{\mu\pi b_{sk}}{2\tau_p}$$

Example: Six pole machine, rotor speed 1500/min, 5/6 chorded coils, q = 2, 85% pole coverage ratio, magnets skewed by one stator slot pitch

Ordinal Number	Stator frequency	Flux density	Winding Factor	Skewing factor	Induced phase voltage	Induced line-line voltage
μ	μf	$B_{\delta\mu}$	$k_{w\mu}$	$\chi_{skew,\mu}$	$U_{i\mu}$	$U_{i\mu,LL}$
1	**75 Hz**	**100 %**	**0.933**	**0.989**	**100 %**	**100 %**
3	**225 Hz**	**-26.1 %**	**-0.50**	**0.900**	**12.73 %**	**0**
5	**375 Hz**	**7.9 %**	**0.067**	**0.738**	**0.42 %**	**0.42 %**
7	**525 Hz**	**1.2 %**	**-0.067**	**0.527**	**0.05 %**	**0.05 %**
9	**675 Hz**	**-6.0 %**	**0.50**	**0.300**	**0.98 %**	**0**
11	**825 Hz**	**8.0 %**	**-0.933**	**0.090**	**0.73 %**	**0.73 %**
13	**975 Hz**	**-8.0 %**	**0.933**	**-0.076**	**0.61 %**	**0.61 %**

Slot harmonic (rows 11 and 13)

Measured line-to-line no-load voltage (back EMF)



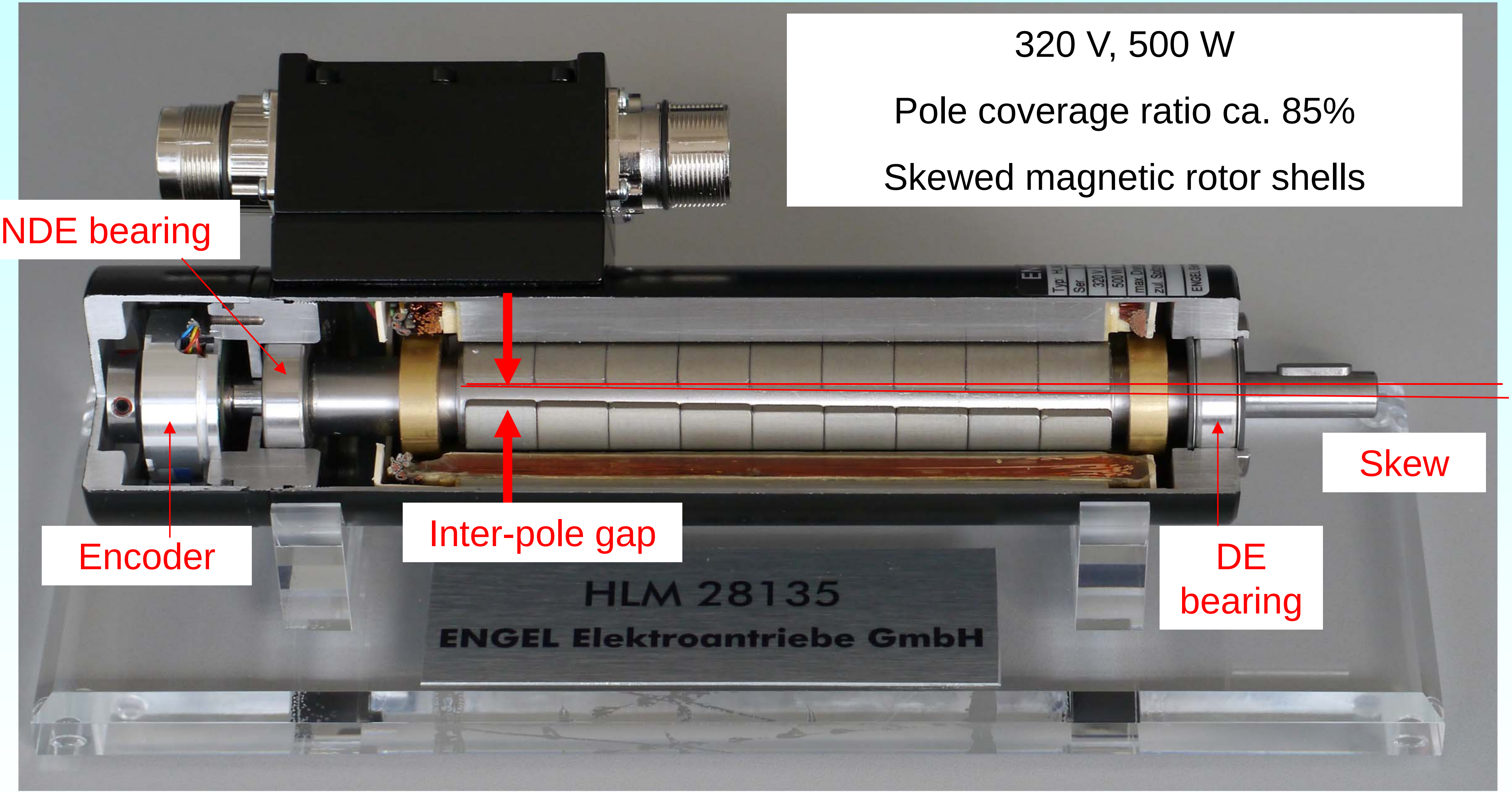
Fourier-analysis of measured no-load voltage:

μ = 1: 33.5 Hz, 74.8 V
μ = 5: 167.0 Hz, 0.34 V

Other amplitudes are negligible !

- 4 pole brushless DC motor (rated data: M_N = 1.3 Nm, n_N = 6000/min)
- designed for sine wave commutation, at 1000/min
- nearly ideal sine wave back EMF

Permanent magnet four-pole synchronous servo motor



Source: Engel Elektroantriebe GmbH, Germany