Energy Converters – CAD and System Dynamics

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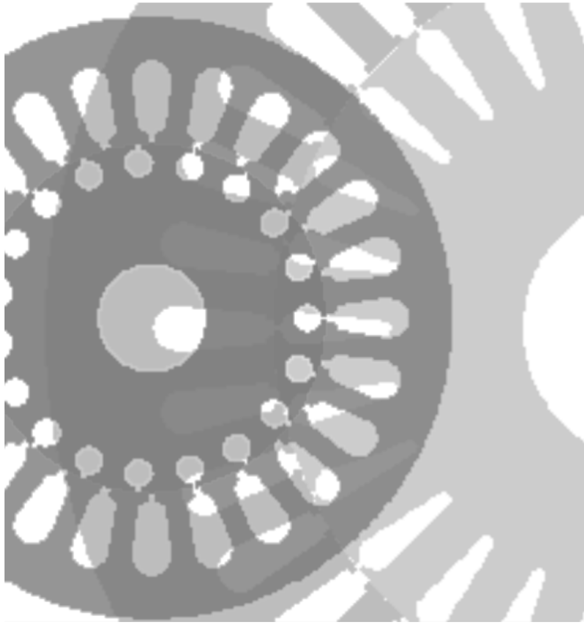
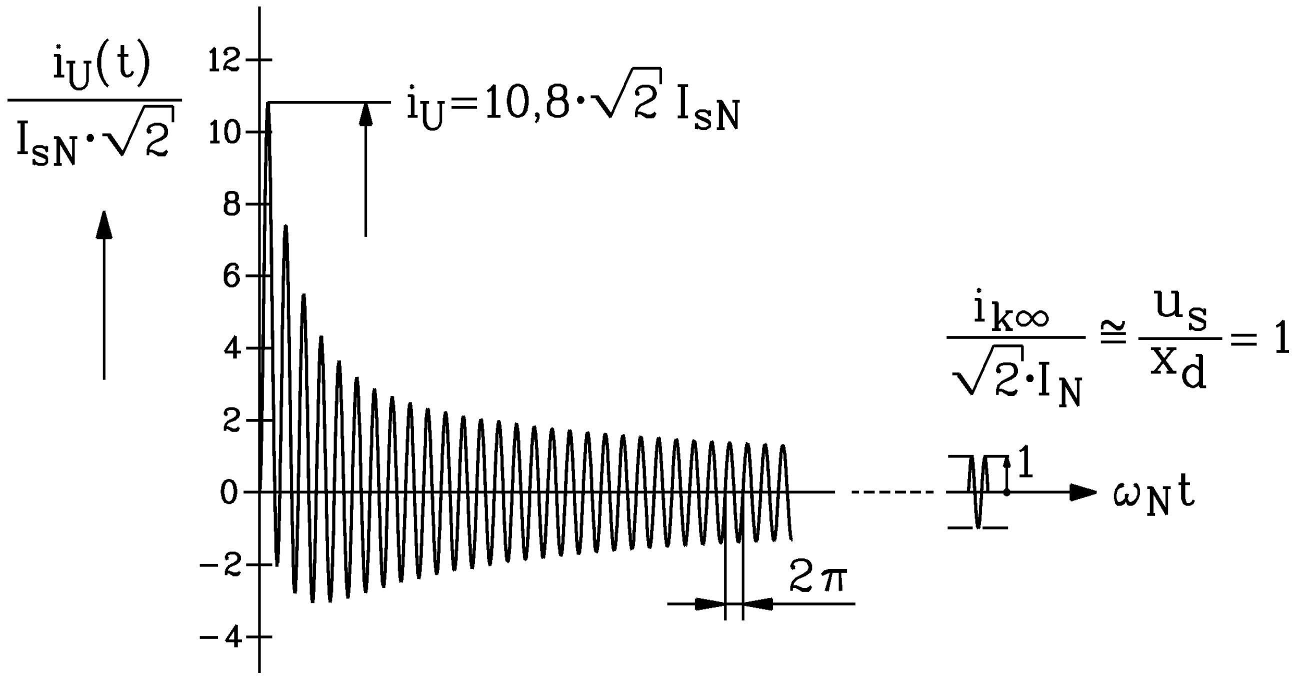
1. Basic design rules for electrical machines
2. Design of Induction Machines
3. Heat transfer and cooling of electrical machines
4. Dynamics of electrical machines
5. Dynamics of DC machines
6. Space vector theory
7. Dynamics of induction machines
8. Dynamics of synchronous machines



Source: SPEED program

8. Dynamics of synchronous machines



Source:
H. Kleinrath, Springer-Verlag

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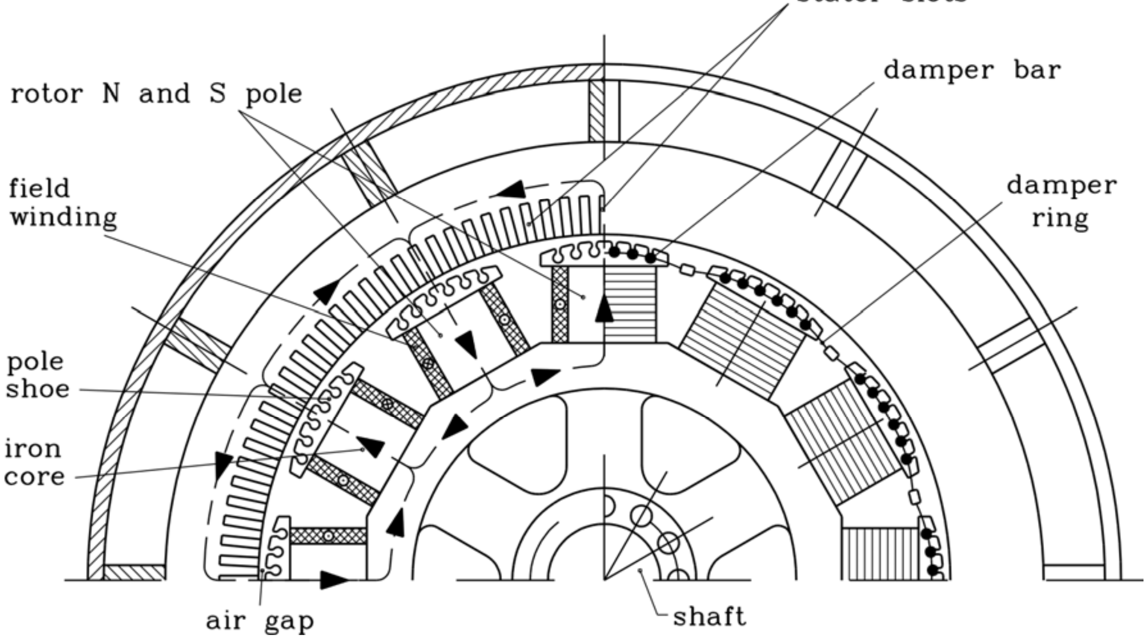
8. Dynamics of synchronous machines

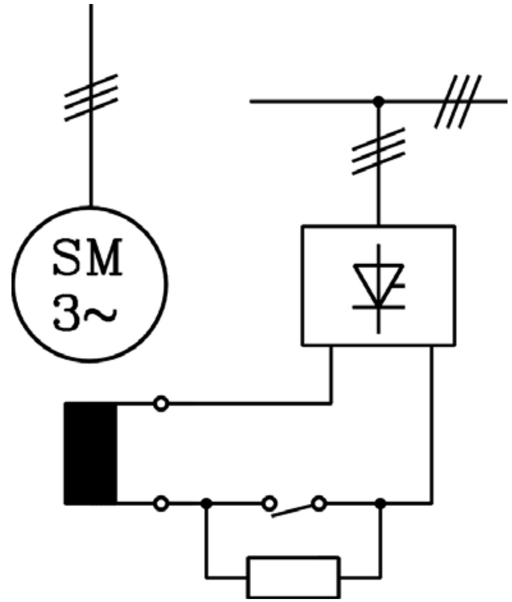
8. Dynamics of synchronous machines

Electrically excited synchronous machine

Source: H. Kleinrath, Studientext



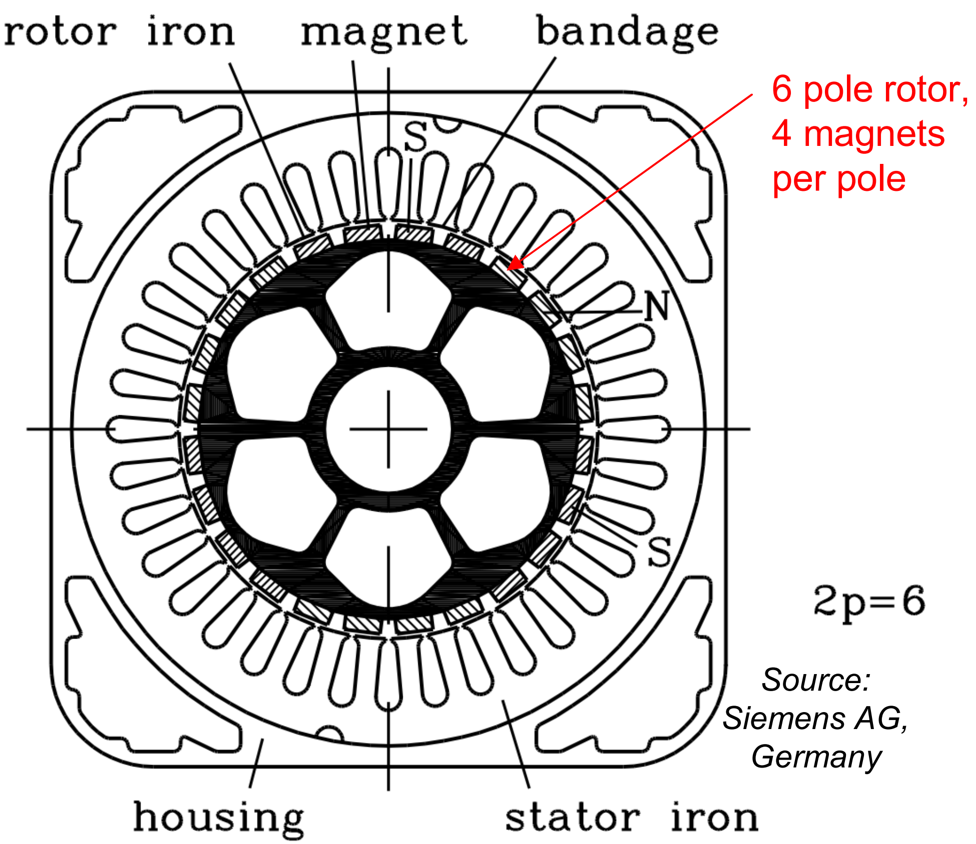
Three phase stator winding,
rotor field and damper winding



DC excitation via slip rings
from a DC voltage
via a controlled rectifier

Repetition

Permanent magnet (PM) excited synchronous machine



Source: Siemens AG, Germany

Cylindrical rotor with surface mounted rotor rare earth high energy magnets

Source: Kuka, Germany

Application of inverter-fed PM synchronous machine as adjustable speed drive for driving robots

Repetition

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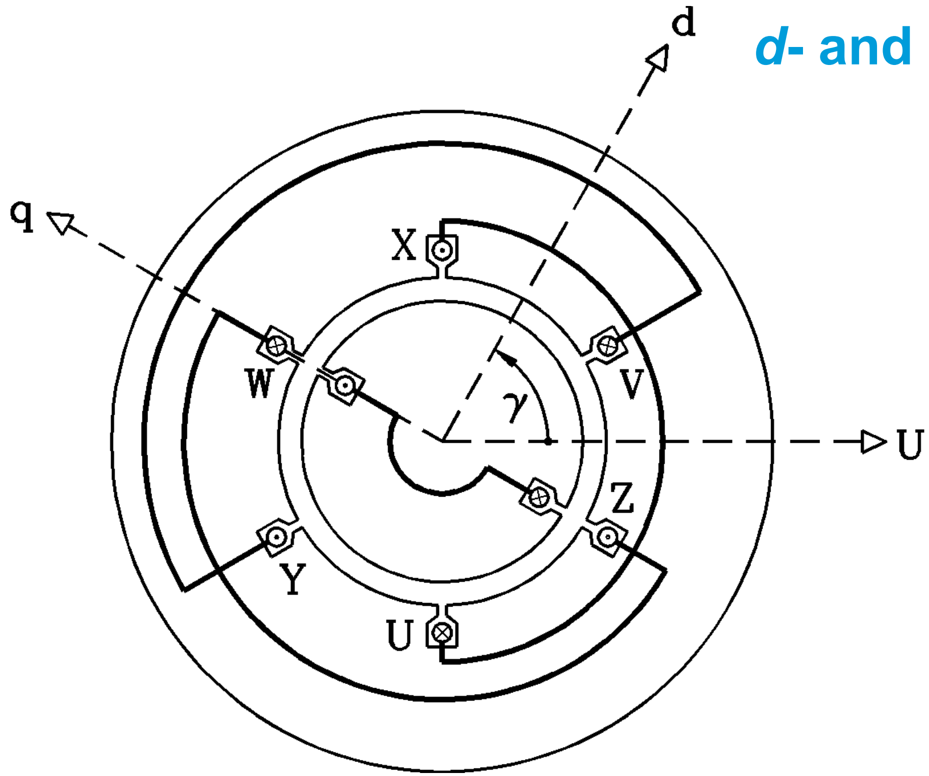
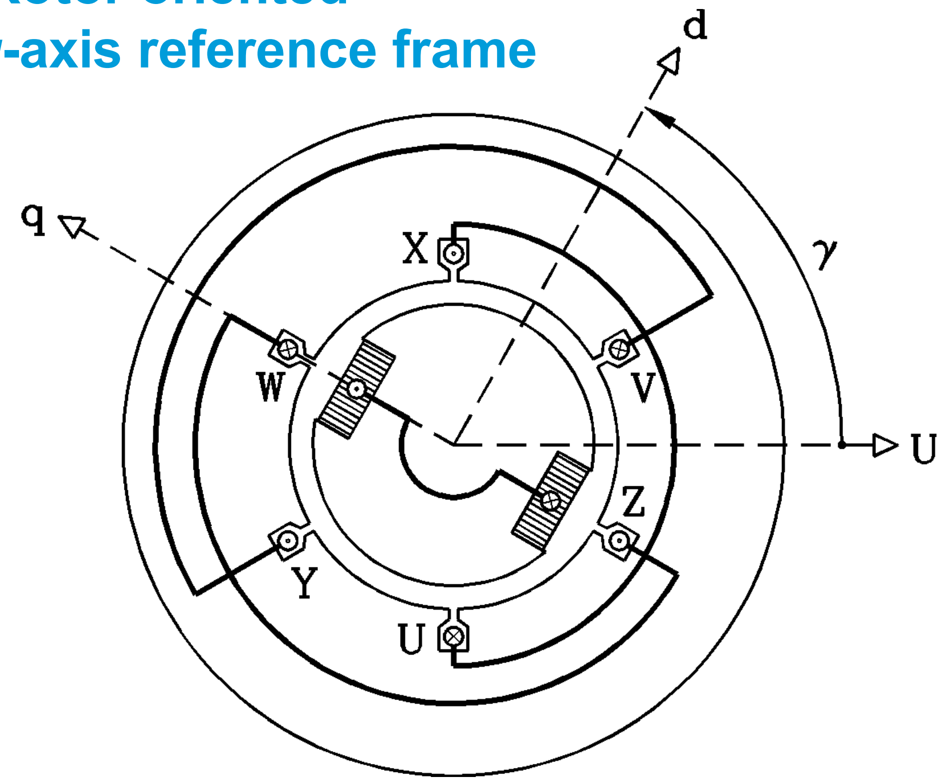
Air gap of synchronous machines

Rotor oriented
***d*- and *q*-axis reference frame**





Cylindrical rotor synchronous machine with constant air gap

Salient pole rotor synchronous machine with non-constant air gap

8. Dynamics of synchronous machines

Repetition

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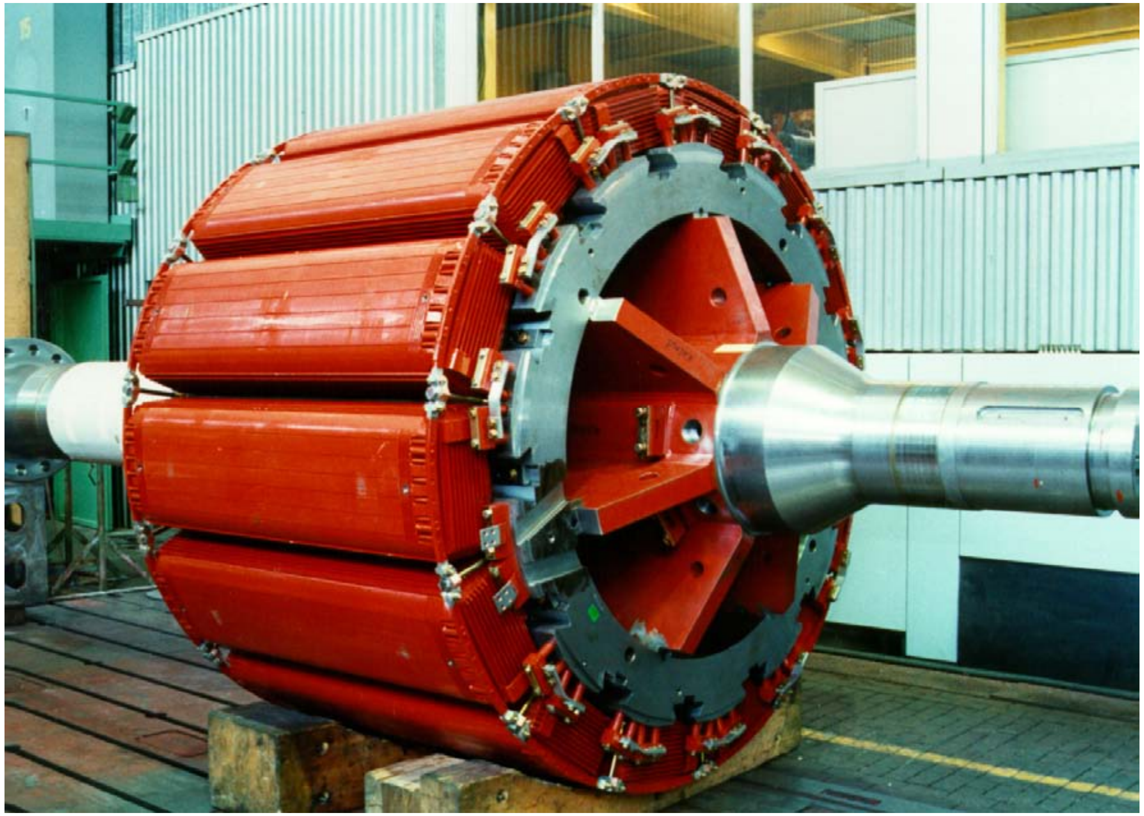
Cylindrical rotor

Source:
Siemens AG, Germany

Salient pole rotor



Source:
Andritz Hydro, Austria

Repetition

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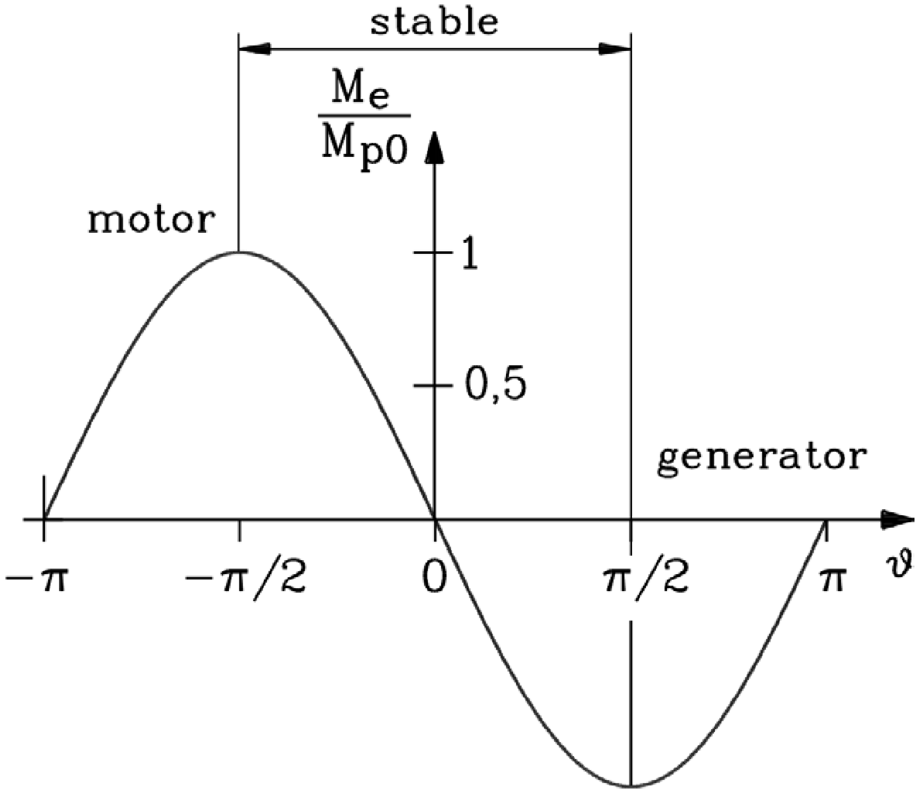
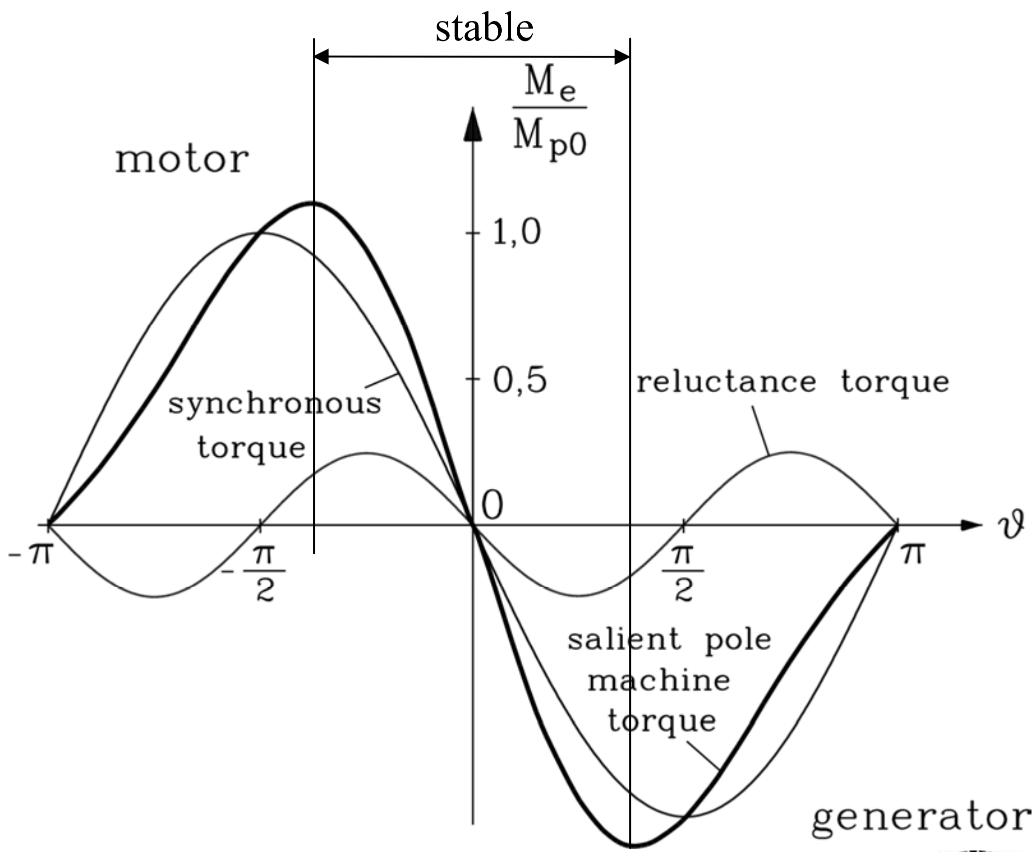
Steady-state $M_e(\vartheta)$-characteristic

Assumptions: Neglected stator losses (R_s = 0), constant r.m.s. phase value U_s, constant stator frequency f_s, constant rotor flux



Cylindrical rotor synchronous machine



Salient pole synchronous machine

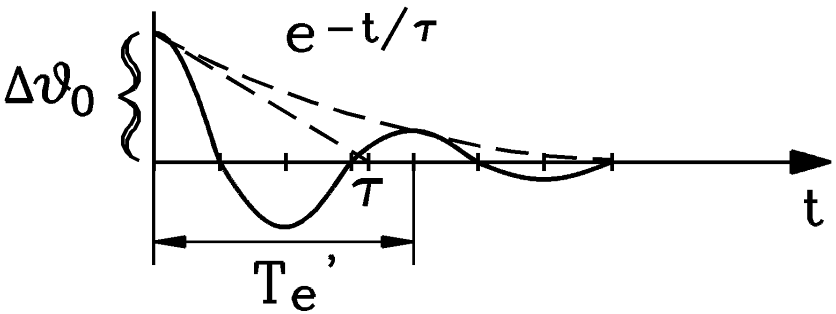
Transient operation of synchronous machine

- Large transient disturbance: e. g. generator operation at the grid:
  - sudden short circuits,
  - voltage dips or oscillations due to switching,
  - load steps on turbine or grid side etc.

- Small transient disturbances lead to natural oscillations of load angle and rotor speed

$$f_{d,m} = \frac{\omega_{d,m}}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{p\cdot|c_\vartheta|}{J} - \alpha^2}$$

Stiffness" of "torsion spring": $c_\vartheta = \frac{dM_e}{d\vartheta}$



$\tau = 1/\alpha$

$T_e' = 1/f_{d,m}$

Damper cage:
Damping of natural oscillations by induced damper bar currents

Summary:
Basics of steady state and significance of dynamic performance of synchronous machines

- Repetition of bachelor course contents
- Salient versus cylindrical pole rotors
- Synchronous versus reluctance torque
- Electrically versus permanent magnet excited rotors
- Pull-out torque and stability limit
- Natural oscillations of rotors damped by damper cage

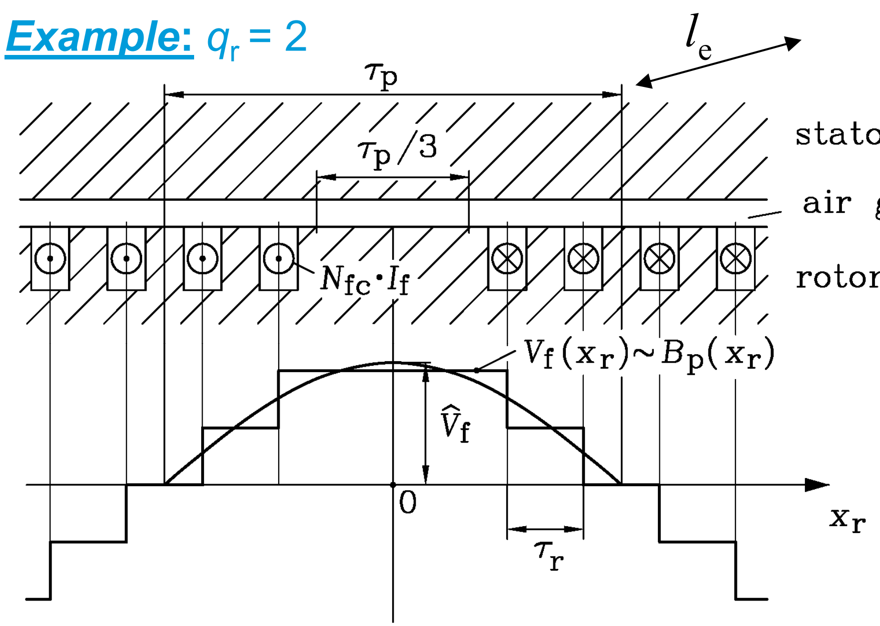
8. Dynamics of synchronous machines

Repetition

Rotor air gap field of cylindrical rotor synchronous machine

- Rotor field winding is excited by the DC field current I_f
- Rotor m.m.f. $V_f(x_r)$ and rotor air gap field distribution $B_p(x_r)$ have steps due to rotor slots
- FOURIER series gives the fundamental rotor field (μ = 1):

Example: $q_r = 2$



$$\mu = 1: \ \hat{V}_f = \frac{2}{\pi} \cdot \frac{N_f}{p} \cdot (k_{p,f} k_{d,f}) \cdot I_f$$

$$\hat{B}_p = \mu_0 \frac{\hat{V}_f}{\delta}, \quad N_f = 2p \cdot q_r \cdot N_{fc}$$

$$k_{p,f} = \sin\left(\frac{W}{\tau_p} \cdot \frac{\pi}{2}\right) = \sin(\pi/3) = \frac{\sqrt{3}}{2}$$

$$k_{d,f} = \frac{\sin(\pi/6)}{q_r \sin(\pi/(6q_r))}, \quad k_{wf} = k_{pf} k_{df}$$

- The fundamental rotor air-gap field is sinusoidal distributed.
- It may described by a space vector, fixed in the rotor reference frame.

Repetition

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Cylindrical rotor synchronous excitation-stator transfer ratio

- Fundamental of rotor m.m.f. $V_f(x_r)$:

$$I'_f = \frac{1}{\ddot{u}_{If}} I_f \qquad \ddot{u}_{If} = \frac{m_s N_s k_{ws} / \sqrt{2}}{N_f k_{wf}} \qquad U'_f = \ddot{u}_{Uf} \cdot U_f \qquad \ddot{u}_{Uf} = \frac{N_s k_{ws}}{\sqrt{2} \cdot N_f k_{wf}}$$

$\ddot{u}_{If}$: field current transfer ratio $\ddot{u}_{Uf}$: field voltage transfer ratio

$$\ddot{u}_{If} = \frac{m_s}{m_f} \cdot \ddot{u}_{Uf} = \frac{m_s}{1} \cdot \ddot{u}_{Uf}$$

- Proof: $I_{f,rms} = I_f / \sqrt{2}$, $\quad U_{f,i,rms} = \omega \cdot L_{fh} \cdot I_{f,rms} = \sqrt{2}\pi f \cdot N_f k_{wf} \cdot \underbrace{\frac{2}{\pi} \tau_p l_e \cdot \underbrace{\frac{\mu_0}{k_C \delta} \cdot \underbrace{\frac{2}{\pi} \cdot \frac{N_f}{p} \cdot k_{wf} \cdot I_f}_{\hat{V}_f}}_{B_f}}_{\Phi_{fh}}$

Rotor field winding self-inductance L_{fh} due to fundamental air-gap field: $L_{fh} = \mu_0 \cdot (N_f k_{wf})^2 \cdot \frac{4}{\pi^2 p} \cdot \frac{\tau_p l_e}{k_C \delta}$

$$L_h = L_{sh} = \ddot{u}_{Uf} \cdot \ddot{u}_{If} \cdot L_{fh} = \frac{m_s (N_s k_{ws})^2}{(\sqrt{2} N_f k_{wf})^2} \cdot \mu_0 \cdot (N_f k_{wf})^2 \cdot \frac{4}{\pi^2 p} \cdot \frac{\tau_p l_e}{k_C \delta} = \mu_0 \cdot (N_s k_{ws})^2 \cdot \frac{2 m_s}{\pi^2 p} \cdot \frac{\tau_p l_e}{k_C \delta}$$

8. Dynamics of synchronous machines

Cylindrical rotor synchronous machine: Rotor magnetic field

$$I'_f = \frac{1}{\ddot{u}_{If}} I_f \qquad \ddot{u}_{If} = \frac{m_s N_s k_{ws} / \sqrt{2}}{N_f k_{wf}}$$

- Fundamental of rotor m.m.f. $V_f(x_r)$:

$$\hat{V}_f = \frac{2}{\pi} \cdot \frac{N_f}{p} \cdot k_{wf} \cdot I_f = \frac{\sqrt{2}}{\pi} \cdot \frac{m_s N_s}{p} \cdot k_{ws} \cdot I'_f$$

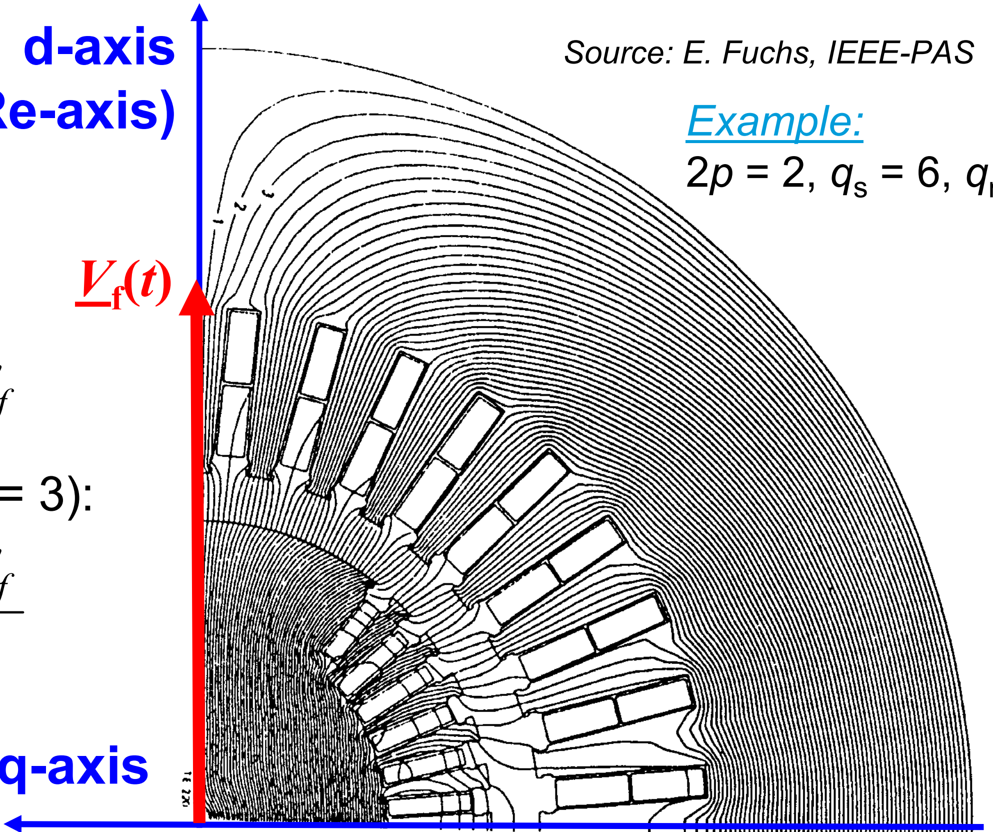
- Expressed with stator winding data ($m_s = 3$):

$$\hat{V}_f = \frac{\sqrt{2}}{\pi} \cdot \frac{3N_s}{p} \cdot k_{ws} \cdot I'_f = \frac{3}{2} \cdot \hat{V}_{1N,ph} \cdot \frac{\sqrt{2} \cdot I'_f}{\hat{I}_N}$$

$$\hat{V}_{1N,ph} = \frac{N_s}{2p} \cdot \hat{I}_N \cdot \frac{4}{\pi} \cdot k_{ws}$$

- **Rotor m.m.f. space vector:**

$$\underline{V}_f(t) = \frac{3}{2} \cdot \hat{V}_{1N,ph} \cdot i'_f(t) \qquad i'_f = \frac{\sqrt{2} \cdot I'_f}{\hat{I}_N}$$



Source: E. Fuchs, IEEE-PAS

Example:
$2p = 2$, $q_s = 6$, $q_r = 6$

Magnetic rotor field, no-load ($I_s = 0, I_f > 0$):
- Field winding excited by I_f
- Stator winding without current **(no-load)**

8. Dynamics of synchronous machines

Stator and rotor m.m.f. and current space vectors

Stator space vectors:

$$\underline{V}_s(t) = \hat{V}_{1N,ph} \cdot \left[i_U(t) + \underline{a} \cdot i_V(t) + \underline{a}^2 \cdot i_W(t) \right]$$

$$\hat{V}_{1,N} = \frac{3}{2} \cdot \hat{V}_{1N,ph}$$

$$\underline{v}_s(t) = \frac{\underline{V}_s(t)}{\hat{V}_{1N}} = \frac{2}{3} \cdot \left[i_U(t) + \underline{a} \cdot i_V(t) + \underline{a}^2 \cdot i_W(t) \right]$$

$$\underline{I}_s(t) = \frac{2}{3} \cdot \left[I_U(t) + \underline{a} \cdot I_V(t) + \underline{a}^2 \cdot I_W(t) \right] \qquad \underline{i}_s(t) = \frac{\underline{I}_s(t)}{\hat{I}_N} = \frac{2}{3} \cdot \left[i_U(t) + \underline{a} \cdot i_V(t) + \underline{a}^2 \cdot i_W(t) \right] = \underline{v}_s(t)$$

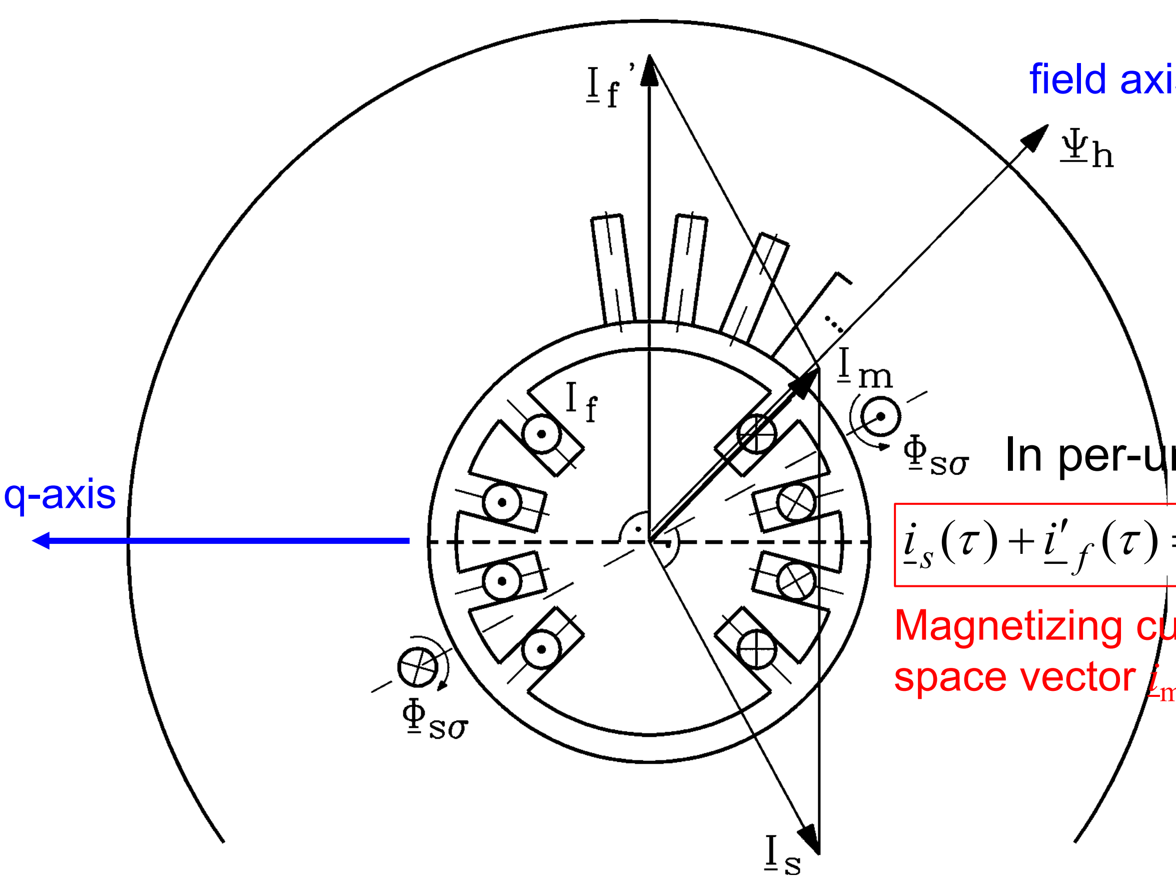
Rotor space vectors: in direction of d-axis! (= Re-axis)

$$\underline{V}_f(t) = \frac{3}{2} \cdot \hat{V}_{1N,ph} \cdot i'_f(t) \qquad \underline{v}_f(t) = \frac{\underline{V}_f(t)}{\hat{V}_{1N}} = i'_f(t) = \underline{i}'_f(t)$$

$$\underline{i}'_f(t) = \frac{\sqrt{2} \cdot \underline{I}'_f(t)}{\hat{I}_N} = \underline{v}_f(t)$$

8. Dynamics of synchronous machines

Magnetizing current space vector



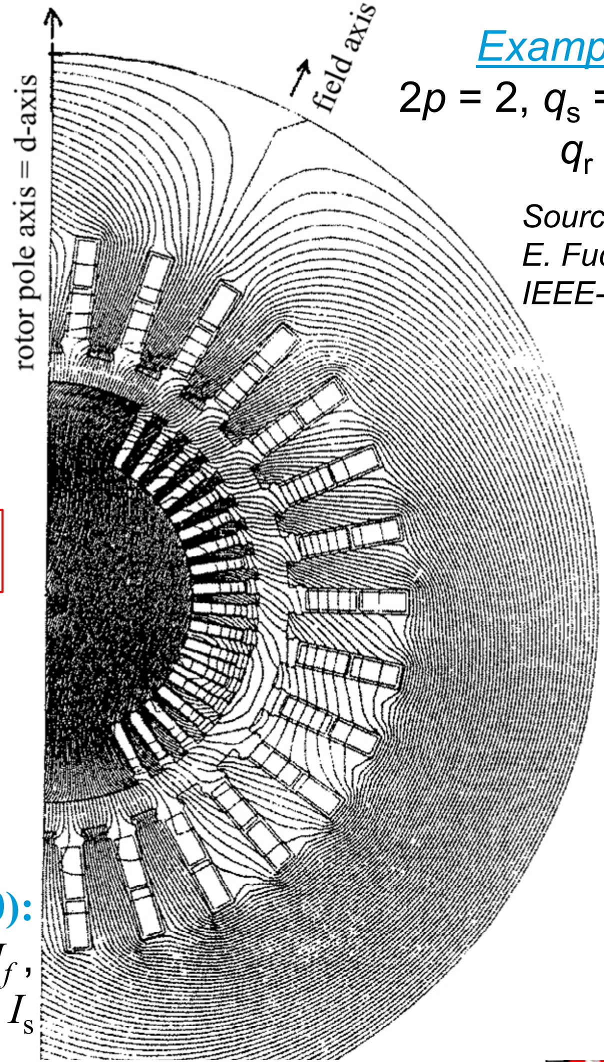
In per-unit:

$$\underline{i}_s(\tau) + \underline{i}'_f(\tau) = \underline{i}_m(\tau)$$

Magnetizing current space vector $\underline{i}_m$

Main stator flux linkage space vector:

$$\underline{\psi}_h(\tau) = x_h \cdot \underline{i}_m(\tau)$$



Example:

$2p = 2$, $q_s = 5$, $q_r = 9$

Source: E. Fuchs, IEEE-PAS

Magnetic field at load ($I_s > 0$, $I_f > 0$):

Field winding excited by I_f,
Stator winding excited by I_s

8. Dynamics of synchronous machines

Stator flux linkage equation in rotor reference frame (d- and q-axis) without damper cage

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In rotor reference frame: $\underline{\psi}_s(\tau) = \underline{\psi}_{s(r)}(\tau) \quad \underline{i}_s(\tau) = \underline{i}_{s(r)}(\tau)$ etc.

$$\left(\underline{\psi}_{s(r)} = \underline{\psi}_{s(s)} \cdot e^{-j\gamma}, \ldots\right)$$

Stator flux linkage:

$$\underline{\psi}_s(\tau) = x_{s\sigma} \cdot \underline{i}_s(\tau) + \underline{\psi}_h(\tau) = x_{s\sigma} \cdot \underline{i}_s(\tau) + x_h \cdot \underline{i}_m(\tau)$$

$$\underline{\psi}_s = x_{s\sigma} \cdot \underline{i}_s + x_h \cdot (\underline{i}_s + \underline{i}'_f) = (x_{s\sigma} + x_h) \cdot \underline{i}_s + x_h \underline{i}'_f \qquad \underline{i}_s = i_d + j \cdot i_q$$

$$\underline{\psi}_s = \psi_d + j\psi_q = x_s \cdot i_d + jx_s \cdot i_q + x_h i'_f$$

$$\psi_d = x_s \cdot i_d + x_h i'_f \qquad \psi_q = x_s \cdot i_q$$

Stator winding inductance: $x_s = x_h + x_{s\sigma} = x_d$
("synchronous inductance")

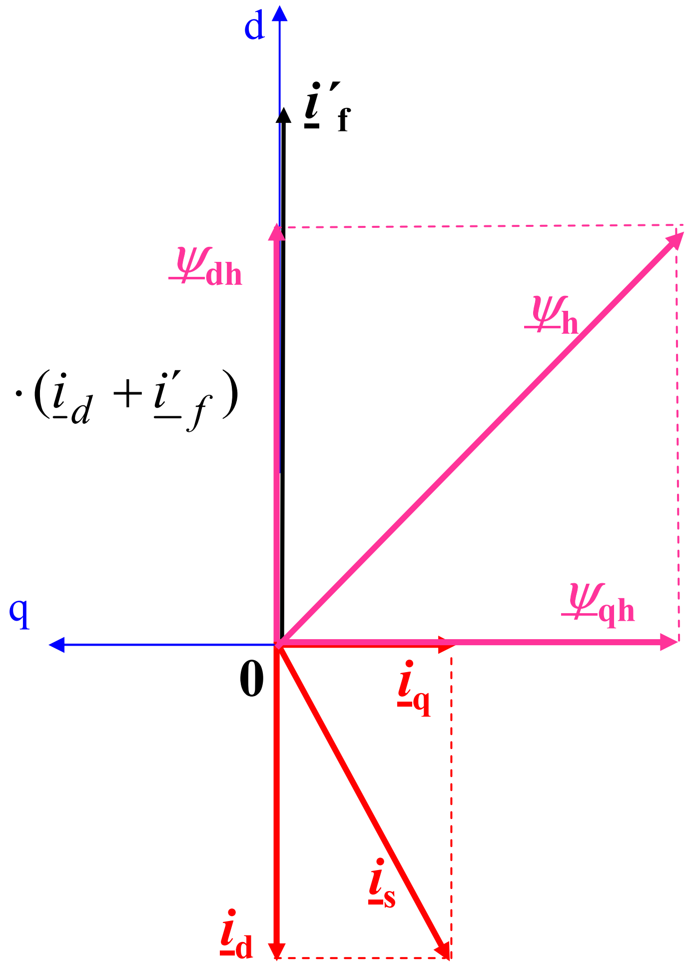
We assume constant iron saturation, so x_h is constant.

8. Dynamics of synchronous machines

Space vector diagram in rotor reference frame (*d*- and *q*-axis)

$$\underline{\psi}_h = \underline{\psi}_{dh} + j \cdot \underline{\psi}_{qh}$$

$$\underline{\psi}_{dh} = x_h \cdot (\underline{i}_d + \underline{i}'_f)$$



$$\underline{\psi}_{qh} = x_h \cdot \underline{i}_q$$

***d*-axis = Re-axis**
***q*-axis = Im-axis**

$$\psi_{dh} = x_h \cdot (i_d + i'_f)$$

$$\psi_{qh} = x_h \cdot i_q$$

$$\psi_d = x_s \cdot i_d + x_h i'_f$$

$$\psi_q = x_s \cdot i_q$$

8. Dynamics of synchronous machines

Dynamic equations of cylindrical rotor synchronous machine without damper cage

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rotor reference frame = *d*- and *q*-axis

$$\underline{i}_s = i_d + j \cdot i_q \qquad \underline{u}_s = u_d + j \cdot u_q$$

$$\underline{\psi}_s = \psi_d + j \cdot \psi_q$$

$$u_d(\tau) = r_s \cdot i_d(\tau) + \frac{d\psi_d(\tau)}{d\tau} - \omega_m(\tau) \cdot \psi_q(\tau)$$

$$u_q(\tau) = r_s \cdot i_q(\tau) + \frac{d\psi_q(\tau)}{d\tau} + \omega_m(\tau) \cdot \psi_d(\tau)$$

$$\psi_d(\tau) = (x_h + x_{s\sigma}) \cdot i_d(\tau) + x_h \cdot i'_f(\tau)$$

$$\psi_q(\tau) = (x_h + x_{s\sigma}) \cdot i_q(\tau)$$

$$\tau_J \cdot \frac{d\omega_m(\tau)}{d\tau} = i_q(\tau) \cdot \psi_d(\tau) - i_d(\tau) \cdot \psi_q(\tau) - m_s(\tau)$$

Unknowns:

$$i_d(\tau), i_q(\tau), \psi_d(\tau), \psi_q(\tau), \omega_m(\tau)$$

Given:

$$u_d(\tau), u_q(\tau), i'_f(\tau), m_s(\tau)$$

Stator zero sequence voltage system: $u_{s0} = r_s \cdot i_{s0} + \frac{d\psi_{s0}}{d\tau}$

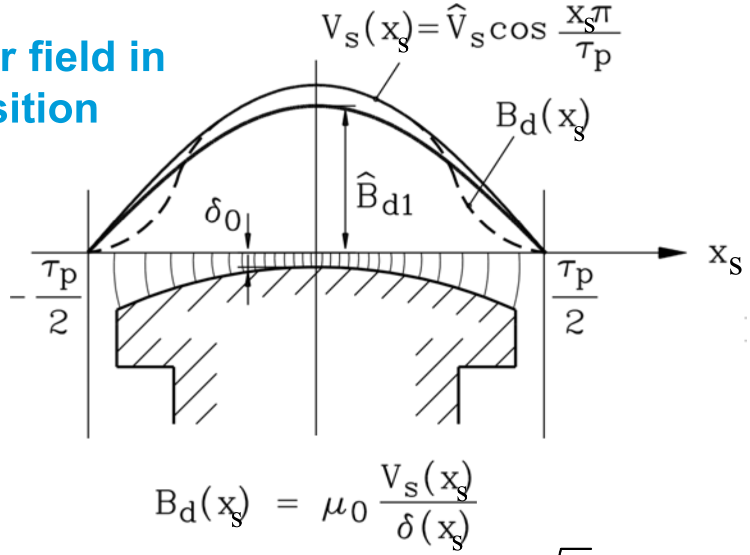
Separate zero-sequence flux linkage equation (not discussed here)

Repetition

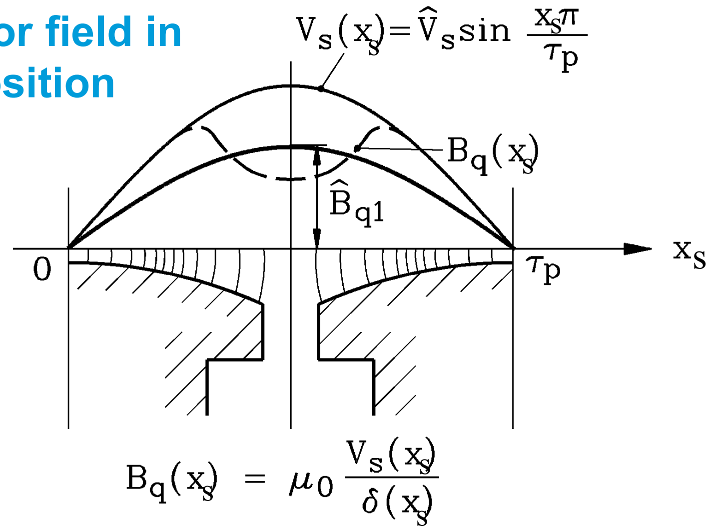
Saliency: Stator air gap field larger in *d*- than in *q*-axis

Air gap is **LARGER in neutral zone (inter-pole gap of *q*-axis)** than in **pole axis (*d*-axis)**.
Hence for equal m.m.f. V_s (sinus fundamental $\nu = 1$) the corresponding air gap field is **SMALLER** in *q*-axis than in *d*-axis and NOT SINUSOIDAL

Stator field in *d*-position



Stator field in *q*-position



- Fundamental stator m.m.f. $\hat{V}_s = \frac{\sqrt{2}}{\pi} \cdot \frac{3}{p} \cdot N_s k_{ws} \cdot I_s$ leads to non-sinus $B_d(x_s)$, $B_q(x_s)$!
- Stator field at cylindrical rotor synchronous machine (air gap δ_0): $\hat{B}_s = \mu_0 \frac{\hat{V}_s}{\delta_0}$
- Amplitudes of FOURIER-fundamental waves of $B_d(x_s)$, $B_q(x_s)$: $\hat{B}_{d1} < \hat{B}_s$, $\hat{B}_{q1} << \hat{B}_s$

Repetition

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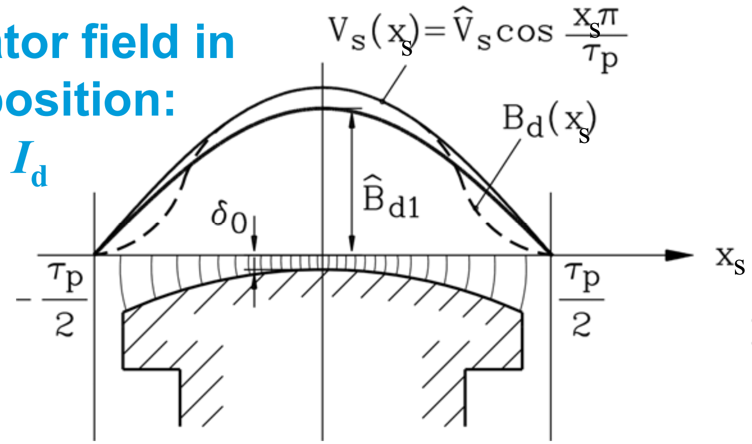
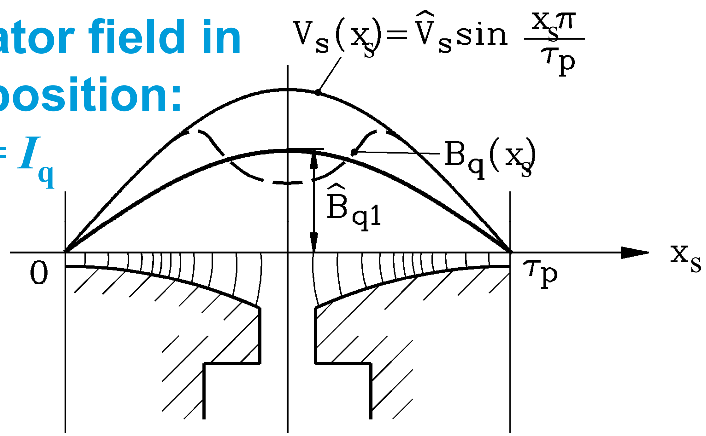
Saliency: Different main inductances in *d*- and *q*-axis

Stator field in *d*-position: $I_s = I_d$



$$c_d = \hat{B}_{d1} / \hat{B}_s \approx 0.95 < 1$$

Stator field in *q*-position: $I_s = I_q$



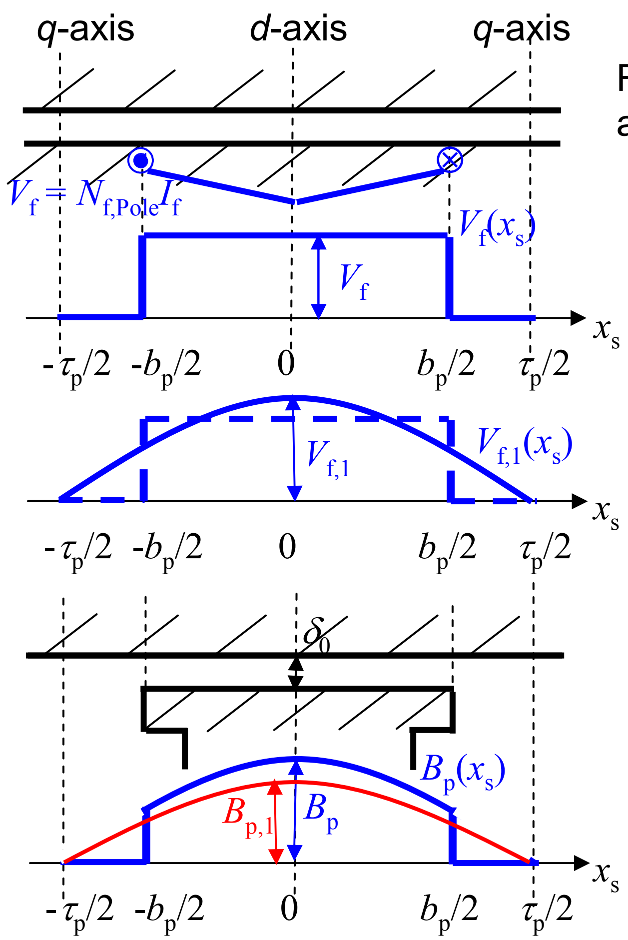
$$c_q = \hat{B}_{q1} / \hat{B}_s \approx 0.5 \dots 0.6 < 1$$

- Stator main inductance of cylindrical rotor synchronous machine (air gap δ_0):

$$\text{at } \mu_{Fe} \to \infty : L_h = \mu_0 \cdot (N_s k_{ws})^2 \cdot \frac{6}{\pi^2 p} \cdot \frac{\tau_p l_e}{k_C \delta_0} = \frac{\Psi_h / \sqrt{2}}{I_s} \sim \frac{\hat{B}_s}{I_s}$$

- Saliency: ***d*-axis:** $L_{dh} = \frac{\Psi_{dh} / \sqrt{2}}{I_d} = \frac{c_d \cdot \Psi_h / \sqrt{2}}{I_s} = c_d \cdot L_h$ ***q*-axis:** $L_{qh} = \frac{\Psi_{qh} / \sqrt{2}}{I_q} = \frac{c_q \cdot \Psi_h / \sqrt{2}}{I_s} = c_q \cdot L_h$

Example: *d*-axis: Constant air-gap δ_0 over b_p: a) Rotor field



Rotor-side m.m.f. $V_f(x_s)$ of field-coil per rotor pole has the fundamental amplitude $V_{f,1}$:

$$N_{f,Pole} = N_f/(2p) \qquad V_{f,1} = \frac{1}{\tau_p} \cdot \int_{-\tau_p}^{\tau_p} V_f(x_s) \cdot \cos(x_s\pi/\tau_p) \cdot dx_s$$

$$V_{f,1} = \frac{2}{\tau_p} \cdot \int_{-b_p/2}^{b_p/2} V_f \cdot \cos(x_s\pi/\tau_p) \cdot dx_s = \frac{4V_f}{\pi} \cdot \underbrace{\sin\left(\frac{b_p}{\tau_p} \cdot \frac{\pi}{2}\right)}_{k_{pf}=k_{wf}} \qquad V_{f,1}(x_s) = V_{f,1} \cdot \cos\left(\frac{x_s\pi}{\tau_p}\right)$$

The fundamental m.m.f. yields along with air-gap $\delta(x_s)$ the radial rotor air-gap field:

$$B_p(x_s) = \mu_0 \cdot V_{f,1}(x_s)/\delta(x_s) \qquad B_p = \mu_0 \cdot V_{f,1}/\delta_0$$

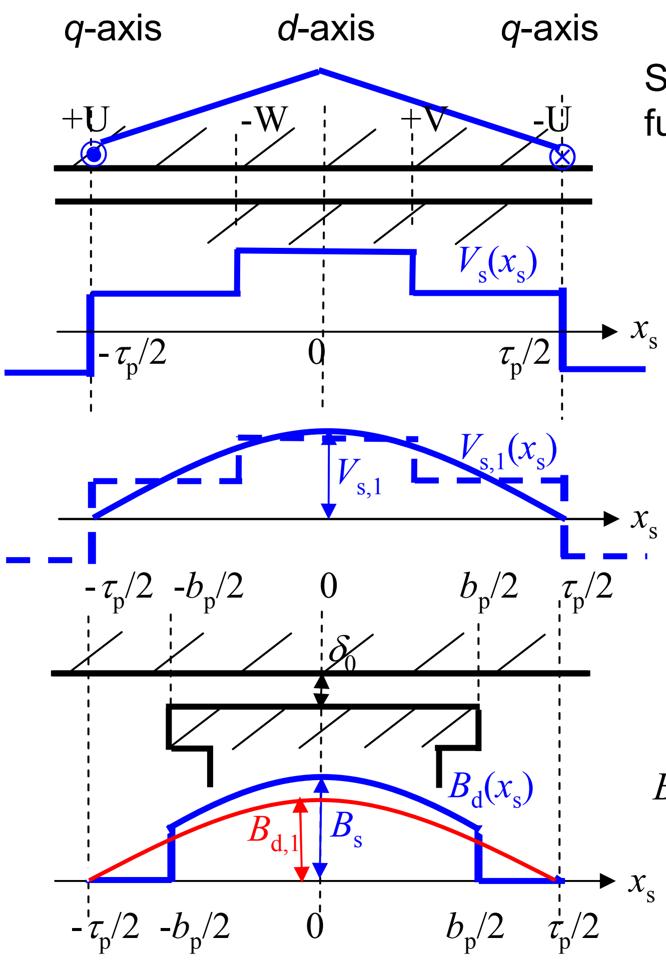
The fundamental amplitude $B_{p,1}$ of the radial rotor air-gap field is:

$$B_{p,1} = \frac{1}{\tau_p} \cdot \int_{-\tau_p}^{\tau_p} B_p(x_s) \cdot \cos(x_s\pi/\tau_p) \cdot dx_s = \frac{2}{\tau_p} \cdot \int_{-b_p/2}^{b_p/2} B_p \cdot \cos^2(x_s\pi/\tau_p) \cdot dx_s = c_d \cdot B_p$$

$$c_d = \frac{1}{\pi} \cdot [a + \sin a], \quad a = \frac{b_p\pi}{\tau_p} \qquad \text{For } b_p = \tau_p : c_d = 1$$

8. Dynamics of synchronous machines

Example: *d*-axis: Constant air-gap δ_0 over b_p: b) Stator field



Stator-side m.m.f. $V_s(x_s)$ of m_s–phase (three-phase) winding has the fundamental amplitude $V_{s,1}$:

$$V_{s,1} = \frac{1}{\tau_p} \cdot \int_{-\tau_p}^{\tau_p} V_s(x_s) \cdot \cos(x_s \pi / \tau_p) \cdot dx_s = \frac{\sqrt{2}}{\pi} \cdot \frac{m_s}{p} \cdot N_s k_{ws} \cdot I_s$$

$$V_{s,1}(x_s) = V_{s,1} \cdot \cos(\frac{x_s \pi}{\tau_p})$$

The fundamental m.m.f. yields along with air-gap $\delta(x_s)$ the radial rotor air-gap field:

$$B_d(x_s) = \mu_0 \cdot V_{s,1}(x_s) / \delta(x_s) \qquad B_s = \mu_0 \cdot V_{s,1} / \delta_0$$

The fundamental amplitude $B_{s,1}$ of the radial rotor air-gap field is:

$$B_{d,1} = \frac{1}{\tau_p} \cdot \int_{-\tau_p}^{\tau_p} B_d(x_s) \cdot \cos(x_s \pi / \tau_p) \cdot dx_s = \frac{2}{\tau_p} \cdot \int_{-b_p/2}^{b_p/2} B_s \cdot \cos^2(x_s \pi / \tau_p) \cdot dx_s = c_d \cdot B_s$$

$$c_d = \frac{1}{\pi} \cdot [a + \sin a], \quad a = \frac{b_p \pi}{\tau_p} \qquad \text{For } b_p = \tau_p : c_d = 1$$

8. Dynamics of synchronous machines

Saliency: Voltage & current transfer ratio $\ddot{u}_{Uf}$ & $\ddot{u}_{If}$

$$B_1(x_s) = B_1 \cdot \cos(\frac{x_s \pi}{\tau_p} - 2\pi f \cdot t) \quad \Phi_1 = \frac{2}{\pi} \cdot \tau_p \cdot l_e \cdot B_1 \Rightarrow U_{i,s} = \sqrt{2} \cdot \pi \cdot f \cdot N_s k_{ws} \cdot \Phi_1$$

$$U_{i,f} = \sqrt{2} \cdot \pi \cdot f \cdot 2p \cdot N_{f,Pole} \cdot k_{wf} \cdot \Phi_1 = \sqrt{2} \cdot \pi \cdot f \cdot N_f \cdot k_{wf} \cdot \Phi_1$$

- Voltage transfer ratio $\ddot{u}_{Uf}$: $\boxed{\ddot{u}_{Uf} = U_{i,s} / U_{i,f} = (N_s \cdot k_{ws}) / (N_f \cdot k_{wf})}$
- Current transfer ratio $\ddot{u}_{If}$: $\ddot{u}_{If} = I_f / I'_f$

$$B_{p,1} = c_d \cdot B_p = c_d \cdot \mu_0 \cdot V_{f,1} / \delta_0 = c_d \cdot \mu_0 \cdot \frac{4 \cdot N_f I_f}{\pi \cdot 2p} \cdot k_{wf} / \delta_0 = B_{d,1}(I'_f) = c_d \cdot \mu_0 \cdot \frac{\sqrt{2}}{\pi} \cdot \frac{m_s}{p} \cdot N_s k_{ws} \cdot I'_f / \delta_0$$

$$\boxed{\ddot{u}_{If} = I_f / I'_f = (m_s \cdot N_s \cdot k_{ws}) / (\sqrt{2} \cdot N_f \cdot k_{wf})}$$

- Stator-side self-inductance due to air-gap field in *d*-axis:

$$L_{dh} = U_{i,s} / (2 \cdot \pi \cdot f \cdot I_s) = c_d \cdot \mu_0 \cdot (N_s k_{ws})^2 \cdot \frac{2 \cdot m_s}{\pi^2 \cdot p} \cdot \frac{\tau_p \cdot l_e}{\delta_0} = c_d \cdot L_h$$

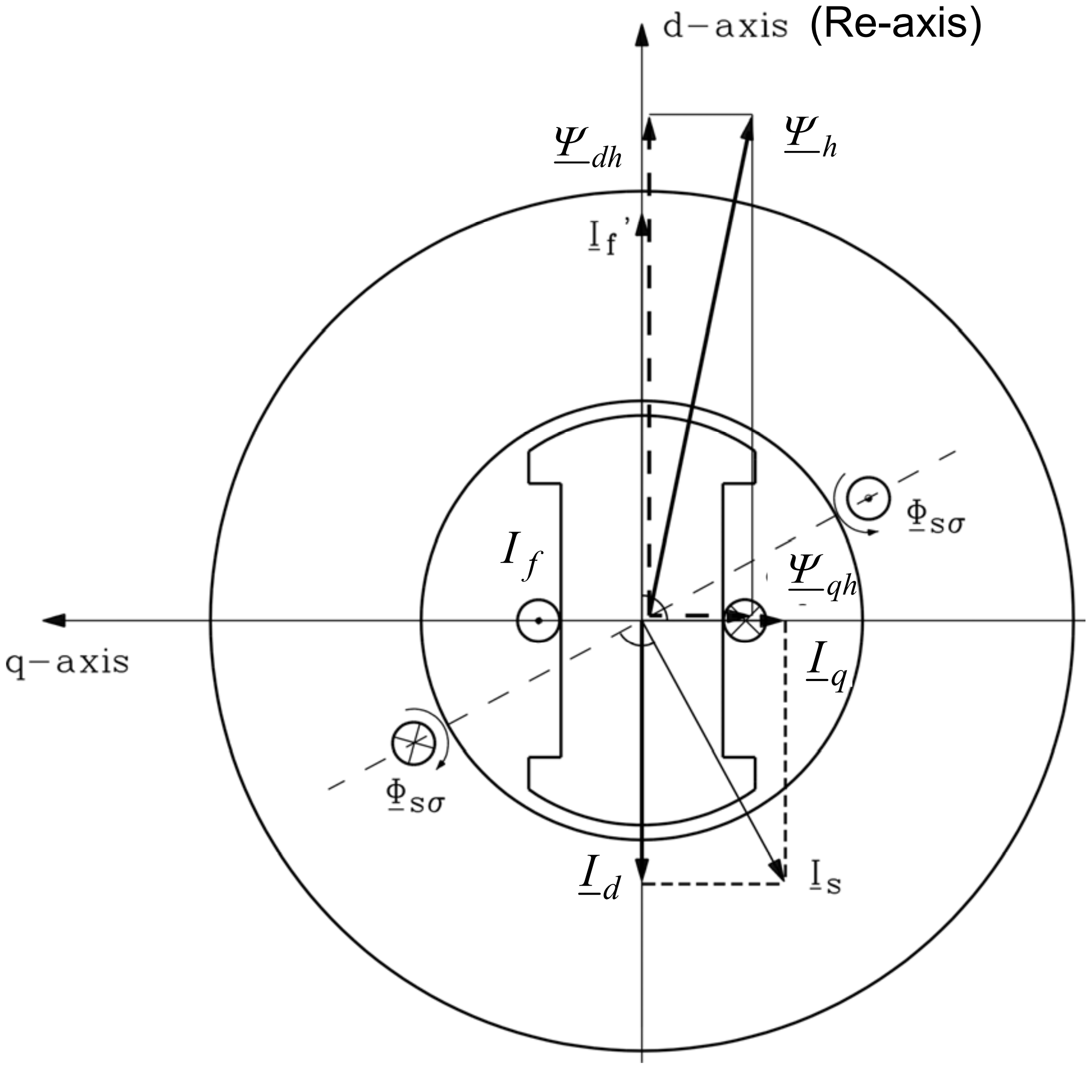
- Rotor-side self-inductance due to air-gap field in *d*-axis:

$$L_{f,h} = U_{i,f} / (2 \cdot \pi \cdot f \cdot I_f) = c_d \cdot \mu_0 \cdot (N_f k_{wf})^2 \cdot \frac{2 \cdot \sqrt{2}}{\pi^2 \cdot p} \cdot \frac{\tau_p \cdot l_e}{\delta_0}$$

$$\boxed{L_{dh} = \ddot{u}_{Uf} \cdot \ddot{u}_{If} \cdot L_{f,h}}$$

8. Dynamics of synchronous machines

Saliency: Stator current and flux linkage space vectors



Stator current space vector:

decomposed: $\underline{I}_s = \underline{I}_d + \underline{I}_q$

Rotor current space vector: $\underline{I}'_f$
(in *d*-axis!)

Magnetizing current space vector:

In *d*-axis: $I_{md} = I'_f + I_d$

In *q*-axis: $I_{mq} = I_q$

Flux linkage space vector: $\underline{\Psi}_h = \underline{\Psi}_{dh} + \underline{\Psi}_{qh}$

$$\Psi_{dh} = L_{dh} \cdot \sqrt{2} \cdot (I'_f + I_{sd})$$
$$\Psi_{qh} = L_{qh} \cdot \sqrt{2} I_{sq}$$

Saliency: Stator flux linkage equations in *d*- and *q*-axis

Stator flux linkage in per-unit:

$$\psi_d = (x_{s\sigma} + x_{dh}) \cdot i_d + x_{dh} i'_f = x_d \cdot i_d + x_{dh} i'_f$$

$$\psi_q = (x_{s\sigma} + x_{qh}) \cdot i_q = x_q i_q$$

Stator inductances:

Synchronous inductance of *d*-axis x_d and of *q*-axis x_q:

$$x_d = x_{dh} + x_{s\sigma} \qquad x_q = x_{qh} + x_{s\sigma} \qquad x_d > x_q$$

We assume constant iron saturation, so x_{dh}, x_{qh} are constant.

By calculating the magnetizing current in d- and q-component, the resulting variable iron saturation due to $\underline{I}_m = \underline{I}_{md} + \underline{I}_{mq}$ *is usually not correctly considered!*

8. Dynamics of synchronous machines

Set of dynamic equations of salient rotor synchronous machine without damper cage

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rotor reference frame = *d*- and *q*-axis

$$u_d(\tau) = r_s \cdot i_d(\tau) + \frac{d\psi_d(\tau)}{d\tau} - \omega_m(\tau) \cdot \psi_q(\tau)$$

$$u_q(\tau) = r_s \cdot i_q(\tau) + \frac{d\psi_q(\tau)}{d\tau} + \omega_m(\tau) \cdot \psi_d(\tau)$$

$$\psi_d(\tau) = (x_{dh} + x_{s\sigma}) \cdot i_d(\tau) + x_{dh} \cdot i'_f(\tau)$$

$$\psi_q(\tau) = (x_{qh} + x_{s\sigma}) \cdot i_q(\tau)$$

$$\tau_J \cdot \frac{d\omega_m(\tau)}{d\tau} = i_q(\tau) \cdot \psi_d(\tau) - i_d(\tau) \cdot \psi_q(\tau) - m_s(\tau)$$

$$\underline{i}_s = i_d + j \cdot i_q \qquad \underline{u}_s = u_d + j \cdot u_q$$

$$\underline{\psi}_s = \psi_d + j \cdot \psi_q$$

Unknowns:

$$i_d(\tau), i_q(\tau), \psi_d(\tau), \psi_q(\tau), \omega_m(\tau)$$

Given:

$$u_d(\tau), u_q(\tau), i'_f(\tau), m_s(\tau)$$

Stator zero sequence voltage system: $u_{s0} = r_s \cdot i_{s0} + \frac{d\psi_{s0}}{d\tau}$

Separate zero-sequence flux linkage equation (not discussed here)

Summary:
Flux linkage in cylindrical and salient rotor synchronous machines

- Repetition of bachelor course contents
- Also with space vectors:
  One magnetizing stator inductance in cylindrical rotor machines
- In salient pole rotors:
  d-axis magnetizing stator inductance bigger than in *q*-axis
- So far: Influence of damper cage neglected

8. Dynamics of synchronous machines

Fluxes in salient pole synchronous machine with damper cage

a) Stator magnetic field component, excited in parallel with rotor ***d*-axis**
b) Stator magnetic field component, excited in parallel with rotor ***q*-axis**

Stray fluxes:

$\Phi_{s\sigma} \Leftrightarrow X_{s\sigma}$

$\Phi_{f\sigma} \Leftrightarrow X_{f\sigma}$

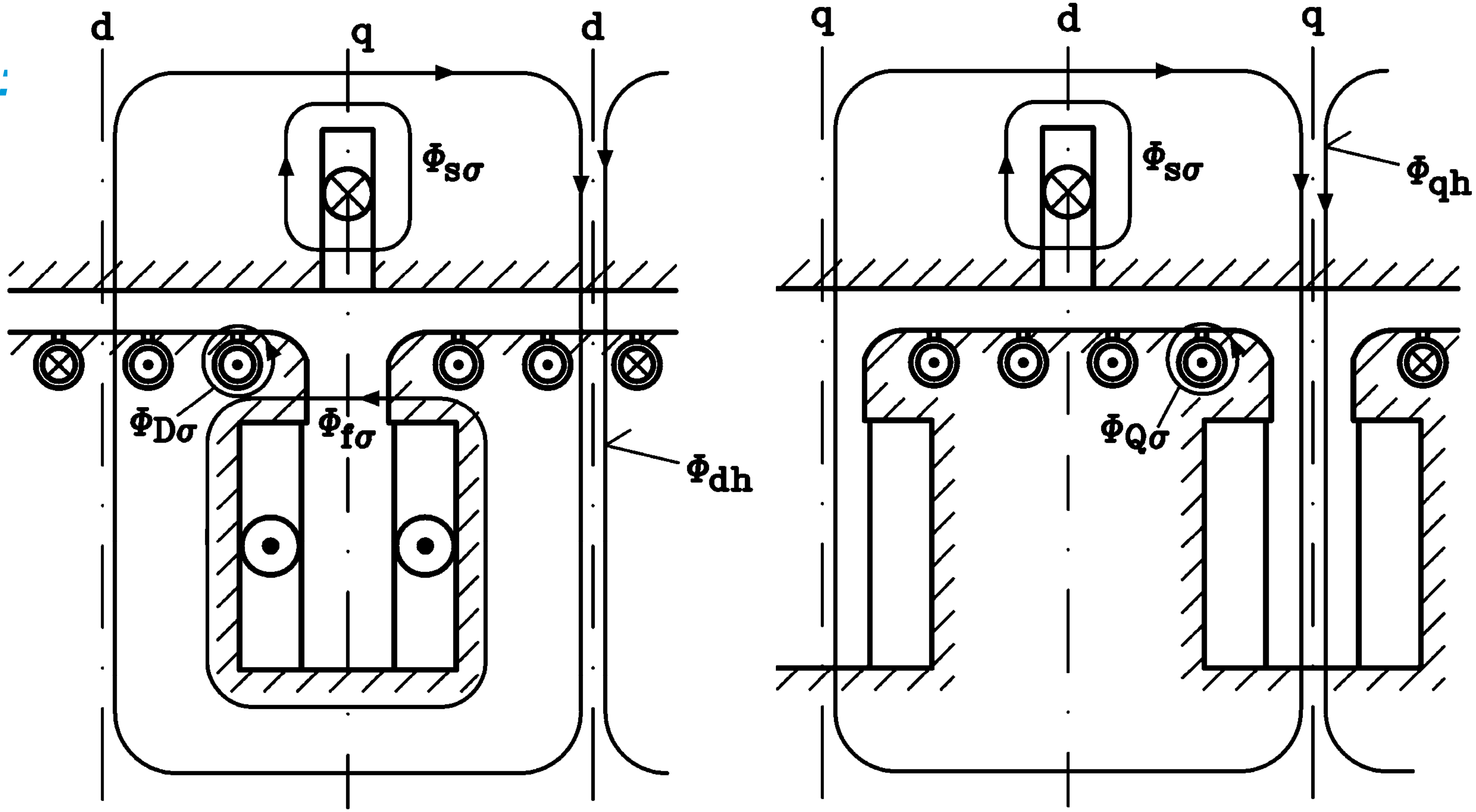
$\Phi_{D\sigma} \Leftrightarrow X_{D\sigma}$

$\Phi_{Q\sigma} \Leftrightarrow X_{Q\sigma}$

Main fluxes:

$\Phi_{dh} \Leftrightarrow X_{dh}$

$\Phi_{qh} \Leftrightarrow X_{qh}$

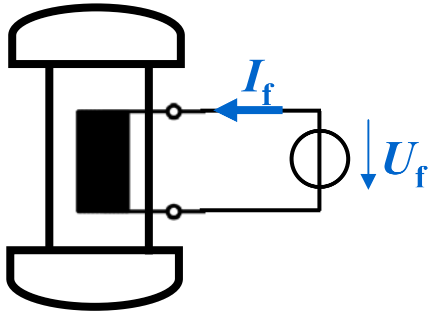


a) *d*-axis flux linkage **b) *q*-axis flux linkage**

Dynamic inductances and reactances

- Magnetic equivalent circuit of synchronous machine in dynamic state of operation:
  - "Transformer": Magnetizing inductance: *d*-axis L_{dh} , *q*-axis L_{qh}
  - Secondary leakage inductance:

    Excitation winding: $\Phi_{f\sigma}$, $X_{f\sigma} = \omega_N L_{f\sigma}$,
  - Damper cage:
    - *d*-axis $\Phi_{D\sigma}$: $X_{D\sigma} = \omega_N L_{D\sigma}$,
    - *q*-axis $\Phi_{Q\sigma}$: $X_{Q\sigma} = \omega_N L_{Q\sigma}$



- Damper bars are short circuited by end rings!
- Exciting DC voltage source of excitation winding has small internal resistance, which may be regarded for induced transient voltages and currents as an ideal voltage source = no internal impedance = field winding is AC short circuited.
- So field and damper secondary windings are (AC) short circuited!

8. Dynamics of synchronous machines

Dynamic voltage equations for synchronous machines

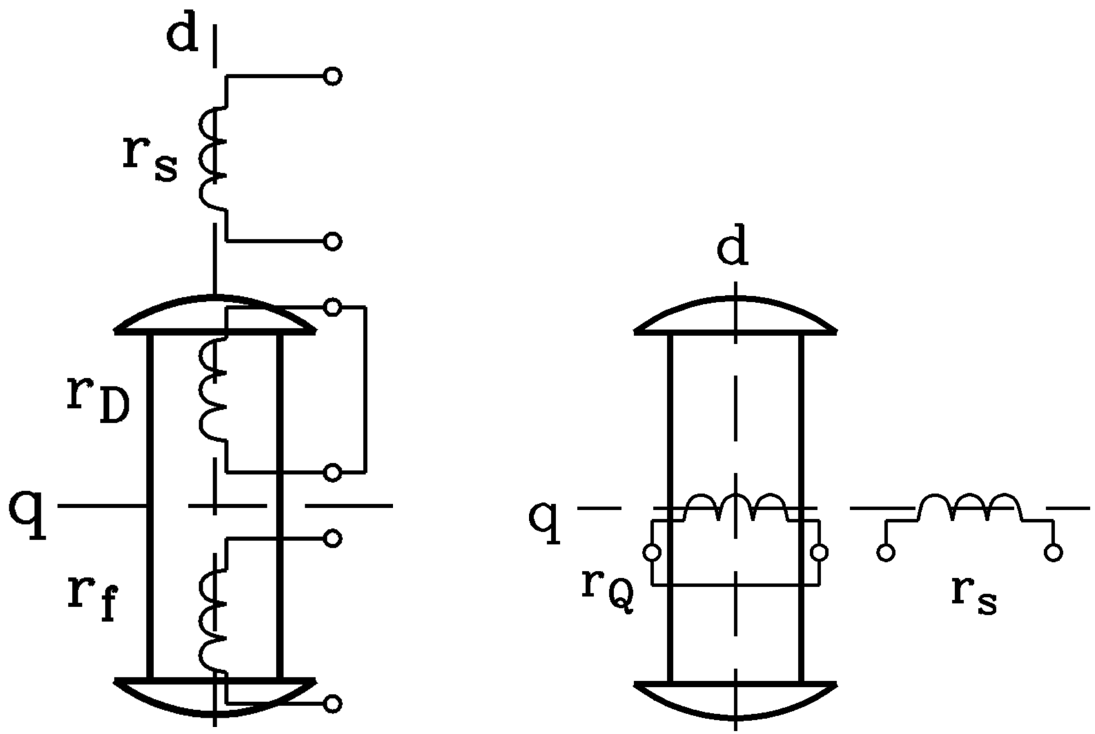
$$u_d = r_s \cdot i_d + \frac{d\psi_d}{d\tau} - \omega_m \cdot \psi_q$$

$$u_q = r_s \cdot i_q + \frac{d\psi_q}{d\tau} + \omega_m \cdot \psi_d$$

$$0 = r_D \cdot i_D + \frac{d\psi_D}{d\tau}$$

$$0 = r_Q \cdot i_Q + \frac{d\psi_Q}{d\tau}$$

$$u_f = r_f \cdot i_f + \frac{d\psi_f}{d\tau}$$



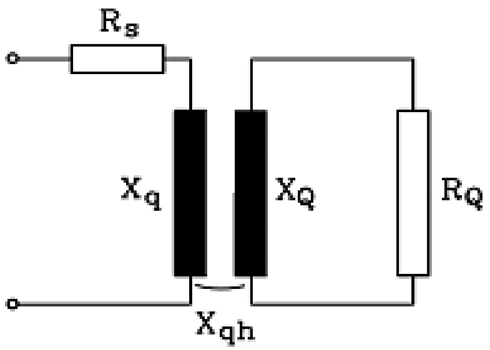
d, q: Stator voltage equations

D, Q: Damper voltage equations

f : Field voltage equation

Equations are valid in <u>rotor reference</u> frame !

q-axis transformer equivalent circuit of synchronous machine



Magnetic flux linkage in *q*-axis of synchronous machine:

2-Winding-transformer:

Stator winding with damper cage = like induction machine

Voltage and current transfer ratios for field and damper winding different!

$$\Psi_q = L_q I_q + M_{sQ} I_Q$$
$$\Psi_Q = M_{Qs} I_q + L_Q I_Q$$

$$\Psi_q = L_q I_q + \ddot{u}_{IQ} M_{sQ} \cdot (I_Q / \ddot{u}_{IQ})$$
$$\ddot{u}_{UQ} \Psi_Q = \ddot{u}_{UQ} M_{Qs} I_q + \ddot{u}_{UQ} \ddot{u}_{IQ} L_Q \cdot (I_Q / \ddot{u}_{IQ})$$

$$\Psi_q = (L_{s\sigma} + L_{qh}) \cdot I_q + L_{qh} \cdot I'_Q$$
$$\Psi'_Q = L_{qh} I_q + (L'_{Q\sigma} + L_{qh}) \cdot I'_Q$$

$$\Psi_q = L_{s\sigma} I_q + L_{qh} \cdot (I_q + I'_Q)$$
$$\Psi'_Q = L'_{Q\sigma} I'_Q + L_{qh} \cdot (I_q + I'_Q)$$

$$\ddot{u}_{UQ} \neq \ddot{u}_{IQ} \qquad \begin{cases} \ddot{u}_{UQ} = \dfrac{N_s k_{ws}}{N_r k_{wr}} = \dfrac{N_s k_{ws}}{(1/2)\cdot 1} \\ \ddot{u}_{IQ} = \ddot{u}_{UQ} \cdot \dfrac{m_s}{m_r} = \ddot{u}_{UQ} \cdot \dfrac{3}{Q_r} \end{cases}$$

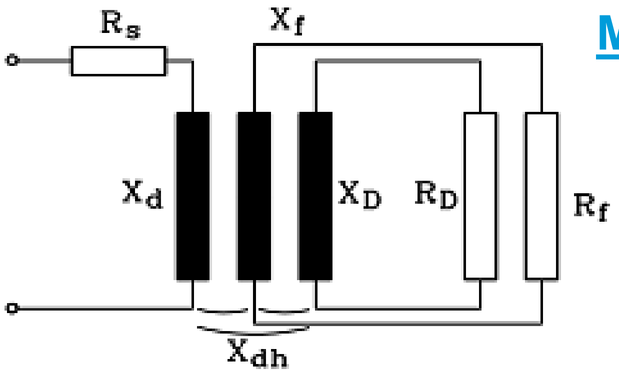
$$R'_Q = \ddot{u}_{UQ} \ddot{u}_{IQ} R_Q$$
$$L'_{Q\sigma} = \ddot{u}_{UQ} \ddot{u}_{IQ} L_{Q\sigma}$$
$$L_Q = L_{Q\sigma} + L_{Qh}$$
$$L_{qh} = \ddot{u}_{IQ} M_{sQ} = \ddot{u}_{UQ} M_{Qs} = \ddot{u}_{UQ} \ddot{u}_{IQ} L_{Qh}$$

We skip the notation ´ in the following for transferred values!

$$\Psi_q = L_{s\sigma} I_q + L_{qh} \cdot (I_q + I_Q)$$
$$\Psi_Q = L_{Q\sigma} I_Q + L_{qh} \cdot (I_q + I_Q)$$

8. Dynamics of synchronous machines

d-axis transformer equivalent circuit of synchronous machine



Magnetic flux linkage in *d*-axis of synchronous machine:

3-Winding-transformer: El. excited rotor with damper cage

Simplification: Bundled flux lines:
Main flux is identical for stator, field and damper winding

Voltage & current transfer ratios for field & damper winding different: $\ddot{u}_{Uf} \neq \ddot{u}_{If} \neq \ddot{u}_{UD} \neq \ddot{u}_{ID}$ $\quad \ddot{u}_{UQ} = \ddot{u}_{UD}, \ddot{u}_{IQ} = \ddot{u}_{ID}$

$$\Psi_d = L_d I_d + M_{sf} I_f + M_{sD} I_D$$
$$\Psi_f = M_{fs} I_d + L_f I_f + M_{fD} I_D$$
$$\Psi_D = M_{Ds} I_d + M_{Df} I_f + L_D I_D$$

$$L_{dh} = \ddot{u}_{ID} M_{sD} = \ddot{u}_{UD} M_{Ds} = \ddot{u}_{If} M_{sf} = \ddot{u}_{Uf} M_{fs}$$
$$L_{dh} = \ddot{u}_{UD} \ddot{u}_{If} M_{Df} = \ddot{u}_{Uf} \ddot{u}_{ID} M_{fD} = \ddot{u}_{Uf} \ddot{u}_{If} L_{fh} = \ddot{u}_{UD} \ddot{u}_{ID} L_{Dh}$$

$$R'_f = \ddot{u}_{Uf} \ddot{u}_{If} R_f \qquad R'_D = \ddot{u}_{UD} \ddot{u}_{ID} R_D$$
$$L'_{f\sigma} = \ddot{u}_{Uf} \ddot{u}_{If} L_{f\sigma} \qquad L'_{D\sigma} = \ddot{u}_{UD} \ddot{u}_{ID} L_{D\sigma}$$
$$L_f = L_{f\sigma} + L_{fh}$$
$$L_D = L_{D\sigma} + L_{Dh}$$

$$\Psi_d = L_d I_d + \ddot{u}_{If} M_{sf} \cdot (I_f / \ddot{u}_{If}) + \ddot{u}_{ID} M_{sD} \cdot (I_D / \ddot{u}_{ID})$$
$$\ddot{u}_{Uf} \Psi_f = \ddot{u}_{Uf} M_{fs} I_d + \ddot{u}_{Uf} \ddot{u}_{If} L_f \cdot (I_f / \ddot{u}_{If}) + \ddot{u}_{Uf} \ddot{u}_{ID} M_{fD} \cdot (I_D / \ddot{u}_{ID})$$
$$\ddot{u}_{UD} \Psi_D = \ddot{u}_{UD} M_{Ds} I_d + \ddot{u}_{UD} \ddot{u}_{If} M_{Df} \cdot (I_f / \ddot{u}_{If}) + \ddot{u}_{UD} \ddot{u}_{ID} L_D \cdot (I_D / \ddot{u}_{ID})$$

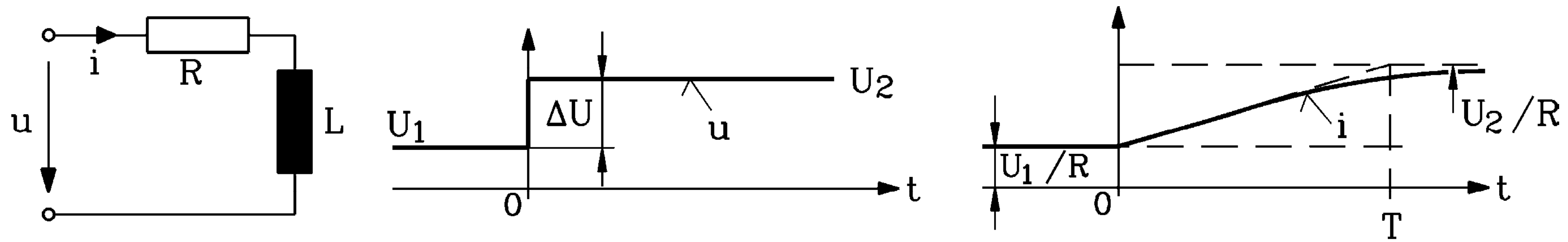
$$\Psi_d = (L_{d\sigma} + L_{dh}) \cdot I_d + L_{dh} I'_f + L_{dh} I'_D$$
$$\Psi'_f = L_{dh} I_d + (L'_{f\sigma} + L_{dh}) \cdot I'_f + L_{dh} I'_D$$
$$\Psi'_D = L_{dh} I_d + L_{dh} I'_f + (L'_{D\sigma} + L_{dh}) \cdot I'_D$$

Skip notation ´ :

$$\Psi_d = L_{s\sigma} I_d + L_{dh} \cdot (I_d + I_f + I_D)$$
$$\Psi_f = L_{f\sigma} I_f + L_{dh} \cdot (I_d + I_f + I_D)$$
$$\Psi_D = L_{D\sigma} I_D + L_{dh} \cdot (I_d + I_f + I_D)$$

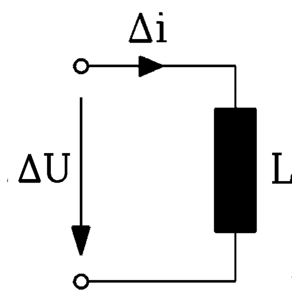
8. Dynamics of synchronous machines

Understanding transients

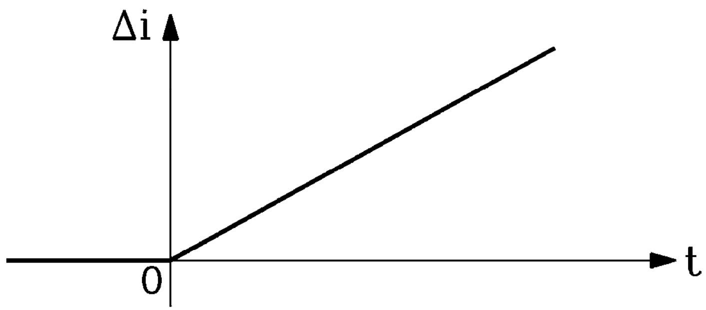


Example: Rotor DC field winding: Voltage step

If only transient change of current **for very short time *t* << *T* after disturbance** is of interest, only the **resulting inductance for dynamic condition must be known!**



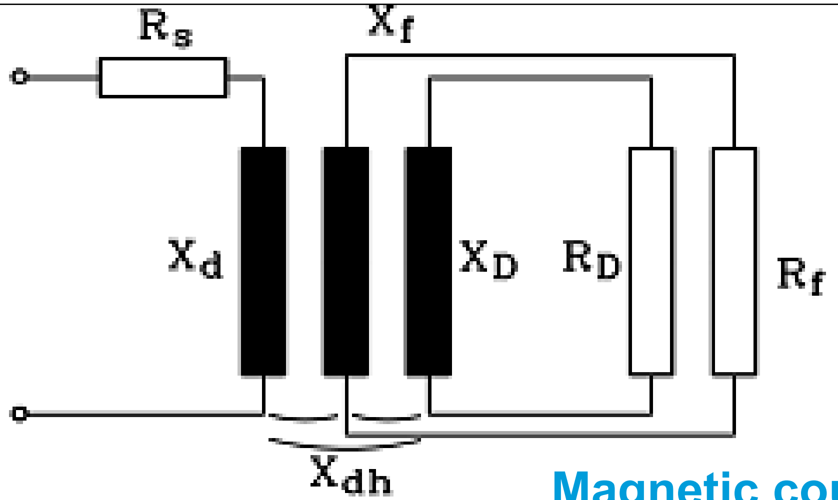
$T = L/R$

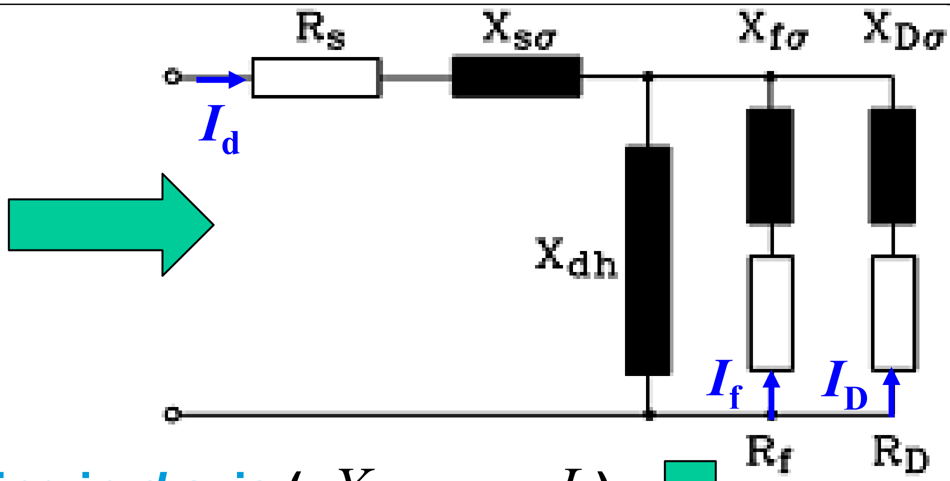


$$i(t) = \frac{U_1}{R} + \frac{\Delta U}{R} \cdot \left(1 - e^{-t/T}\right) \longrightarrow \Delta i(t) = i(t) - \left.\frac{U_1}{R}\right|_{t<<T} \approx \frac{\Delta U}{L} \cdot t$$

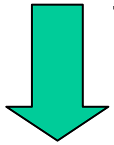
$$e^{-t/T} \approx 1 - \frac{t}{T}, \; t << T$$

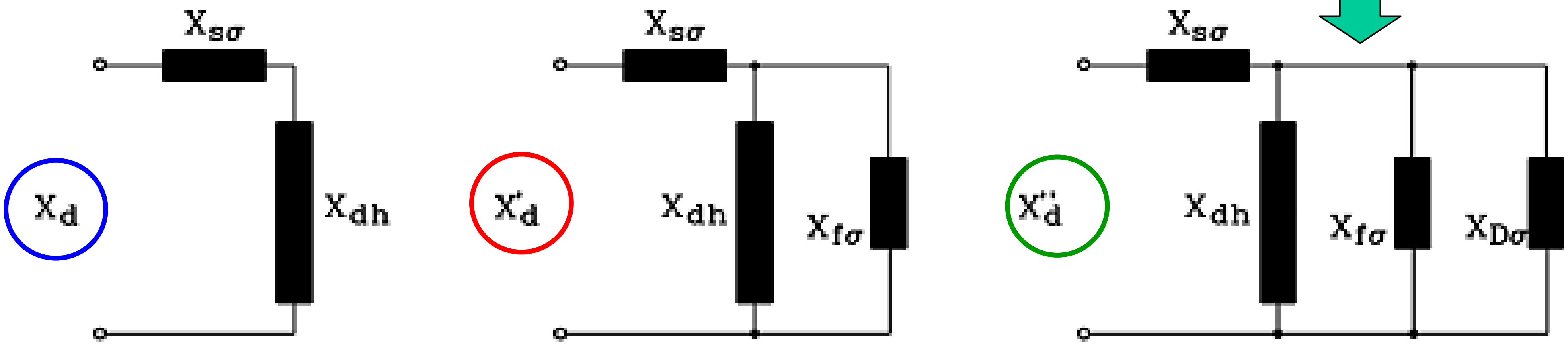
8. Dynamics of synchronous machines





Magnetic coupling in *d*-axis ($X = \omega_N \cdot L$)



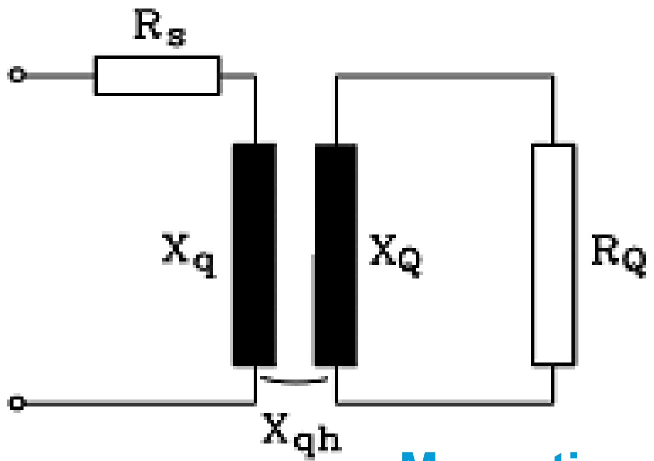


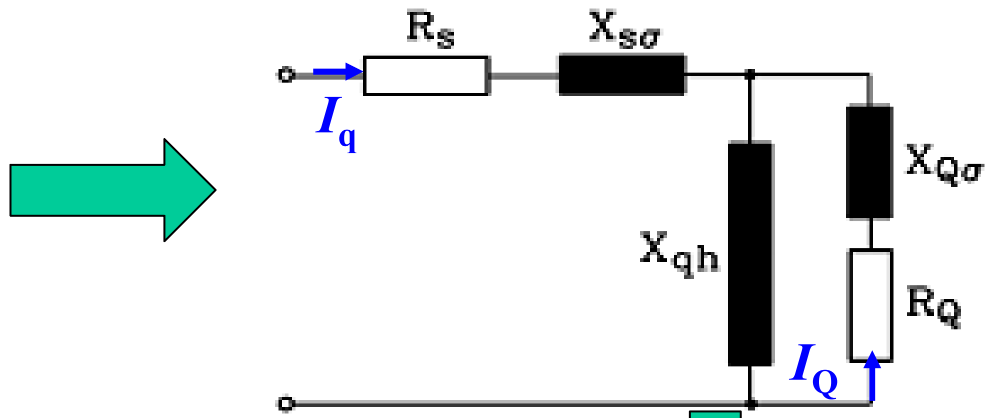
No damper cage, no field winding: synchronous *d*-reactance X_d

No damper cage: transient reactance X'_d

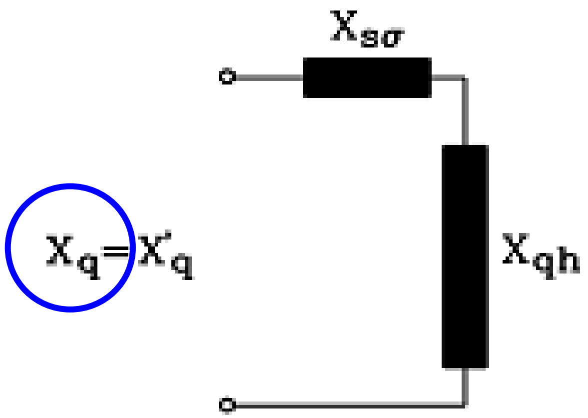
Damper cage and field winding: subtransient *d*-reactance X''_d

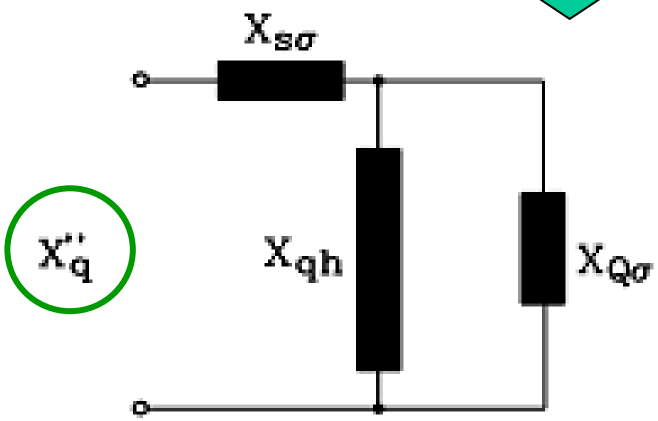
8. Dynamics of synchronous machines





Magnetic coupling in *q*-axis ($X = \omega_N \cdot L$)





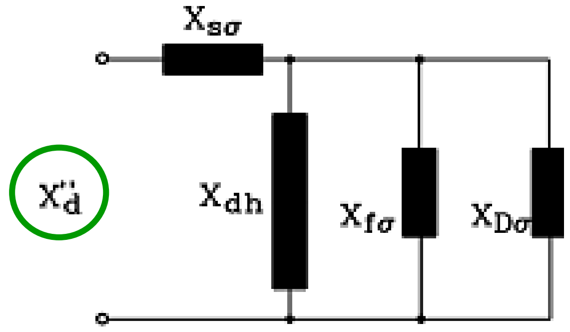
No damper cage, no field winding:
synchronous *q*-reactance X_q

Damper cage:
subtransient *q*-reactance X''_q

Subtransient inductances and reactances

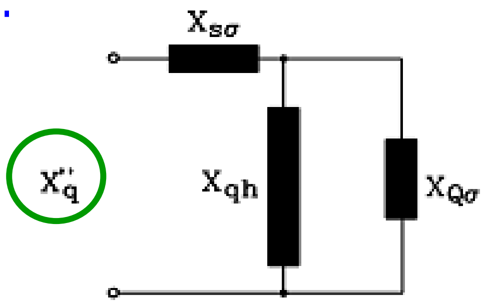
- **„Subtransient reactance" of *d*-axis**
  (secondary resistances neglected):

$$X''_d = \omega_N L''_d = \omega_N \cdot \left(L_{s\sigma} + \frac{L_{dh} L_{f\sigma} L_{D\sigma}}{L_{dh} L_{f\sigma} + L_{dh} L_{D\sigma} + L_{f\sigma} L_{D\sigma}}\right)$$



- **"Subtransient reactance of *q*-axis"** :

$$X''_q = \omega_N L''_q = \omega_N \cdot \left(L_{s\sigma} + \frac{L_{qh} L_{Q\sigma}}{L_{qh} + L_{Q\sigma}}\right)$$



$x_d = 1 > x_q = 0.6$, but :

$x''_d = 0.15 < x''_q = 0.18$

$x''_d \approx x''_q$: subtransient symmetry

Example: Salient poles:

$$X_\mathrm{d} / Z_\mathrm{N} = x_\mathrm{d} = 1\,\mathrm{p.u.},\ X_\mathrm{q} / Z_\mathrm{N} = x_\mathrm{q} = 0.6\,\mathrm{p.u.},\ X_{\mathrm{s}\sigma} = X_{\mathrm{f}\sigma} = X_{\mathrm{D}\sigma} = X_{\mathrm{Q}\sigma} = 0.1 \cdot Z_\mathrm{N} :$$

$$X''_d / Z_N = x''_d = 0.1 + \frac{0.9 \cdot 0.1 \cdot 0.1}{0.9 \cdot 0.1 + 0.9 \cdot 0.1 + 0.1 \cdot 0.1} = \underline{\underline{0.15}} \qquad X''_q / Z_N = x''_q = 0.1 + \frac{0.5 \cdot 0.1}{0.5 + 0.1} = \underline{\underline{0.18}}$$

8. Dynamics of synchronous machines

Subtransient performance of synchronous machine

- During subtransient state of synchronous machine ***not*** the ***synchronous reactances*** X_d, X_q are active, but the much smaller ***subtransient reactances***: X''_d, X''_q

- Note: Even if $X_d > X_q$ like in salient pole machines, the subtransient reactances are nearly the SAME: $X''_d < X''_q$, but $X''_d \approx X''_q$

- Winding resistances R_s, R_f, R_D and R_Q cause a **decay** of dynamic currents, which flow due to induced transient voltages in the stator and rotor windings

- In the damper cage the dynamic current in the damper bars decays much faster than in the field winding.

- **Typical time constant of decay:**

  in damper winding **20 ... 50 ms,**

  in excitation winding **0.5 s ... 2 s.**

8. Dynamics of synchronous machines

Example: Calculation of r_s, r_f, r_D (1)

Two-pole turbine generator (= cylindrical rotor) with complete damper cage:
Copper wedges of rotor field winding slots are damper bars

$$S_N = 125\,\text{MVA}, U_N = 10.5\,\text{kV Y}, I_N = 6880\,\text{A}, f_N = 50\,\text{Hz}, d_{ra} = 920\,\text{mm}, l_{Fe,r} = 2.9\,\text{m}$$

$$Z_N = U_{Nph} / I_{Nph} = 0.88\,\Omega$$

Stator winding resistance per phase:

$$R_{s,75°C} = 1.56\,\text{m}\Omega, Q_s = 66, q_s = 11, m_s = 3, N_{sc} = 1, a_s = 2, 2p = 2 \Rightarrow N_s = 2p \cdot q_s \cdot N_{sc} / a_s = 11$$

$$W / \tau_p = 27/33, k_{ps} = 0.9595, k_{ds} = 0.9553, k_{ws} = 0.9166$$

$$R_{s,AC} = k_{AC} \cdot R_{s,DC} = 1.33 \cdot 1.56 = 2.078\,\text{m}\Omega, \boxed{r_s = R_{s,AC} / Z_N = 0.00236\,\text{p.u.}}$$

Field winding resistance:

$$R_{f,75°C} = 94.2\,\text{m}\Omega, Q_f = 32, q_r = 8, m_f = 1, N_{fc} = 9, a_f = 1, \Rightarrow N_f = 2p \cdot q_r \cdot N_{fc} / a_f = 144$$

$$k_{pf} = \sqrt{3}/2, k_{df} = 0.9556, k_{wf} = 0.8276$$

$$\ddot{u}_{Uf} = \frac{k_{ws} N_s}{k_{wf} N_f} = \frac{0.9166 \cdot 11}{0.8276 \cdot 144} = 0.0846, \quad \ddot{u}_{If} = \ddot{u}_{Uf} \cdot \frac{m_s}{m_f \cdot \sqrt{2}} = 0.1795, \quad \ddot{u}_{If} \cdot \ddot{u}_{Uf} = 0.01518$$

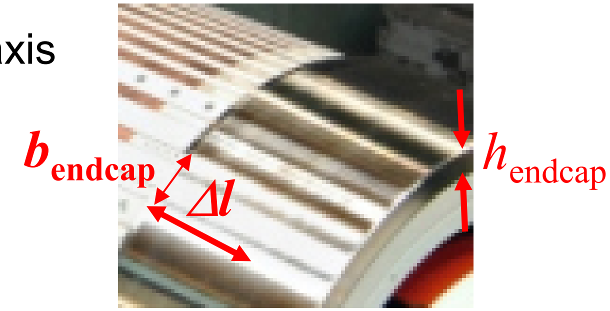
$$R'_f = \ddot{u}_{If} \cdot \ddot{u}_{Uf} \cdot R_f = 1.43\,\text{m}\Omega, \boxed{r_f = R'_f / Z_N = 0.00162\,\text{p.u.}}$$

8. Dynamics of synchronous machines

Example: Calculation of r_s, r_f, r_D (2)

Damper winding resistance:

Here: Ideal symmetrical damper cage = no difference between *d*- and *q*-axis



$$h_D = 20\text{ mm}, b_D = (32+49)/2 = 40.5\text{ mm},$$

$$l_{bar} = l_{Fe,r} + 2\cdot\Delta l = 2.9 + 2\cdot 0.1 = 3.1\text{ m}$$

$$R_{bar,75°C} = l_{bar}/(h_D b_D \kappa_{Cu}) = 3.1/(0.02\cdot 0.0405\cdot 44\cdot 10^6) = 87\,\mu\Omega$$

$$Q_r = 48 = m_r = m_D,\ N_r = 1/2 = N_D,\ k_{wr} = 1 = k_{wD}$$

$$h_{endcap} = 35\text{ mm}, \tau_{Qr} = d_{ra}\pi/Q_r = 60.2\text{ mm}, \Delta b_{endcap} \approx \tau_{Qr}/2$$

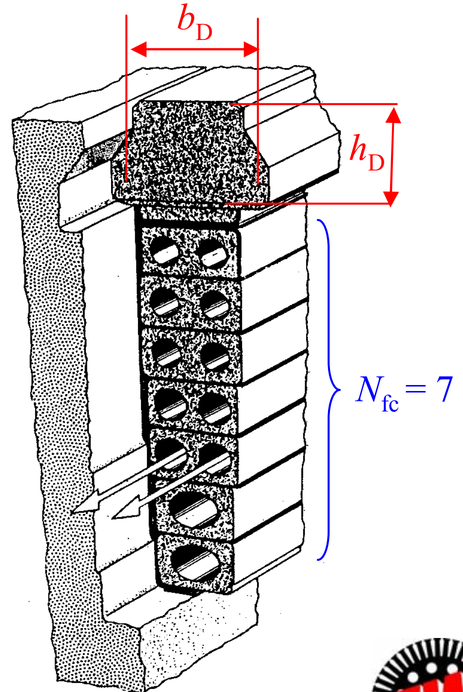
$$\Delta R_{endcap,75°C} = \frac{\Delta b_{endcap}}{\Delta l\cdot h_{endcap}\cdot\kappa_{stainless\ steel}} = \frac{0.0301}{0.1\cdot 0.035\cdot 1.4\cdot 10^6} = 6.15\,\mu\Omega$$

$$\Delta R^*_{endcap,75°C} = \frac{\Delta R_{endcap,75°C}}{2\cdot(\sin(\pi p/Q_r))^2} = \frac{6.15}{2\cdot(\sin(\pi/48))^2} = 718\,\mu\Omega$$

$$R_D = R_{bar,75\,°C} + \Delta R^*_{endcap,75\,°C} = 87 + 718 = 805\,\mu\Omega$$

$$\ddot{u}_{UD} = \frac{k_{ws}N_s}{k_{wD}N_D} = \frac{0.9166\cdot 11}{1\cdot 0.5} = 20.165,\quad \ddot{u}_{ID} = \ddot{u}_{UD}\cdot\frac{m_s}{m_D} = 1.26,\quad \ddot{u}_{ID}\cdot\ddot{u}_{UD} = 25.414$$

$$R'_D = \ddot{u}_{ID}\cdot\ddot{u}_{UD}\cdot R_D = 18.247\text{ m}\Omega,\ \boxed{r_D = R'_D/Z_N = 0.020736\text{ p.u.}}$$



4-pole Turbine generator: Rotor insertion in stator

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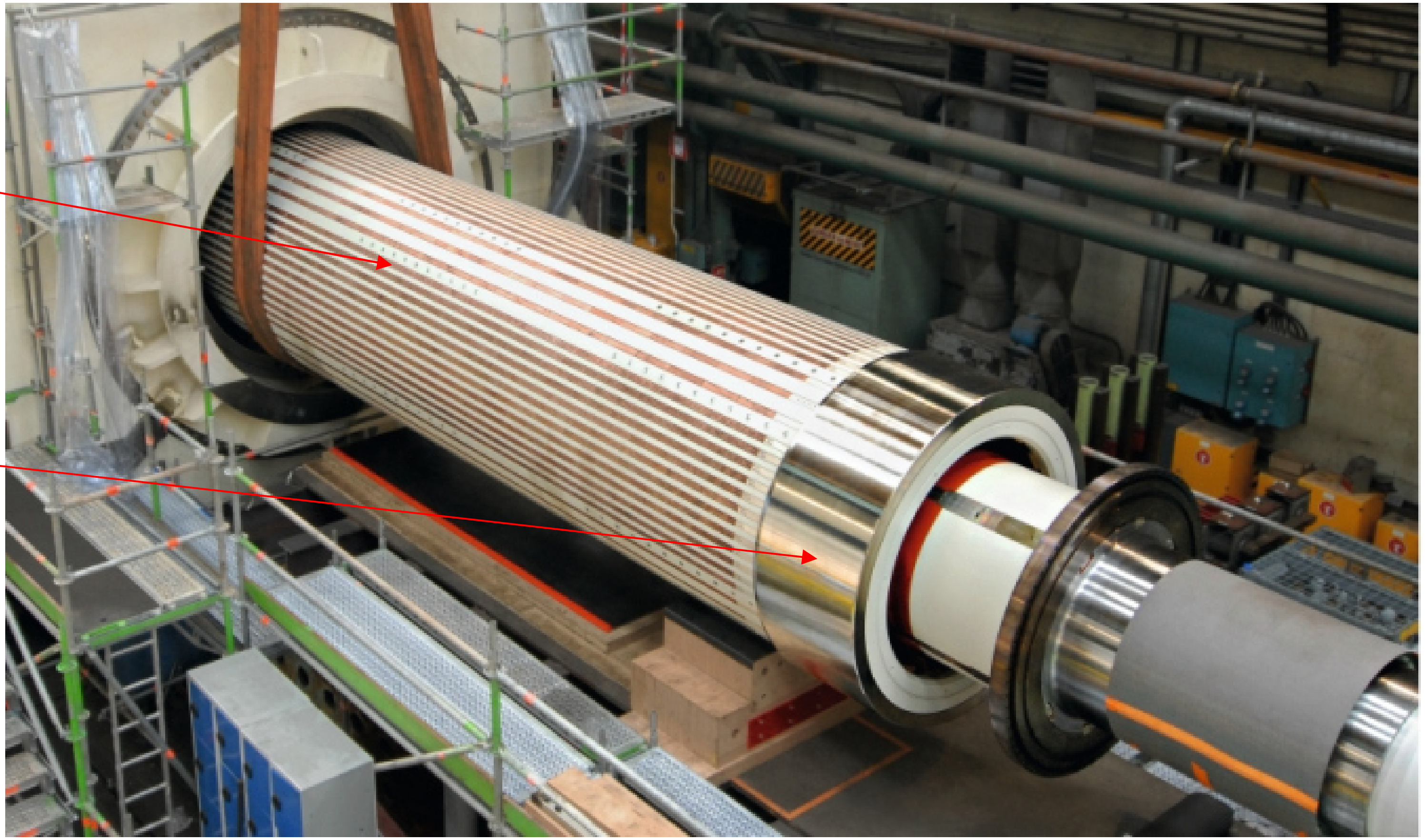
Copper wedges as complete damper cage

Stainless steel end-caps as retaining rings



Source: ALSTOM

8. Dynamics of synchronous machines

Example: Calculation of r_s, r_f, r_D (3)

Stator / field / damper winding resistance in physical units:

$$R_s = 2.078\ \text{m}\Omega,\ R_f = 94.2\ \text{m}\Omega,\ R_D = 805\mu\Omega$$

Per-unit stator / field / damper winding resistance, with respect to stator-side:

$$r_s = 0.00237,\ r_f = 0.00162,\ r_D = 0.02074$$

- In physical units the field winding has the biggest ohmic value
- The damper bar has the smallest ohmic value, although the retaining stainless steel cap (as "end-ring") increases the resistance strongly.
- In p.u. the damper resistance is the biggest value, the field winding resistance is the smallest.
- **As the p.u. inductance values x_s, x_f, x_D (with respect to the stator side) are similar, the time constant for the separated windings**

$$\tau_f = x_f / r_f > \tau_s = x_s / r_s > \tau_D = x_D / r_D$$

is for the damper the shortest ⇒ The dynamic damper currents decay fastest!

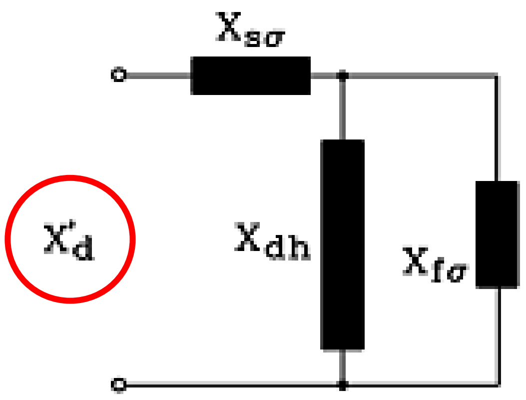
- *Example:* $x_f = x_D = 1.8$ p.u.: $\tau_f = 1111$ p.u. (3.5 s), $\tau_D = 87$ p.u. (277 ms)

8. Dynamics of synchronous machines

„Transient“ state of synchronous machine

- Intermediate state in *d*-axis: Dynamic currents in the damper cage have already decayed, whereas in the field winding still a big dynamic field current is flowing.
- Equivalent reactance for this “transient”: **“Transient reactance” of *d*-axis X_d'!**
  Is only slightly bigger than subtransient reactance !
- After decay of all dynamic currents the stator reactance becomes again X_d resp. X_q

$$X_d' = \omega_N L_d' = \omega_N \cdot (L_{s\sigma} + \frac{L_{dh} L_{f\sigma}}{L_{dh} + L_{f\sigma}})$$



Example: $x_{dh} = 0.9,\ x_{s\sigma} = x_{f\sigma} = 0.1:\ X_d' / Z_N = x_d' = \underline{\underline{0.19}}$

$$x_d' = 0.1 + \frac{0.9 \cdot 0.1}{0.9 + 0.1} = \underline{\underline{0.19}}$$

$$x_d'' = 0.15 < x_d' = 0.19$$

8. Dynamics of synchronous machines

Subtransient and transient state

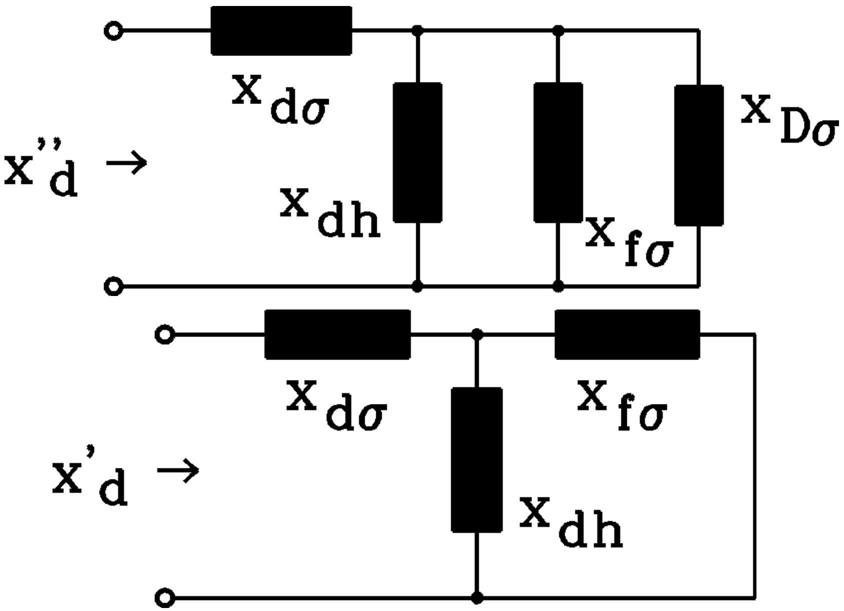
- Time constant of damper winding for decay of transient damper current is <u>much shorter</u> (factor 10) than time constant of field winding for decay of transient field current
- So we can distinguish "transient" and "sub-transient" state

Subtransient time section **0 .... ≈ 0.5 s**	**<u>Dynamic</u> current flow in stator, damper and field winding**
Transient time section **0.5 s .... ≈ 2 s**	**<u>Dynamic</u> current flow in stator and field winding**
Steady state (synchronous) **> 2 ... 3 s**	**<u>Steady state</u> current flow in stator and field winding**

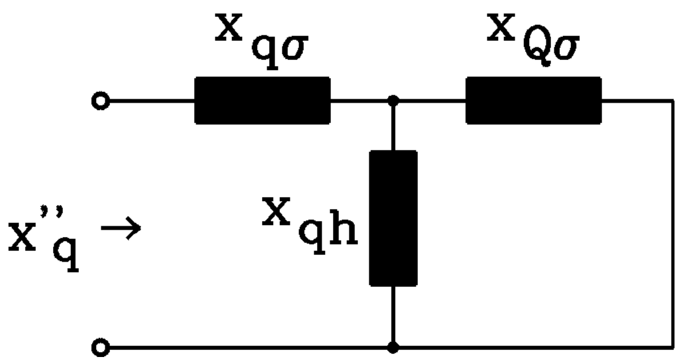
Stator inductance per phase for steady state and dynamics (1)

	Steady state	Transient	Sub-transient
Direct axis	Synchronous inductance L_d , x_d	Transient inductance L'_d , x'_d	Subtransient inductance L''_d , x''_d
Quadrature axis	Synchronous inductance L_q , x_q	Synchronous inductance L_q , x_q	Subtransient inductance L''_q , x''_q

d-axis



q-axis



$$x'_\mathrm{q} = x_\mathrm{q}$$

Stator inductance per phase for steady state and dynamics (2)

Subtransient inductance of *d*-axis: (physical units: *L*, per unit value: *x*)

$$L''_d = L_{s\sigma} + \frac{L_{dh}L_{f\sigma}L_{D\sigma}}{L_{dh}L_{f\sigma} + L_{dh}L_{D\sigma} + L_{f\sigma}L_{D\sigma}} \qquad x''_d = x_{s\sigma} + \frac{x_{dh}x_{f\sigma}x_{D\sigma}}{x_{dh}x_{f\sigma} + x_{dh}x_{D\sigma} + x_{f\sigma}x_{D\sigma}}$$

Subtransient inductance of *q*-axis:

$$L''_q = L_{s\sigma} + \frac{L_{qh}L_{Q\sigma}}{L_{qh} + L_{Q\sigma}} \qquad x''_q = x_{s\sigma} + \frac{x_{qh}x_{Q\sigma}}{x_{qh} + x_{Q\sigma}}$$

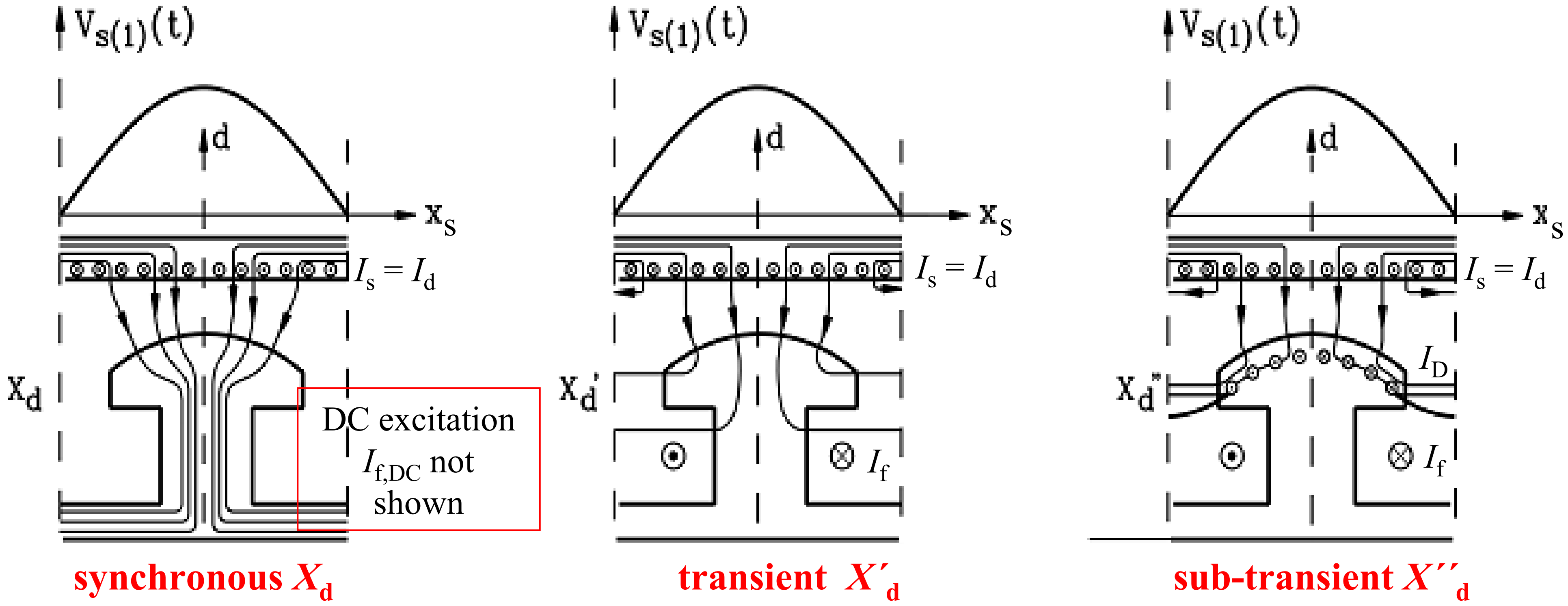
Transient inductance of *d*-axis:

$$L'_d = L_{s\sigma} + \frac{L_{dh}L_{f\sigma}}{L_{dh} + L_{f\sigma}} \qquad x'_d = x_{s\sigma} + \frac{x_{dh}x_{f\sigma}}{x_{dh} + x_{f\sigma}}$$

Transient inductance of *q*-axis:

$$L'_q = L_q \qquad x'_q = x_q$$

d-axis: Field lines for dynamic and steady state

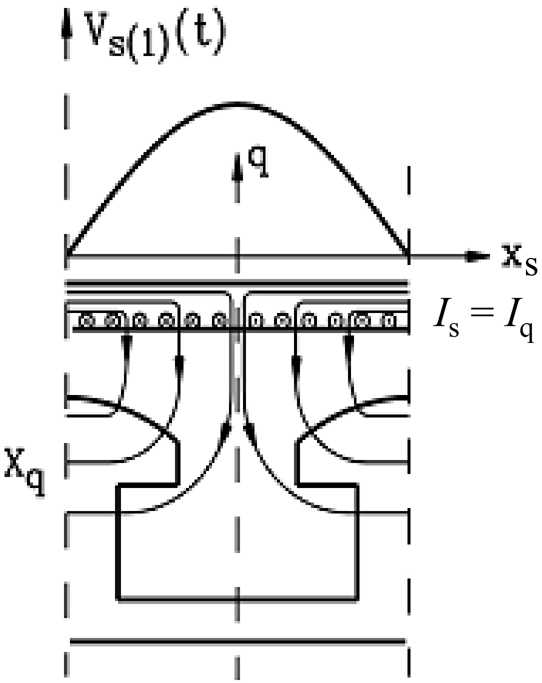


Sinusoidal distributed (= fundamental) stator m.m.f. $V_{s(1)}(x)$

- synchronous: No reaction of rotor windings
- transient: Reaction of rotor field winding
- sub-transient: Reaction of rotor field and damper winding

Source: K. Bonfert, Springer-Verlag

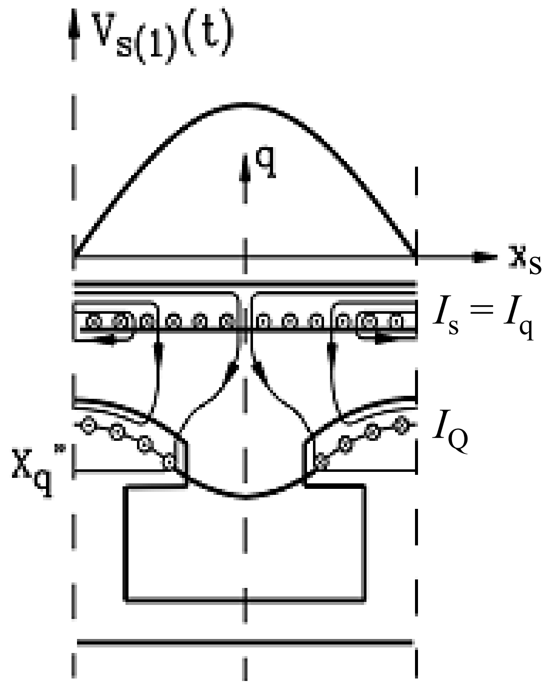
q-axis: Field lines for dynamic and steady state



synchronous X_q

DC excitation
$I_{f,DC}$ not shown

NO transient ⇒ $X'_q = X_q$

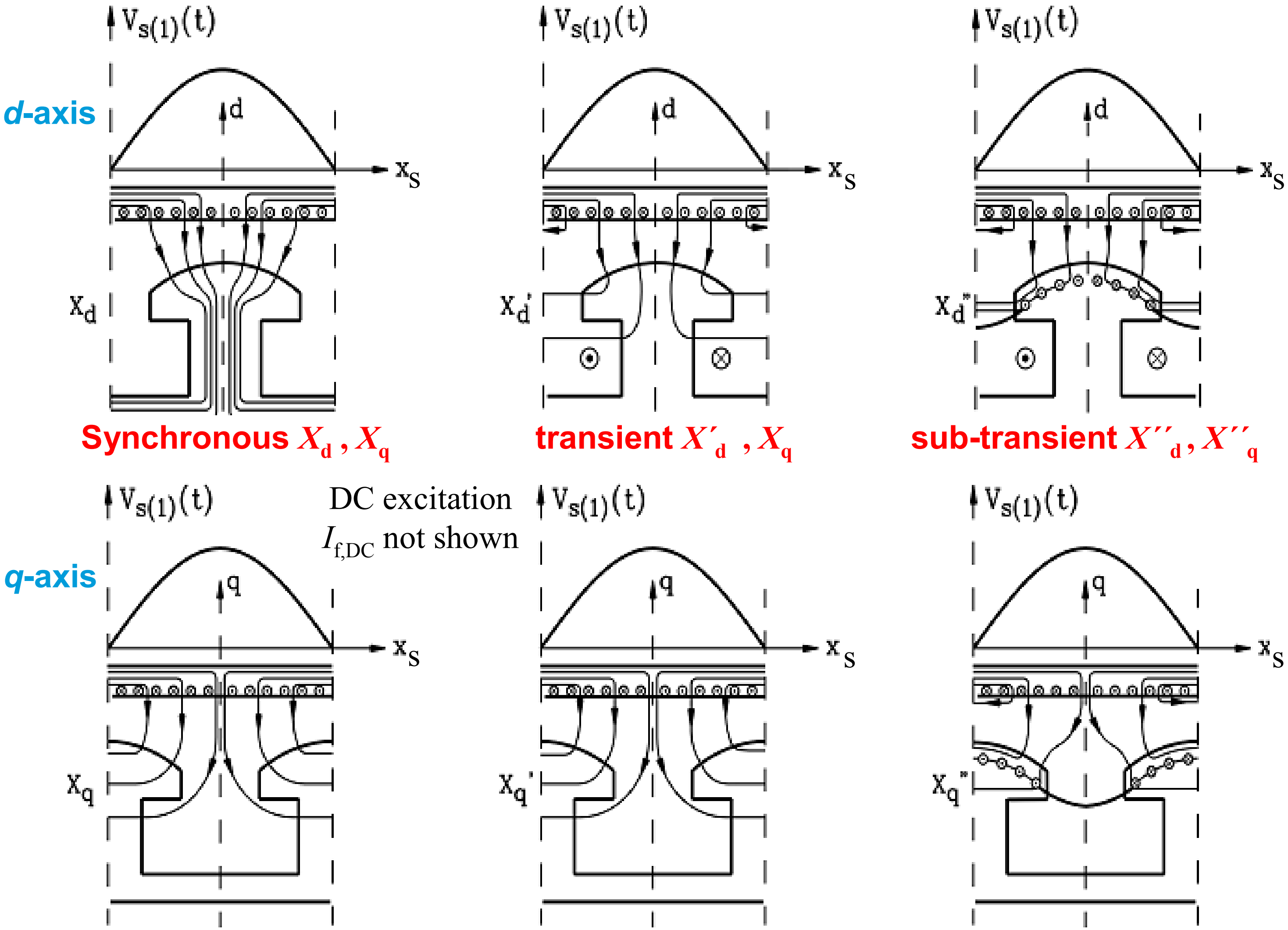


sub-transient X''_q

- synchronous: No reaction of rotor windings
- NO transient state, because no linkage with rotor field winding
- sub-transient: Reaction of damper winding

Source:
K. Bonfert,
Springer-Verlag

8. Dynamics of synchronous machines



Field lines for dynamic and steady state

Sinusoidal distributed (= fundamental) stator m.m.f. $V_{s(1)}(x_s)$

- synchronous:
  No reaction of rotor windings
- transient:
  Reaction of rotor field winding
- sub-transient:
  Reaction of rotor field and damper winding

Source: K. Bonfert, Springer-Verlag

p.u. reactances of synchronous machines

Synchronous reactance of direct axis	$x_d = X_d / Z_N$ $X_d = \omega_N L_d$	0.8 ... 1.2: Salient pole synchronous machines with high pole count 1.2 ... 2.5: 4- and 2-pole cylindrical rotor synchronous machines with high utilization
synchronous reactance of quadrature axis	$x_q = X_q / Z_N$ $X_q = \omega_N L_q$	$(0.5...0.6) \cdot x_d$: Salient pole synchronous machines with high pole count $(0.8...0.9) \cdot x_d$: 2- and 4-pole cylindrical rotor synchronous machines
transient reactance of direct axis	$x'_d = X'_d / Z_N$ $X'_d = \omega_N L'_d$	0.2-0.25 ... 0.35-0.4
transient reactance of quadrature axis	$x'_q = x_q =$ $= X_q / Z_N$	Identical with synchronous reactance
subtransient reactance of direct axis	$x''_d = X''_d / Z_N$ $X''_d = \omega_N L''_d$	0.1-0.12 ... 0.2-0.3
subtransient reactance of quadrature axis	$x''_q = X''_q / Z_N$ $X''_q = \omega_N L''_q$	usually $x''_q > x''_d$, as field winding is missing in *q*-axis: 0.1 ... 0.3, but $x''_q \approx x''_d$

Summary (1):
Dynamic flux linkages of synchronous machines

- *d*- and *q*-axes separation of flux linkages in synchronous machines
- Rotor cage described only by two short circuited coils in d- and q-axes each: D and Q
- Electrically excited synchronous machines:
    - Three-winding transformer in *d*-axis
    - Two-winding transformer in *q*-axis
- Sudden change in flux linkage leads to transient rotor currents in damper and field winding
- Subtransient *d*- and *q*-axis inductance (in p.u.: „reactance“)
- Transient *d*-axis inductance, when transient rotor current only in field winding

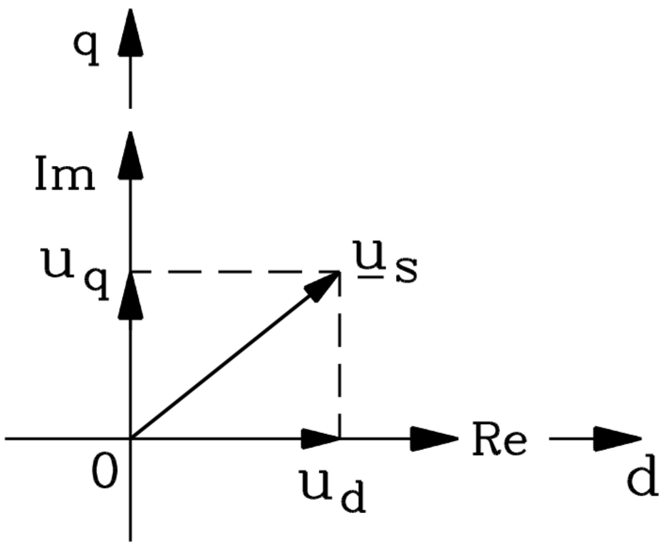
Summary (2):
Dynamic inductances in synchronous machines

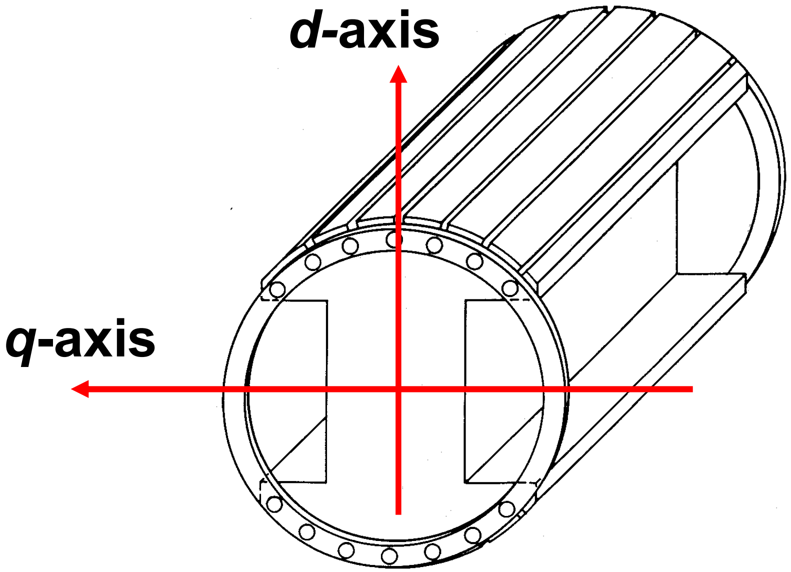
- Steady-state and dynamic inductances are mostly given in p.u.
- p.u.-values as inductances or reactances identical
- „p.u.-reactances“ is the standardized wording
- Subtransient reactances smaller than transient reactance
- Transient reactance smaller than steady-state reactance
- Sub-transient *q*-reactance slightly larger than subtransient *d*-reactance
- In subtransient reactances machines are nearly symmetrical, although at steady-state (esp. salient poles!) not!

Space vectors in rotor reference frame

Equations are formulated in rotor reference frame:

Real axis = *d*-axis, Imaginary axis = *q*-axis





In synchronous steady state condition ($\omega_s = \omega_m$) stator voltage / current space vectors at sinus stator three-phase voltage feeding **DO NOT MOVE** in **rotor reference frame:**

$$\underline{u}_{s(r)} = \underline{u}_{s(s)} \cdot e^{-j\cdot\gamma(\tau)} = \underline{u}_{s(s)} \cdot e^{-j\cdot\omega_m \tau} = \underline{u}_s \cdot e^{j\cdot\omega_s \tau} \cdot e^{-j\cdot\omega_m \tau} = \underline{u}_s$$

Voltage equations of stator and damper winding in rotor reference frame

- **Stator :** $\underline{u}_{s(r)} = r_s \cdot \underline{i}_{s(r)} + \frac{d\underline{\psi}_{s(r)}}{d\tau} + j \cdot \omega_m \cdot \underline{\psi}_{s(r)}$
- **Damper :** $0 = r'_r \cdot \underline{i}'_{r(r)} + \frac{d\underline{\psi}'_{r(r)}}{d\tau}$

- Subscript (r) denotes: rotor reference frame
- We skip subscript (r) for simplicity

Decomposition of stator space vectors in *d*- and *q*-components:

$$\underline{u}_s = u_d + j \cdot u_q, \quad \underline{i}_s = i_d + j \cdot i_q \quad \underline{\psi}_s = \psi_d + j \cdot \psi_q$$

Damper current space vector: $\underline{i}'_r = i_D + j \cdot i_Q \quad \underline{\psi}'_r = \psi_D + j \cdot \psi_Q$

Transformation ratio for voltage and current:

$$\ddot{u}_{ID} = \ddot{u}_{IQ} = \frac{k_{ws} \cdot N_s \cdot m_s}{(1/2) \cdot Q_r}, \quad \ddot{u}_{UD} = \ddot{u}_{UQ} = \frac{k_{ws} \cdot N_s}{1/2}$$

Field winding: Different transformation ratio: **WE SKIP THE ´ FURTHER**

$$i'_f = i_f / \ddot{u}_{If} \quad \psi'_f = \ddot{u}_{Uf} \cdot \psi_f \quad r'_f = \ddot{u}_{Uf} \cdot \ddot{u}_{If} \cdot r_f$$

Dynamic voltage equations for synchronous machines

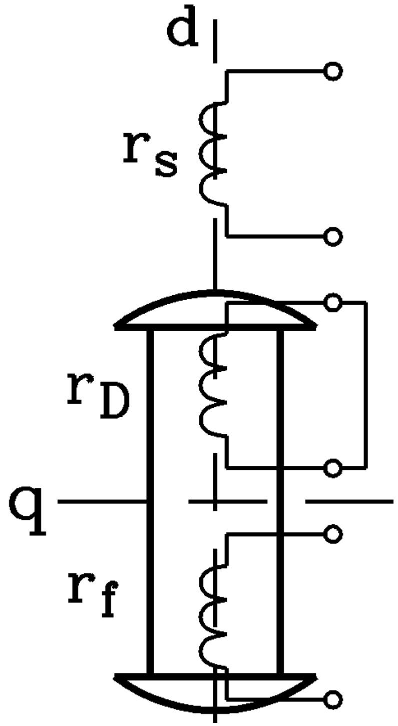
$$u_d = r_s \cdot i_d + \frac{d\psi_d}{d\tau} - \omega_m \cdot \psi_q$$

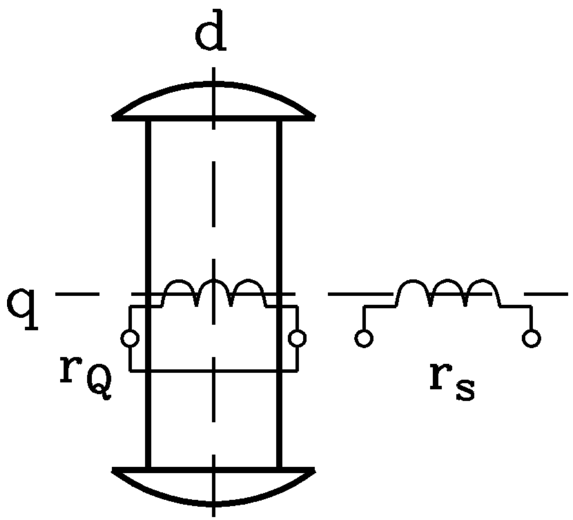
$$u_q = r_s \cdot i_q + \frac{d\psi_q}{d\tau} + \omega_m \cdot \psi_d$$

$$0 = r_D \cdot i_D + \frac{d\psi_D}{d\tau}$$

$$0 = r_Q \cdot i_Q + \frac{d\psi_Q}{d\tau}$$

$$u_f = r_f \cdot i_f + \frac{d\psi_f}{d\tau}$$





d, q: Stator voltage equations

D, Q: Damper voltage equations

f : Field voltage equation

Equations are valid in <u>rotor reference</u> frame !

Flux linkage equations in *d*- and *q*-axis

$$\psi_d = x_d i_d + x_{dh} i_D + x_{dh} i_f$$

$$\psi_q = x_q i_q + x_{qh} i_Q$$

Stator flux linkage

$$\psi_D = x_{dh} i_d + x_D i_D + x_{dh} i_f$$

$$\psi_Q = x_{qh} i_q + x_Q i_Q$$

Damper flux linkage

$$\psi_f = x_{dh} i_d + x_{dh} i_D + x_f i_f$$

Field winding flux linkage

Stator inductance $x_d = x_{dh} + x_{s\sigma}$ $\quad x_q = x_{qh} + x_{s\sigma}$

Damper inductance $x_D = x_{dh} + x_{D\sigma}$ $\quad x_Q = x_{qh} + x_{Q\sigma}$

Field inductance $x_f = x_{dh} + x_{f\sigma}$

We assume constant iron saturation, so x_{dh}, x_{qh} are constant

8. Dynamics of synchronous machines

Electromagnetic torque

$$m_e = \mathrm{Im}\{\underline{i}_s \cdot \underline{\psi}_s^*\} = i_q \cdot \psi_d - i_d \cdot \psi_q$$

Stator current and flux linkage space vector components in *d*-*q*-reference frame

$$\tau_J \cdot \frac{d\omega_m(\tau)}{d\tau} = i_q(\tau)\cdot\psi_d(\tau) - i_d(\tau)\cdot\psi_q(\tau) - m_s(\tau)$$

Mechanical equation

Complete **set of dynamic equations in rotor reference frame for**

a) electrically excited synchronous machines with damper cage (and rotor saliency):

11 equations, 11 unknowns

$$i_d, i_q, i_D, i_Q, i_f, \psi_d, \psi_q, \psi_D, \psi_Q, \psi_f, \omega_m \leftarrow u_d, u_q, u_f, m_s$$

b) permanent magnet synchronous machines: NO damper cage, NO field winding:

5 equations, 5 unknowns

$$i_d, i_q, \psi_d, \psi_q, \omega_m \leftarrow u_d, u_q, \mathrm{Perm.Flux}\ \ \psi_\mathrm{f} = \psi_p, m_s$$

8. Dynamics of synchronous machines

Electrically excited synchronous machines
- with damper cage and rotor saliency

$$u_d(\tau) = r_s \cdot i_d(\tau) + \frac{d\psi_d(\tau)}{d\tau} - \omega_m(\tau) \cdot \psi_q(\tau) \qquad u_q(\tau) = r_s \cdot i_q(\tau) + \frac{d\psi_q(\tau)}{d\tau} + \omega_m(\tau) \cdot \psi_d(\tau)$$

$$0 = r_D \cdot i_D(\tau) + \frac{d\psi_D(\tau)}{d\tau} \qquad 0 = r_Q \cdot i_Q(\tau) + \frac{d\psi_Q(\tau)}{d\tau}$$

$$u_f(\tau) = r_f \cdot i_f(\tau) + \frac{d\psi_f(\tau)}{d\tau}$$

$$\psi_d(\tau) = (x_{dh} + x_{s\sigma}) \cdot i_d(\tau) + x_{dh} i_D(\tau) + x_{dh} i_f(\tau) \qquad \psi_q(\tau) = (x_{qh} + x_{s\sigma}) \cdot i_q(\tau) + x_{qh} \cdot i_Q(\tau)$$

$$\psi_D(\tau) = x_{dh} i_d(\tau) + (x_{dh} + x_{D\sigma}) \cdot i_D(\tau) + x_{dh} i_f(\tau) \qquad \psi_Q(\tau) = x_{qh} i_q(\tau) + (x_{qh} + x_{Q\sigma}) \cdot i_Q(\tau)$$

$$\psi_f(\tau) = x_{dh} i_d(\tau) + x_{dh} i_D(\tau) + (x_{dh} + x_{f\sigma}) \cdot i_f(\tau)$$

$$\tau_J \cdot \frac{d\omega_m(\tau)}{d\tau} = i_q(\tau) \cdot \psi_d(\tau) - i_d(\tau) \cdot \psi_q(\tau) - m_s(\tau)$$

8. Dynamics of synchronous machines

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Permanent magnet synchronous machines
no damper / field winding

$$u_d(\tau) = r_s \cdot i_d(\tau) + \frac{d\psi_d(\tau)}{d\tau} - \omega_m(\tau) \cdot \psi_q(\tau)$$

$$u_q(\tau) = r_s \cdot i_q(\tau) + \frac{d\psi_q(\tau)}{d\tau} + \omega_m(\tau) \cdot \psi_d(\tau)$$

$$\psi_d(\tau) = (x_{dh} + x_{s\sigma}) \cdot i_d(\tau) + \psi_p$$

$$\psi_q(\tau) = (x_{qh} + x_{s\sigma}) \cdot i_q(\tau)$$

$$\tau_J \cdot \frac{d\omega_m(\tau)}{d\tau} = i_q(\tau) \cdot \psi_d(\tau) - i_d(\tau) \cdot \psi_q(\tau) - m_s(\tau)$$

Stator zero sequence voltage system: $u_{s0} = r_s \cdot i_{s0} + \frac{d\psi_{s0}}{d\tau}$

Separate zero-sequence flux linkage equations (not discussed here)

Steady state operation of synchronous machine

- In rotor reference frame: steady state: $d./d\tau = 0$
- **DC values = *d*- and *q*-phasor amplitudes of *d-q*-phasor diagram**

$$u_d = r_s \cdot i_d - \omega_m \cdot \psi_q$$

$$u_q = r_s \cdot i_q + \omega_m \cdot \psi_d$$

$$0 = r_D \cdot i_D$$

$$0 = r_Q \cdot i_Q$$

$$u_f = r_f \cdot i_f$$

$$i_D = i_Q = 0, \quad i_f = u_f / r_f$$

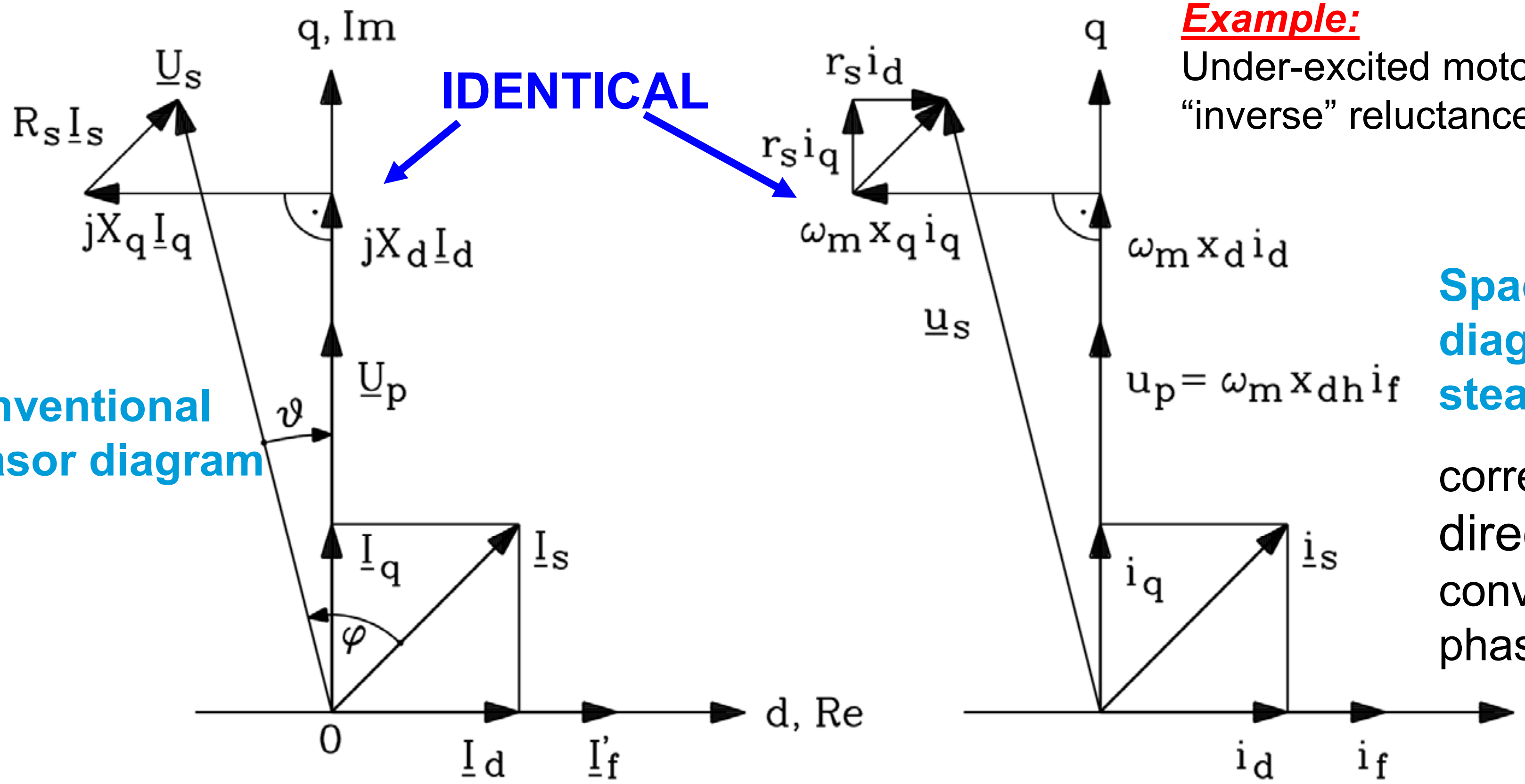
$$\psi_d = x_d i_d + x_{dh} i_f \qquad \psi_q = x_q i_q$$

The ***steady state back emf*** u_p is directed in *q*-axis:

$$u_p = \omega_m \cdot x_{dh} i_f$$

$$u_d = r_s \cdot i_d - \omega_m \cdot x_q i_q \qquad u_q = r_s \cdot i_q + \omega_m \cdot x_d i_d + \omega_m \cdot x_{dh} i_f$$

Steady state DC values diagram



Conventional phasor diagram

Example:
Under-excited motor operation, "inverse" reluctance $x_q = 2x_d$

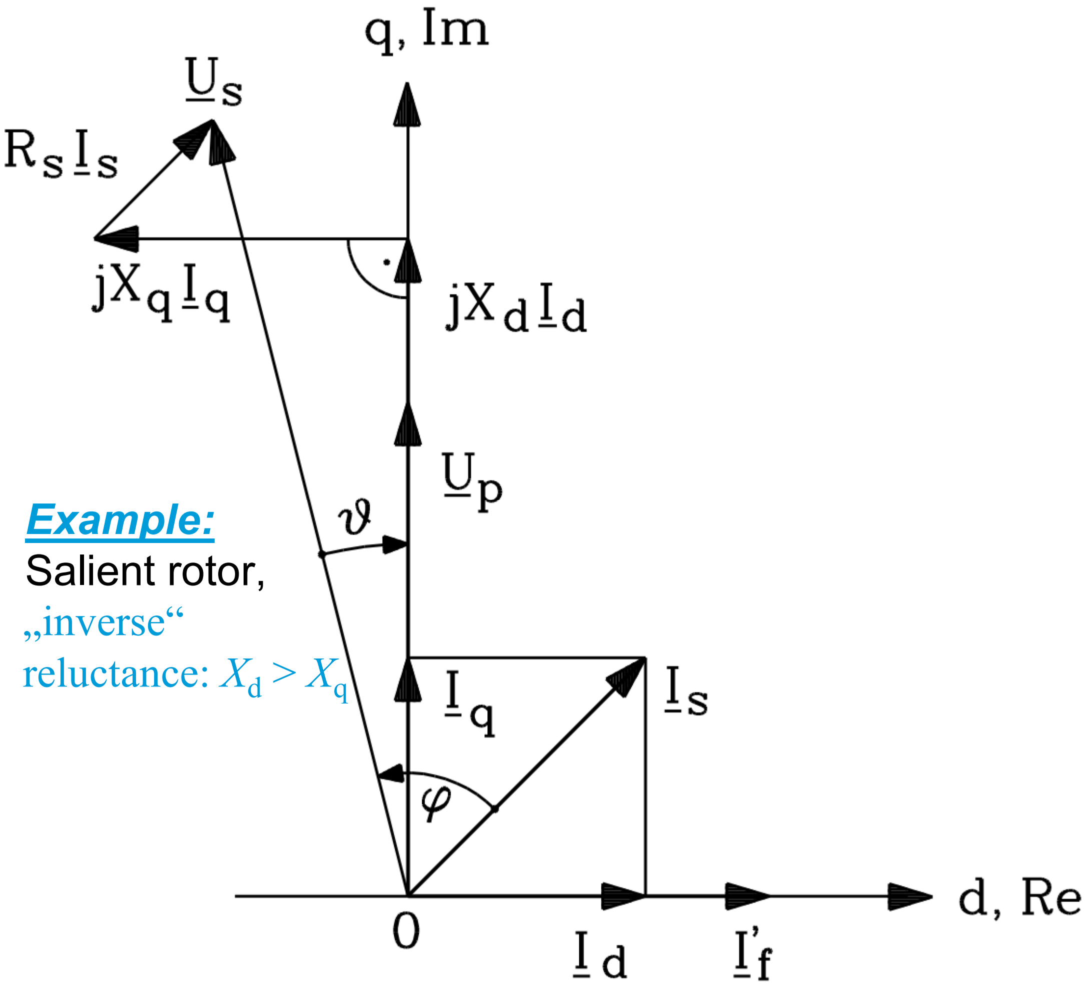
Space vector diagram for steady state:

corresponds directly to conventional phasor diagram

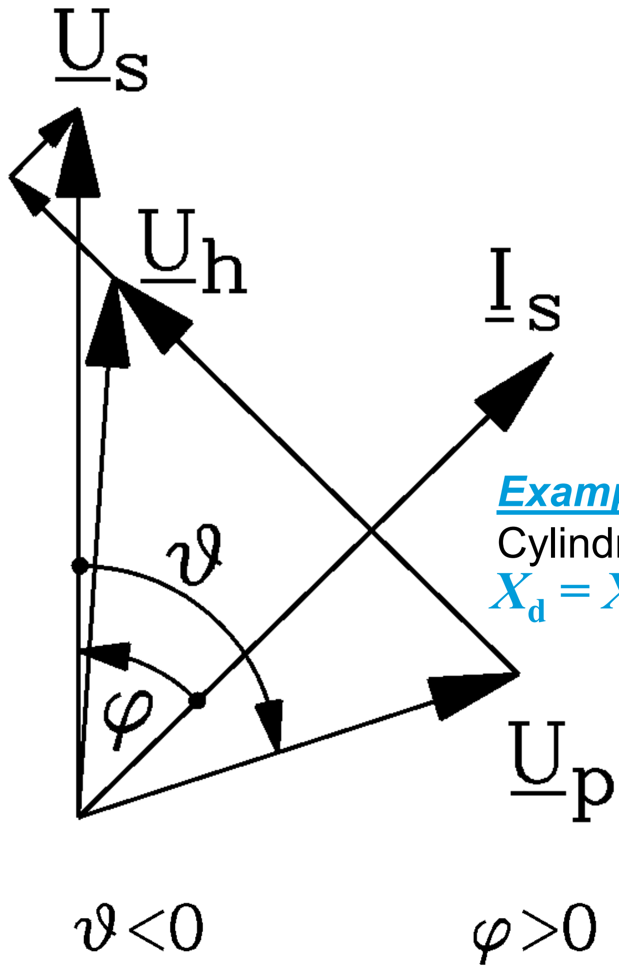
$$u_d = r_s \cdot i_d - \omega_m \cdot x_q i_q \qquad u_q = r_s \cdot i_q + \omega_m \cdot x_d i_d + \omega_m \cdot x_{dh} i_f$$

8. Dynamics of synchronous machines

Do you remember ? Motor operation – under-excited



Example:
Salient rotor,
„inverse“
reluctance: $X_d > X_q$



Example:
Cylindrical rotor,
$X_d = X_q$

Summary:
Set of dynamic equations for synchronous machines

- Formulation in rotor reference frame
- Due to damper and field winding:
  11 dynamic equations for *d*- and *q*-axis
- PM machines:
  Only 5 dynamic equations, if rotor eddy currents are neglected
- Steady state solutions are in rotor reference frame DC values
- Steady state DC values correspond
  with
  Re- and Im-part of steady state AC complex phasor results
  (see: Bachelor´s course)

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8. Dynamics of synchronous machines

Park transformation (1)

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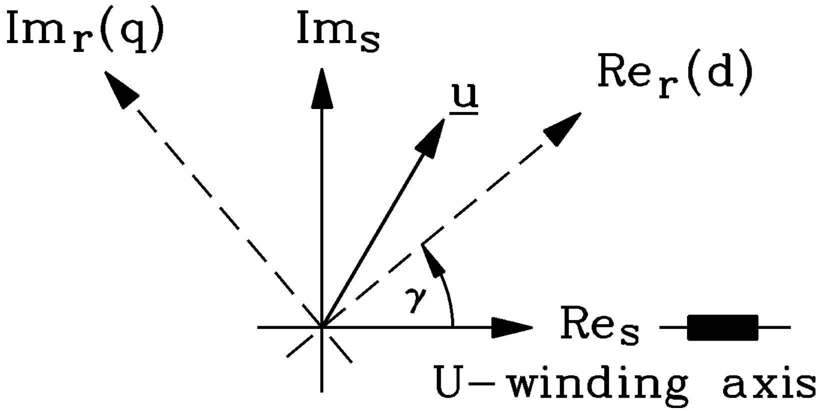
Transformation from stator (s) three-phase U, V, W-system
into rotor (r) two-axis *d*-*q*-system: e.g. stator voltage space vector $\underline{u}_s$:



$$u_d + j\cdot u_q = \underline{u}_{s(r)} = \underline{u}_{s(s)}\cdot e^{-j\cdot\gamma} =$$

$$= \frac{2}{3}\cdot\left(u_U + \underline{a}\cdot u_V + \underline{a}^2\cdot u_W\right)\cdot e^{-j\cdot\gamma}$$

$$u_{s0} = \frac{1}{3}\cdot\left(u_U + u_V + u_W\right) = u_0$$

$$\begin{pmatrix} u_d \\ u_q \\ u_0 \end{pmatrix} = \begin{pmatrix} \frac{2}{3}\cdot\cos\gamma & \frac{2}{3}\cdot\cos(\gamma - \frac{2\pi}{3}) & \frac{2}{3}\cdot\cos(\gamma - \frac{4\pi}{3}) \\ -\frac{2}{3}\cdot\sin\gamma & -\frac{2}{3}\cdot\sin(\gamma - \frac{2\pi}{3}) & -\frac{2}{3}\cdot\sin(\gamma - \frac{4\pi}{3}) \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix}\cdot\begin{pmatrix} u_U \\ u_V \\ u_W \end{pmatrix}$$

$$\underline{a} = e^{j\cdot 2\pi/3}$$

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Park transformation (2)

$$\begin{pmatrix} u_d \\ u_q \\ u_0 \end{pmatrix} = \begin{pmatrix} \frac{2}{3}\cdot\cos\gamma & \frac{2}{3}\cdot\cos(\gamma-\frac{2\pi}{3}) & \frac{2}{3}\cdot\cos(\gamma-\frac{4\pi}{3}) \\ -\frac{2}{3}\cdot\sin\gamma & -\frac{2}{3}\cdot\sin(\gamma-\frac{2\pi}{3}) & -\frac{2}{3}\cdot\sin(\gamma-\frac{4\pi}{3}) \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix} \cdot \begin{pmatrix} u_U \\ u_V \\ u_W \end{pmatrix} = (T)\cdot\begin{pmatrix} u_U \\ u_V \\ u_W \end{pmatrix}$$

$$\begin{pmatrix} u_U \\ u_V \\ u_W \end{pmatrix} = \begin{pmatrix} \cos\gamma & -\sin\gamma & 1 \\ \cos(\gamma-\frac{2\pi}{3}) & -\sin(\gamma-\frac{2\pi}{3}) & 1 \\ \cos(\gamma-\frac{4\pi}{3}) & -\sin(\gamma-\frac{4\pi}{3}) & 1 \end{pmatrix} \cdot \begin{pmatrix} u_d \\ u_q \\ u_0 \end{pmatrix} = (T)^{-1}\cdot\begin{pmatrix} u_d \\ u_q \\ u_0 \end{pmatrix}$$

Summary:
Park transformation

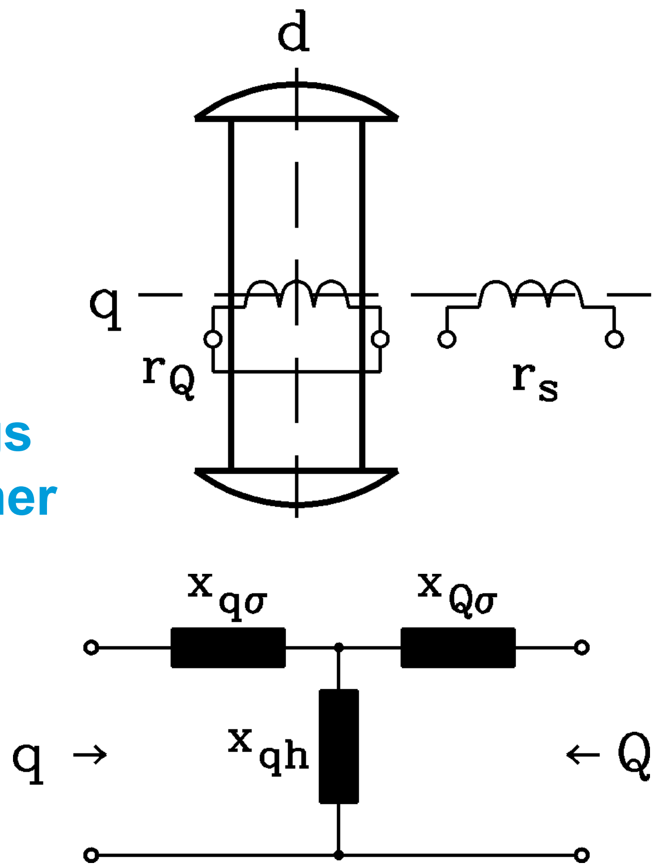
- Alternative formulation to KOVAC´s complex space vectors
- Matrix transformation from U,V,W to d,q,0-system
- Historically older, since ca. 1930
- *CLARKE*´s transformation:
  From U,V,W to stator α,β,0-system
- *PARK*´s transformation:
  From U,V,W to rotor d,q,0-system

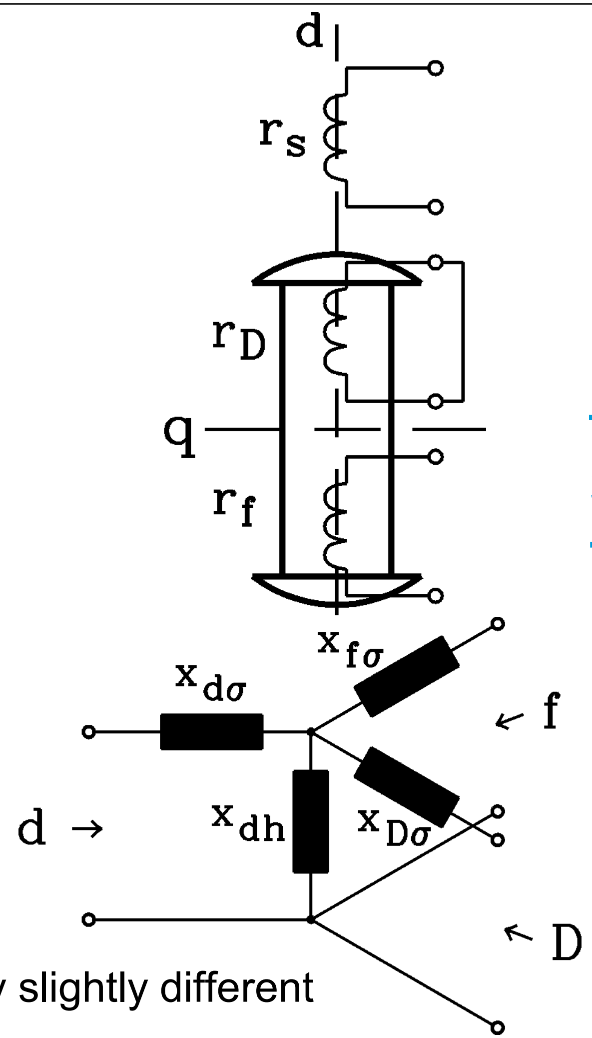
8. Dynamics of synchronous machines

Magnetic coupling in *d*- and *q*-axis

***q*-axis:**

2 windings transformer





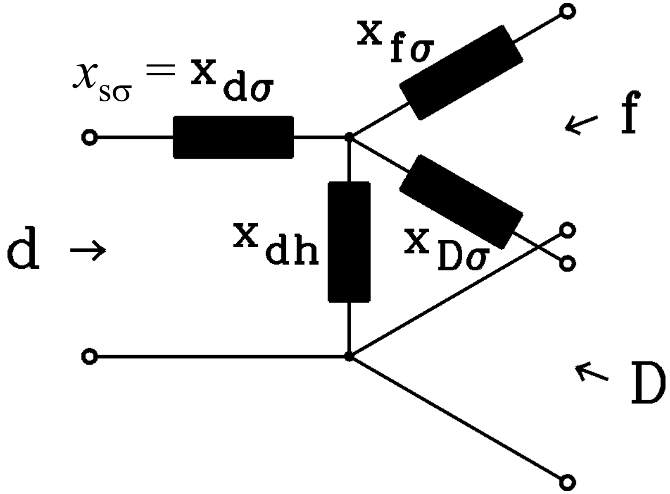
***d*-axis:**

3 windings transformer

Stator leakage inductance $x_{d\sigma} \neq x_{q\sigma}$ due to saliency slightly different

Here: Simplification: $x_{d\sigma} = x_{q\sigma} = x_{s\sigma}$

Equivalent circuits for magnetic coupling in *d*- and *q*-axis in per-unit

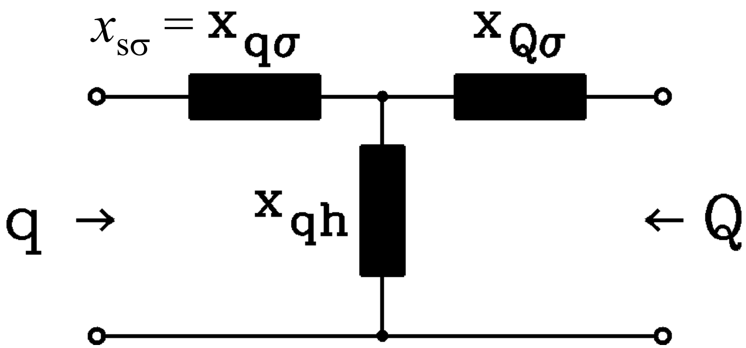


d-axis: 3 windings transformer

$$\psi_d = (x_{dh} + x_{s\sigma}) \cdot i_d + x_{dh} i_D + x_{dh} i_f$$

$$\psi_D = x_{dh} i_d + (x_{dh} + x_{D\sigma}) \cdot i_D + x_{dh} i_f$$

$$\psi_f = x_{dh} i_d + x_{dh} i_D + (x_{dh} + x_{f\sigma}) \cdot i_f$$



q-axis: 2 windings transformer

$$\psi_q = (x_{qh} + x_{s\sigma}) \cdot i_q + x_{qh} \cdot i_Q$$

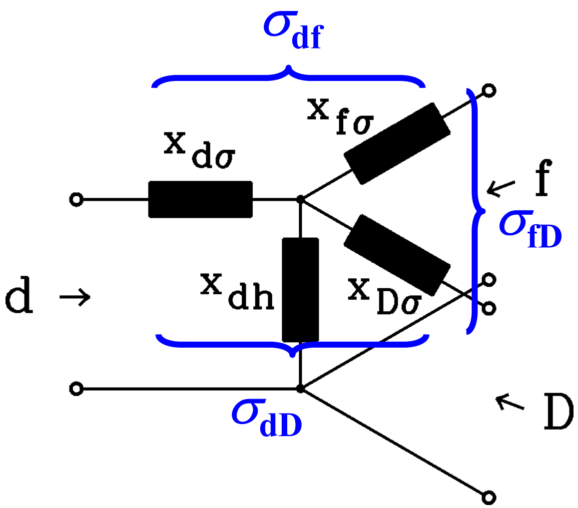
$$\psi_Q = x_{qh} \cdot i_q + (x_{qh} + x_{Q\sigma}) \cdot i_Q$$

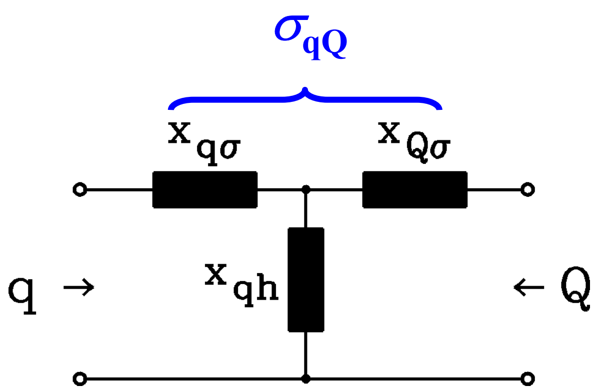
Blondel stray coefficients $0 \le \sigma \le 1$

Describes degree of magnetic coupling between two windings.

***d*-axis:** $$\sigma_{dD} = 1 - \frac{x_{dh}^2}{x_d x_D}, \quad \sigma_{df} = 1 - \frac{x_{dh}^2}{x_d x_f}, \quad \sigma_{fD} = 1 - \frac{x_{dh}^2}{x_f x_D}$$

***q*-axis:** $$\sigma_{qQ} = 1 - \frac{x_{qh}^2}{x_q x_Q}$$





$$\psi_q = x_q \cdot i_q + x_{qh} \cdot i_Q$$
$$\psi_Q = x_{qh} \cdot i_q + x_Q \cdot i_Q$$

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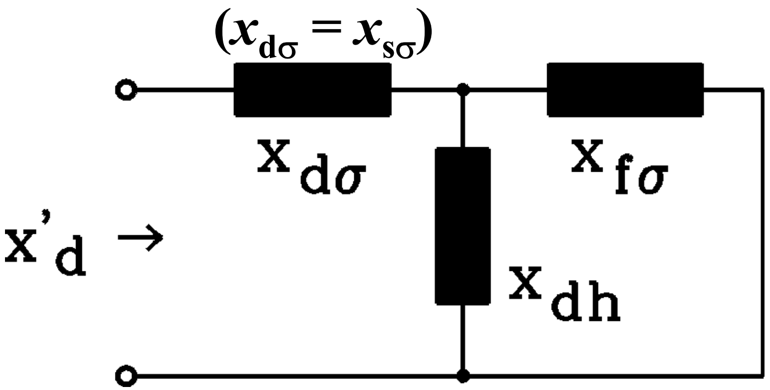
Expression of dynamic inductances with *BLONDEL* coefficients

$$x'_d = x_d \cdot \sigma_{df} = x_d \cdot \left(1 - \frac{x_{dh}^2}{x_d x_f}\right) =$$

$$= x_d - \frac{x_{dh}^2}{x_f} = x_{s\sigma} + \frac{x_{dh} x_f}{x_f} - \frac{x_{dh}^2}{x_f} =$$

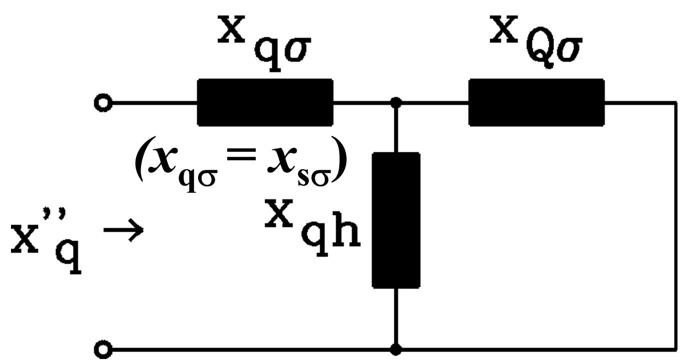
$$= x_{s\sigma} + \frac{x_{dh} x_{f\sigma}}{x_{dh} + x_{f\sigma}}$$



$$x''_q = x_q \cdot \sigma_{qQ} = x_q \cdot \left(1 - \frac{x_{qh}^2}{x_q x_Q}\right) =$$

$$= x_q - \frac{x_{qh}^2}{x_Q} = x_{s\sigma} + \frac{x_{qh} x_Q}{x_Q} - \frac{x_{qh}^2}{x_Q} =$$

$$= x_{s\sigma} + \frac{x_{qh} x_{Q\sigma}}{x_{qh} + x_{Q\sigma}}$$



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Example: Calculation via *BLONDEL* coefficients

Electrically excited salient pole synchronous machine with damper cage:

$x_{dh} = 1.2,\ x_{qh} = 0.6,\ x_{s\sigma} = 0.15,\ x_{f\sigma} = 0.2,\ x_{D\sigma} = 0.1,\ x_{Q\sigma} = 0.1$

$$x_d'' = x_{s\sigma} + \frac{x_{dh}x_{f\sigma}x_{D\sigma}}{x_{dh}x_{f\sigma} + x_{dh}x_{D\sigma} + x_{f\sigma}x_{D\sigma}} = 0.15 + \frac{1.2\cdot 0.2\cdot 0.1}{1.2\cdot 0.2 + 1.2\cdot 0.1 + 0.2\cdot 0.1} = \underline{\underline{0.21}}$$

$$x_q'' = x_{s\sigma} + \frac{x_{qh}x_{Q\sigma}}{x_{qh} + x_{Q\sigma}} = 0.15 + \frac{0.6\cdot 0.1}{0.6 + 0.1} = \underline{\underline{0.24}} = \sigma_{qQ}x_q = 0.31\cdot 0.75 = \underline{\underline{0.24}}$$

$$\left(\sigma_{qQ} = 1 - \frac{x_{qh}^2}{x_q x_Q} = 1 - \frac{0.6^2}{0.75\cdot 0.7} = 0.31\right)$$

$$x_d' = x_{s\sigma} + \frac{x_{dh}x_{f\sigma}}{x_{dh} + x_{f\sigma}} = 0.15 + \frac{1.2\cdot 0.2}{1.2 + 0.2} = \underline{\underline{0.32}}$$

$$x_d' = \sigma_{df}\cdot x_d = 0.238\cdot 1.35 = \underline{\underline{0.32}}$$

$$\left(\sigma_{df} = 1 - \frac{x_{dh}^2}{x_d x_f} = 1 - \frac{1.2^2}{1.35\cdot 1.4} = 0.238\right)$$

$$x_q' = x_q = x_{qh} + x_{s\sigma} = 0.6 + 0.15 = \underline{\underline{0.75}}$$

$$x_d = x_{dh} + x_{s\sigma} = 1.2 + 0.15 = \underline{\underline{1.35}}$$

Summary:
Equivalent circuits for magnetic coupling in synchronous machines

- Three-winding transformer in *d*-axis yields three *BLONDEL* stray coefficients
- Two-winding transformer in *q*-axis yields one *BLONDEL* stray coefficient
- Subtransient *q*-axis and transient *d*-axis reactance:
  May be calculated with *BLONDEL* coefficients
- *BLONDEL* coefficients are extensively used to write formulas shorter
- Here:
  Only ONE main flux in *d*-axis, but in reality:
  Main fluxes between *d* and *f* resp. *d* and *D* different (see: 8.9.3)

Dynamic performance at constant speed operation

$$\omega_m = \text{const.}$$

- Electrical time constants of stator and rotor windings are much shorter than mechanical time constant
- **Operation at constant speed during electrical transient is assumed.**
- *Examples:* - Sudden short circuits,
  - Electric load steps with only change of reactive power,
  - Switching during inverter operation
- **10 LINEAR DYNAMIC EQUATIONS:** 5^{th} order differential equation system; *Laplace* transformation may be used !
- **Only electrical equations are needed, as *n* = const.**
- Initial conditions for $d\psi/d\tau$ for flux linkages: $\psi_{d0}, \psi_{q0}, \psi_{f0}, \psi_{D0}, \psi_{Q0}$
  e. g.: $L\{d\psi_d(\tau)/d\tau\} = s \cdot \breve{\psi}_d(s) - \psi_{d0}$

Linear dynamic equations in *Laplace* domain, *n* = const.

(1) Voltage equations:

Unknowns: $i_d, i_q, i_D, i_Q, i_f, \psi_d, \psi_q, \psi_D, \psi_Q, \psi_f$

$$\breve{u}_d + \psi_{d0} = r_s \cdot \breve{i}_d + s \cdot \breve{\psi}_d - \omega_m \cdot \breve{\psi}_q$$

$$\breve{u}_q + \psi_{q0} = r_s \cdot \breve{i}_q + s \cdot \breve{\psi}_q + \omega_m \cdot \breve{\psi}_d$$

$$\psi_{D0} = r_D \cdot \breve{i}_D + s \cdot \breve{\psi}_D$$

$$\psi_{Q0} = r_Q \cdot \breve{i}_Q + s \cdot \breve{\psi}_Q$$

$$\breve{u}_f + \psi_{f0} = r_f \cdot \breve{i}_f + s \cdot \breve{\psi}_f$$

(2) Flux linkage equations:

$$\begin{pmatrix} \breve{\psi}_d \\ \breve{\psi}_D \\ \breve{\psi}_f \end{pmatrix} = \begin{pmatrix} x_d & x_{dh} & x_{dh} \\ x_{dh} & x_D & x_{dh} \\ x_{dh} & x_{dh} & x_f \end{pmatrix} \cdot \begin{pmatrix} \breve{i}_d \\ \breve{i}_D \\ \breve{i}_f \end{pmatrix}$$

$$\begin{pmatrix} \breve{\psi}_q \\ \breve{\psi}_Q \end{pmatrix} = \begin{pmatrix} x_q & x_{qh} \\ x_{qh} & x_Q \end{pmatrix} \cdot \begin{pmatrix} \breve{i}_q \\ \breve{i}_Q \end{pmatrix}$$

(3) Initial conditions:

$$\begin{pmatrix} \psi_{d0} \\ \psi_{D0} \\ \psi_{f0} \end{pmatrix} = \begin{pmatrix} x_d & x_{dh} & x_{dh} \\ x_{dh} & x_D & x_{dh} \\ x_{dh} & x_{dh} & x_f \end{pmatrix} \cdot \begin{pmatrix} i_{d0} \\ i_{D0} \\ i_{f0} \end{pmatrix}$$

$$\begin{pmatrix} \psi_{q0} \\ \psi_{Q0} \end{pmatrix} = \begin{pmatrix} x_q & x_{qh} \\ x_{qh} & x_Q \end{pmatrix} \cdot \begin{pmatrix} i_{q0} \\ i_{Q0} \end{pmatrix}$$

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10 – 6 = 4 remaining stator equations!

$$\begin{pmatrix} \breve{\psi}_d \\ \breve{\psi}_D \\ \breve{\psi}_f \end{pmatrix} = \begin{pmatrix} x_d & x_{dh} & x_{dh} \\ x_{dh} & x_D & x_{dh} \\ x_{dh} & x_{dh} & x_f \end{pmatrix} \cdot \begin{pmatrix} \breve{i}_d \\ \breve{i}_D \\ \breve{i}_f \end{pmatrix}$$

$$\begin{pmatrix} \breve{\psi}_q \\ \breve{\psi}_Q \end{pmatrix} = \begin{pmatrix} x_q & x_{qh} \\ x_{qh} & x_Q \end{pmatrix} \cdot \begin{pmatrix} \breve{i}_q \\ \breve{i}_Q \end{pmatrix}$$

$$\psi_{D0} = r_D \cdot \breve{i}_D + s \cdot \breve{\psi}_D$$

$$\psi_{Q0} = r_Q \cdot \breve{i}_Q + s \cdot \breve{\psi}_Q$$

$$\breve{u}_f + \psi_{f0} = r_f \cdot \breve{i}_f + s \cdot \breve{\psi}_f$$

$$\breve{\psi}_d - \frac{\psi_{d0}}{s} = x_d(s) \cdot \left(\breve{i}_d - \frac{i_{d0}}{s} \right) + x_f(s) \cdot \left(\frac{\breve{u}_f}{r_f} - \frac{i_{f0}}{s} \right) - x_D(s) \cdot \frac{i_{D0}}{s}$$

$$\breve{\psi}_q - \frac{\psi_{q0}}{s} = x_q(s) \cdot \left(\breve{i}_q - \frac{i_{q0}}{s} \right) - x_Q(s) \cdot \frac{i_{Q0}}{s}$$

"Reactance operators"

8. Dynamics of synchronous machines

8.6.1 “Reactance operators”

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$$x_d(s) = \frac{s^2 \cdot x_d'' \cdot \sigma_{fD} \cdot \tau_f \cdot \tau_D + s \cdot x_d \cdot (\sigma_{df} \cdot \tau_f + \sigma_{dD} \cdot \tau_D) + x_d}{s^2 \cdot \sigma_{fD} \cdot \tau_f \cdot \tau_D + s \cdot (\tau_f + \tau_D) + 1}$$

$$x_q(s) = x_q'' \cdot \frac{s + \frac{1}{\sigma_{qQ} \cdot \tau_Q}}{s + \frac{1}{\tau_Q}}$$

Important: $x_d(s)$, $x_q(s)$

Open-circuit time constants of damper and field winding:

$$\tau_D = \frac{x_D}{r_D}, \quad \tau_Q = \frac{x_Q}{r_Q}, \quad \tau_f = \frac{x_f}{r_f}$$

$$x_D(s) = \frac{s \cdot \frac{x_{f\sigma}}{r_f} + 1}{s^2 \cdot \sigma_{fD} \cdot \tau_f \cdot \tau_D + s \cdot (\tau_f + \tau_D) + 1} \cdot x_{dh}$$

$$x_f(s) = \frac{s \cdot \frac{x_{D\sigma}}{r_D} + 1}{s^2 \cdot \sigma_{fD} \cdot \tau_f \cdot \tau_D + s \cdot (\tau_f + \tau_D) + 1} \cdot x_{dh}$$

$$x_Q(s) = \frac{\frac{1}{\tau_Q}}{s + \frac{1}{\tau_Q}} \cdot x_{qh}$$

8. Dynamics of synchronous machines

"Reactance operators": $x_d(s), x_q(s), x_D(s), x_Q(s), x_f(s)$

Initial condition: Synchronous operation:

$$\frac{\breve{u}_f}{r_f} = \frac{u_{f0}}{s \cdot r_f} = \frac{i_{f0}}{s} \qquad \frac{\breve{u}_f}{r_f} - \frac{i_{f0}}{s} = 0$$

$$i_{D0} = 0, i_{Q0} = 0$$

At transient operation:

$$\breve{\psi}_d - \frac{\psi_{d0}}{s} = x_d(s) \cdot \left(\breve{i}_d - \frac{i_{d0}}{s} \right)$$

$$\breve{\psi}_q - \frac{\psi_{q0}}{s} = x_q(s) \cdot \left(\breve{i}_q - \frac{i_{q0}}{s} \right)$$

$$x_d(s) = \frac{s^2 \cdot x''_d \cdot \sigma_{fD} \cdot \tau_f \cdot \tau_D + s \cdot x_d \cdot \left(\sigma_{df} \cdot \tau_f + \sigma_{dD} \cdot \tau_D \right) + x_d}{s^2 \cdot \sigma_{fD} \cdot \tau_f \cdot \tau_D + s \cdot \left(\tau_f + \tau_D \right) + 1}$$

- **Most important are "the reactance operators" $x_d(s), x_q(s)$!**
- The others $x_f(s), x_D(s), x_Q(s)$ are only needed, if $i_{D0} \neq 0$, $i_{Q0} \neq 0$, $i_{f0} \neq u_{f0}/r_f$

$$x_q(s) = x''_q \cdot \frac{s + \frac{1}{\sigma_{qQ} \cdot \tau_Q}}{s + \frac{1}{\tau_Q}}$$

Determination of “reactance operator” $x_q(s)$ (1)

Voltage equation:

$$\psi_{Q0} = r_Q \cdot \breve{i}_Q + s \cdot \breve{\psi}_Q$$

Flux linkage equation:

$$\breve{\psi}_Q = x_{qh}\breve{i}_q + x_Q\breve{i}_Q$$

Calculation of damper current:

$$\psi_{Q0} = r_Q \cdot \breve{i}_Q + s \cdot \left(x_{qh}\breve{i}_q + x_Q\breve{i}_Q\right) \quad \Rightarrow \quad \breve{i}_Q = \frac{\psi_{Q0} - s \cdot x_{qh} \cdot \breve{i}_q}{r_Q + s \cdot x_Q}$$

Calculation of q-flux linkage:

$$\breve{\psi}_q = x_q\breve{i}_q + x_{qh}\breve{i}_Q = x_q\breve{i}_q + x_{qh} \cdot \frac{\psi_{Q0} - s \cdot x_{qh} \cdot \breve{i}_q}{r_Q + s \cdot x_Q} = \left(x_q - \frac{s \cdot x_{qh}^2}{r_Q + s \cdot x_Q}\right) \cdot \breve{i}_q + \frac{x_{qh} \cdot \psi_{Q0}}{r_Q + s \cdot x_Q}$$

$x_q(s)$

Determination of reactance operator $x_q(s)$:

$$x_q(s) = \frac{x_q(r_Q + s \cdot x_Q) - s \cdot x_{qh}^2}{r_Q + s \cdot x_Q} = \frac{x_q r_Q + s \cdot \sigma_{qQ} \cdot x_q x_Q}{r_Q + s \cdot x_Q} = \frac{x_q r_Q + s \cdot x_q'' x_Q}{r_Q + s \cdot x_Q}$$

8. Dynamics of synchronous machines

Determination of "reactance operator" $x_q(s)$ (2)

$$x_q(s) = \frac{x_q r_Q + s \cdot x_q'' x_Q}{r_Q + s \cdot x_Q}$$

Reactance operator in a more convenient form: $\left(\tau_{Q\sigma} = \sigma_{qQ} \cdot \frac{x_Q}{r_Q} = \sigma_{qQ} \cdot \tau_Q\right)$

$$x_q(s) = \frac{\frac{x_q''}{\sigma_{qQ}} r_Q + s \cdot x_q'' x_Q}{r_Q + s \cdot x_Q} = \frac{s + \frac{r_Q}{\sigma_{qQ} x_Q}}{s + \frac{r_Q}{x_Q}} \cdot x_q'' = \frac{s + \frac{1}{\tau_{Q\sigma}}}{s + \frac{1}{\tau_Q}} \cdot x_q''$$

By using $\psi_{Q0} = x_{qh} i_{q0} + x_Q i_{Q0}$ we get with the abbreviation $x_Q(s) = \frac{\frac{1}{\tau_Q}}{s + \frac{1}{\tau_Q}} \cdot x_{qh}$:

$$\breve{\psi}_q - \frac{\psi_{q0}}{s} = x_q(s) \cdot \left(\breve{i}_q - \frac{i_{q0}}{s}\right) - x_Q(s) \cdot \frac{i_{Q0}}{s}$$

“Reactance operator of *q*-axis damper winding” $x_Q(s)$

$$\psi_{Q0} = x_{qh} i_{q0} + x_Q i_{Q0} \qquad \psi_{q0} = x_q i_{q0} + x_{qh} i_{Q0} \qquad x_q(s) = \frac{x_q (r_Q + s \cdot x_Q) - s \cdot x_{qh}^2}{r_Q + s \cdot x_Q}$$

$$\breve{\psi}_q - \frac{\psi_{q0}}{s} = x_q(s) \cdot \left(\breve{i}_q - \frac{i_{q0}}{s} \right) + \frac{x_{qh} \cdot \psi_{Q0}}{r_Q + s \cdot x_Q} - \frac{\psi_{q0}}{s} + x_q(s) \cdot \frac{i_{q0}}{s}$$

$$\frac{x_{qh} \cdot (x_{qh} i_{q0} + x_Q i_{Q0})}{r_Q + s \cdot x_Q} - \frac{x_q i_{q0} + x_{qh} i_{Q0}}{s} \cdot \frac{r_Q + s \cdot x_Q}{r_Q + s \cdot x_Q} + \frac{x_q \cdot (r_Q + s \cdot x_Q) - s \cdot x_{qh}^2}{r_Q + s \cdot x_Q} \cdot \frac{i_{q0}}{s}$$

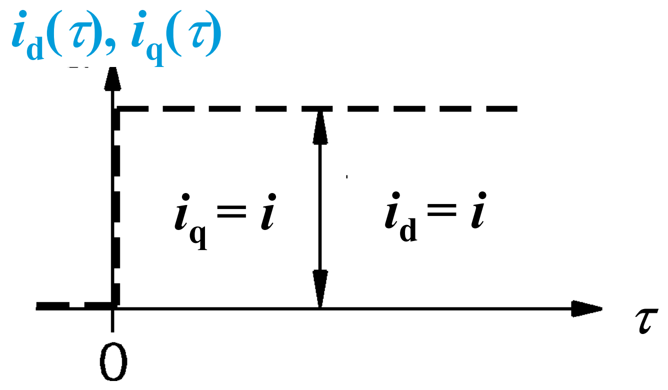
$$\frac{x_{qh}^2 i_{q0} + x_{qh} x_Q i_{Q0} - \frac{r_Q x_q i_{q0}}{s} - \frac{r_Q x_{qh} i_{Q0}}{s} - x_Q x_q i_{q0} - x_Q x_{qh} i_{Q0} + \frac{r_Q x_q i_{q0}}{s} + x_Q x_q i_{q0} + x_{qh}^2 i_{q0}}{r_Q + s \cdot x_Q}$$

$$-\frac{i_{Q0}}{s} \cdot \frac{r_Q \cdot x_{qh}}{r_Q + s \cdot x_Q} := -\frac{i_{Q0}}{s} \cdot x_Q(s)$$

$$x_Q(s) = \frac{r_Q x_{qh}}{r_Q + s \cdot x_Q} = \frac{\frac{r_Q}{x_Q}}{\frac{r_Q}{x_Q} + s} \cdot x_{qh} = \frac{\frac{1}{\tau_Q}}{\frac{1}{\tau_Q} + s} \cdot x_{qh}$$

Reactance operators at short and long time scale

We assume $\psi_{d0} = 0, i_{d0} = 0, \psi_{q0} = 0, i_{q0} = 0$



Reactance operators are flux transfer functions !

$$\breve{\psi}_d = x_d(s) \cdot \breve{i}_d \qquad \breve{\psi}_q = x_q(s) \cdot \breve{i}_q$$

Example: **Current steps**

***d*-axis:** $L(i_d) = i/s \quad \breve{\psi}_d(s) = x_d(s) \cdot i/s$

For τ = 0: $\psi_d(0) = \lim\limits_{s\to\infty} s \cdot \breve{\psi}_d(s) = \lim\limits_{s\to\infty} x_d(s) \cdot i = x''_d \cdot i$

For $\tau \to \infty$: $\psi_d(\infty) = \lim\limits_{s\to 0+} s \cdot \breve{\psi}_d(s) = \lim\limits_{s\to 0+} x_d(s) \cdot i = x_d \cdot i$

***q*-axis:** $L(i_q) = i/s \quad \breve{\psi}_q(s) = x_q(s) \cdot i/s$

For τ = 0: $\psi_q(0) = \lim\limits_{s\to\infty} s \cdot \breve{\psi}_q(s) = \lim\limits_{s\to\infty} x_q(s) \cdot i = x''_q \cdot i$

For $\tau \to \infty$: $\psi_q(\infty) = \lim\limits_{s\to 0+} s \cdot \breve{\psi}_q(s) = \lim\limits_{s\to 0+} x_q(s) \cdot i = x_q \cdot i$

8.6.2 Electrical rotor time constants for flux change in *d*- and *q*-axis

$$\breve{\psi}_d = x_d(s)\cdot \breve{i}_d$$

$$\breve{\psi}_q = x_q(s)\cdot \breve{i}_q$$

$$\breve{\psi}_d / x_d(s) = \breve{i}_d$$

$$\breve{\psi}_q / x_q(s) = \breve{i}_q$$

- Usually voltages are given, defining the flux linkages.
- The currents have to be calculated !

$$x_d(s) = x''_d \cdot \frac{(s+\frac{1}{\tau'_d})\cdot(s+\frac{1}{\tau''_d})}{(s+\alpha_{d1})\cdot(s+\alpha_{d2})}$$

Numerator (upper part of the fraction), Denominator (lower part of the fraction)

Numerator:

$$s^2 + s\cdot\frac{\sigma_{df}\tau_f + \sigma_{dD}\tau_D}{\sigma_{fD}\tau_f\tau_D}\cdot\frac{x_d}{x''_d} + \frac{1}{\sigma_{fD}\tau_f\tau_D}\cdot\frac{x_d}{x''_d} = 0 \Rightarrow Roots: s_1 = -\frac{1}{\tau'_d},\ s_2 = -\frac{1}{\tau''_d}$$

Denominator:

$$s^2 + s\cdot\frac{\tau_f + \tau_D}{\sigma_{fD}\tau_f\tau_D} + \frac{1}{\sigma_{fD}\tau_f\tau_D} = 0 \Rightarrow Roots: s_1 = -\alpha_{d1},\ s_2 = -\alpha_{d2}$$

8. Dynamics of synchronous machines

Electrical rotor time constants for the *d*-axis

$$x_d(s) = x_d'' \cdot \frac{(s+\frac{1}{\tau_d'})\cdot(s+\frac{1}{\tau_d''})}{(s+\alpha_{d1})\cdot(s+\alpha_{d2})}$$

$$\frac{1}{x_d(s)} = \frac{1}{x_d''} \cdot \frac{(s+\alpha_{d1})\cdot(s+\alpha_{d2})}{(s+\frac{1}{\tau_d'})\cdot(s+\frac{1}{\tau_d''})} \quad \Rightarrow \quad \frac{1}{x_d(s)} = A + \frac{B\cdot s}{s+\frac{1}{\tau_d'}} + \frac{C\cdot s}{s+\frac{1}{\tau_d''}}$$

Determine *A*, *B*, *C* by comparison of numerators !

Result:

$$\frac{1}{x_d(s)} = \frac{1}{x_d} + \left(\frac{1}{x_d'} - \frac{1}{x_d}\right)\cdot\frac{s}{s+\frac{1}{\tau_d'}} + \left(\frac{1}{x_d''} - \frac{1}{x_d'}\right)\cdot\frac{s}{s+\frac{1}{\tau_d''}}$$

where:

$$x_d' = x_d'' \cdot \frac{\frac{1}{\tau_d''} - \frac{1}{\tau_d'}}{\alpha_{d1} + \alpha_{d2} - \frac{1}{\tau_d'}\cdot\left(1+\frac{x_d''}{x_d}\right)}$$

8. Dynamics of synchronous machines

Inverse “reactance operators” and electrical rotor time constants

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“Reactance operators” are flux transfer functions:

$$\breve{\psi}_d / x_d(s) = \breve{i}_d$$

$$\breve{\psi}_q / x_q(s) = \breve{i}_q$$

$$\frac{1}{x_d(s)} = \frac{1}{x_d} + \left(\frac{1}{x'_d} - \frac{1}{x_d}\right) \cdot \frac{s}{s + \frac{1}{\tau'_d}} + \left(\frac{1}{x''_d} - \frac{1}{x'_d}\right) \cdot \frac{s}{s + \frac{1}{\tau''_d}}$$

$$\frac{1}{x_q(s)} = \frac{1}{x_q} + \left(\frac{1}{x''_q} - \frac{1}{x_q}\right) \cdot \frac{s}{s + \frac{1}{\tau''_q}}$$

- The stator current in *d*- and *q*-axis is changing with changing stator flux linkage with a subtransient (very short) and a transient (short) time constant, caused by rotor winding transients ! a) **transient time constant** τ'_d in *d*-axis,

  b) **subtransient time constants** τ''_d, τ''_q in *d*- and *q*-axis.

- The stator flux linkage in *d*- and *q*-axis is changing with changing stator current with a short and a long “open-circuit” time constant, caused by these rotor winding transients !

  a) **Open-circuit transient time constant** $\tau'_{d0} = 1/\alpha_{d1} = \tau_f$ in *d*-axis,

  b) **Open-circuit subtransient time constants** $\tau''_{d0} = 1/\alpha_{d2}, \tau''_{q0} = \tau_Q$ in *d*- and *q*-axis

Calculation of inverse reactance operator $1/x_q(s)$

$\tau_{Q\sigma} = \tau''_q$: Subtransient time constant of q-axis

$$\frac{1}{x_q(s)} = \frac{1}{x''_q} \cdot \frac{s + \frac{1}{\tau_Q}}{s + \frac{1}{\tau_{Q\sigma}}} = \frac{1}{x''_q} \cdot \frac{s}{s + \frac{1}{\tau''_q}} + \frac{1}{x''_q} \cdot \frac{\frac{1}{\tau_Q}}{s + \frac{1}{\tau''_q}} \underbrace{- \frac{1}{x_q} \cdot \frac{s}{s + \frac{1}{\tau''_q}} + \frac{1}{x_q} \cdot \frac{s}{s + \frac{1}{\tau''_q}}}_{0}$$

$$\frac{1}{x_q(s)} = \left(\frac{1}{x''_q} - \frac{1}{x_q} \right) \cdot \frac{s}{s + \frac{1}{\tau''_q}} + \underbrace{\frac{1}{\sigma_{qQ} x_q} \cdot \frac{\frac{1}{\tau_Q}}{s + \frac{1}{\sigma_{qQ}\tau_Q}} + \frac{1}{x_q} \cdot \frac{s}{s + \frac{1}{\sigma_{qQ}\tau_Q}}}_{\frac{1}{x_q}}$$

$$\boxed{\frac{1}{x_q(s)} = \frac{1}{x_q} + \left(\frac{1}{x''_q} - \frac{1}{x_q} \right) \cdot \frac{s}{s + \frac{1}{\tau''_q}}}$$

8. Dynamics of synchronous machines

Calculation of *d*-axis "short circuit time constants" τ'_d, τ''_d

$$s^2 + s \cdot \frac{\sigma_{df}\tau_f + \sigma_{dD}\tau_D}{\sigma_{fD}\tau_f\tau_D} \cdot \frac{x_d}{x''_d} + \frac{1}{\sigma_{fD}\tau_f\tau_D} \cdot \frac{x_d}{x''_d} = 0$$

$$s^2 + s \cdot p + q = 0 = (s - s_1) \cdot (s - s_2) \qquad s_1, s_2 = -p/2 \pm \sqrt{(p/2)^2 - q}$$

$$\frac{1}{\tau'_d}, \frac{1}{\tau''_d} = \frac{p}{2} \cdot \left(1 \mp \sqrt{1 - \frac{4q}{p^2}}\right) \qquad \frac{1}{\tau'_d} = -s_1, \ \frac{1}{\tau''_d} = -s_2$$

$$\frac{1}{\tau'_d}, \frac{1}{\tau''_d} = \frac{x_d \cdot (\sigma_{df}\tau_f + \sigma_{dD}\tau_D)}{x''_d \cdot 2 \cdot \sigma_{fD} \cdot \tau_f \cdot \tau_D} \cdot \left[1 \mp \sqrt{1 - \frac{4x''_d \cdot \sigma_{fD} \cdot \tau_f \cdot \tau_D}{x_d \cdot (\sigma_{df}\tau_f + \sigma_{dD}\tau_D)^2}}\right]$$

8. Dynamics of synchronous machines

Calculation of *d*-axis “short circuit time constants” for $\tau_D << \tau_f$

$$\frac{1}{\tau'_d},\frac{1}{\tau''_d}=\frac{x_d\cdot(\sigma_{df}\tau_f+\sigma_{dD}\tau_D)}{x''_d\cdot 2\cdot\sigma_{fD}\cdot\tau_f\cdot\tau_D}\cdot\left[1\mp\sqrt{1-\frac{4x''_d\cdot\sigma_{fD}\cdot\tau_f\cdot\tau_D}{x_d\cdot(\sigma_{df}\tau_f+\sigma_{dD}\tau_D)^2}}\right]\overset{\tau_D<<\tau_f}{=}\approx$$

$$\approx\frac{x_d\cdot\sigma_{df}}{x''_d\cdot 2\cdot\sigma_{fD}\cdot\tau_D}\cdot\left[1\mp\sqrt{1-\underbrace{\frac{4x''_d\cdot\sigma_{fD}\cdot\tau_D}{x_d\cdot\sigma_{df}^2\tau_f}}_{a}}\right]\approx\frac{x_d\cdot\sigma_{df}}{x''_d\cdot 2\cdot\sigma_{fD}\cdot\tau_D}\cdot\left[1\mp\left(1-\frac{2x''_d\cdot\sigma_{fD}\cdot\tau_D}{x_d\cdot\sigma_{df}^2\tau_f}\right)\right]=$$

$$=\frac{1}{\sigma_{df}\cdot\tau_f},\quad\cong\frac{x_d\cdot\sigma_{df}}{x''_d\cdot\sigma_{fD}\cdot\tau_D}$$

(with $\sqrt{1-a}\cong 1-a/2$ for $a << 1$)

a) Short circuit time constant of field winding =
= transient time constant of *d*-axis

$$\tau'_d\cong\sigma_{df}\cdot\tau_f$$

b) Short circuit time constant of damper winding in *d*-axis =
= subtransient time constant of *d*-axis

$$\tau''_d\cong\frac{x''_d\cdot\sigma_{fD}\cdot\tau_D}{x_d\cdot\sigma_{df}}$$

8. Dynamics of synchronous machines

d-axis: Sub-transient and transient time constants τ'_d, τ''_d

- Subtransient time constant of *d*-axis: $\tau''_d \cong \frac{x''_d \cdot \sigma_{fD} \cdot \tau_D}{x_d \cdot \sigma_{df}}$

- Transient time constant of *d*-axis: $\tau'_d \cong \sigma_{df} \cdot \tau_f$

Example:

$\sigma_{fD} = \sigma_{df} = 0.1,\ x_d = 1,\ x''_d = 0.15,\ \tau_f = 10\tau_D$

$$\tau''_d = \frac{x''_d \cdot \sigma_{fD} \cdot \tau_D}{x_d \cdot \sigma_{df}} = \frac{0.15 \cdot 0.1 \cdot \tau_D}{1 \cdot 0.1} = 0.15 \cdot \tau_D$$

$$\tau'_d = \sigma_{df} \cdot \tau_f = 0.1 \cdot 10 \cdot \tau_D = \tau_D$$

$$\tau'_d / \tau''_d = 1/0.15 = 7$$

Exact values:

$\tau''_d = 0.16 \cdot \tau_D$

$\tau'_d = 0.94 \cdot \tau_D$

If open-circuit time constant of the field winding τ_f is much bigger than of the damper winding $\tau_f >> \tau_D$, then also transient time constant is much bigger than subtransient time constant: $\tau'_d >> \tau''_d$

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q-axis: Sub-transient time constant τ''_q

- Subtransient time constant of *q*-axis $\tau''_q = \sigma_{qQ} \cdot \tau_Q = \tau_{Q\sigma} = \frac{\sigma_{qQ} \cdot x_Q}{r_Q}$

Example: $\sigma_{qQ} = 0.1$, $\tau_Q = \tau_D$ $\quad \tau''_q = \sigma_{qQ} \cdot \tau_Q = 0.1 \cdot \tau_D$

Result: $\tau''_d = 0.15 \cdot \tau_D \qquad \tau''_q = 0.1 \cdot \tau_D$

$$\tau''_d / \tau'_d = 0.15 / 1 = 1/7 \qquad \tau''_q / \tau'_d = 0.1 / 1 = 1/10$$

Subtransient time constants are much shorter than the transient time constant !

"Open circuit" *d*-axis time constants τ'_{d0}, τ''_{d0} for $\tau_D << \tau_f$

$\tau_D << \tau_f$: *d*-axis damper time constant τ_D much shorter than τ_f of field winding:

$$s^2 + s\cdot\frac{\tau_f+\tau_D}{\sigma_{fD}\tau_f\tau_D} + \frac{1}{\sigma_{fD}\tau_f\tau_D} = s^2 + s\cdot p + q = (s+\alpha_{d1})\cdot(s+\alpha_{d2}) = 0$$

$$\alpha_{d1},\alpha_{d2} = \frac{p}{2}\cdot\left(1\mp\sqrt{1-\frac{4q}{p^2}}\right) = \frac{\tau_f+\tau_D}{2\cdot\sigma_{fD}\cdot\tau_f\cdot\tau_D}\left[1\mp\sqrt{1-\frac{4\sigma_{fD}\cdot\tau_f\cdot\tau_D}{(\tau_f+\tau_D)^2}}\right] \underset{\tau_D << \tau_f}{\approx} \frac{1}{2\cdot\sigma_{fD}\cdot\tau_D}\left[1\mp\sqrt{1-\frac{4\sigma_{fD}\cdot\tau_D}{\tau_f}}\right]$$

With $\sqrt{1-a} \cong 1-a/2$ for *a* << 1:

$$\alpha_{d1,2} \approx \frac{1}{2\cdot\sigma_{fD}\cdot\tau_D}\left[1\mp\sqrt{1-\underbrace{\frac{4\sigma_{fD}\cdot\tau_D}{\tau_f}}_{a}}\right] \approx \frac{1}{2\cdot\sigma_{fD}\cdot\tau_D}\left[1\mp\left(1-\frac{2\sigma_{fD}\cdot\tau_D}{\tau_f}\right)\right] = \frac{1}{\tau_f}, \cong \frac{1}{\sigma_{fD}\cdot\tau_D}.$$

a) Open circuit time constant of field (= field winding time constant) $\tau'_{d0} = \tau_f = 1/\alpha_{d1}$

b) Open circuit time constant of damper in *d*-axis $\tau''_{d0} = \sigma_{fD}\cdot\tau_D = 1/\alpha_{d2}$

8. Dynamics of synchronous machines

Example: Rotor time constants for *d*-axis

Given: $x_{dh} = 1.2,\ x_{s\sigma} = 0.15,\ x_{f\sigma} = 0.2,\ x_{D\sigma} = 0.1,\ r_f = 0.002,\ r_D = 0.02$

$$x''_d = 0.21,\ x_d = 1.2 + 0.15 = 1.35,\ x_f = 1.2 + 0.2 = 1.4,\ x_D = 1.2 + 0.1 = 1.3$$

$$\sigma_{df} = 1 - 1.2^2/(1.35 \cdot 1.4) = 0.238,\ \sigma_{fD} = 1 - 1.2^2/(1.3 \cdot 1.4) = 0.209,\ \sigma_{dD} = 1 - 1.2^2/(1.35 \cdot 1.3) = 0.179$$

$$\tau_f = 1.4/0.002 = 700,\ \tau_D = 1.3/0.02 = 65 \rightarrow \tau_D = 65 << \tau_f = 700$$

A) For $\tau_f >> \tau_D$: $\tau'_d \cong 0.238 \cdot 700 = 166.6,\ \tau'_{d0} \cong \tau_f = 700$

$$\tau''_d \cong \frac{0.21 \cdot 0.209 \cdot 65}{1.35 \cdot 0.238} = 8.879,\ \tau''_{d0} \cong 0.209 \cdot 65 = 13.59$$

B) For arbitrary τ_f, τ_D:

$$\frac{1}{\tau'_d}, \frac{1}{\tau''_d} = \frac{1.35 \cdot (0.238 \cdot 700 + 0.179 \cdot 65)}{0.21 \cdot 2 \cdot 0.209 \cdot 700 \cdot 65} \cdot \left[1 \mp \sqrt{1 - \frac{4 \cdot 0.21 \cdot 0.209 \cdot 700 \cdot 65}{1.35 \cdot (0.238 \cdot 700 + 0.179 \cdot 65)^2}}\right] = 0.00619/0.12032$$

$$\frac{1}{\tau'_{d0}}, \frac{1}{\tau''_{d0}} = \frac{700 + 65}{2 \cdot 0.209 \cdot 700 \cdot 65} \cdot \left[1 \mp \sqrt{1 - \frac{4 \cdot 0.209 \cdot 700 \cdot 65}{(700 + 65)^2}}\right] = 0.00133/0.07091$$

$$\tau'_d = 161.5,\ \tau'_{d0} = 752.4,\ \tau''_d = 8.31,\ \tau''_{d0} = 12.64$$

	A)	B)
τ'_{d0}	700.0	752.4
τ'_d	166.6	161.5
τ''_{d0}	13.59	12.64
τ''_d	8.88	8.31

For $\tau_f >> \tau_D$: Simple formulas A) sufficient !

Example: Rotor time constants for *q*-axis

Given: $x_{dh} = 1.2,\ x_{s\sigma} = 0.15,\ x_{f\sigma} = 0.2,\ x_{D\sigma} = 0.1,\ r_f = 0.002,\ r_D = 0.02$

$x_{qh} = 0.6 \qquad x_{Q\sigma} = 0.1 \qquad r_Q = 0.08$

$$x''_q = 0.314 \cdot 0.75 = 0.236,\ x_q = 0.6 + 0.15 = 0.75$$

$$\sigma_{qQ} = 1 - 0.6^2 / (0.75 \cdot 0.7) = 0.314 \qquad \tau_Q = 0.7 / 0.08 = 8.75$$

For arbitrary τ_Q:

$$\tau''_{q0} = \tau_Q = 8.75,\ \tau''_q = 0.314 \cdot 8.75 = 2.75$$

τ''_{d0} , τ''_{q0}	12.64	8.75
τ''_d , τ''_q	8.31	2.75

In the *q*-axis the damper current i_Q vanishes faster than the damper current i_D in the *d*-axis !

8. Dynamics of synchronous machines

Proof: Transient inductance of stator winding in *d*-axis may be described by a simple equivalent circuit !

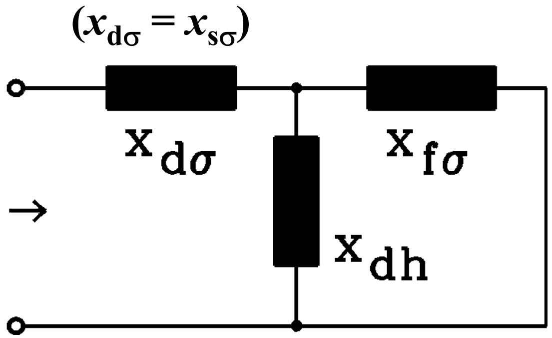
$$x'_d = x''_d \cdot \frac{\frac{1}{\tau''_d} - \frac{1}{\tau'_d}}{\alpha_{d1} + \alpha_{d2} - \frac{1}{\tau'_d}\cdot\left(1 + \frac{x''_d}{x_d}\right)} \underset{\tau_D \ll \tau_f}{\approx} x''_d \cdot \frac{\frac{x_d \cdot \sigma_{df}}{x''_d \cdot \sigma_{fD} \cdot \tau_D} - \frac{1}{\sigma_{df} \cdot \tau_f}}{\frac{1}{\tau_f} + \frac{1}{\sigma_{fD}\tau_D} - \frac{1}{\sigma_{df} \cdot \tau_f}\cdot\left(1 + \frac{x''_d}{x_d}\right)} \underset{\tau_D \ll \tau_f}{\approx} x''_d \cdot \frac{\frac{x_d \cdot \sigma_{df}}{x''_d \cdot \sigma_{fD} \cdot \tau_D}}{\frac{1}{\sigma_{fD}\tau_D}} =$$

$$= x_d \cdot \sigma_{df} = x_{s\sigma} + \frac{x_{dh} x_{f\sigma}}{x_{dh} + x_{f\sigma}}$$

$$x'_d = x_d \cdot \sigma_{df} = x_d \cdot \left(1 - \frac{x_{dh}^2}{x_d x_f}\right) = x_d - \frac{x_{dh}^2}{x_f} = x_{s\sigma} + \frac{x_{dh} x_f}{x_f} - \frac{x_{dh}^2}{x_f} =$$

$$= x_{s\sigma} + \frac{x_{dh} x_{f\sigma}}{x_{dh} + x_{f\sigma}}$$

$x'_d \rightarrow$



Due to $\tau_f \gg \tau_D$ the transient inductance may be easily calculated from an equivalent circuit, where the transient current in the damper winding has already vanished

Example: Transient reactance x'_d

Given: $x_{dh} = 1.2, x_{s\sigma} = 0.15, x_{f\sigma} = 0.2, x_{D\sigma} = 0.1, r_f = 0.002, r_D = 0.02$

$x''_d = 0.21, x_d = 1.2 + 0.15 = 1.35$ $\sigma_{df} = 0.238, \sigma_{fD} = 0.209, \sigma_{dD} = 0.179$

A) For $\tau_f >> \tau_D$:

$$x'_d \cong 0.238 \cdot 1.35 = 0.3213$$

B) For arbitrary τ_f, τ_D:

$$x'_d = 0.21 \cdot \frac{\frac{1}{8.31} - \frac{1}{161.5}}{0.001329 + 0.07091 - \frac{1}{161.5} \cdot \left(1 + \frac{0.21}{1.35}\right)} = 0.3683$$

	A)	B)
x'_d	0.3213	0.3683

Due to $\tau_f >> \tau_D$ the transient inductance may be calculated from an equivalent circuit with sufficient accuracy!

8. Dynamics of synchronous machines

8.6.3 Electric excitation: Dynamic equations for constant speed

$$\breve{u}_d + \psi_{d0} = r_s \cdot \breve{i}_d + s \cdot \breve{\psi}_d - \omega_m \cdot \breve{\psi}_q$$

$$\breve{u}_q + \psi_{q0} = r_s \cdot \breve{i}_q + s \cdot \breve{\psi}_q + \omega_m \cdot \breve{\psi}_d$$

$$\breve{\psi}_d - \frac{\psi_{d0}}{s} = x_d(s) \cdot \left(\breve{i}_d - \frac{i_{d0}}{s} \right) + x_f(s) \cdot \left(\frac{\breve{u}_f}{r_f} - \frac{i_{f0}}{s} \right) - x_D(s) \cdot \frac{i_{D0}}{s}$$

$$\breve{\psi}_q - \frac{\psi_{q0}}{s} = x_q(s) \cdot \left(\breve{i}_q - \frac{i_{q0}}{s} \right) - x_Q(s) \cdot \frac{i_{Q0}}{s}$$

Electrically excited synchronous machine

- **Field winding**
- **Damper cage**
- **Saliency**

Initial conditions:

$\psi_{d0}, \psi_{q0}, i_{d0}, i_{q0}, i_{D0}, i_{Q0}, i_{f0}$

4 unknowns: i_d, i_q, ψ_d, ψ_q

3 given values: u_d, u_q, u_f

"Reactance operators" ***for*** $i_{D0} = i_{Q0} = 0, i_{f0} = u_f / r_f$

$$\begin{cases} \dfrac{1}{x_d(s)} = \dfrac{1}{x_d} + \left(\dfrac{1}{x'_d} - \dfrac{1}{x_d} \right) \cdot \dfrac{s}{s + \dfrac{1}{\tau'_d}} + \left(\dfrac{1}{x''_d} - \dfrac{1}{x'_d} \right) \cdot \dfrac{s}{s + \dfrac{1}{\tau''_d}} \\ \dfrac{1}{x_q(s)} = \dfrac{1}{x_q} + \left(\dfrac{1}{x''_q} - \dfrac{1}{x_q} \right) \cdot \dfrac{s}{s + \dfrac{1}{\tau''_q}} \end{cases}$$

8. Dynamics of synchronous machines

Five reactance operators for electrically excited synchronous machines with damper cage

Reactance operators:

$$\begin{cases} \dfrac{1}{x_d(s)} = \dfrac{1}{x_d} + \left(\dfrac{1}{x'_d} - \dfrac{1}{x_d}\right) \cdot \dfrac{s}{s + \dfrac{1}{\tau'_d}} + \left(\dfrac{1}{x''_d} - \dfrac{1}{x'_d}\right) \cdot \dfrac{s}{s + \dfrac{1}{\tau''_d}} \\ \dfrac{1}{x_q(s)} = \dfrac{1}{x_q} + \left(\dfrac{1}{x''_q} - \dfrac{1}{x_q}\right) \cdot \dfrac{s}{s + \dfrac{1}{\tau''_q}} \end{cases}$$

$$x_d(s) = \frac{s^2 \cdot x''_d \cdot \sigma_{fD} \cdot \tau_f \cdot \tau_D + s \cdot x_d \cdot \left(\sigma_{df} \cdot \tau_f + \sigma_{dD} \cdot \tau_D\right) + x_d}{s^2 \cdot \sigma_{fD} \cdot \tau_f \cdot \tau_D + s \cdot \left(\tau_f + \tau_D\right) + 1}$$

$$x_q(s) = x''_q \cdot \frac{s + \dfrac{1}{\sigma_{qQ} \cdot \tau_Q}}{s + \dfrac{1}{\tau_Q}}$$

$$x_D(s) = \frac{s \cdot \dfrac{x_{f\sigma}}{r_f} + 1}{s^2 \cdot \sigma_{fD} \cdot \tau_f \cdot \tau_D + s \cdot \left(\tau_f + \tau_D\right) + 1} \cdot x_{dh}$$

$$x_Q(s) = \frac{\dfrac{1}{\tau_Q}}{s + \dfrac{1}{\tau_Q}} \cdot x_{qh}$$

$$x_f(s) = \frac{s \cdot \dfrac{x_{D\sigma}}{r_D} + 1}{s^2 \cdot \sigma_{fD} \cdot \tau_f \cdot \tau_D + s \cdot \left(\tau_f + \tau_D\right) + 1} \cdot x_{dh}$$

8.6.4 Permanent magnets: Dynamic equations for constant speed

$$\breve{u}_d + \psi_{d0} = r_s \cdot \breve{i}_d + s \cdot \breve{\psi}_d - \omega_m \cdot \breve{\psi}_q$$

$$\breve{u}_q + \psi_{q0} = r_s \cdot \breve{i}_q + s \cdot \breve{\psi}_q + \omega_m \cdot \breve{\psi}_d$$

$$\breve{\psi}_d - \frac{\psi_p}{s} = x_d \cdot \breve{i}_d \qquad \breve{\psi}_q = x_q \cdot \breve{i}_q$$

$$\psi_{d0} = x_d \cdot i_{d0} + \psi_p \qquad \psi_{q0} = x_q \cdot i_{q0}$$

Permanent magnet excited synchronous machine

- **No field winding**
- **Often no damper cage**
- **Often no saliency** $x_q = x_d$
- **Permanent rotor flux linkage due to permanent magnets** $\psi_p \Leftrightarrow \psi_{f0} = x_{dh} \cdot i_{f0}$

Initial conditions:

$\psi_{d0}, \psi_{q0}, \quad i_{d0}, i_{q0}$

"Reactance operators":

(if no damper cage)

$$\begin{cases} x_d(s) = x_d'' = x_d' = x_d \\ x_q(s) = x_q'' = x_q \end{cases}$$

3 given values: u_d, u_q, ψ_p

4 unknowns: i_d, i_q, ψ_d, ψ_q

8. Dynamics of synchronous machines

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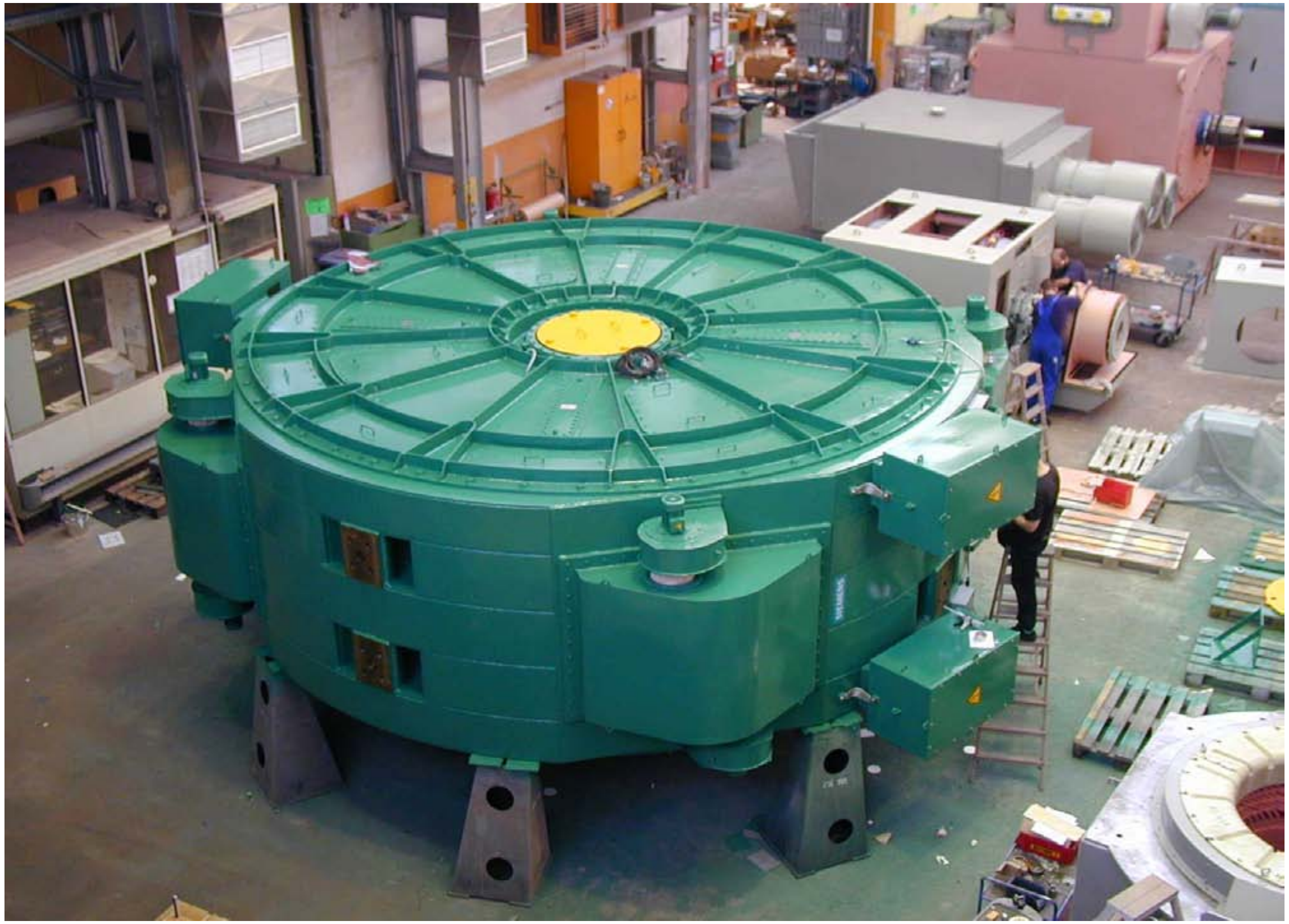
Gearless permanent magnet wind generator *Scanwind / Norway* 3 MW, 17/min



Wind rotor diameter 90 m

Three-blade rotor

Pitch control

Variable speed operation

10 ... 20/min

Gearless drive

IGBT inverter 690 V

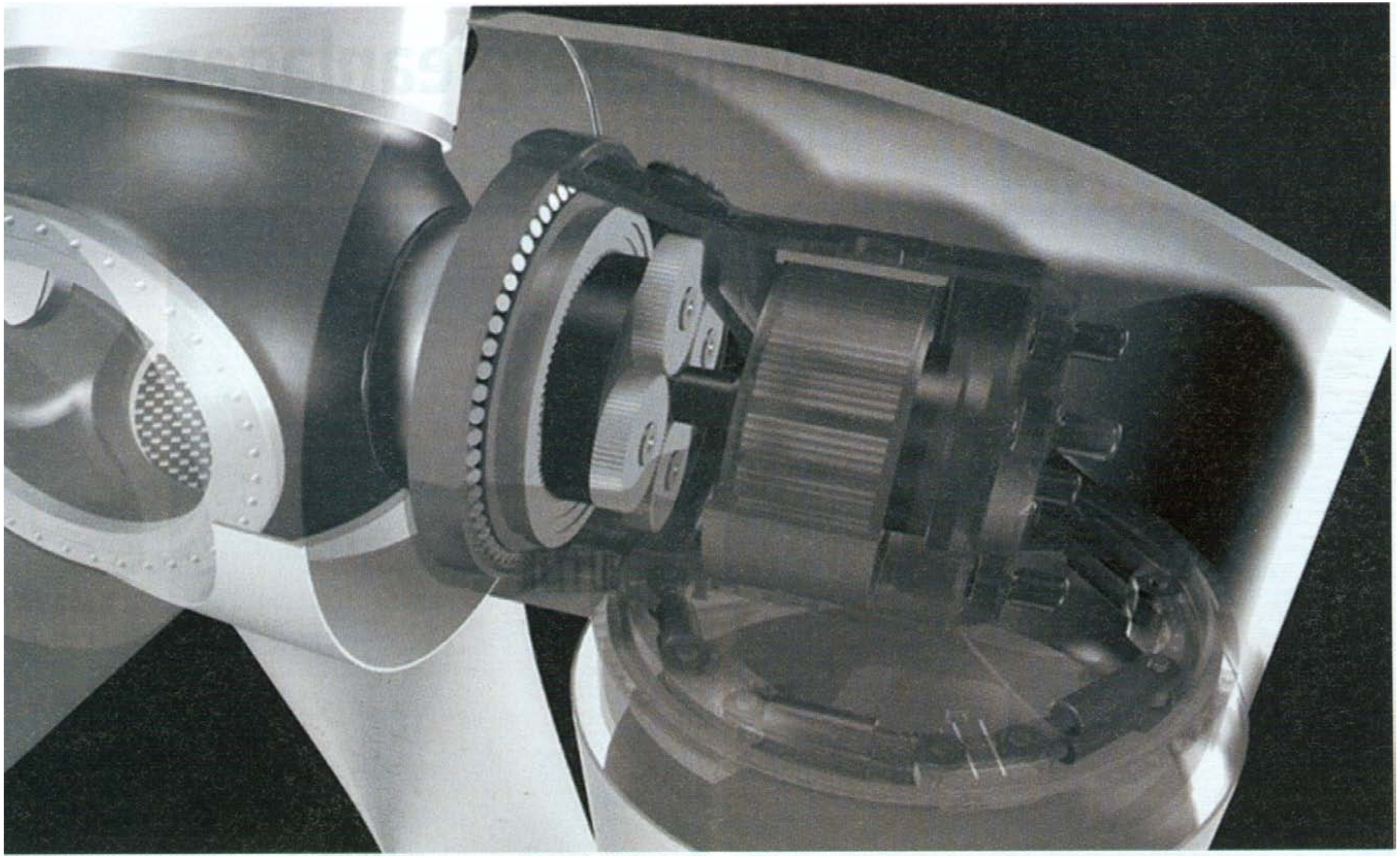
Source:

Siemens AG

Germany

8. Dynamics of synchronous machines

“Multibrid” - permanent magnet wind generator – dual stage gear



Source: Multibrid, Germany

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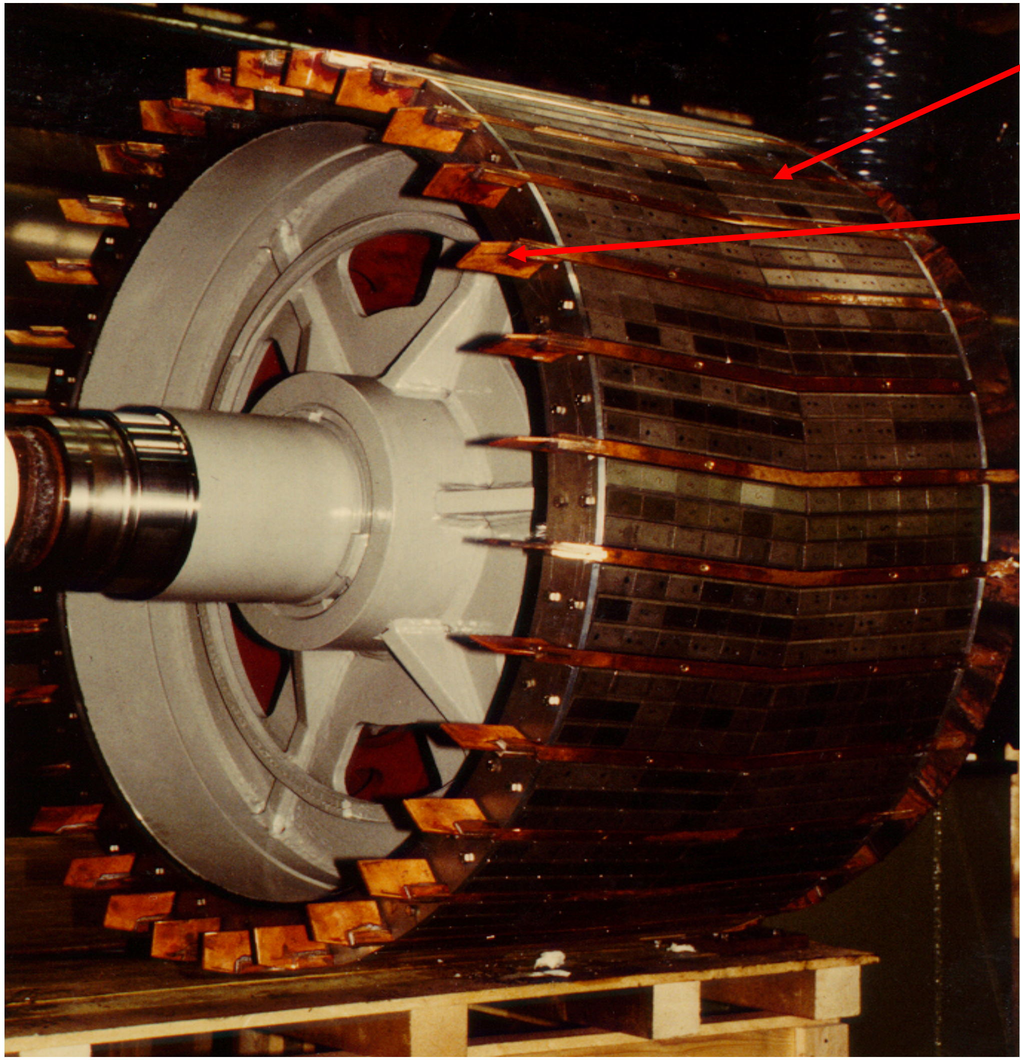
Prototype rotor of PM Synchronous Motor for Submarine propulsion



Skewed rotor magnets to reduce cogging torque

Cooling fins to dissipate rotor magnet eddy current losses and to enhance internal air flow

High pole count (32 poles) for slow speed operation

Source:

Siemens AG, Germany

Summary:
Transient performance of synchronous machines at constant speed operation

- At constant speed and constant iron saturation:
  Linear voltage and flux linkage equations $\Rightarrow$ *Laplace*-transform possible
- Eliminating of rotor flux linkages leads to “reactance operators” as abbreviation
- Five reactance operators, but only two (stator d- and q-axis) mainly of interest
- „Reactance operators“ give the dynamic change of e.g. stator inductances from the small subtransient to the big steady state values
- In PM machines:
  Reactance operators are simply the steady-state inductances
  (if eddy currents in the rotor magnets are negligible)
- In voltage-fed machines the inverse reactance operators are needed
- „Reactance operators“ are flux-current transfer functions

8. Dynamics of synchronous machines

8.1 Basics of steady state and significance of dynamic performance of synchronous machines

8.2 Transient flux linkages of synchronous machines

8.3 Set of dynamic equations for synchronous machines

8.4 *Park* transformation

8.5 Equivalent circuits for magnetic coupling in synchronous machines

8.6 Transient performance of synchronous machines at constant speed operation

8.7 Time constants of electrically excited synchronous machines with damper cage

8.8 Sudden short circuit of electrically excited synchronous machine with damper cage

8.9 Sudden short circuit torque and measurement of transient machine parameters

8.10 Transient stability of electrically excited synchronous machines

8. Dynamics of synchronous machines

Rotor time constants for *d*-axis (for $\tau_f >> \tau_D$)

	From dynamic equations	From equivalent circuit
τ'_{d0}	$\tau'_{d0} \cong \tau_f$	$\tau'_{d0} \cong x_f / r_f = \tau_f$
τ''_{d0}	$\tau''_{d0} \cong \sigma_{fD} \cdot \tau_D$	$\tau''_{d0} \cong \dfrac{x_{D\sigma} + x_{dh} \cdot (x_{f\sigma} / x_f)}{r_D}$
τ'_d	$\tau'_d \cong \sigma_{df} \cdot \tau_f$	$\tau'_d \cong \left(x_{f\sigma} + \dfrac{x_{dh} x_{s\sigma}}{x_{dh} + x_{s\sigma}} \right) / r_f = \dfrac{x'_d}{x_d} \cdot \tau'_{d0}$
τ''_d	$\tau''_d \cong \dfrac{x''_d \cdot \sigma_{fD} \cdot \tau_D}{x_d \cdot \sigma_{df}}$	$\tau''_d \cong \dfrac{x_{D\sigma} + \dfrac{x_{dh} x_{f\sigma} x_{s\sigma}}{x_{dh} x_{f\sigma} + x_{dh} x_{s\sigma} + x_{f\sigma} x_{s\sigma}}}{r_D}$

- Although looking different, these expressions are IDENTICAL relations !

Subtransient time constant of *d*-axis ($\tau_D << \tau_f$)

a) Short-circuit time constant:

$$\underline{\underline{\tau''_d}} \cong \frac{x''_d \cdot \sigma_{fD} \cdot \tau_D}{x_d \cdot \sigma_{df}} = x''_d \cdot \frac{1 - \frac{x_{dh}^2}{x_f x_D}}{1 - \frac{x_{dh}^2}{x_f x_d}} \cdot \frac{x_D}{x_d} \cdot \frac{1}{r_D} = x''_d \cdot \frac{x_f x_D - x_{dh}^2}{x_f x_d - x_{dh}^2} \cdot \frac{1}{r_D} =$$

$$= \left(x_{s\sigma} + \frac{1}{\frac{1}{x_{f\sigma}} + \frac{1}{x_{D\sigma}} + \frac{1}{x_{dh}}} \right) \cdot \frac{\frac{1}{x_{f\sigma}} + \frac{1}{x_{D\sigma}} + \frac{1}{x_{dh}}}{\frac{1}{x_{f\sigma}} + \frac{1}{x_{s\sigma}} + \frac{1}{x_{dh}}} \cdot \frac{x_{D\sigma}}{x_{s\sigma}} \cdot \frac{1}{r_D} = \left(x_{D\sigma} + \frac{1}{\frac{1}{x_{f\sigma}} + \frac{1}{x_{s\sigma}} + \frac{1}{x_{dh}}} \right) \cdot \frac{1}{r_D} =$$

$$\underline{\underline{= (x_{D\sigma} + \frac{x_{dh} x_{f\sigma} x_{s\sigma}}{x_{dh} x_{f\sigma} + x_{dh} x_{s\sigma} + x_{f\sigma} x_{s\sigma}}) \cdot \frac{1}{r_D}}}$$

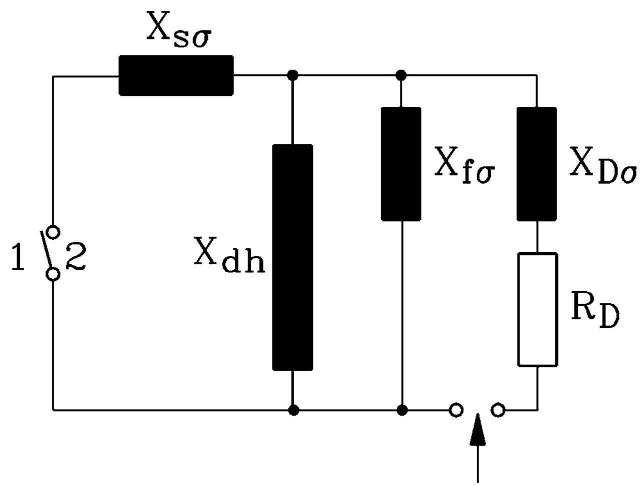
b) Open-circuit time constant:

$$\tau''_{d0} \cong \sigma_{fD} \cdot \tau_D = \left(1 - \frac{x_{dh}^2}{x_f x_D} \right) \cdot \frac{x_D}{r_D} = \left(x_D - \frac{x_{dh}^2}{x_f} \right) \cdot \frac{1}{r_D} = \left(x_{D\sigma} + \frac{x_{dh} x_f}{x_f} - \frac{x_{dh}^2}{x_f} \right) \cdot \frac{1}{r_D} = \underline{\underline{\left(x_{D\sigma} + \frac{x_{dh} x_{f\sigma}}{x_f} \right) \cdot \frac{1}{r_D}}}$$

8. Dynamics of synchronous machines

Subtransient time constant of *d*-axis ($\tau_D << \tau_f$)

- Influence of damper winding on i_s (decay of damper current)
- Stator winding connected to grid (switch position 2): Grid internal impedance is assumed zero
- Stator winding is open-circuited (switch position 1): time constant changes to τ''_{d0}
- Resistance of considered winding: r_D
- Resultant inductance: Coupling of stator, field and damper winding



Resultant inductance x_{res}

$$x_{res} = x_{D\sigma} + \frac{x_{dh} x_{f\sigma} x_{s\sigma}}{x_{dh} x_{f\sigma} + x_{dh} x_{s\sigma} + x_{f\sigma} x_{s\sigma}}$$

$$\tau''_d \cong \frac{x_{D\sigma} + \dfrac{x_{dh} x_{f\sigma} x_{s\sigma}}{x_{dh} x_{f\sigma} + x_{dh} x_{s\sigma} + x_{f\sigma} x_{s\sigma}}}{r_D}$$

Switch position 2

$$\tau''_{d0} \cong \frac{x_{D\sigma} + x_{dh} \cdot (x_{f\sigma} / x_f)}{r_D}$$

Switch position 1

8. Dynamics of synchronous machines

Transient short-circuit time constant of *d*-axis ($\tau_D \ll \tau_f$)

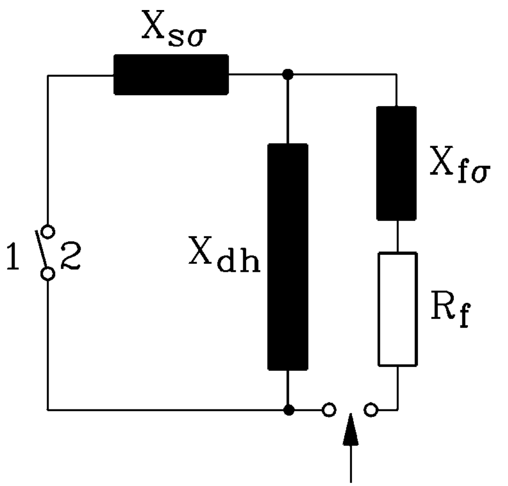
$$\tau'_d \cong \sigma_{df} \cdot \tau_f = \left(1 - \frac{x_{dh}^2}{x_d x_f}\right) \cdot \frac{x_f}{r_f} = \left(x_f - \frac{x_{dh}^2}{x_d}\right) \cdot \frac{1}{r_f} =$$

$$= \left(x_{f\sigma} + \frac{x_{dh} x_d}{x_d} - \frac{x_{dh}^2}{x_d}\right) \cdot \frac{1}{r_f} = \underline{\underline{\left(x_{f\sigma} + \frac{x_{dh} x_{s\sigma}}{x_d}\right) \cdot \frac{1}{r_f}}}$$

or

$$\underline{\underline{\tau'_d}} \cong \frac{x'_d}{x_d} \cdot \tau'_{d0} = \frac{\sigma_{df} x_d}{x_d} \cdot \tau'_{d0} = \frac{\sigma_{df} x_d}{x_d} \cdot \frac{x_f}{r_f} = \sigma_{df} \cdot \frac{x_f}{r_f} = \underline{\underline{\sigma_{df} \cdot \tau_f}}$$

Transient time constant of *d*-axis ($\tau_D << \tau_f$)



Resultant inductance x_{res}

- Influence of field winding on i_s (decay of field current)
- Damper current already zero !
- Stator winding connected to grid ("Switch in position 2"): Grid internal impedance is assumed zero
- If stator winding is open-circuited ("Switch: position 1"): Time constant changes to τ'_{d0}
- Resistance of considered winding: r_f
- Resultant inductance: Coupling of stator and field winding

$$x_{res} = x_{f\sigma} + \frac{x_{dh} x_{s\sigma}}{x_{dh} + x_{s\sigma}}$$

$$\tau'_d \cong \sigma_{df} \cdot \tau_f = \left(x_{f\sigma} + \frac{x_{dh} x_{s\sigma}}{x_{dh} + x_{s\sigma}} \right) / r_f$$

Switch position 2

$$\tau'_{d0} \cong x_f / r_f = \tau_f$$

Switch position 1

8. Dynamics of synchronous machines

Rotor time constant for *q*-axis (exact formulas!)

	From dynamic equations	From equivalent circuit
τ''_{q0}	$\tau''_{q0} = \tau_Q$	$\tau''_{q0} = \frac{x_Q}{r_Q}$
τ''_q	$\tau''_q = \sigma_{qQ} \cdot \tau_Q$	$\tau''_q = \frac{x_{Q\sigma} + \frac{x_{qh} x_{s\sigma}}{x_{qh} + x_{s\sigma}}}{r_Q}$

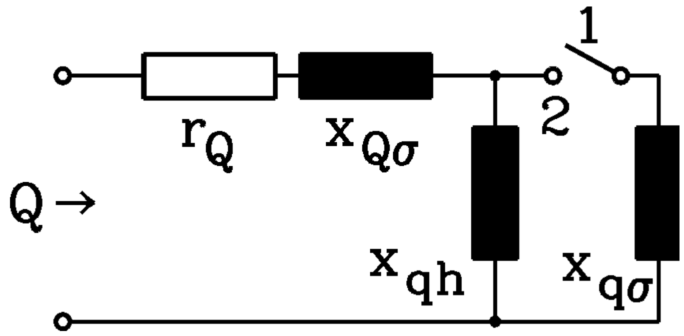
- Although looking different, these expressions are IDENTICAL relations !

- NO influence of field winding in q-axis ! $\Rightarrow$ No transient time constant of q-axis !

8. Dynamics of synchronous machines

Subtransient short- & open-circuit time constants of *q*-axis (exact)

- Influence of damper winding on i_s:
  Decay of q-component of damper current
- Stator winding connected to grid ("Switch in position 2"):
  Grid internal impedance is assumed zero
- If stator winding is **open-circuited or** current source operation: ("switch in position 1") ⇒
  Time constant changes to $\tau''_{q0} = \tau_Q = x_Q / r_Q$
- Resistance of considered winding: r_Q
- Resultant inductance: Coupling of stator and damper winding



$(x_{q\sigma} = x_{s\sigma})$

Resultant inductance x_{res}

$$x_{res} = x_{Q\sigma} + \frac{x_{qh} x_{s\sigma}}{x_{qh} + x_{s\sigma}}$$

$$\tau''_q = \tau_{Q\sigma} = \sigma_{qQ} \cdot \tau_Q = \left(1 - \frac{x_{qh}^2}{x_q x_Q}\right) \cdot \frac{x_Q}{r_Q} = \left(x_Q - \frac{x_{qh}^2}{x_q}\right) \cdot \frac{1}{r_Q} =$$

$$= \left(x_{Q\sigma} + \frac{x_{qh} x_q}{x_q} - \frac{x_{qh}^2}{x_q}\right) \cdot \frac{1}{r_Q} = \underline{\underline{\left(x_{Q\sigma} + \frac{x_{qh} x_{s\sigma}}{x_q}\right) \cdot \frac{1}{r_Q}}}$$

$$\boxed{\tau''_q = \left(x_{Q\sigma} + \frac{x_{qh} x_{s\sigma}}{x_{qh} + x_{s\sigma}}\right) / r_Q}$$ Switch position 2

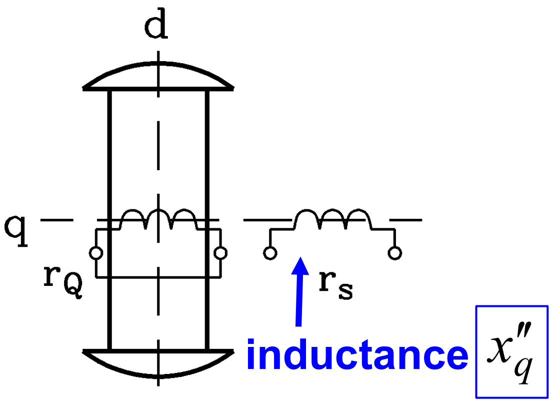
$$\boxed{\tau''_{q0} = x_Q / r_Q}$$ Switch position 1

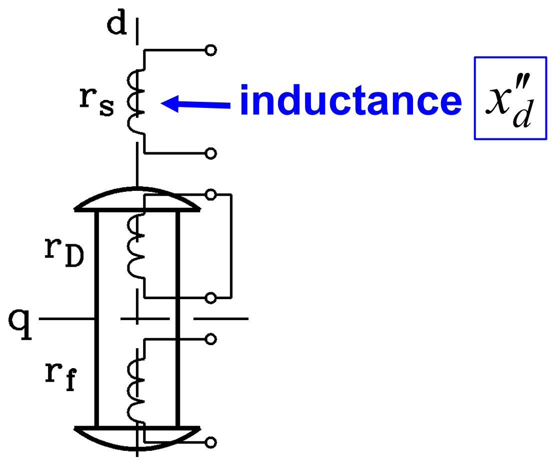
8. Dynamics of synchronous machines

Armature time constant τ_a - both for *d*- and *q*-axis

- Influence of stator winding on transient DC component of i_s
- Resistance of considered winding: r_s (<< 1)
- Resultant stator inductance immediately after disturbance is "average" of *d*- and *q*-axis subtransient inductance:

  "Average": Electrical paralleling of *d*- and *q*-axis: $x_{res} = \dfrac{2x''_d \cdot x''_q}{x''_d + x''_q}$





$$\tau_a = \frac{2x''_d \cdot x''_q}{(x''_d + x''_q) \cdot r_s}$$

Note: If *d*- and *q*-axis are identical $x''_d = x''_q$, then we get: $x_{res} = \dfrac{2x''_d \cdot x''_d}{x''_d + x''_d} = x''_d$

Summary: Stator and rotor time constants

d-axis time constants (τ_D << τ_f)

- **Time constant: "resultant inductance / resistance of considered winding"**
- **d-axis three winding transformer: Winding of stator + rotor damper + rotor field**

- **Three time constants for variation e.g. of stator current i_s:**

Subtransient time constant τ_d'' **:** Influence of decay of current in damper winding on i_s

Transient time constant τ_d' **:** Influence of decay of transient field current on i_s
Damper current already zero !

Armature time constant τ_a **:** Influence of stator winding on transient DC component of i_s

$$\tau_d'' = \frac{x_{D\sigma} + \dfrac{x_{dh}x_{f\sigma}x_{s\sigma}}{x_{dh}x_{f\sigma} + x_{dh}x_{s\sigma} + x_{f\sigma}x_{s\sigma}}}{r_D}$$

$$\tau_d' = \frac{x_{f\sigma} + \dfrac{x_{dh} \cdot x_{s\sigma}}{x_{dh} + x_{s\sigma}}}{r_f}$$

$$\tau_a = \frac{2x_d'' \cdot x_q''}{(x_d'' + x_q'') \cdot r_s}$$

8. Dynamics of synchronous machines

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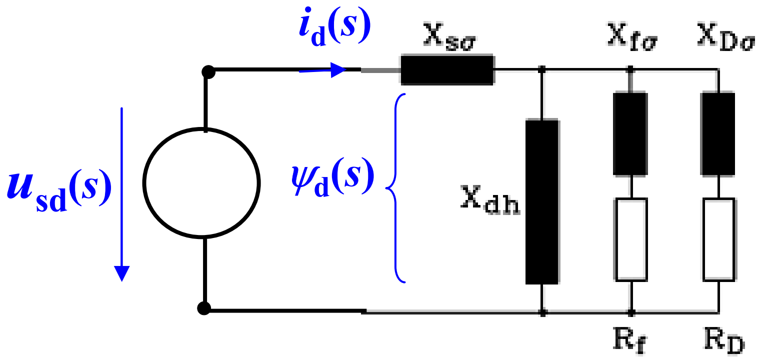
Magnitudes of time constants of electrically excited synchronous machines with damper cage ($\tau_D << \tau_f$)

Range of values for small (1 MVA)... big (2000 MVA) machines:

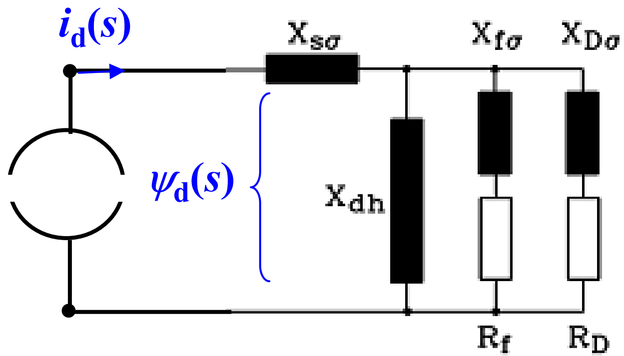
Small ... big

Transient open circuit time constant of *d*-axis	$T'_{d0} = L_f / R_f = T_f$	2 ...7 ... 10 s
Transient short circuit time constant of *d*-axis	$T'_d = (L'_d / L_d) \cdot T'_{d0}$	0.6-0.8 ... 1-2 s
Subtransient open circuit time constant of *d*-axis	T''_{d0}	$T''_{d0} \approx (1.1 - 1.5) \cdot T''_d$
Subtransient short circuit time constant of *d*-axis	$T''_d \approx T''_{d0} \cdot L''_d / L'_d$	0.02 ... 0.1 ... 0.5 s
Subtransient short circuit time constant of *q*-axis	$T''_q \approx T''_d$	0.02 ... 0.1 ... 0.5 s
Armature time constant	$T_a \approx L''_d / R_s$	0.1 ... 0.4-0.5 s
Acceleration time constant (starting time constant)	T_J	3 ... 8-10 s

"Reactance operator" $x_d(s)$ = Flux-current transfer function *d*-axis



$\omega_m = 0$



Voltage source feeding (internal resistance zero): „stator short circuit operation"

$$\breve{i}_d = \breve{\psi}_d / x_d(s)$$

$$\frac{1}{x_d(s)} = \frac{1}{x''_d} \cdot \frac{(s + \frac{1}{\tau'_{d0}}) \cdot (s + \frac{1}{\tau''_{d0}})}{(s + \frac{1}{\tau'_d}) \cdot (s + \frac{1}{\tau''_d})}$$

Time constants for voltage step response:

$$\tau'_d, \tau''_d$$

Current source feeding (internal resistance infinite): „stator open circuit operation"

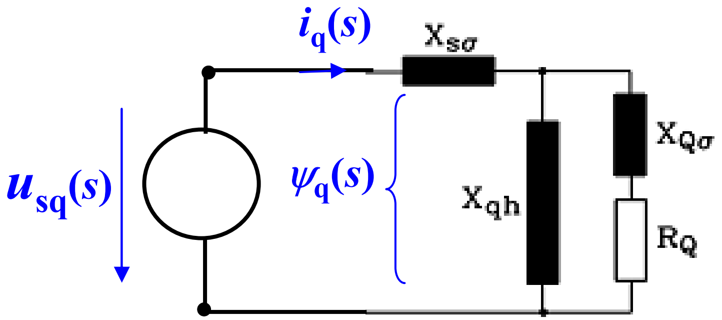
$$\breve{\psi}_d = x_d(s) \cdot \breve{i}_d$$

$$x_d(s) = x''_d \cdot \frac{(s + \frac{1}{\tau'_d}) \cdot (s + \frac{1}{\tau''_d})}{(s + \frac{1}{\tau'_{d0}}) \cdot (s + \frac{1}{\tau''_{d0}})}$$

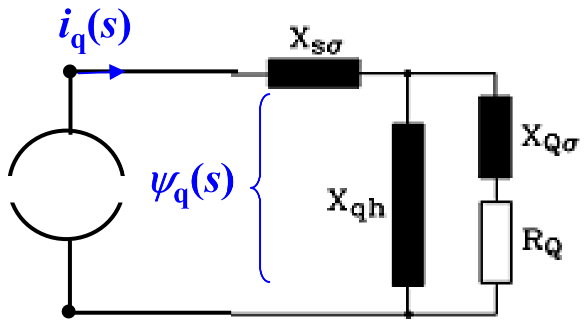
Time constants for current step response:

$$\tau'_{d0}, \tau''_{d0}$$

"Reactance operator" $x_q(s)$ = Flux-current transfer function q-axis



$$\omega_m = 0$$



Voltage source feeding (internal resistance <u>zero</u>): „stator <u>short</u> circuit operation"

$$\breve{i}_q = \breve{\psi}_q / x_q(s)$$

$$\frac{1}{x_q(s)} = \frac{1}{x''_q} \cdot \frac{s + \frac{1}{\tau''_{q0}}}{s + \frac{1}{\tau''_q}}$$

Time constant for voltage step response:

$$\tau''_q$$

Current source feeding (internal resistance <u>infinite</u>): „stator <u>open</u> circuit operation"

$$\breve{\psi}_q = x_q(s) \cdot \breve{i}_q$$

$$x_q(s) = x''_q \cdot \frac{s + \frac{1}{\tau''_q}}{s + \frac{1}{\tau''_{q0}}}$$

Time constant for current step response:

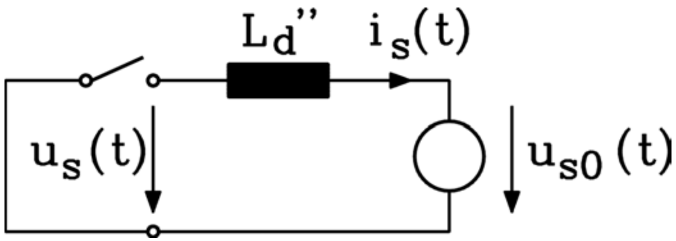
$$\tau''_{q0}$$

Summary: Time constants of electrically excited synchronous machines with damper cage

- Sub-transient and (in *d*-axis) transient rotor time constants for two cases:
- Case A: Stator is open circuit = no-load time constants T_{d0}'', T_{d0}', T_{q0}''
- Case B: Stator connected to (ideal stiff) grid = = Short-circuit time constants T_d'', T_d', T_q''
- Equivalent circuits for *d*- and *q*-axis explain the time constants, but:
  d-axis equivalent circuits for x_d', T_d'', T_d', T_{d0}'', T_{d0}' are only valid for $T_D << T_f$,
  BUT: for x_d'' and *q*-axis equivalent circuits for x_q'', T_q'', T_{q0}'' values are exact!
- Stator: Armature time constant T_a only valid for $r_s << 1$
- In addition: Mechanical time constant T_m in case of variable speed operation (here not presented)
- Starting time constant T_J represents rotor inertia

Non-damped sudden short circuit of synchronous machine

- Synchronous generator at no-load (= open stator circuit)
- **Sudden short circuited** at all three stator terminals at t = 0, calculated with physical units
- Induced no-load voltage causes a dynamic short circuit current $i_s(t)$
- Due to rotor transient currents the subtransient inductance is active: $L''_d = X''_d / \omega_s$
- Non-damped: $R_s = 0$, $L_s = L_d''$



No-load voltage: (e.g. phase U):

$$u_s(t) = \hat{U} \cdot \sin(\omega_s t + \varphi_0) = u_{s0}(t),\ \omega_s = 2\pi f_s$$

$R_s = 0,\ L''_d = L''_q$, Initial condition: $i_s(0) = 0$:

$$u_s(t) = 0 = u_{s0}(t) + L''_d \cdot di_s / dt \qquad i_s(t) = -\frac{1}{L''_d}\int_0^t u_{s0}(t) \cdot dt = \frac{\hat{U}}{\omega_s L''_d} \cdot (\cos(\omega_s t + \varphi_0) - \cos\varphi_0)$$

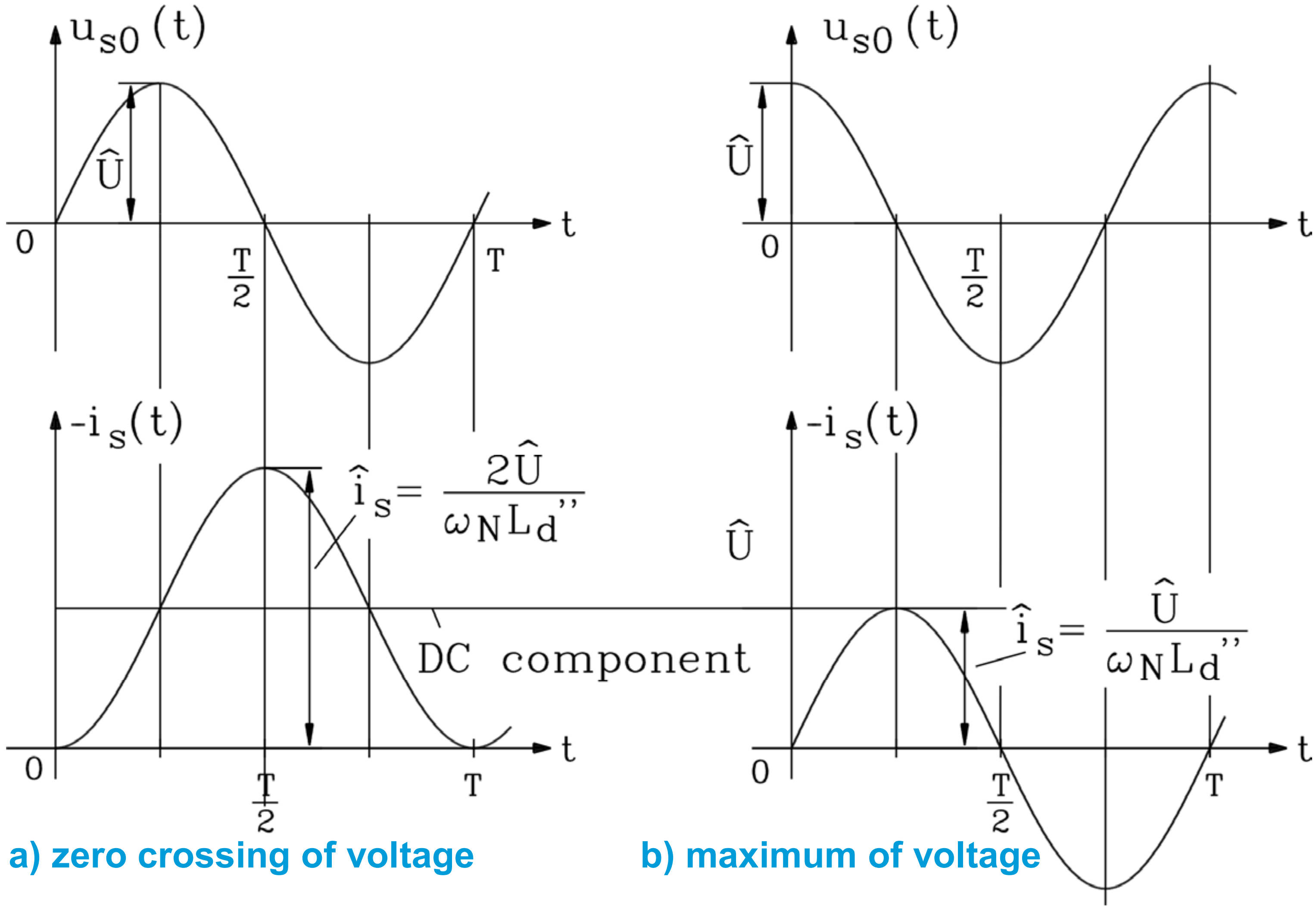
a) Short circuit at zero crossing of voltage: $\varphi_0 = 0: \quad i_s(t) = \frac{\hat{U}}{\omega_s L''_d} \cdot (\cos(\omega_s t) - 1)$

b) Short circuit at maximum voltage: $\varphi_0 = \pi / 2 : i_s(t) = -\frac{\hat{U}}{\omega_s L''_d} \cdot \sin(\omega_s t) = -\hat{I}''_k \cdot \sin(\omega_s t)$

Sudden short circuit current at R_s = 0 (1)

Example:

$\omega_s = \omega_N$



Short circuit at a) zero crossing of voltage b) maximum of voltage

8. Dynamics of synchronous machines

Sudden short circuit current at R_s = 0 (2)

a) Short circuit at **zero crossing of stator voltage:**
The current rises from 0 at t = 0 to double value of AC amplitude $\hat{I}''_k$ at time $t = \pi / \omega_s$

A subtransient DC current with same amplitude as AC current flows in the stator winding:

$$\hat{U} = \sqrt{2}U, \quad I_{DC} = \hat{I}''_k \Rightarrow \hat{I}_k = 2\hat{I}''_k = 2\sqrt{2}I''_k = \frac{2\sqrt{2}U}{\omega_s L''_d} = \frac{2\sqrt{2}U}{X''_d}$$

b) Short circuit at **maximum of stator voltage**:
No subtransient DC current occurs, so current peak $\hat{I}_k$ is half the previous value.

$$I_{DC} = 0 \Rightarrow \hat{I}_k = \hat{I}''_k = \sqrt{2}I''_k = \frac{\sqrt{2}U}{\omega_s L''_d} = \frac{\sqrt{2}U}{X''_d}$$

Example: $x''_d = 0.15\text{p.u.}, \hat{U}_U / \hat{U}_N = 1$

$$a) \quad \frac{\hat{I}_k}{\sqrt{2}I_N} = \frac{2U_U / U_N}{X''_d / Z_N} = \frac{2 \cdot 1}{0.15} = \underline{\underline{13.33}} \quad \text{VERY HIGH} \qquad b) \quad \frac{\hat{I}_k}{\sqrt{2}I_N} = \frac{1}{0.15} = \underline{\underline{6.66}}$$

8. Dynamics of synchronous machines

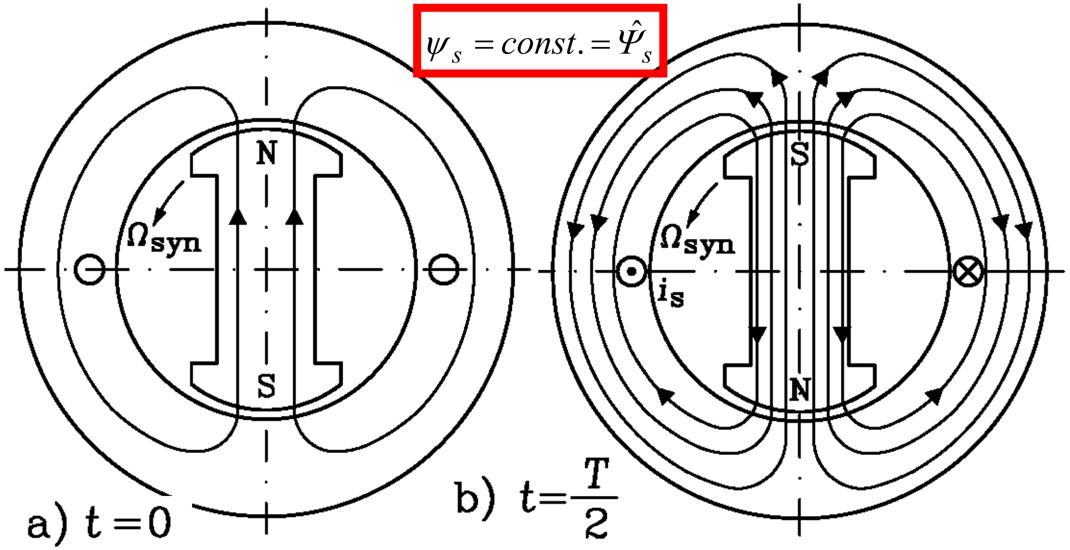
Sudden short circuit at zero voltage crossing at no-load (R_s = 0, rotor PM excitation) (1)

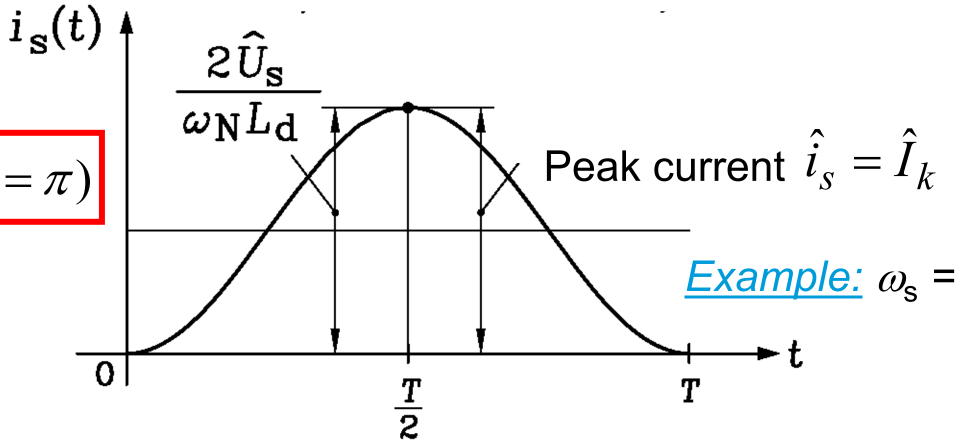
$t < 0 : \hat{U}_s = \omega_s \hat{\Psi}_s$

$t \geq 0 :$

$u_s = d\psi_s / dt = 0$

$\psi_s = const. = \hat{\Psi}_s$





$\hat{i}_s = 2\hat{\Psi}_s / L_d = 2\hat{U}_s / (\omega_s L_d) = i_s(\omega_s t = \pi)$

Example: $\omega_s = \omega_N$

8. Dynamics of synchronous machines

Sudden short circuit at zero voltage crossing at no-load (R_s = 0, rotor PM excitation) (2)

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Physics explanation of sudden short circuit situation:

- Rotor turns with speed Ω_{syn}, and is PM-excited
- Open-circuit stator coil voltage is induced $\hat{U}_s = \omega_s \hat{\Psi}_s$

a) At t = 0: Flux linkage of stator coil is maximum: $\psi_s = \hat{\Psi}_s$, so induced voltage is ZERO.
Now short circuit occurs.
Flux linkage stays constant: $u_s = d\psi_s / dt = 0$ $\psi_s = const. = \hat{\Psi}_s$

b) After half rotor turn ($\omega_s t = \pi$) rotor field is linked to stator with inverse polarity: $\psi_s = -\hat{\Psi}_s$
In the stator coil short circuit current i_s must flow to excite additional flux linkage $2\hat{\Psi}_s$
to keep total flux linkage constant. $2\hat{\Psi}_s - \hat{\Psi}_s = \hat{\Psi}_s = const.$
PM machine without damper: $L''_d = L_d$

So we get the current: $L_d \hat{i}_s = 2\hat{\Psi}_s$ $\hat{i}_s = 2\hat{\Psi}_s / L_d = 2\hat{U}_s / (\omega_s L_d) = i_s(\omega_s t = \pi)$

8. Dynamics of synchronous machines

Physics of transient inductance L_d'

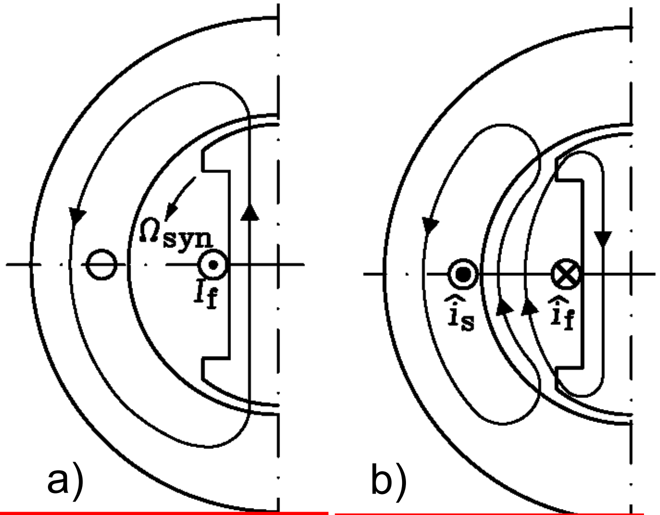
$$t < 0: \hat{U}_s = \omega_s \hat{\Psi}_s$$

$$t \geq 0:$$

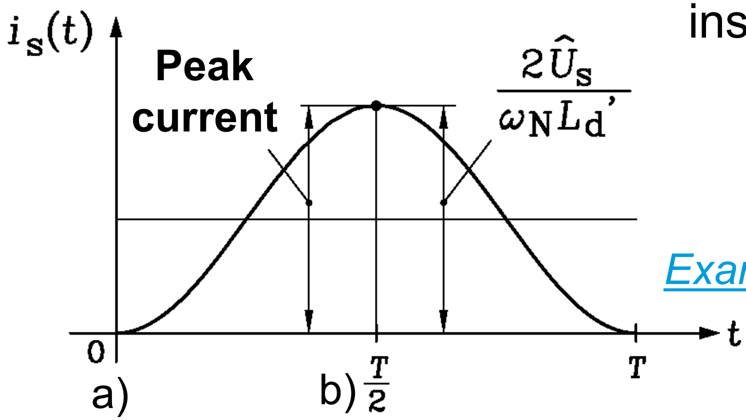
$$u_s = d\psi_s / dt = 0 \rightarrow \psi_s = const.$$

Rotor is electrically excited with field current I_f, thus short-circuited by u_f-voltage source

$$u_f = d\psi_f / dt = 0 \rightarrow \psi_f = const.$$



$\psi_s = const. = \hat{\Psi}_s$ | $\psi_f = const. = \hat{\Psi}_f$



Stator flux is rejected from rotor via transient rotor field,

must pass via air-gap,

yields reduced inductance L_d' instead of L_d

Example: $\omega_s = \omega_N$

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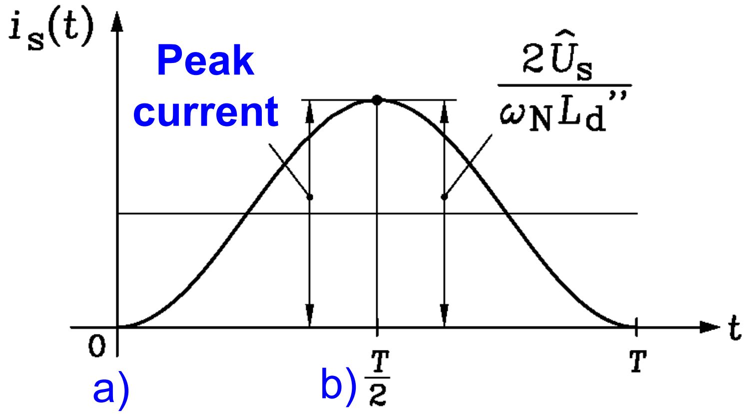
Physics of subtransient inductance L_d', L_d''

- **Rotor is electrically excited with field current I_f**
- Open-circuit stator coil voltage: $\hat{U}_s = \omega_s \hat{\Psi}_s$

a) At $t = 0$ flux linkage of stator coil is maximum $\psi_s = \hat{\Psi}_s$, so induced voltage is ZERO. Now short circuit occurs. Flux linkage stays constant: $u_s = d\psi_s / dt = 0 \rightarrow \psi_s = const.$

b) After half a rotor turn ($\omega_s t = \pi$) rotor field has reversed. Rotor winding nearly short-circuited by feeding DC voltage source: $u_f = d\psi_f / dt = 0 \rightarrow \psi_f = const.$

In the stator coil short circuit current i_s must flow to keep $\psi_s = const. = \hat{\Psi}_s$, but stator and rotor current oppose, so stator and rotor field must close via air-gap, which decreases stator inductance: $L_d \rightarrow L_d' << L_d$



With add. damper cage: $L_d \rightarrow L_d'' << L_d$

Big stator current $L_d'' \hat{i}_s = 2\hat{\Psi}_s$ needed for $\psi_s = const.$

Dynamic field current is big to keep $\psi_f = \hat{\Psi}_f = const.: \hat{i}_f >> I_f$

$$\hat{i}_s = 2\hat{\Psi}_s / L_d'' = 2\hat{U}_s / (\omega_s L_d'') = i_s(\omega_s t = \pi)$$

8. Dynamics of synchronous machines

Sudden short circuit with damping

Sudden short circuit in stator winding after no-load generator (at constant speed $\omega_m = 1$)

$$\breve{u}_d + \psi_{d0} = r_s \cdot \breve{i}_d + s \cdot \breve{\psi}_d - \omega_m \cdot \breve{\psi}_q \Rightarrow \psi_{d0} = r_s \cdot \breve{i}_d + s \cdot \breve{\psi}_d - \omega_m \cdot \breve{\psi}_q$$

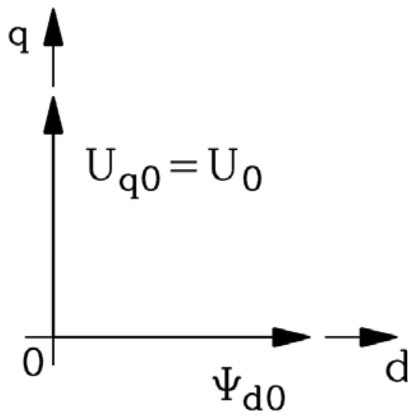
$$\breve{u}_q + \psi_{q0} = r_s \cdot \breve{i}_q + s \cdot \breve{\psi}_q + \omega_m \cdot \breve{\psi}_d \Rightarrow 0 = r_s \cdot \breve{i}_q + s \cdot \breve{\psi}_q + \omega_m \cdot \breve{\psi}_d$$

$$\breve{\psi}_d - \frac{\psi_{d0}}{s} = x_d(s) \cdot \breve{i}_d$$

$$\breve{\psi}_q = x_q(s) \cdot \breve{i}_q$$

Sudden short circuit in stator winding $\Rightarrow u_d = u_q = 0$

after no-load: Generator no-load = zero stator & damper currents!



Initial conditions = Generator no-load:

$$u_{f0} = r_f \cdot i_{f0} \qquad i_{D0} = 0 \qquad i_{Q0} = 0 \qquad i_{d0} = i_{q0} = 0$$

$$\psi_{d0} = x_d i_{d0} + x_{dh} i_{f0} = x_{dh} i_{f0}, \qquad \psi_{q0} = x_q i_{q0} = 0$$

$$u_{d0} = r_s \cdot i_{d0} - \omega_m \cdot x_q i_{q0} = 0$$

$$u_{q0} = r_s \cdot i_{q0} + \omega_m \cdot x_d i_{d0} + \omega_m \cdot x_{dh} i_{f0} = \omega_m \cdot x_{dh} i_{f0} = u_0$$

Solution of stator voltage equations using reactance operators

$$(r_s + s \cdot x_d(s)) \cdot \breve{i}_d + \psi_{d0} - \omega_m x_q(s) \cdot \breve{i}_q = \psi_{d0}$$

$$(r_s + s \cdot x_q(s)) \cdot \breve{i}_q + \omega_m x_d(s) \cdot \breve{i}_d = -\omega_m \cdot \psi_{d0} / s = -u_0 / s$$

$$\begin{pmatrix} r_s + s \cdot x_d(s) & -\omega_m x_q(s) \\ \omega_m x_d(s) & r_s + s \cdot x_q(s) \end{pmatrix} \cdot \begin{pmatrix} \breve{i}_d \\ \breve{i}_q \end{pmatrix} = \begin{pmatrix} 0 \\ -u_0 / s \end{pmatrix}$$

$$\breve{i}_d = -\frac{u_0}{s} \cdot \frac{\omega_m x_q(s)}{Det}, \quad \breve{i}_q = -\frac{u_0}{s} \cdot \frac{r_s + s \cdot x_d(s)}{Det}$$

$$Det = (r_s + s \cdot x_d(s)) \cdot (r_s + s \cdot x_q(s)) + \omega_m^2 x_d(s) x_q(s)$$

We use for $t = 0$:

$$\lim_{t \to 0+} f(t) \Leftrightarrow \lim_{\underline{s} \to \infty} \underline{s} \cdot F(\underline{s})$$

$$\underline{\tau \to 0:} \quad \frac{r_s}{x_d(s)} \approx \frac{r_s}{x_d(s \to \infty)} \approx \frac{r_s}{x''_d}$$

$$\frac{r_s}{x_q(s)} \approx \frac{r_s}{x_q(s \to \infty)} \approx \frac{r_s}{x''_q}$$

$$\underline{r_s << 1:} \quad Det = x_d(s) x_q(s) \left[\left(\frac{r_s}{x_d(s)} + s\right) \cdot \left(\frac{r_s}{x_q(s)} + s\right) + \omega_m^2 \right] \approx x_d(s) x_q(s) \left[\left(s + \frac{(x''_d + x''_q) r_s}{2 x''_d x''_q} \right)^2 + \omega_m^2 \right]$$

We introduce: $\tau_a = \dfrac{2 x''_d \cdot x''_q}{(x''_d + x''_q) \cdot r_s}$

8. Dynamics of synchronous machines

Voltage-current-transfer function at q-voltage step u_{q0} at constant speed ω_m = const.

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$$\left.\begin{aligned}\breve{i}_d &= -\frac{u_{q0}}{s}\cdot\frac{\omega_m x_q(s)}{(r_s + s\cdot x_d(s))\cdot(r_s + s\cdot x_q(s)) + \omega_m^2 x_d(s)x_q(s)}\\ \breve{i}_q &= -\frac{u_{q0}}{s}\cdot\frac{r_s + s\cdot x_d(s)}{(r_s + s\cdot x_d(s))\cdot(r_s + s\cdot x_q(s)) + \omega_m^2 x_d(s)x_q(s)}\end{aligned}\right\}$$

$$x_d(s) = x_d''\cdot\frac{(s+\frac{1}{\tau_d'})\cdot(s+\frac{1}{\tau_d''})}{(s+\frac{1}{\tau_{d0}'})\cdot(s+\frac{1}{\tau_{d0}''})} = \frac{P_{d,2}}{P_{d0,2}}$$

$$x_q(s) = x_q''\cdot\frac{s+\frac{1}{\tau_q''}}{s+\frac{1}{\tau_{q0}''}} = \frac{P_{q,1}}{P_{q0,1}}$$

$P_n(s)$: Polynomial in s of order n

$$\text{e.g.:}\quad \breve{i}_d = -\frac{u_{q0}}{s}\cdot\frac{\omega_m\cdot P_{q,1}/P_{q0,1}}{(r_s + s\cdot(P_{d,2}/P_{d0,2}))\cdot(r_s + s\cdot(P_{q,1}/P_{q0,1})) + \omega_m^2\cdot(P_{d,2}/P_{d0,2})\cdot(P_{q,1}/P_{q0,1})}$$

$$\breve{i}_d = -\frac{u_{q0}}{s}\cdot\frac{\omega_m\cdot P_{q,1}\cdot P_{d0,2}}{(r_s\cdot P_{d,02} + s\cdot P_{d,2})\cdot(r_s\cdot P_{q0,1} + s\cdot P_{q,1}) + \omega_m^2\cdot P_{d,2}\cdot P_{q,1}} = -\frac{u_{q0}}{s}\cdot\frac{\omega_m\cdot P_3(s)}{P_5(s)}$$

$$\breve{i}_d = -\frac{u_{q0}}{s}\cdot\frac{\omega_m\cdot P_3(s)}{P_5(s)} = -\frac{u_{q0}}{s}\cdot\frac{\omega_m\cdot\tilde{P}_3(s)}{\left((s+\frac{1}{\tilde{\tau}_a})^2 + \omega_m^2\right)\cdot(s+\frac{1}{\tilde{\tau}_d''})\cdot(s+\frac{1}{\tilde{\tau}_d'})\cdot(s+\frac{1}{\tilde{\tau}_q''})}$$

$$\left\{\begin{aligned}&(s+\frac{1}{\tilde{\tau}_a})^2 + \omega_m^2 = (s-\underline{s}_A)\cdot(s-\underline{s}_B)\\ &\underline{s}_{A,B} = -\frac{1}{\tilde{\tau}_a} \pm j\cdot\omega_m\end{aligned}\right.$$

Transfer function has characteristic polynomial $P_5(s)$ of 5th order !

8. Dynamics of synchronous machines

Time constants as roots of characteristic polynomial $P_5(s)$ (1)

$$\breve{u}_d + \psi_{d0} = r_s \cdot \breve{i}_d + s \cdot \breve{\psi}_d - \omega_m \cdot \breve{\psi}_q$$

$$\breve{u}_q + \psi_{q0} = r_s \cdot \breve{i}_q + s \cdot \breve{\psi}_q + \omega_m \cdot \breve{\psi}_d$$

- Via induction of motion d- and q-axis are coupled at $\omega_m \neq 0$!
- d-axis (3rd order system: s, f, D) and q-axis (2nd-order system: s, Q) are coupled as total 5th-order system
  $\Rightarrow$ Transfer function has characteristic polynomial $P_5(s)$ of 5th order !
- At variable speed ω_m stator voltage and mechanical equation give a non-linear differential equation system! Linearization in a stable equilibrium operation point ω_{m0} gives six linearized differential equations: Voltage equations: stator d, q, rotor: f, D, Q; mech. equation!
- At variable speed ω_m the transfer function of the linearized system (e.g. $\omega_m(u_d, u_q)$) has as denominator a characteristic polynomial $P_6(s)$ of 6th order, yielding also a mechanical time constant τ_m !

$$P_6(s) = (s + \frac{1}{\tau_m}) \cdot \left((s + \frac{1}{\hat{\tau}_a})^2 + \omega_{m0}^2 \right) \cdot (s + \frac{1}{\hat{\tau}''_d}) \cdot (s + \frac{1}{\hat{\tau}'_d}) \cdot (s + \frac{1}{\hat{\tau}''_q})$$

- The time constants $\hat{\tau}_a, \hat{\tau}''_d, \hat{\tau}'_d, \hat{\tau}''_q$ depend also on r_s and on the operation point quantities ω_{m0} and differ from the time constants $\tilde{\tau}_a, \tilde{\tau}''_d, \tilde{\tau}'_d, \tilde{\tau}''_q$.
- The time constants $\tilde{\tau}_a, \tilde{\tau}''_d, \tilde{\tau}'_d, \tilde{\tau}''_q$ depend also on r_s, ω_m and differ from the time constants $\tau_a, \tau''_d, \tau'_d, \tau''_q$.
  The values $\tau''_d, \tau'_d, \tau''_q$ are only valid for $r_s = 0$.

8. Dynamics of synchronous machines

Time constants as roots of characteristic polynomial $P_5(s)$ (2)

- Roots for $P_n(s)$, $n > 4$, cannot be determined analytically (*N.H. Abel*) $\Rightarrow$ No exact formulas exist for $\tau_m, \hat{\tau}_a, \hat{\tau}''_d, \hat{\tau}'_d, \hat{\tau}''_q$ and $\tilde{\tau}_a, \tilde{\tau}''_d, \tilde{\tau}'_d, \tilde{\tau}''_q$
- For $r_s << 1$ the value $\tau_a = \dfrac{2x''_d \cdot x''_q}{(x''_d + x''_q) \cdot r_s}$ is derived as an approximation.
- Then we obtain:
  $\tilde{\tau}_a \approx \tau_a, \tilde{\tau}''_d \approx \tau''_d, \tilde{\tau}'_d \approx \tau'_d, \tilde{\tau}''_q \approx \tau''_q$

- Summary:
  1) At $\omega_m = 0$ the d- and q-circuit are decoupled.
     a) At $r_s = 0$ the time constants are $\tau''_d, \tau'_d, \tau''_q$
     b) At $r_s > 0$ the time constants are $\tau_{ad}, \tau''_{ds}, \tau'_{ds}$ and τ_{aq}, τ''_{qs}

  2) At $\omega_m = \text{const.} \neq 0$ the d- and q-circuit are coupled.
     a) At $r_s = 0$ the time constants are $\tau''_d, \tau'_d, \tau''_q$
     b) At $r_s > 0$ the time constants are $\tilde{\tau}_a, \tilde{\tau}''_d, \tilde{\tau}'_d, \tilde{\tau}''_q$

Mostly we have $r_s << 1$. Hence we can use: $\tilde{\tau}_a \approx \tau_a = \dfrac{2x''_d \cdot x''_q}{(x''_d + x''_q) \cdot r_s}$ $\quad \tilde{\tau}''_d \approx \tau''_d, \tilde{\tau}'_d \approx \tau'_d, \tilde{\tau}''_q \approx \tau''_q$

8. Dynamics of synchronous machines

Introduction of armature time constant τ_a ($r_s << 1$, $x_d'' \approx x_q''$)

1) $$\left(\frac{r_s}{x_d(s)}+s\right)\cdot\left(\frac{r_s}{x_q(s)}+s\right)\approx\left(\frac{r_s}{x_d''}+s\right)\cdot\left(\frac{r_s}{x_q''}+s\right)\quad,\quad if\ r_s<<1$$

2) $$\left(\frac{r_s}{x_d''}+s\right)\cdot\left(\frac{r_s}{x_q''}+s\right)=\underbrace{\frac{r_s}{x_d''}\cdot\frac{r_s}{x_q''}}+s\cdot\left(\frac{r_s}{x_d''}+\frac{r_s}{x_q''}\right)+s^2\approx\underbrace{\frac{1}{4}\cdot\left(\frac{r_s}{x_d''}+\frac{r_s}{x_q''}\right)^2}+s\cdot\left(\frac{r_s}{x_d''}+\frac{r_s}{x_q''}\right)+s^2$$

$$a\approx b:(1):a\cdot b\approx a^2,(2):(a+b)^2/4\approx(2a)^2/4=a^2$$

3) $$\left(\frac{r_s}{x_d''}+s\right)\cdot\left(\frac{r_s}{x_q''}+s\right)\approx\left(s+\frac{(1/x_d''+1/x_q'')\cdot r_s}{2}\right)^2=\left(s+\frac{(x_d''+x_q'')\cdot r_s}{2x_d''x_q''}\right)^2=\left(s+\frac{1}{\tau_a}\right)^2$$

We introduce: $$\tau_a=\frac{2x_d''\cdot x_q''}{(x_d''+x_q'')\cdot r_s}$$

armature time constant τ_a

Solution of stator current space vector *d*- and *q*-component

Solution in *Laplace* domain:

$$\breve{i}_d \cong -\frac{u_0}{s}\cdot\frac{\omega_m x_q(s)}{x_d(s)x_q(s)\left[\left(s+\frac{1}{\tau_a}\right)^2+\omega_m^2\right]} = -\frac{u_0}{s}\cdot\frac{\omega_m}{x_d(s)\left[\left(s+\frac{1}{\tau_a}\right)^2+\omega_m^2\right]}$$

$$\breve{i}_q \cong -\frac{u_0}{s}\cdot\frac{r_s+s\cdot x_d(s)}{x_d(s)x_q(s)\left[\left(s+\frac{1}{\tau_a}\right)^2+\omega_m^2\right]} \approx -\frac{u_0}{s}\cdot\frac{s}{x_q(s)\left[\left(s+\frac{1}{\tau_a}\right)^2+\omega_m^2\right]}$$

Solution in time domain:

$$\underline{\underline{i_d(\tau) = -\frac{u_0}{\omega_m}\cdot\left[\frac{1}{x_d}+\left(\frac{1}{x_d'}-\frac{1}{x_d}\right)\cdot e^{-\tau/\tau_d'}+\left(\frac{1}{x_d''}-\frac{1}{x_d'}\right)\cdot e^{-\tau/\tau_d''}-\frac{1}{x_d''}\cdot e^{-\tau/\tau_a}\cdot\cos(\omega_m\tau)\right]}}$$

$$\underline{\underline{i_q(\tau) = -\frac{u_0}{\omega_m\cdot x_q''}\cdot e^{-\tau/\tau_a}\cdot\sin(\omega_m\tau)}}$$

8. Dynamics of synchronous machines

Inverse *Laplace* transform of stator current solution (1)

d :

$$\frac{1}{s\cdot x_d(s)}=\frac{1}{s\cdot x_d}+\left(\frac{1}{x'_d}-\frac{1}{x_d}\right)\cdot\frac{1}{s+\frac{1}{\tau'_d}}+\left(\frac{1}{x''_d}-\frac{1}{x'_d}\right)\cdot\frac{1}{s+\frac{1}{\tau''_d}}\Rightarrow$$

$$\Rightarrow f_d(\tau)=\frac{1}{x_d}+\left(\frac{1}{x'_d}-\frac{1}{x_d}\right)\cdot e^{-\tau/\tau'_d}+\left(\frac{1}{x''_d}-\frac{1}{x'_d}\right)\cdot e^{-\tau/\tau''_d}$$

q :

$$\frac{1}{s\cdot x_q(s)}=\frac{1}{s\cdot x_q}+\left(\frac{1}{x''_q}-\frac{1}{x_q}\right)\cdot\frac{1}{s+\frac{1}{\tau''_q}}\Rightarrow f_q(\tau)=\frac{1}{x_q}+\left(\frac{1}{x''_q}-\frac{1}{x_q}\right)\cdot e^{-\tau/\tau''_q}$$

d :

$$\frac{\omega_m}{\left(s+\frac{1}{\tau_a}\right)^2+\omega_m^2}\Rightarrow g_d(\tau)=e^{-\tau/\tau_a}\cdot\sin(\omega_m\tau)$$

q :

$$\frac{s}{\left(s+\frac{1}{\tau_a}\right)^2+\omega_m^2}\underset{r_s<<1}{\approx}\frac{s+1/\tau_a}{\left(s+\frac{1}{\tau_a}\right)^2+\omega_m^2}\Rightarrow g_q(\tau)=e^{-\tau/\tau_a}\cdot\cos(\omega_m\tau)$$

8. Dynamics of synchronous machines

Inverse *Laplace* transform of stator current solution (2)

Product in *Laplace* domain is "convolution" in time domain:

$$\breve{f}(s)\cdot\breve{g}(s)\Rightarrow f(\tau)*g(\tau)=\int_0^\tau f(\tau-\xi)\cdot g(\xi)\cdot d\xi$$

Abbreviations: $\alpha=1/\tau_a\sim r_s$, $\beta=1/\tau''_d\sim r_D$ or $\beta=1/\tau'_d\sim r_f$: $\alpha,\ \beta<<1$

or $\beta=1/\tau''_q\sim r_Q$

Solution of "convolution" integrals: e.g.: $1/\tau_a, 1/\tau'_d, 1/\tau''_d, 1/\tau''_q\sim 0.01...0.1<<1$

d : $e^{-\beta\tau}*e^{-\alpha\tau}\cdot\sin(\omega_m\tau)=\int_0^\tau e^{-\beta\cdot(\tau-\xi)}e^{-\alpha\xi}\cdot\sin(\omega_m\xi)\cdot d\xi=$

$$=\frac{1}{\omega_m}\cdot\frac{e^{-\beta\tau}}{1+((\beta-\alpha)/\omega_m)^2}\cdot\left[1-e^{(\beta-\alpha)\cdot\tau}\cdot\cos\omega_m\tau+\frac{\beta-\alpha}{\omega_m}\cdot e^{(\beta-\alpha)\cdot\tau}\cdot\sin\omega_m\tau\right]_{\alpha,\beta<<1}\approx\frac{e^{-\beta\tau}-e^{-\alpha\tau}\cdot\cos\omega_m\tau}{\omega_m}$$

q : $e^{-\beta\tau}*e^{-\alpha\tau}\cdot\cos(\omega_m\tau)=\int_0^\tau e^{-\beta\cdot(\tau-\xi)}\cdot e^{-\alpha\xi}\cdot\cos(\omega_m\xi)\cdot d\xi\underset{\alpha,\beta<<1}{\approx}\frac{e^{-\alpha\tau}\cdot\sin(\omega_m\tau)}{\omega_m}$

8. Dynamics of synchronous machines

Inverse *Laplace* transform of stator current solution (3)

d : $$f_d(\tau)*g_d(\tau)=\left\{\frac{1}{x_d}+\left(\frac{1}{x'_d}-\frac{1}{x_d}\right)\cdot e^{-\tau/\tau'_d}+\left(\frac{1}{x''_d}-\frac{1}{x'_d}\right)\cdot e^{-\tau/\tau''_d}\right\}*e^{-\tau/\tau_a}\cdot\sin(\omega_m\tau)$$

$$f_d(\tau)*g_d(\tau)\approx\frac{1}{\omega_m x_d}\cdot\left(1-e^{-\tau/\tau_a}\cdot\cos\omega_m\tau\right)+\frac{1}{\omega_m}\cdot\left(\frac{1}{x'_d}-\frac{1}{x_d}\right)\cdot\left(e^{-\tau/\tau'_d}-e^{-\tau/\tau_a}\cdot\cos\omega_m\tau\right)+$$

$$+\frac{1}{\omega_m}\cdot\left(\frac{1}{x''_d}-\frac{1}{x'_d}\right)\cdot\left(e^{-\tau/\tau''_d}-e^{-\tau/\tau_a}\cdot\cos\omega_m\tau\right)=$$

$$=\frac{1}{\omega_m x_d}+\frac{1}{\omega_m}\cdot\left(\frac{1}{x'_d}-\frac{1}{x_d}\right)\cdot e^{-\tau/\tau'_d}+\frac{1}{\omega_m}\cdot\left(\frac{1}{x''_d}-\frac{1}{x'_d}\right)\cdot e^{-\tau/\tau''_d}-\frac{1}{\omega_m x''_d}e^{-\tau/\tau_a}\cdot\cos\omega_m\tau$$

q : $$f_q(\tau)*g_q(\tau)=\left\{\frac{1}{x_q}+\left(\frac{1}{x''_q}-\frac{1}{x_q}\right)\cdot e^{-\tau/\tau''_q}\right\}*e^{-\tau/\tau_a}\cdot\cos(\omega_m\tau)$$

$$f_q(\tau)*g_q(\tau)\approx\left\{\frac{1}{x_q}+\left(\frac{1}{x''_q}-\frac{1}{x_q}\right)\right\}\cdot\frac{1}{\omega_m}\cdot e^{-\tau/\tau_a}\cdot\sin(\omega_m\tau)=\frac{1}{\omega_m x''_q}\cdot e^{-\tau/\tau_a}\cdot\sin(\omega_m\tau)$$

8. Dynamics of synchronous machines

Inverse *Laplace* transform of stator current solution (4)

d : $f_d(\tau) * g_d(\tau) \approx \frac{1}{\omega_m x_d} + \frac{1}{\omega_m} \cdot \left(\frac{1}{x'_d} - \frac{1}{x_d} \right) \cdot e^{-\tau/\tau'_d} + \frac{1}{\omega_m} \cdot \left(\frac{1}{x''_d} - \frac{1}{x'_d} \right) \cdot e^{-\tau/\tau''_d} - \frac{1}{\omega_m x''_d} \cdot e^{-\tau/\tau_a} \cdot \cos \omega_m \tau$

$$i_d(\tau) = -\frac{u_0}{\omega_m} \cdot \left[\frac{1}{x_d} + \left(\frac{1}{x'_d} - \frac{1}{x_d} \right) \cdot e^{-\tau/\tau'_d} + \left(\frac{1}{x''_d} - \frac{1}{x'_d} \right) \cdot e^{-\tau/\tau''_d} - \frac{1}{x''_d} \cdot e^{-\tau/\tau_a} \cdot \cos(\omega_m \tau) \right]$$

q : $f_q(\tau) * g_q(\tau) \approx \frac{1}{\omega_m x''_q} \cdot e^{\tau/\tau_a} \cdot \sin(\omega_m \tau)$

$$i_q(\tau) = -\frac{u_0}{\omega_m x''_q} \cdot e^{-\tau/\tau_a} \cdot \sin(\omega_m \tau)$$

Without damping: $i_d(\tau) = -\frac{u_0}{\omega_m} \cdot \left(\frac{1}{x''_d} - \frac{1}{x''_d} \cdot \cos(\omega_m \tau) \right)$ $\quad i_q(\tau) = -\frac{u_0}{\omega_m x''_q} \cdot \sin(\omega_m \tau)$

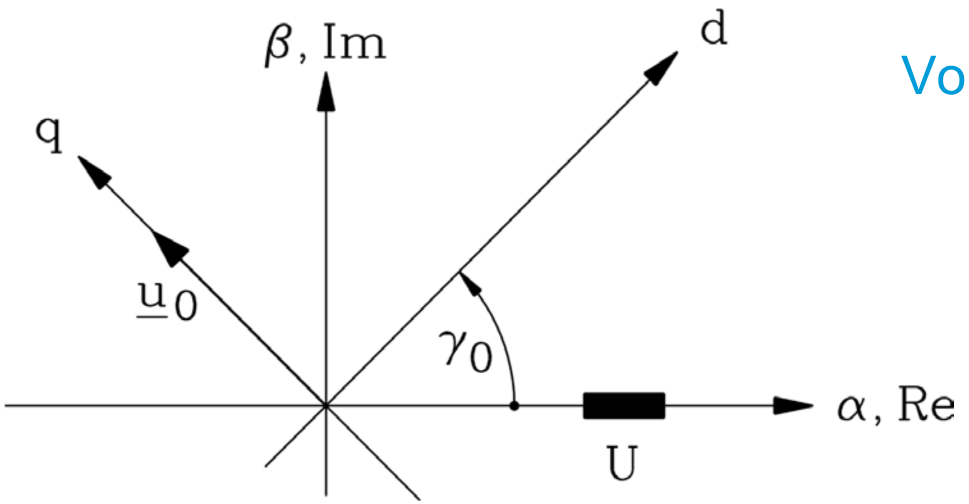
8. Dynamics of synchronous machines

Transformation from rotor into stator reference frame ($\omega_m = 1$)

$\underline{i}_{s(r)} = i_d + j \cdot i_q$ ⇒ transformed into stator reference frame: $\underline{i}_{s(s)} = \underline{i}_{s(r)} \cdot e^{j\gamma}$

$$\gamma = \omega_m \tau + \gamma_0 = \tau + \gamma_0$$

γ_0: defines the rotor position at sudden short circuit



Voltage in phase U at sudden short circuit:

$$\underline{u}_{s(r)} = u_d + j \cdot u_q = j \cdot u_0$$

$$u_{s,U}(\tau) = \mathrm{Re}\{\underline{u}_{s(r)} \cdot e^{j\gamma}\} =$$

$$= \mathrm{Re}\{j \cdot u_0 \cdot e^{j(\omega_m \tau + \gamma_0)}\} = -u_0 \cdot \sin(\omega_m \tau + \gamma_0)$$

γ_0 = 0: Maximum DC current component,
γ_0 = $\pi/2$: No DC current component

Stator sudden short-circuit phase current U:

$$i_U(\tau) = \mathrm{Re}\{\underline{i}_{s(s)}(\tau)\} = \mathrm{Re}\{(i_d(\tau) + j \cdot i_q(\tau)) \cdot (\cos\gamma + j \cdot \sin\gamma)\} =$$

$$= i_d(\tau) \cdot \cos(\omega_m \tau + \gamma_0) - i_q(\tau) \cdot \sin(\omega_m \tau + \gamma_0)$$

8. Dynamics of synchronous machines

Stator current solution in phase U

$$i_U(\tau) = i_d(\tau)\cdot\cos(\omega_m\tau+\gamma_0) - i_q(\tau)\cdot\sin(\omega_m\tau+\gamma_0)$$

$$i_U(\tau) = -\frac{u_0}{\omega_m}\cdot\left[\frac{1}{x_d}+\left(\frac{1}{x'_d}-\frac{1}{x_d}\right)\cdot e^{-\tau/\tau'_d}+\left(\frac{1}{x''_d}-\frac{1}{x'_d}\right)\cdot e^{-\tau/\tau''_d}\right]\cdot\cos(\omega_m\tau+\gamma_0)+$$

$$\underbrace{+\frac{u_0}{\omega_m x''_d}\cdot e^{-\tau/\tau_a}\cdot\cos(\omega_m\tau)\cdot\cos(\omega_m\tau+\gamma_0)+\frac{u_0}{\omega_m x''_q}\cdot e^{-\tau/\tau_a}\cdot\sin(\omega_m\tau)\cdot\sin(\omega_m\tau+\gamma_0)}$$

$$\frac{u_0}{\omega_m}\cdot\left[\frac{1}{2}\cdot\left(\frac{1}{x''_d}+\frac{1}{x''_q}\right)\cdot\cos\gamma_0+\frac{1}{2}\cdot\left(\frac{1}{x''_d}-\frac{1}{x''_q}\right)\cdot\cos(2\omega_m\tau+\gamma_0)\right]\cdot e^{-\tau/\tau_a}$$

$$i_U(\tau) = -\frac{u_0}{\omega_m}\cdot\left[\frac{1}{x_d}+\left(\frac{1}{x'_d}-\frac{1}{x_d}\right)\cdot e^{-\tau/\tau'_d}+\left(\frac{1}{x''_d}-\frac{1}{x'_d}\right)\cdot e^{-\tau/\tau''_d}\right]\cdot\cos(\omega_m\tau+\gamma_0)+$$

$$+\frac{u_0}{\omega_m}\cdot\left[\frac{1}{2}\cdot\left(\frac{1}{x''_d}+\frac{1}{x''_q}\right)\cdot\cos\gamma_0+\frac{1}{2}\cdot\left(\frac{1}{x''_d}-\frac{1}{x''_q}\right)\cdot\cos(2\omega_m\tau+\gamma_0)\right]\cdot e^{-\tau/\tau_a}$$

Sudden short circuit stator current in phase U

$$i_U(\tau) = -\frac{u_0}{\omega_m} \cdot \left[\frac{1}{x_d} + \left(\frac{1}{x'_d} - \frac{1}{x_d} \right) \cdot e^{-\tau/\tau'_d} + \left(\frac{1}{x''_d} - \frac{1}{x'_d} \right) \cdot e^{-\tau/\tau''_d} \right] \cdot \cos(\omega_m \tau + \gamma_0) +$$

$$+ \frac{u_0}{\omega_m} \cdot \left[\frac{1}{2} \cdot \left(\frac{1}{x''_d} + \frac{1}{x''_q} \right) \cdot \cos\gamma_0 + \frac{1}{2} \cdot \left(\frac{1}{x''_d} - \frac{1}{x''_q} \right) \cdot \cos(2\omega_m \tau + \gamma_0) \right] \cdot e^{-\tau/\tau_a}$$

- First part $[.] \cdot \cos(\omega_m \tau + \gamma_0)$: **AC short circuit current,** frequency $\omega_m = 1$:
  Starting with big amplitude u_0 / x''_d at $\tau = 0$, decaying after three time constants $3\tau''_d$ to the intermediate amplitude u_0 / x'_d.
  After three time constants $3\tau'_d$ it decays to steady state short circuit current u_0 / x_d.
- Second part $[.] \cdot \cos\gamma_0$: **DC short circuit current**: Decaying with armature time constant τ_a, depends on γ_0.
- Third part $[.] \cdot \cos(2\omega_m \tau + \gamma_0)$ is AC short circuit current with **double frequency** $2\omega_m$, which occurs only, if $x''_d \neq x''_q$, and usually is small.

8. Dynamics of synchronous machines

No influence of τ_q'' on stator sudden short circuit current

$$i_U(\tau) = -\frac{u_0}{\omega_m} \cdot \left[\frac{1}{x_d} + \left(\frac{1}{x'_d} - \frac{1}{x_d} \right) \cdot e^{-\tau/\tau'_d} + \left(\frac{1}{x''_d} - \frac{1}{x'_d} \right) \cdot e^{-\tau/\tau''_d} \right] \cdot \cos(\omega_m \tau + \gamma_0) +$$
$$+ \frac{u_0}{\omega_m} \cdot \left[\frac{1}{2} \cdot \left(\frac{1}{x''_d} + \frac{1}{x''_q} \right) \cdot \cos\gamma_0 + \frac{1}{2} \cdot \left(\frac{1}{x''_d} - \frac{1}{x''_q} \right) \cdot \cos(2\omega_m \tau + \gamma_0) \right] \cdot e^{-\tau/\tau_a}$$

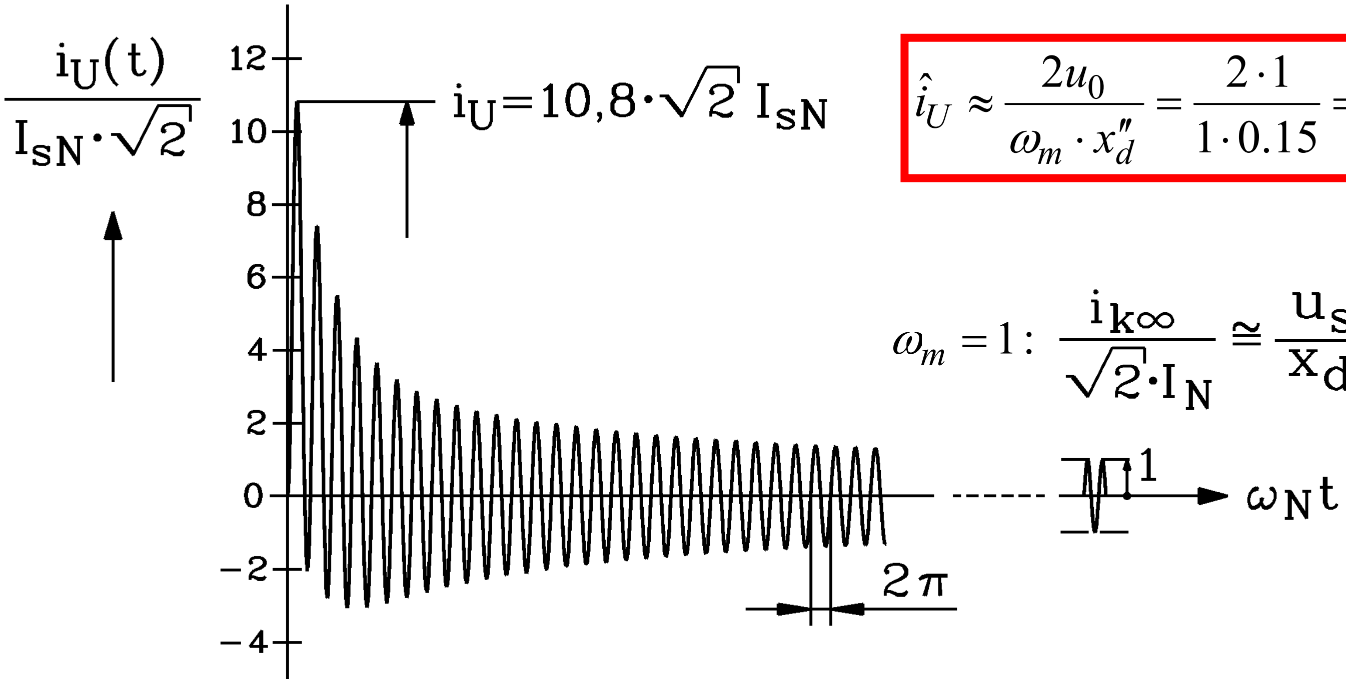
Due to $r_s << 1, r_D << 1, r_Q << 1$: No influence of τ''_q, x_q on $i_s(\tau)$!

Example: Sudden short circuit current per phase

24-pole synchronous generator: Sudden short circuit after no-load at rated voltage $u_0 = 1$ and rated speed $\omega_m = 1$, yielding stator frequency $f_N = 50$ Hz:

Machine data: $S_N = 300$ MVA, $U_N = 24$ kV, $I_{sN} = 7217$ A, $x_d = 1, x'_d = 0.3, x''_d = x''_q = 0.15$

Time constants: $T_a = 0.03s, T'_d = 0.3s, T''_d = 0.05s \Rightarrow \tau_a = 9.4, \tau'_d = 94.2, \tau''_d = 15.7$



$$\hat{i}_U \approx \frac{2u_0}{\omega_m \cdot x''_d} = \frac{2 \cdot 1}{1 \cdot 0.15} = 13.3$$

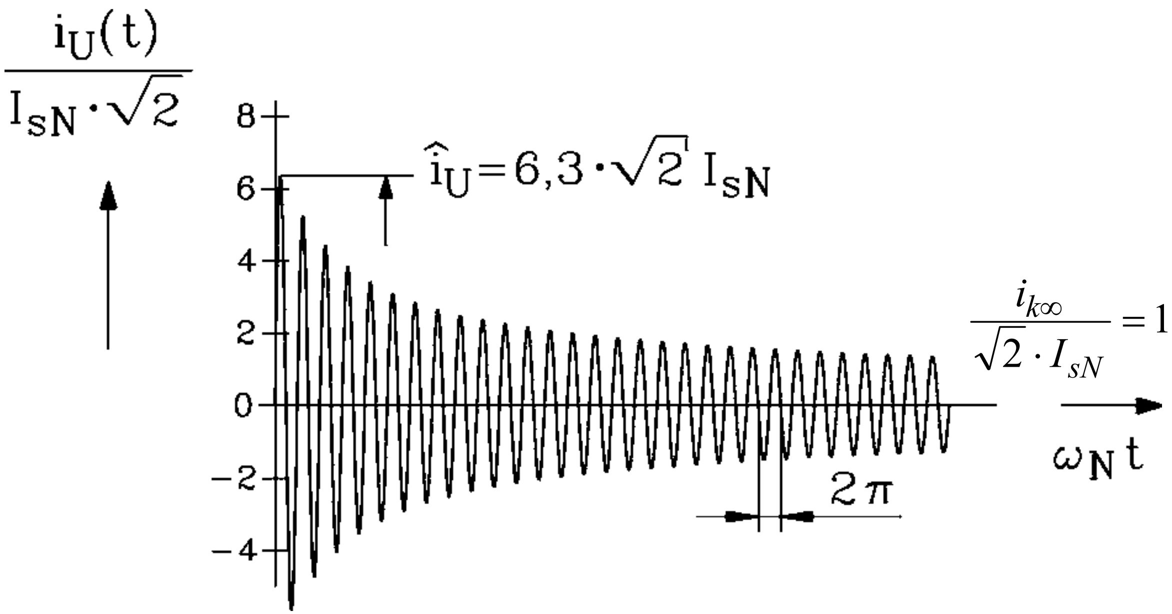
Worst case: Phase voltage is zero at short circuit: $\gamma_0 = 0$

Maximum peak current: 10.8 times rated current, at $\tau \approx \pi$

Source: H. Kleinrath, Springer-Verlag

8. Dynamics of synchronous machines

Best case sudden short circuit current per phase



Best case:
Phase voltage is maximum at short circuit: γ_0 = 90°

Minimum peak current:
6.3 times rated current, at $\tau \approx \pi/2$

Source:
H. Kleinrath, Springer-Verlag

At $\gamma_0 = \pi / 2$ the short circuit occurs in phase U at $\tau = 0$,

when voltage is maximum. DC component is zero due to $\cos\gamma_0 = 0$.

Amplitude of AC component : $\hat{i}_U \approx \frac{u_0}{\omega_m \cdot x_d''} = \frac{1}{1 \cdot 0.15} = 6.67$

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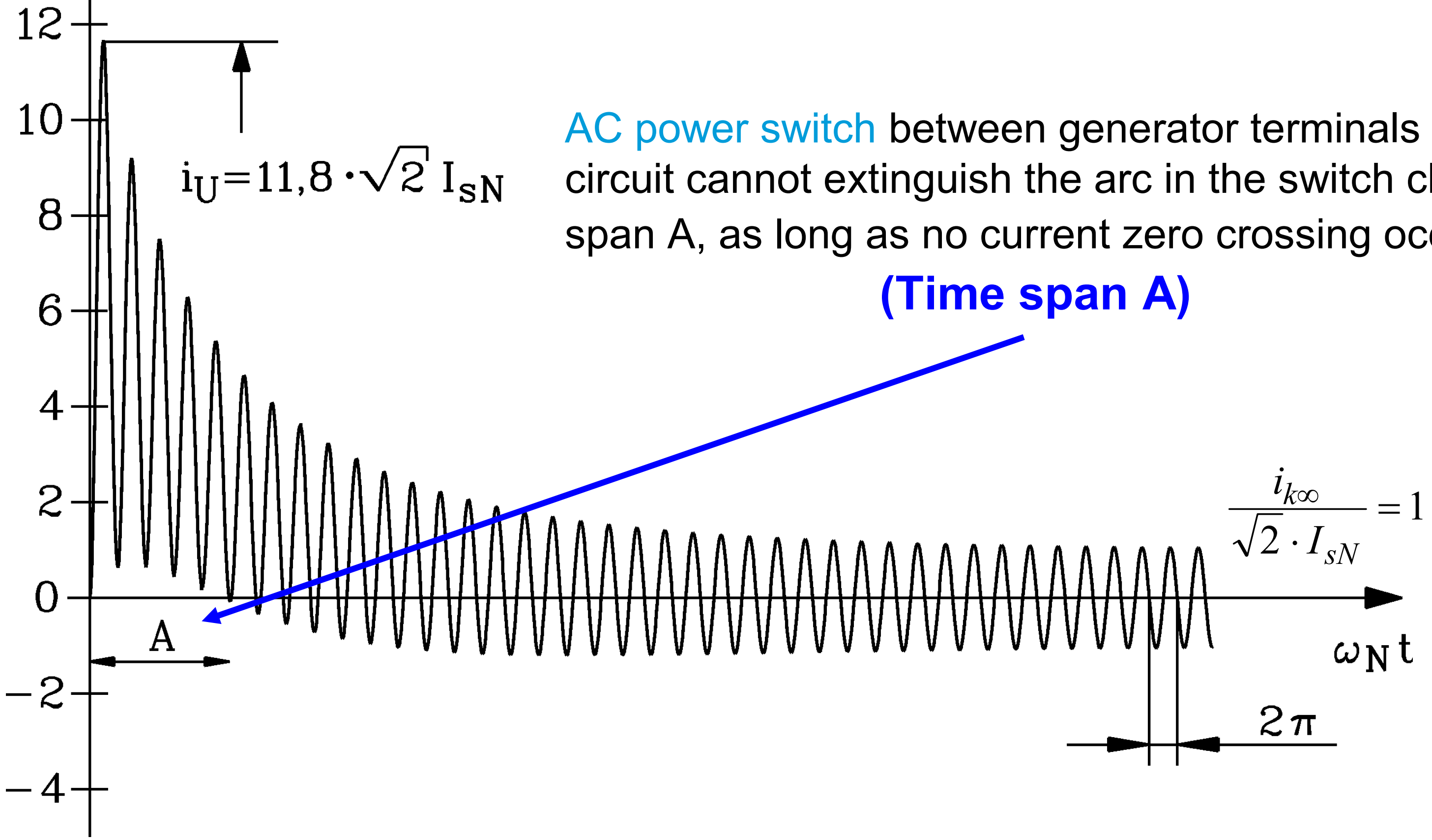
Sudden short circuit current at long armature time constant

Short circuit of synchronous generator (like previous Example), but 3-times increased armature time constant and by 2/3 decreased transient / subtransient time constant:

Time constants: $T_a = 0.1s, T'_d = 0.2s, T''_d = 0.03s \quad \Rightarrow \quad \tau_a = 31.4, \tau'_d = 62.8, \tau''_d = 9.4$

AC power switch between generator terminals and location of short circuit cannot extinguish the arc in the switch chamber during time span A, as long as no current zero crossing occurs

(Time span A)



Worst case:
Phase voltage is zero at short circuit: $\gamma_0 = 0$

Maximum peak current:
11.8 times rated current,
at $\tau \approx \pi$

Source:
H. Kleinrath, Springer-Verlag

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Summary:
Sudden short circuit of electrically excited synchronous machine with damper cage

- Sub-transient reactance x''_d and τ''_d rule the AC sudden short circuit current
- No influence of τ''_q
- DC component due to switching increases the amplitude in the worst-case by nearly factor 2
- Decay of DC component with armature time constant τ_{a}
- Increase of inductance of subtransient via transient to steady state

$$x''_d \to x'_d \to x_d$$

  is governed by subtransient and transient rotor time constants τ''_d, τ'_d
- Huge short circuit current amplitude of factor 10 to 15
- Calculation valid for <u>constant speed</u> and <u>small values</u> $1/\tau_a, 1/\tau'_d, 1/\tau''_d, 1/\tau''_q << 1$

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8. Dynamics of synchronous machines

8. Dynamics of synchronous machines

8.9.1 Sudden short circuit torque

- **Sudden short circuit torque:** We neglect damping of short circuit current !

For $\tau \to 0$ we get from the reactance operators for $s \to \infty$

with $i_{d0} = 0, i_{q0} = 0$ and $x_d(s \to \infty) = x''_d$, $x_q(s \to \infty) = x''_q$

$$\breve{\psi}_d - \frac{\psi_{d0}}{s} = x_d(s) \cdot \left(\breve{i}_d - \frac{i_{d0}}{s} \right) \Rightarrow \psi_d(\tau) \approx \psi_{d0} + x''_d \cdot i_d(\tau)$$

$$\breve{\psi}_q - \frac{\psi_{q0}}{s} = x_q(s) \cdot \left(\breve{i}_q - \frac{i_{q0}}{s} \right) \Rightarrow \psi_q(\tau) \approx \psi_{q0} + x''_q \cdot i_q(\tau)$$

- **Flux linkage equations** with neglected damping of short circuit current:

$$\left. \begin{array}{l} \psi_{d0} = u_{q0} / \omega_m = \\ = u_0 / \omega_m \\ \psi_{q0} = 0 \end{array} \right\} \Rightarrow \begin{array}{|l|} \hline \psi_d(\tau) \approx (u_0 / \omega_m) + x''_d \cdot i_d(\tau) \\ \psi_q(\tau) \approx x''_q \cdot i_q(\tau) \\ \hline \end{array} \quad r_s, r_D, r_Q, r_f = 0$$

Calculation of non-damped sudden short circuit torque (1)

- **Short circuit current,** neglecting damping:

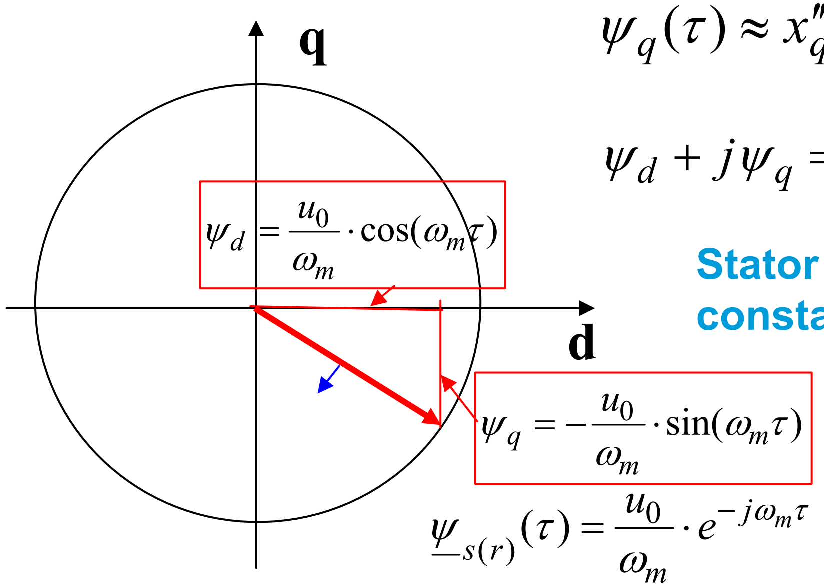
$$i_d(\tau) = -\frac{u_0}{\omega_m x_d''}\cdot\left[1-\cos(\omega_m\tau)\right], \quad i_q(\tau) = -\frac{u_0}{\omega_m x_q''}\cdot\sin(\omega_m\tau)$$

- **Flux linkage equations** with neglected damping of short circuit current:

$$\psi_d(\tau) \approx \frac{u_0}{\omega_m} + x_d''\cdot\left(-\frac{u_0}{\omega_m x_d''}\right)\cdot\left[1-\cos(\omega_m\tau)\right] = \frac{u_0}{\omega_m}\cdot\cos(\omega_m\tau)$$

$$\psi_q(\tau) \approx x_q''\cdot\left(-\frac{u_0}{\omega_m x_q''}\right)\cdot\sin(\omega_m\tau) = -\frac{u_0}{\omega_m}\cdot\sin(\omega_m\tau)$$

$$\psi_d + j\psi_q = \frac{u_0}{\omega_m}\cdot\left(\cos(\omega_m\tau) - j\cdot\sin(\omega_m\tau)\right) = \frac{u_0}{\omega_m}\cdot e^{-j\omega_m\tau} = \underline{\psi}_{s(r)}(\tau)$$



Stator flux linkage space vector is in stator reference frame constant:

$$\underline{\psi}_{s(s)} = \underline{\psi}_{s(r)}\cdot e^{j\cdot(\omega_m\tau+\gamma_0)} = \frac{u_0}{\omega_m}\cdot e^{j\cdot\gamma_0} = const.$$

Compare the ideal short-circuit condition:

$$u_s = 0 = d\psi_s / d\tau = 0 : \psi_s = const.$$

Calculation of non-damped sudden short circuit torque (2)

- **Short circuit torque** with neglected damping of short circuit current:

$$m_e(\tau) = i_q(\tau)\cdot\psi_d(\tau) - i_d(\tau)\cdot\psi_q(\tau) \cong i_q\cdot\frac{u_0}{\omega_m} + i_q\cdot x''_d\cdot i_d - i_d\cdot x''_q\cdot i_q =$$

$$= i_q\cdot\frac{u_0}{\omega_m} + \left(x''_d - x''_q\right)\cdot i_d\cdot i_q \approx -\frac{u_0}{\omega_m x''_q}\cdot\sin(\omega_m\tau)\cdot\frac{u_0}{\omega_m} + \left(x''_d - x''_q\right)\cdot\frac{u_0}{\omega_m x''_d}\cdot[1-\cos(\omega_m\tau)]\cdot\frac{u_0}{\omega_m x''_q}\cdot\sin(\omega_m\tau) =$$

$$= -\frac{u_0}{\omega_m x''_q}\cdot\sin(\omega_m\tau)\cdot\frac{u_0}{\omega_m}\cdot\cos(\omega_m\tau) - \frac{u_0}{\omega_m x''_d}\cdot[1-\cos(\omega_m\tau)]\cdot\frac{u_0}{\omega_m}\cdot\sin(\omega_m\tau)$$

$$\approx 0 \quad (x''_d \approx x''_q)$$

$$m_e(\tau) \approx -\frac{u_0^2}{\omega_m^2\cdot x''_d}\cdot\sin(\omega_m\tau) + \frac{u_0^2}{2\omega_m^2}\cdot\left(\frac{1}{x''_d} - \frac{1}{x''_q}\right)\cdot\sin(2\omega_m\tau)$$

- ***Example:*** 7-fold rated torque as AC torque with rotational frequency: $\omega_m = 1$

$$m_e \cong -\frac{u_0^2}{\omega_m^2\cdot x''_d}\cdot\sin(\omega_m\tau) = -\frac{1^2}{1^2\cdot 0.15}\cdot\sin(1\cdot\tau) = 7\cdot\sin(\tau)$$

8. Dynamics of synchronous machines

Short-circuit torque oscillation with ω_m

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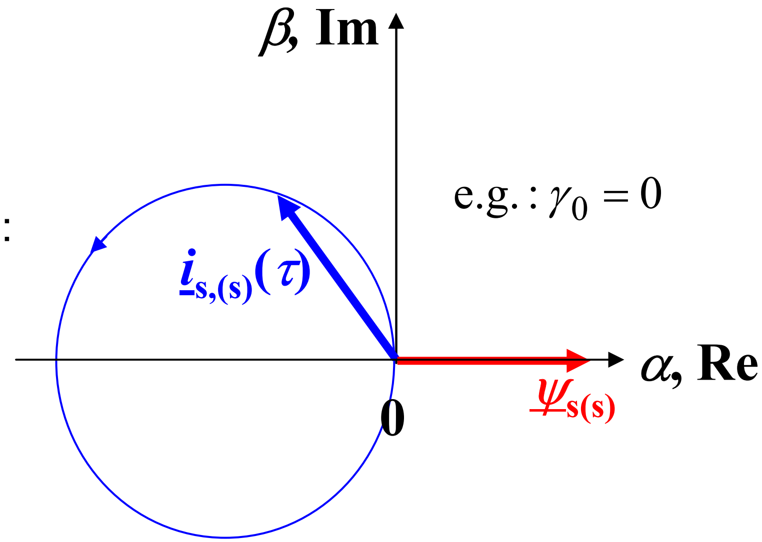
- Undamped stator flux linkage space vector is in stator reference frame constant:

$$\underline{\psi}_{s(s)} = \frac{u_0}{\omega_m} \cdot e^{j\cdot\gamma_0} = const.$$

- Undamped stator short-circuit current in rotor reference frame:

$$i_d(\tau) = -\frac{u_0}{\omega_m} \cdot \left(\frac{1}{x''_d} - \frac{1}{x''_d} \cdot \cos(\omega_m \tau) \right) \qquad i_q(\tau) = -\frac{u_0}{\omega_m x''_q} \cdot \sin(\omega_m \tau)$$

$$i_d + j \cdot i_q = \frac{u_0}{\omega_m x''_d} \cdot \left(-1 + e^{-j\omega_m \tau} \right) = \underline{i}_{s(r)}(\tau)$$



- In stator reference frame:

$$\underline{i}_{s(s)} = \underline{i}_{s(r)} \cdot e^{j\cdot(\omega_m \tau + \gamma_0)} = \frac{u_0}{\omega_m x''_d} \cdot \left[1 - e^{-j\cdot\omega_m \tau} \right] \cdot e^{j\cdot(\omega_m \tau + \gamma_0)} = \frac{u_0}{\omega_m x''_d} \cdot \left[e^{j\cdot(\omega_m \tau + \gamma_0)} - e^{j\cdot\gamma_0} \right]$$

$$m_e(\tau) = \mathrm{Im}\left\{ \underline{i}_s \cdot \underline{\psi}_s^* \right\} = \frac{u_0^2}{\omega_m^2 x''_d} \cdot \mathrm{Im}\left\{ \left(e^{j\cdot(\omega_m \tau + \gamma_0)} - e^{j\cdot\gamma_0} \right) \cdot e^{-j\cdot\gamma_0} \right\} = \frac{u_0^2}{\omega_m^2 x''_d} \cdot \mathrm{Im}\left\{ e^{j\cdot\omega_m \tau} \right\} = \frac{u_0^2}{\omega_m^2 x''_d} \cdot \sin \omega_m \tau$$

- **Result:**

  Short-circuit torque m_e pulsates with ω_m due to DC stator flux linkage $\underline{\psi}_s$

8. Dynamics of synchronous machines

Non-damped electromagnetic short circuit torque (r_s , r_f , $r_{D,Q}$ = 0)

Three-phase sudden short circuit: $$m_e(\tau) \approx -\frac{u_0^2}{\omega_m^2 \cdot x_d''} \cdot \sin(\omega_m \tau) + \frac{u_0^2}{2\omega_m^2} \cdot \left(\frac{1}{x_d''} - \frac{1}{x_q''} \right) \cdot \sin(2\omega_m \tau)$$

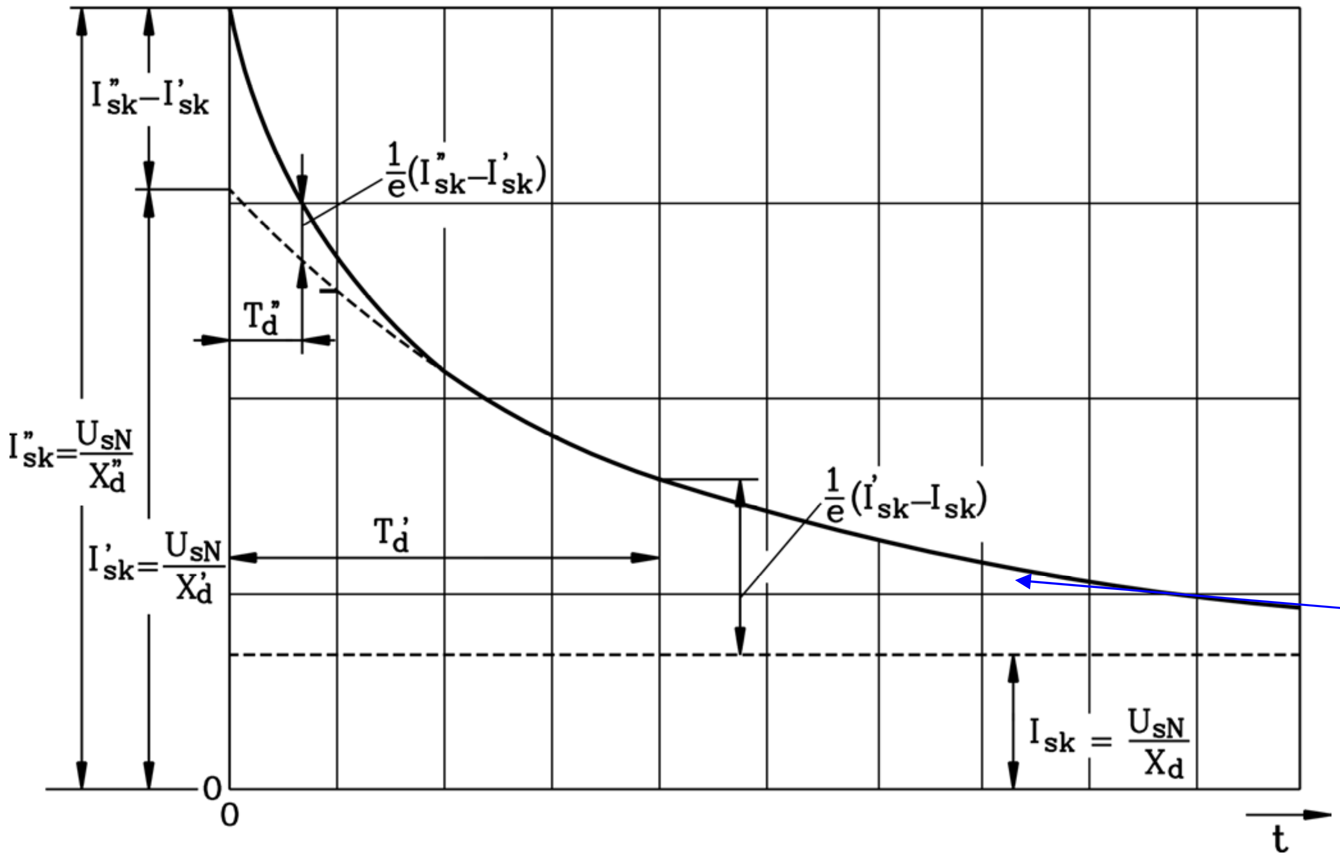
- For subtransient symmetrical machines $x_d'' = x_q''$ the dynamic short circuit pulsates with angular frequency ω_m with big amplitude $u_0^2 / (\omega_m^2 \cdot x_d'')$
- Average value of torque is (nearly) zero: $M_{e,av} \approx P_{Cu,s} / \Omega_m \rightarrow m_{e,av} \approx r_s \cdot i_s^2 / (2 \cdot \omega_m)$
- With damping (e.g. r_s > 0) torque decays with time constant $\tau_a / 2$ due to $m_e \sim i \cdot \psi$
- With damping: Average torque $m_{e,av}$ is bigger than zero: Mechanical input power via torque is converted into the losses mainly in the stator winding (minor: damper & field)
- Ratio peak torque/average torque is very big $u_0^2 / (\omega_m^2 \cdot x_d'' \cdot m_{e,av})$: Endangers machine shaft
- Short circuit at full load overexcited: i_{f0}, ψ_0 bigger $\Rightarrow$ short circuit current bigger ca. + 10%
- Two-phase sudden short circuit: Peak current ca. -15%, but peak torque by +30% bigger

8.9.2 Measurement of transient machine parameters

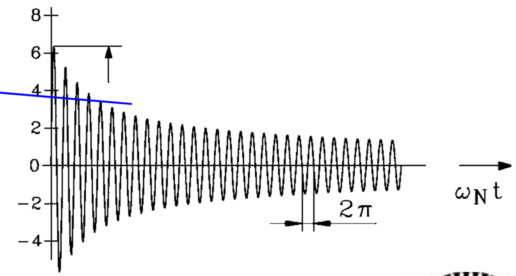
AC envelope:

$$I_{s,U}(t) = U_{s,N} \cdot \left[\frac{1}{X_d} + \left(\frac{1}{X'_d} - \frac{1}{X_d} \right) \cdot e^{-t/T'_d} + \left(\frac{1}{X''_d} - \frac{1}{X'_d} \right) \cdot e^{-t/T''_d} \right]$$

I_s



From the measured envelope of the symmetrical AC short circuit current time function AT REDUCED STATOR NO-LOAD VOLTAGE the time constants T'_d, T''_d and the reactances X_d, X'_d, X''_d are determined.



8. Dynamics of synchronous machines

8.9.3 Synchronous machine transient equations for RUNGE-KUTTA solution

$$d\psi_d / d\tau = u_d - r_s i_d + \omega_m \psi_q$$

$$d\psi_q / d\tau = u_q - r_s i_q - \omega_m \psi_d$$

$$d\psi_D / d\tau = -r_D i_D$$

$$d\psi_Q / d\tau = -r_Q i_Q$$

$$d\psi_f / d\tau = u_f - r_f i_f$$

$$d\omega_m / d\tau = (i_q \psi_d - i_d \psi_q - m_s) / \tau_J$$

Six 1st order differential equations

Given external quantities:

m_s, u_d, u_q, u_f

$$\psi_d = x_d i_d + x_{dh} i_D + x_{dh} i_f$$

$$\psi_D = x_{dh} i_d + x_D i_D + x_{dh} i_f$$

$$\psi_f = x_{dh} i_d + x_{dh} i_D + x_f i_f$$

$$\psi_q = x_q i_q + x_{qh} i_Q$$

$$\psi_Q = x_{qh} i_q + x_Q i_Q$$

Five algebraic flux linkage equations

Initial conditions: $\psi_{d0}, \psi_{q0}, \psi_{D0}, \psi_{Q0}, \psi_{f0}$

taken from: $i_{d0}, i_{q0}, i_{D0}, i_{Q0}, i_{f0}$

and: ω_{m0}

For inverse PARK-Transformation: γ_0

$$\omega_{m0} = 1,\ \gamma_0 = 0$$

$$\psi_{d0} = x_{dh} i_{f0}$$

$$\psi_{q0} = 0$$

$$\psi_{D0} = x_{dh} i_{f0}$$

$$\psi_{Q0} = 0$$

$$\psi_{f0} = x_f i_{f0}$$

Generator no-load condition:

$$u_{d0} = 0, u_{q0} = 1, i_{d0} = 0, i_{q0} = 0$$

$$i_{D0} = 0, i_{Q0} = 0$$

From that we get with the stationary equations:

$$i_{f0} = \psi_{d0} / x_{dh} = u_{q0} / (\omega_{m0} x_{dh}) =$$

$$= 1/(1 \cdot x_{dh}) = 1/x_{dh}$$

$$1 = u_{q0} = \omega_m \cdot x_{dh} \cdot i_{f0} = 1 \cdot x_{dh} \cdot i_{f0}$$

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Example: Turbine generator: Reactances

2-pole turbine generator 600 MVA, 26 kV Y, 13.32 kA, 50 Hz, 3000/min, $I_{fN} = 1800$ A, $U_{fN} = 146$ V:

Hydrogen-gas cooled (H_2) *(ABB Birr, Switzerland (now GE))*

Data set: $r_s = 0.004,\ r_f = 0.001,\ r_D = 0.0187,\ r_Q = 0.0867,\ T_J = 3.8\,\mathrm{s} \rightarrow \tau_J = 1200$

$x_{s\sigma} = 0.19,\ x_{dh} = 1.73,\ x_{qh} = 1.66,\ x_{D\sigma} = 0.1313,\ x_{Q\sigma} = 0.0731,\ x_{f\sigma} = 0.1642$

$$x''_d = x_{s\sigma} + \frac{x_{dh}x_{f\sigma}x_{D\sigma}}{x_{dh}x_{f\sigma} + x_{dh}x_{D\sigma} + x_{f\sigma}x_{D\sigma}} = 0.19 + \frac{1.73\cdot 0.16\cdot 0.13}{1.73\cdot(0.16+0.13)+0.16\cdot 0.13} = \underline{\underline{0.26}}$$

$$x''_q = x_{s\sigma} + \frac{x_{qh}x_{Q\sigma}}{x_{qh}+x_{Q\sigma}} = 0.19 + \frac{1.66\cdot 0.07}{1.66+0.07} = \underline{\underline{0.257}}$$

$$x_d = x_{s\sigma} + x_{dh} = 0.19 + 1.73 = \underline{\underline{1.92}}$$

$$x'_d = x_{s\sigma} + \frac{x_{dh}x_{f\sigma}}{x_{dh}+x_{f\sigma}} = 0.19 + \frac{1.73\cdot 0.16}{1.73+0.16} = \underline{\underline{0.34}}$$

$$x_q = x_{s\sigma} + x_{qh} = 0.19 + 1.66 = \underline{\underline{1.85}}$$

$$i_{f0} = \frac{u_{q0}}{\omega_{m0}x_{dh}} = \frac{1}{1\cdot 1.73} = \underline{\underline{0.58}}$$

8. Dynamics of synchronous machines

Example: Turbine generator: Time constants & short circuit current

$$\tau_a = \frac{2x''_d \cdot x''_q}{(x''_d + x''_q)\cdot r_s} \approx \frac{x''_d}{r_s} = \frac{0.26}{0.004} = \underline{\underline{65}}, \quad T_a = \frac{\tau_a}{2\pi f_N} = \frac{65}{2\pi 50} = \underline{\underline{0.2s}}$$

$$\tau''_d = \frac{x_{D\sigma} + \dfrac{x_{dh}x_{f\sigma}x_{s\sigma}}{x_{dh}x_{f\sigma} + x_{dh}x_{s\sigma} + x_{f\sigma}x_{s\sigma}}}{r_D} = \frac{0.13 + \dfrac{1.73\cdot 0.16\cdot 0.13}{1.73\cdot(0.16+0.13)+0.16\cdot 0.13}}{0.0187} = \underline{\underline{11.4}}, \quad T''_d = \frac{11.4}{2\pi 50} = \underline{\underline{36ms}}$$

$$\tau''_q = \frac{x_{Q\sigma} + \dfrac{x_{qh}x_{s\sigma}}{x_{qh} + x_{s\sigma}}}{r_Q} = \frac{0.073 + \dfrac{1.66\cdot 0.19}{1.66+0.19}}{0.0867} = \underline{\underline{2.8}}, \quad T''_q = \frac{2.8}{2\pi 50} = \underline{\underline{8.9ms}}$$

$$\tau_f = \frac{x_{dh} + x_{f\sigma}}{r_f} = \frac{1.73+0.16}{0.001} = \underline{\underline{1890}}, \quad T_f = \frac{1890}{2\pi 50} = \underline{\underline{6.0s}}$$

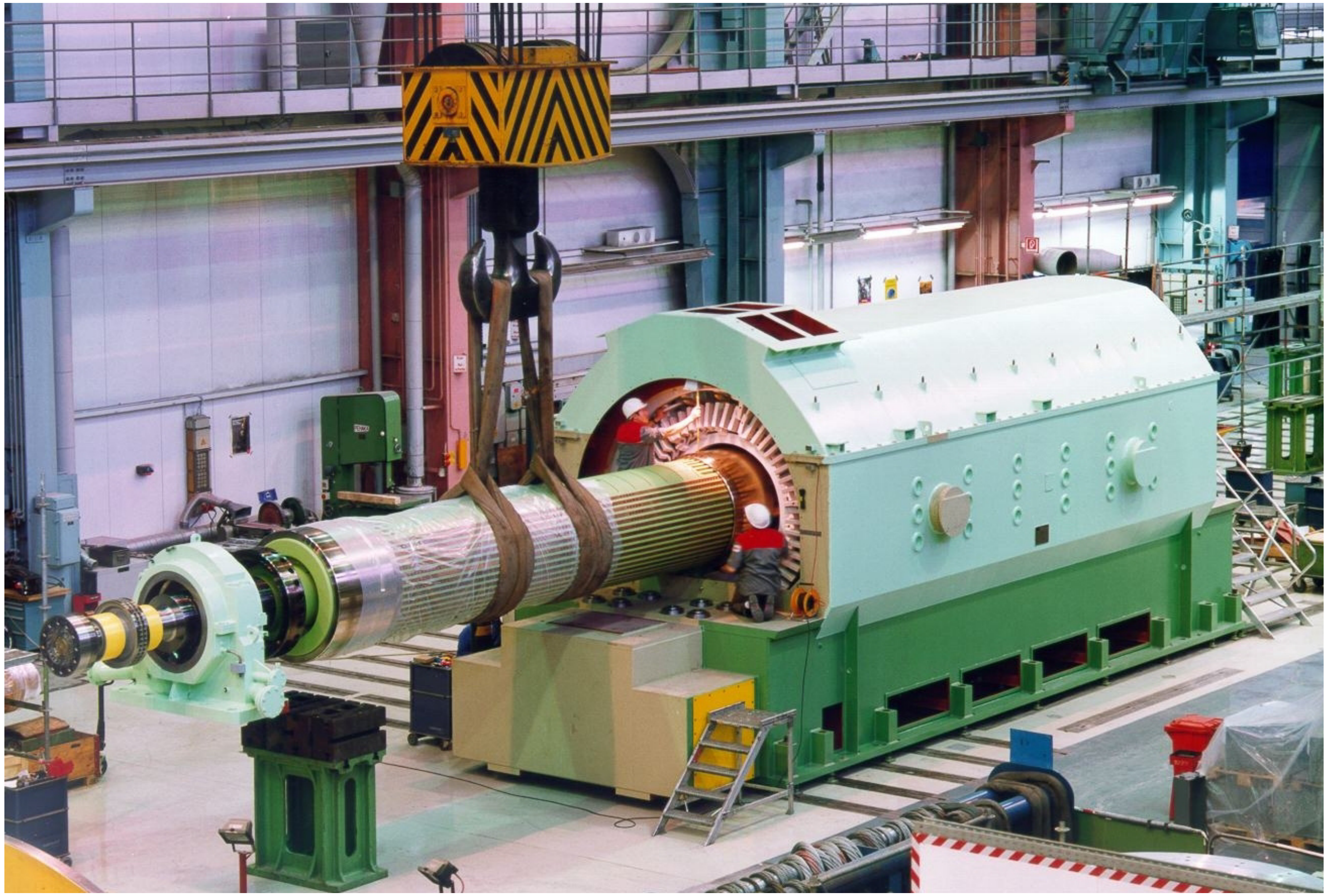
$$\tau'_d = \frac{x'_d}{x_d}\cdot\tau_f = \frac{0.34}{1.92}\cdot 1890 = \underline{\underline{334.7}}, \quad T'_d = \frac{334.7}{2\pi 50} = \underline{\underline{1.07s}}$$

Worst-case sudden short circuit current at voltage zero crossing: $\hat{i}_{s,k} = \dfrac{2u_0}{x''_d} = \dfrac{2\cdot 1}{0.26} = \underline{\underline{7.7}}$

$$(\hat{I}_{s,k} = 7.7\cdot\sqrt{2}\cdot 13323 = 145085 \text{ A})$$

8. Dynamics of synchronous machines

Turbine generator assembly in the test bay before short-circuit test at 10% no-load voltage



Air-cooled two-pole turbine generator

400 MVA

3000/min

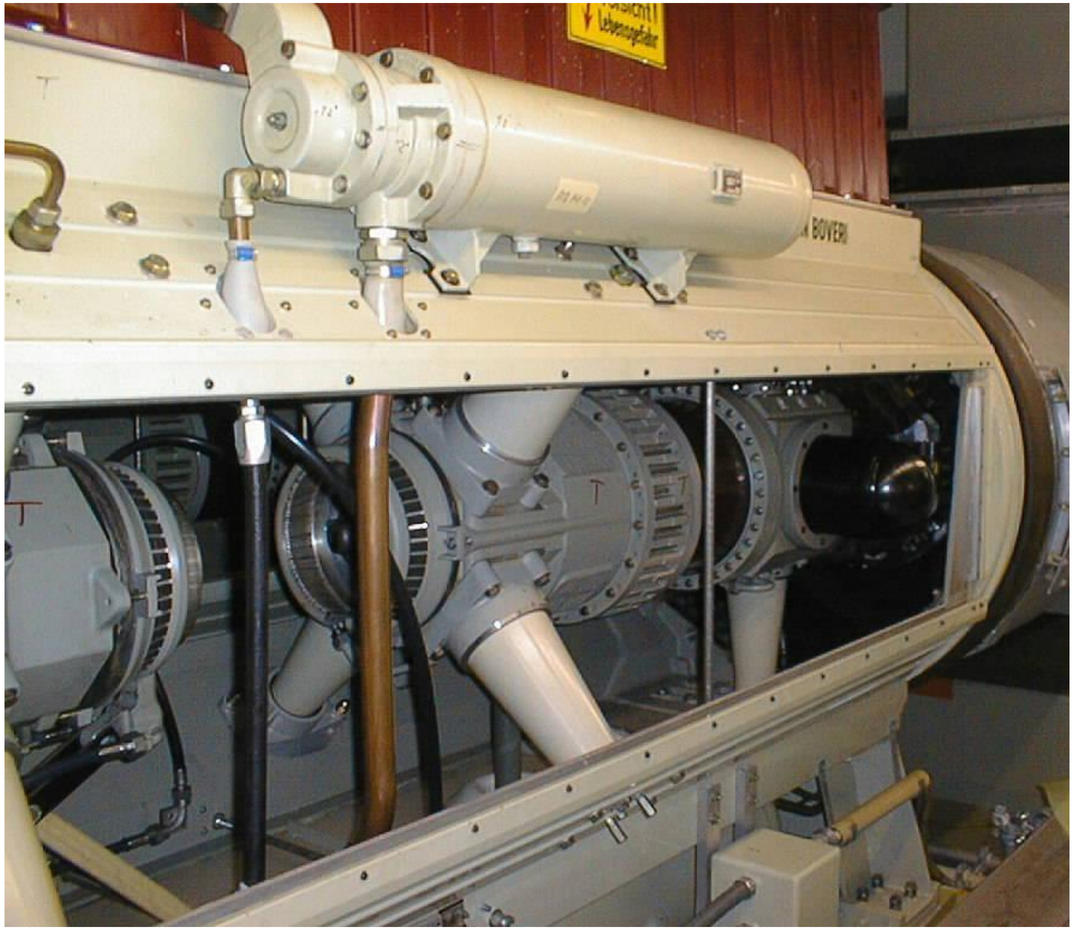
50 Hz

Source: ABB (now GE), Birr, Switzerland

8. Dynamics of synchronous machines

Power switch between generator and transformer

"Pressurized gas generator current switch" between generator winding and transformer for short-circuit current switching-off



One phase of three-phase current switch of four-pole turbine generator in nuclear power plant *Krümmel, D*

Opened during revision

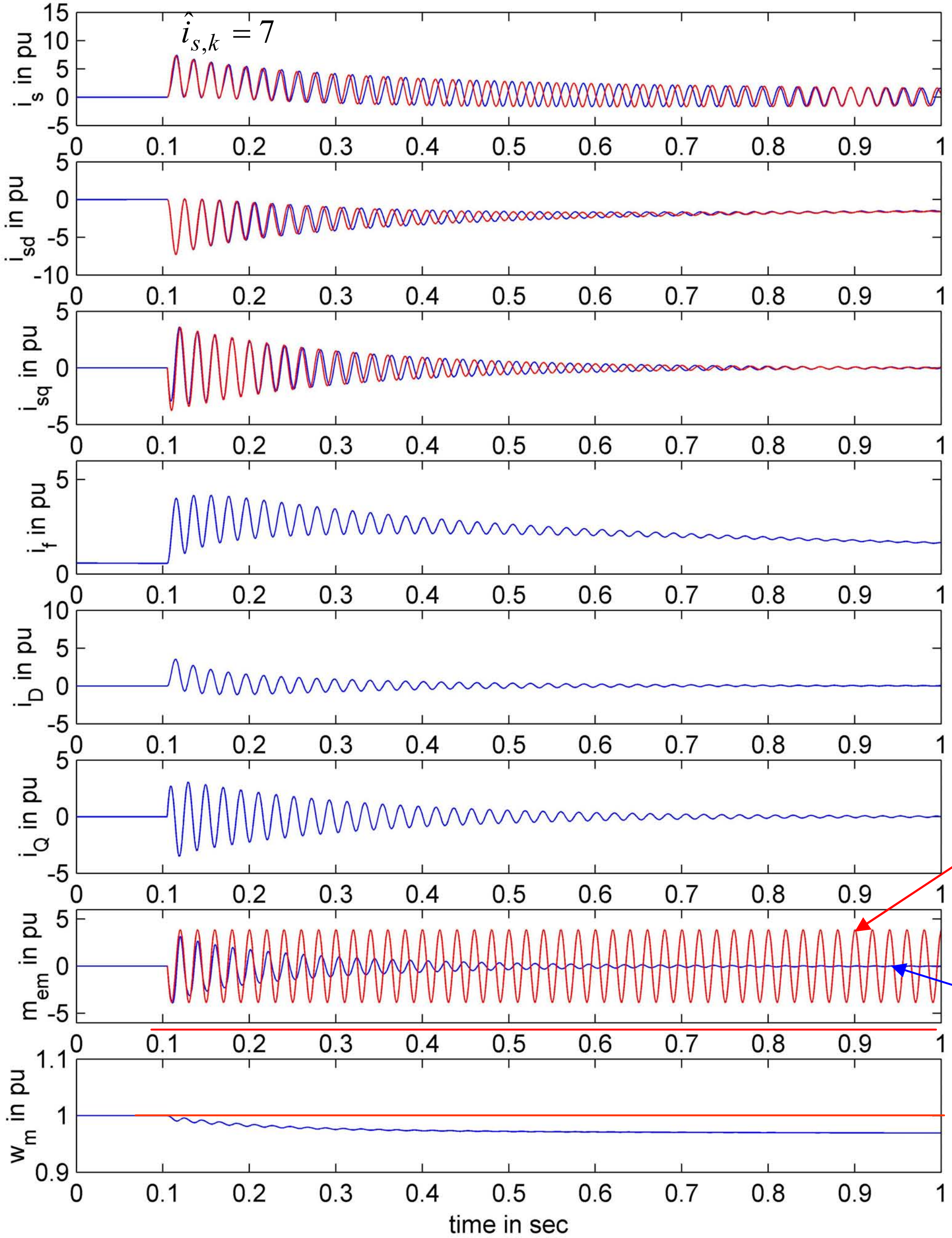
Generator data: 1.4 GW el. power, 1500/min, 50 Hz

Estimation of short circuit peak current:

$$\cos\varphi = 0.85\ \text{o.e.};\ U_N = 26\ \text{kV},\ I_N = 31.1\ \text{kA}$$

$$\hat{I}_k \approx 7 \cdot \sqrt{2} \cdot I_N = 307.8\ \text{kA}$$

Source: Wikipedia.de



Example: Sudden short circuit at zero voltage crossing (1)

2-pole turbine generator 600 MVA, 26 kV, 50 Hz, 3000/min, $I_{fN} = 1800$ A, $U_{fN} = 146$ V:

(ABB Birr, Switzerland (now GE))

$$r_s = 0.004, r_f = 0.001, r_D = 0.0187, r_Q = 0.0867$$

$$\tau_J = 1200$$

$$x_{s\sigma} = 0.19, x_{dh} = 1.73, x_{qh} = 1.66,$$

$$x_{D\sigma} = 0.1313, x_{Q\sigma} = 0.0731, x_{f\sigma} = 0.1642$$

Analytical
(for $\omega_m = 1 = \text{const.}$ and small r_s)

vs.

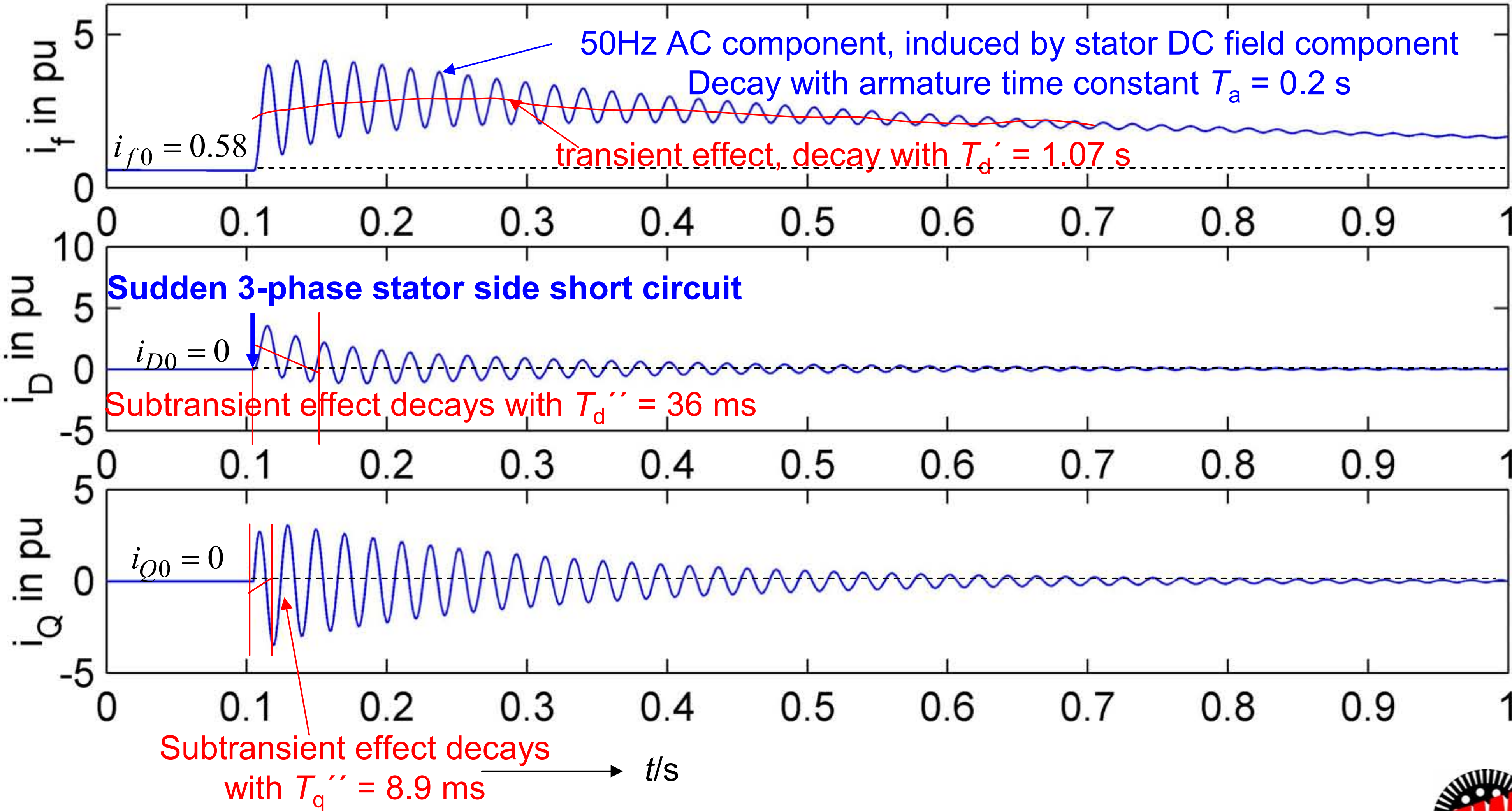
numerical calculation

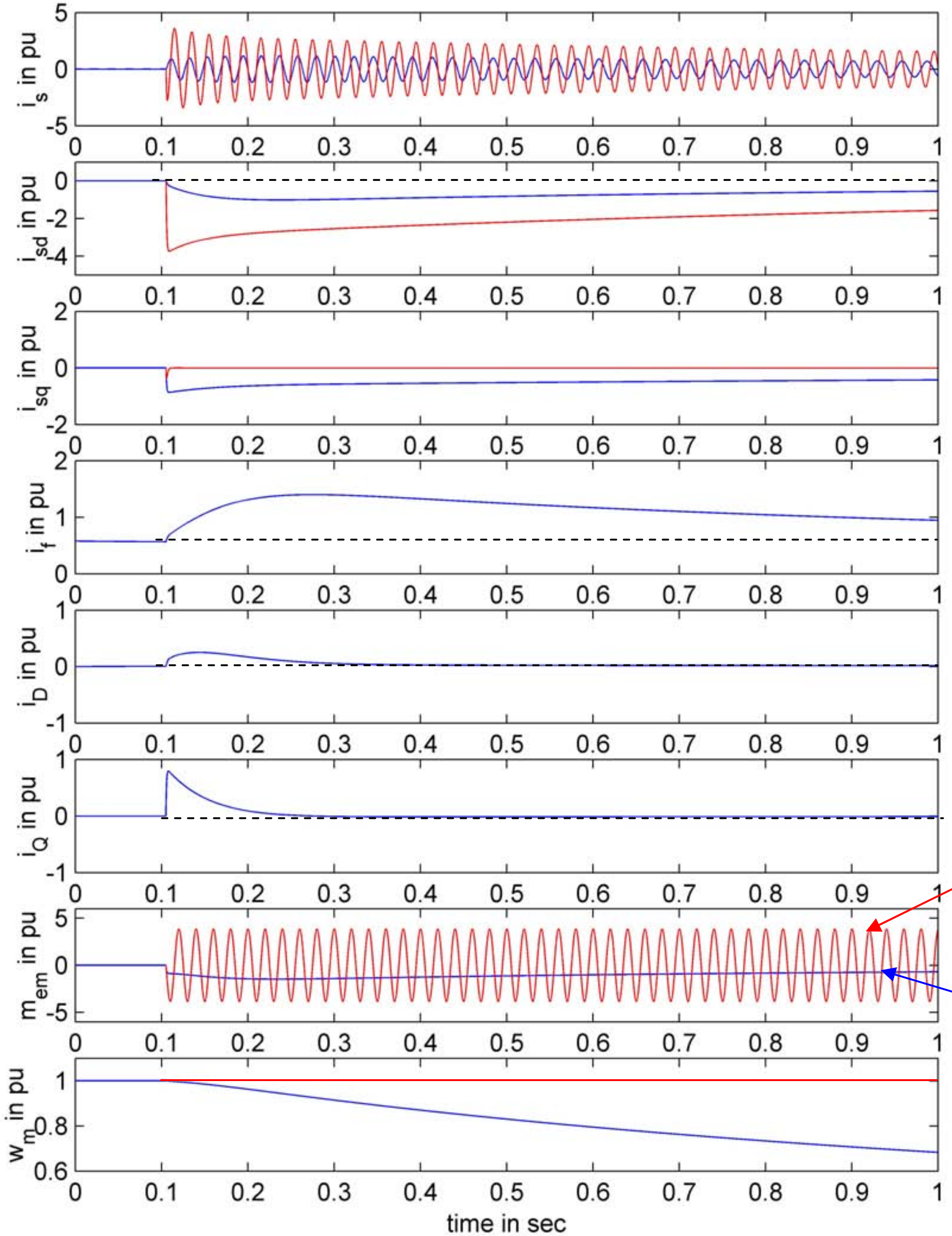
Difference in stator currents negligible, as stator resistance is small!

Difference in torque, as damping is analytically neglected!

8. Dynamics of synchronous machines

Example: Sudden stator-side 3-phase short circuit (2)





Example: Sudden short circuit at zero voltage crossing with very big stator resistance (250-fold!)

$$r_s = 1.0, r_f = 0.001, r_D = 0.0187, r_Q = 0.0867$$

$$\tau_J = 1200, x_{s\sigma} = 0.19, x_{dh} = 1.73, x_{qh} = 1.66,$$

$$x_{D\sigma} = 0.1313, x_{Q\sigma} = 0.0731, x_{f\sigma} = 0.1642$$

$$\tau_a \approx \frac{x''_d}{r_s} = 0.26/1.0 = 0.26 \quad T_a = \tau_a / \omega_N = 0.8ms$$

Difference in calculated stator currents big, as stator resistance is big!

Analytical (for ω_m = const. and small r_s)

vs.

numerical

calculation

Very fast decay of stator DC current component = no oscillations on rotor side.

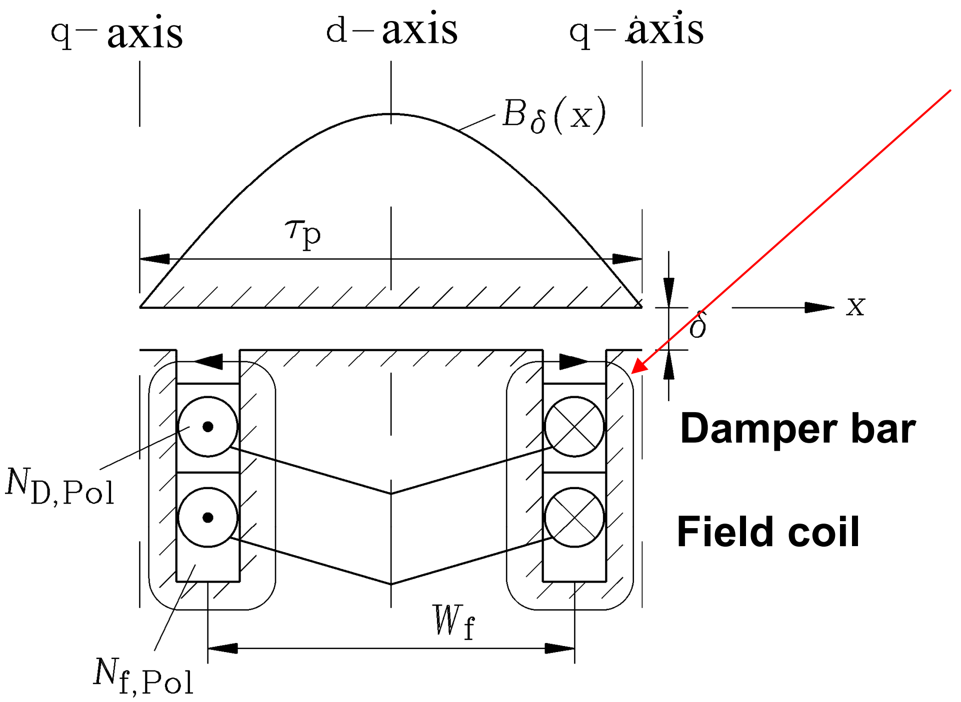
Big difference in calculated torque, as damping is analytically neglected!

Strong decay in speed due to high stator losses!

Enhanced flux linkage model for synchronous machines (1)

Example:

Cylindrical rotor synchronous machine:



Common field lines between damper and field winding in the rotor slots

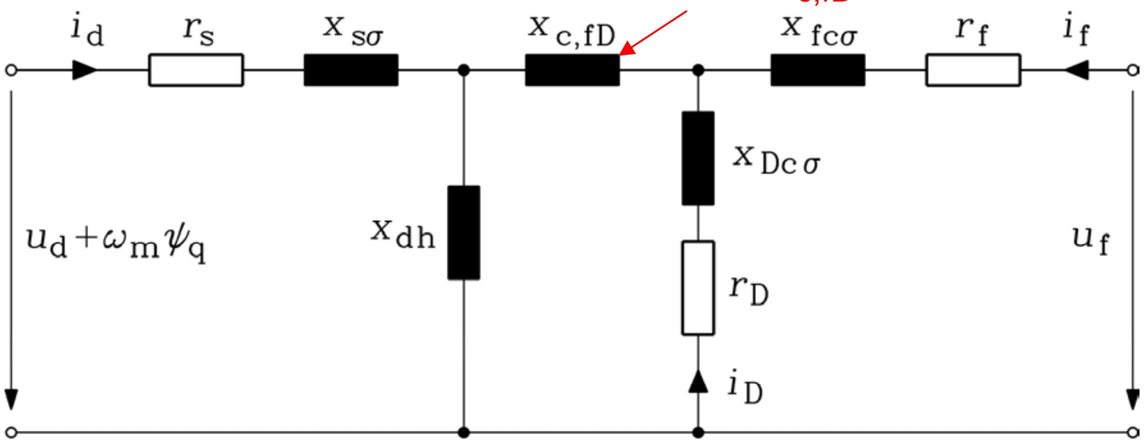
$\Rightarrow$ **Flux linkage between damper and field winding is in cylindrical rotor synchronous machines in the *d*-axis <u>bigger</u> than**

a) between field and stator winding

b) between damper and stator winding

8. Dynamics of synchronous machines

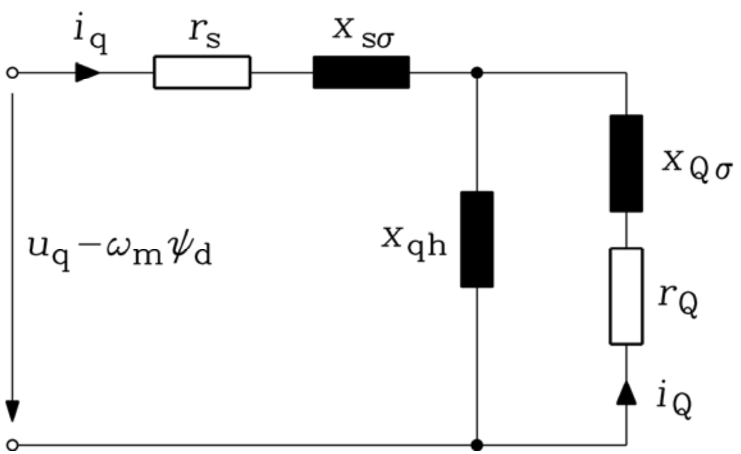
Enhanced flux linkage model for synchronous machines (2)



Modified *d*-axis equivalent circuit model:

Increased flux linkage between damper and field winding is represented by the “coupling” reactance $x_{c,fD}$!

(Dr. M. Canay, BBC, Baden, Switzerland)

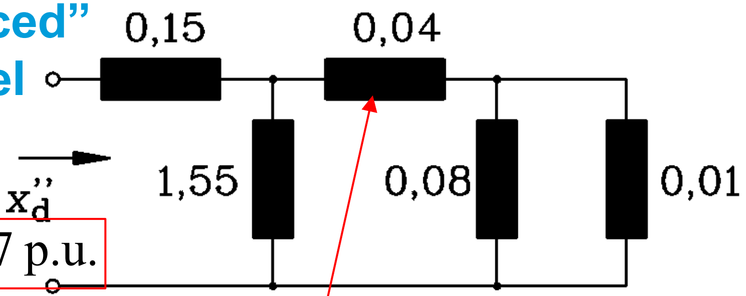


No changes in the *q*-axis equivalent circuit model !

8. Dynamics of synchronous machines

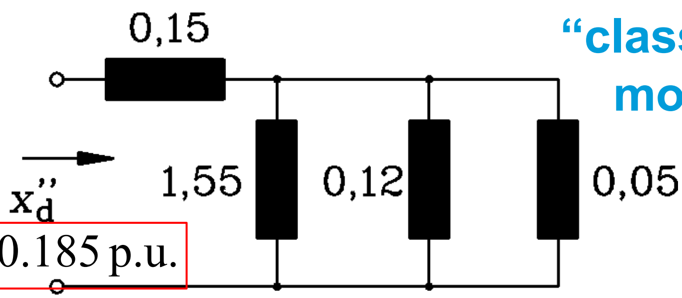
Enhanced flux linkage model for synchronous machines (3)

"enhanced" model



$= 0.197\,\text{p.u.}$

"classical" model



$= 0.185\,\text{p.u.}$

"coupling" reactance $x_{\text{c,fD}}$

Source:
M. Canay, PhD thesis, EPFL Lausanne

$$x = 0.04 + \frac{0.08 \cdot 0.01}{0.08 + 0.01} = 0.0488\,\text{p.u.}$$

$$x''_{\text{d}} = x_{\text{s}\sigma} + \frac{x_{\text{dh}}x}{x_{\text{dh}} + x} =$$

$$= 0.15 + \frac{1.55 \cdot 0.0488}{1.55 + 0.0488} = 0.197\,\text{p.u.}$$

$$x''_{\text{d}} = 0.15 + \frac{1.55 \cdot 0.12 \cdot 0.05}{1.55 \cdot 0.12 + 1.55 \cdot 0.05 + 0.12 \cdot 0.05} =$$

$$= 0.185\,\text{p.u.}$$

Result: **For subtransient reactance**

- **Only small increase of x_{d}'' between "enhanced" and "classical" flux linkage model**
- **Only slight decrease of stator-side short-circuit current!**

8. Dynamics of synchronous machines

Enhanced flux linkage model for synchronous machines (4)

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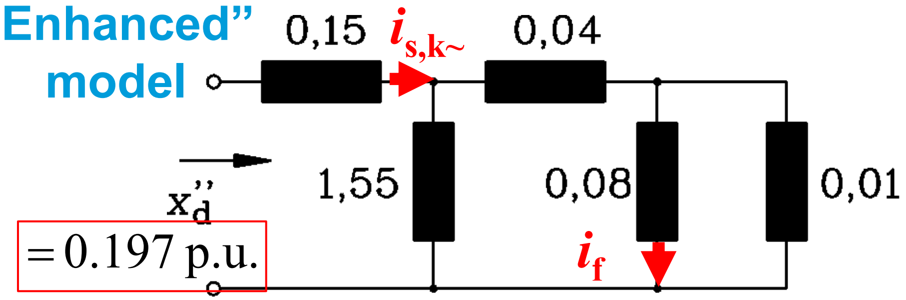
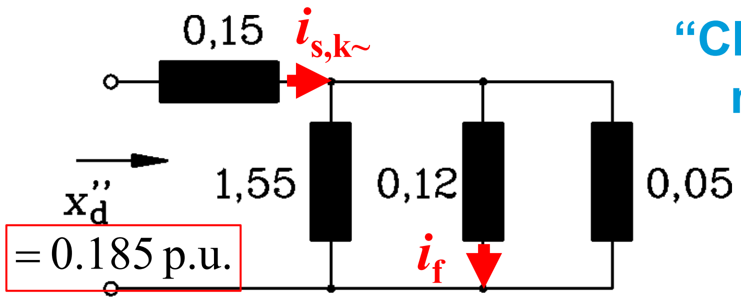
"Enhanced" model





"Classical" model

Source:
M. Canay, PhD thesis, EPFL Lausanne

Stator s.c. AC current:

$$i_{s,k\sim} = \frac{u_s}{x''_d} = \frac{1}{0.197} = 5.07$$

Error with "classical" model:

$$i_{s,k\sim} = \frac{u_s}{x''_d} = \frac{1}{0.185} = 5.42$$ **-6%**

Rotor s.c. AC field current:

$$i_f = i_{s,k\sim} \cdot \frac{1.55}{0.08 + (0.04 + 1.55) \cdot \left(1 + \frac{0.08}{0.01}\right)} = 0.546$$

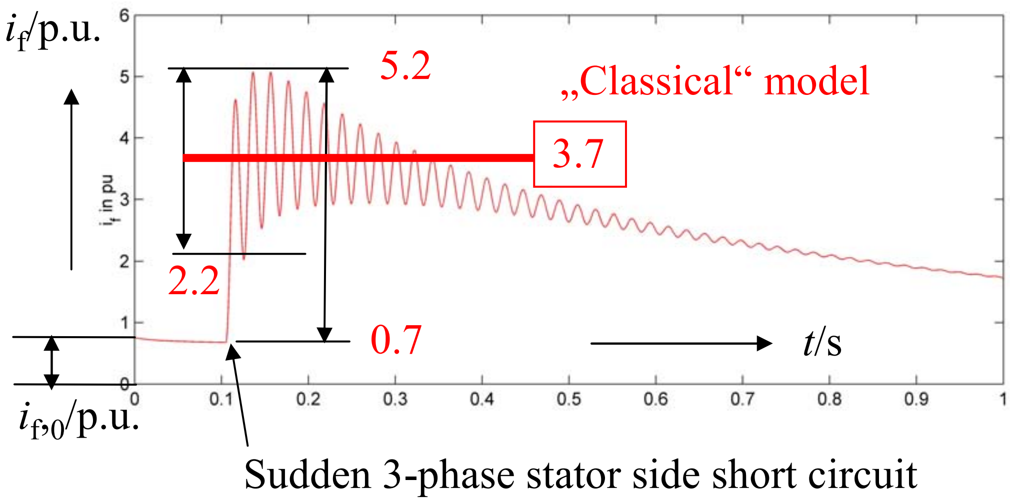
$$i_f = i_{s,k\sim} \cdot \frac{1.55}{0.12 + 1.55 \cdot \left(1 + \frac{0.12}{0.05}\right)} = 1.559$$ **-65%**

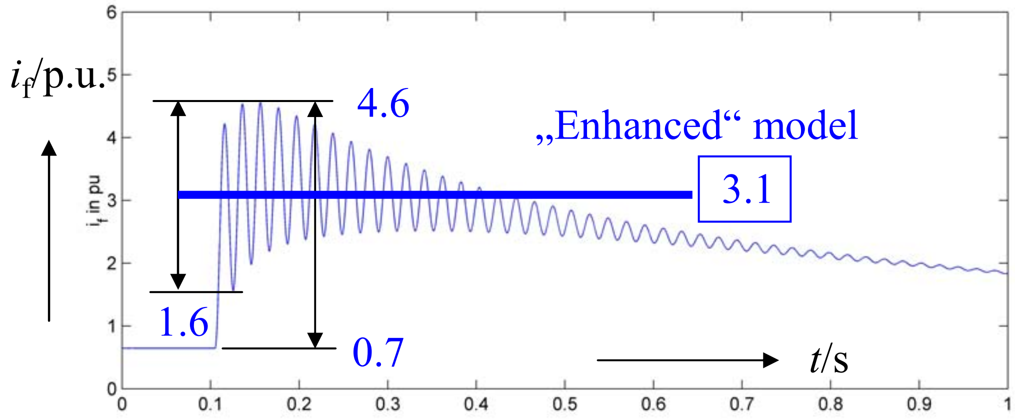
Result:

- **"Classical" model predicts too big rotor side transient field current!**
- **For dynamical rotor side quantities the "enhanced" flux linkage model must be used!**

8. Dynamics of synchronous machines

Example: Transient field current in rotor exciter winding!





$i_{f0} = 0.7$

Numerical RUNGE-KUTTA calculation of the rotor field current due to stator side sudden short circuit after no-load operation.

Data: 2-pole turbine generator

$f_N = 50Hz, \tau_J = 1200, u_{s0} = 1, \omega_{m0} = 1$

$r_s = 0.004, r_f = 0.001, r_D = 0.0187, r_Q = 0.0867$

$x_{s\sigma} = 0.15, x_{dh} = 1.55, x_{qh} = 1.48, x_{Q\sigma} = 0.05,$

$a)$ Classical flux model: $x_{D\sigma} = 0.05, x_{f\sigma} = 0.12$

$b)$ Enhanced flux model: $x_{D\sigma} = 0.01, x_{f\sigma} = 0.08,$

$x_{c,fD} = 0.04$

Transient current overshoot difference:

Enhanced vs. classical flux model:

$$\frac{\Delta i_{f,enhanced}}{\Delta i_{f,classical}} = \frac{3.1 - 0.7}{3.7 - 0.7} = 0.80 \quad -20\%$$

Summary:
Sudden short circuit torque and measurement of transient machine parameters

- Stator DC current component $i_{s,DC}$ causes alternating short-circuit torque with big amplitude (factor 6 … 8), decaying with ca. 50% armature time constant $\tau_a/2$
- Measurement of dynamic inductances and rotor time constants from sudden short circuit test (at reduced stator voltage, usually at 10%)
- Numerical calculation of sudden short circuit for non-constant speed $\omega_m \downarrow$
- Transient rotor currents in damper and field winding visible
- For correct rotor current calculation the more detailed flux linkage model of *M. CANAY* is needed: $x_{c,fD}$

Quasi-static stability

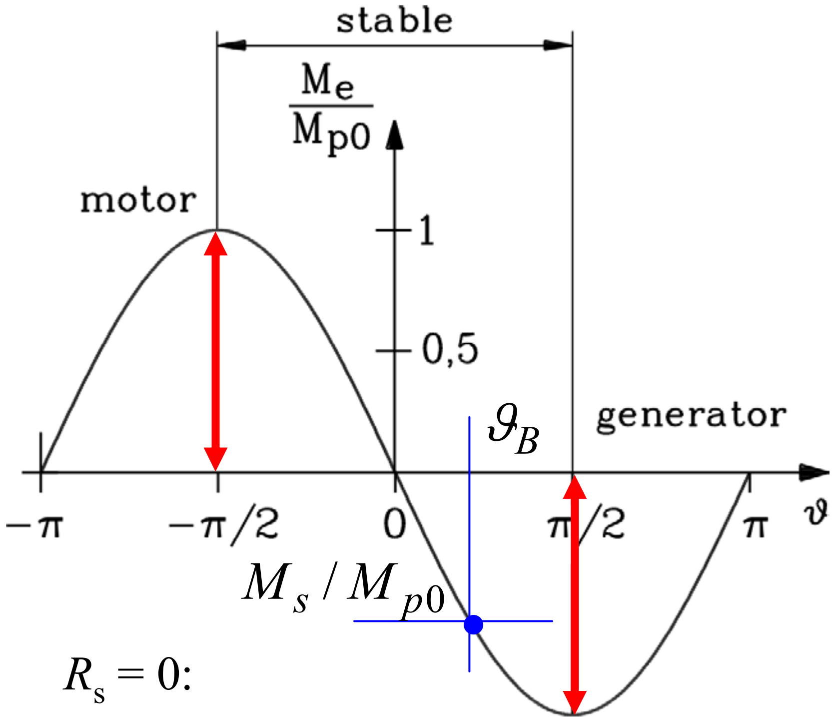
Motivation:

- At steady state operation:
  Limit of quasi-static stability for cylindrical rotor synchronous machines is a max. load angle ϑ of $\pm\pi/2$ and pull-out torque M_{p0}

$$\frac{P_{e,\max}}{S_N} = \pm\frac{P_{e,p0}}{S_N} = \pm\frac{M_{p0}\cdot\Omega_{syn}}{3U_N I_N} = \pm\frac{u_s u_p}{x_d}$$

- At a sudden load step an electrically excited synchronous machine shows for **"transient" time scale** $0 < \tau < 3\tau'_d$ a higher load angle limit $\vartheta > \pi/2$ **and a higher dynamic pull-out torque** $M_{p,dyn} > M_{p,0}$
- This is due to the increased transient field current $i_f > i_{f0}$!

Quasi-static stability range and steady state torque of cylindrical rotor synchronous machine



$R_s = 0$:

$$\pm M_{p0} = \pm 3U_s U_p / (X_d \Omega_{syn})$$

Steady state operation in rotor reference frame

$$u_{d0} = r_s i_{d0} - \omega_s \psi_{q0} = r_s i_{d0} - \omega_s x_q i_{q0}$$

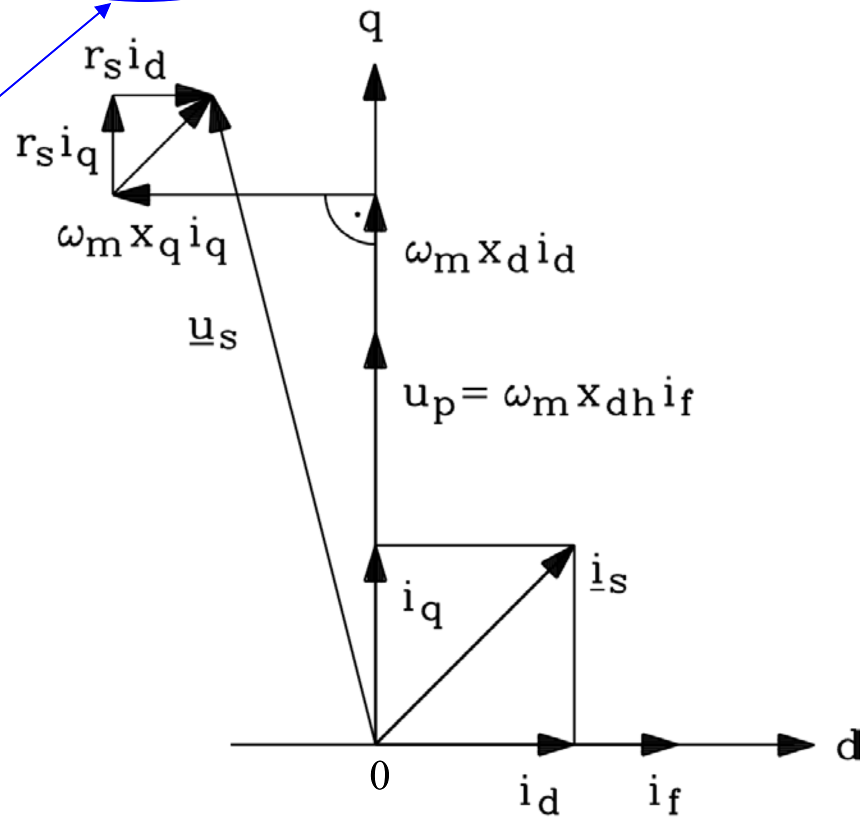
$$u_{q0} = r_s i_{q0} + \omega_s \psi_{d0} = r_s i_{q0} + \omega_s x_d i_{d0} + \omega_s x_{dh} i_{f0}$$

$$\psi_{d0} = x_d i_{d0} + x_{dh} i_{f0}$$

$$\psi_{q0} = x_q i_{q0}$$

Synchronous back EMF u_p

$$u_p = \omega_s x_{dh} i_{f0}$$



Synchronous operation: $\omega_m = \omega_s$

e.g. motor, under-excited

Assumption of transient constant rotor flux ψ_f

- Steady state operation: $u_{f0} = r_f \cdot i_{f0} = \text{const.}$
- Sudden load step at electrically excited synchronous machine: $i_f(\tau) = i_{f0} + \Delta i_f(\tau)$
- *After a sudden load step:* Damper bar currents already vanished, transient field current still flows = **"transient" time scale:** $3\tau''_d < \tau < 3\tau'_d$

$$u_f = \cancel{u_{f0}} = r_f \cdot i_f(\tau) + d\psi_f(\tau)/d\tau = \cancel{r_f \cdot i_{f0}} + r_f \cdot \Delta i_f(\tau) + d\psi_f(\tau)/d\tau$$

$$0 = \underbrace{r_f \cdot \Delta i_f(\tau)}_{\approx 0} + d\psi_f(\tau)/d\tau$$

→ **r_f neglected**: $d\psi_f / d\tau = 0 \Rightarrow \psi_f = \text{const.}$

$$\psi_f = x_{dh} i_d + x_f i_f = \text{const.} \longrightarrow i_f = (\psi_f - x_{dh} i_d)/x_f$$

- Stator flux linkage of *d*-axis during transient state:

$$\psi_d = x_d i_d + x_{dh} i_f = x_d i_d - (x_{dh}^2 / x_f) \cdot i_d + (x_{dh}/x_f) \cdot \psi_f = x'_d i_d + (x_{dh}/x_f) \cdot \psi_f$$

- Transient reactance: $x'_d = x_d - (x_{dh}^2 / x_f) = \sigma_{df} \cdot x_d$

Transient back EMF u'_p (= damping of transient i_f is neglected)

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- Comparing stator *d*-axis flux linkage (for $r_s = 0$) before and after load step **in *d*-axis:**

Before: $u_{q0} = \omega_s \psi_{d0} = \omega_s \cdot (x_d i_{d0} + x_{dh} i_{f0})$

After: $u_q = \omega_s \psi_d = \omega_s \cdot (x'_d i_d + (x_{dh} / x_f) \cdot \psi_f)$

a) Instead of x_d now the **transient inductance** x'_d is acting.

b) Instead of $\omega_s x_{dh} i_{f0}$ (= stationary back EMF u_p) the smaller value $(x_{dh} / x_f) \cdot \omega_s \psi_f$ has to be taken.

Transient back EMF:

$$u'_p = \frac{x_{dh}}{x_f} \cdot \omega_s \psi_f$$

- **In quadrature axis** due to $x'_q = x_q$ stationary and transient conditions are identical.

- $X_d \to X'_d$, $U_p \to U'_p$ and use of complex calculus for sine wave $u_s(t)$, $i_s(t)$

8. Dynamics of synchronous machines

Transient parameters x'_d and u'_p

- ***Result:***
  For synchronous machines in transient state: $\tau < 3\tau'_d$

a) Calculate phasor diagram in rotor reference frame like in synchronous state,

b) BUT

- take instead of u_p the transient back EMF u'_p

- take instead of synchronous reactance x_d the transient reactance x'_d .

- Transient back EMF: (For $r_f \approx 0$)
  Induced voltage from rotor side flux of DC and transient field current i_f is considered to have constant amplitude during transient state: $\tau < 3\tau'_d$

Transient stability of cylindrical rotor synchronous machine

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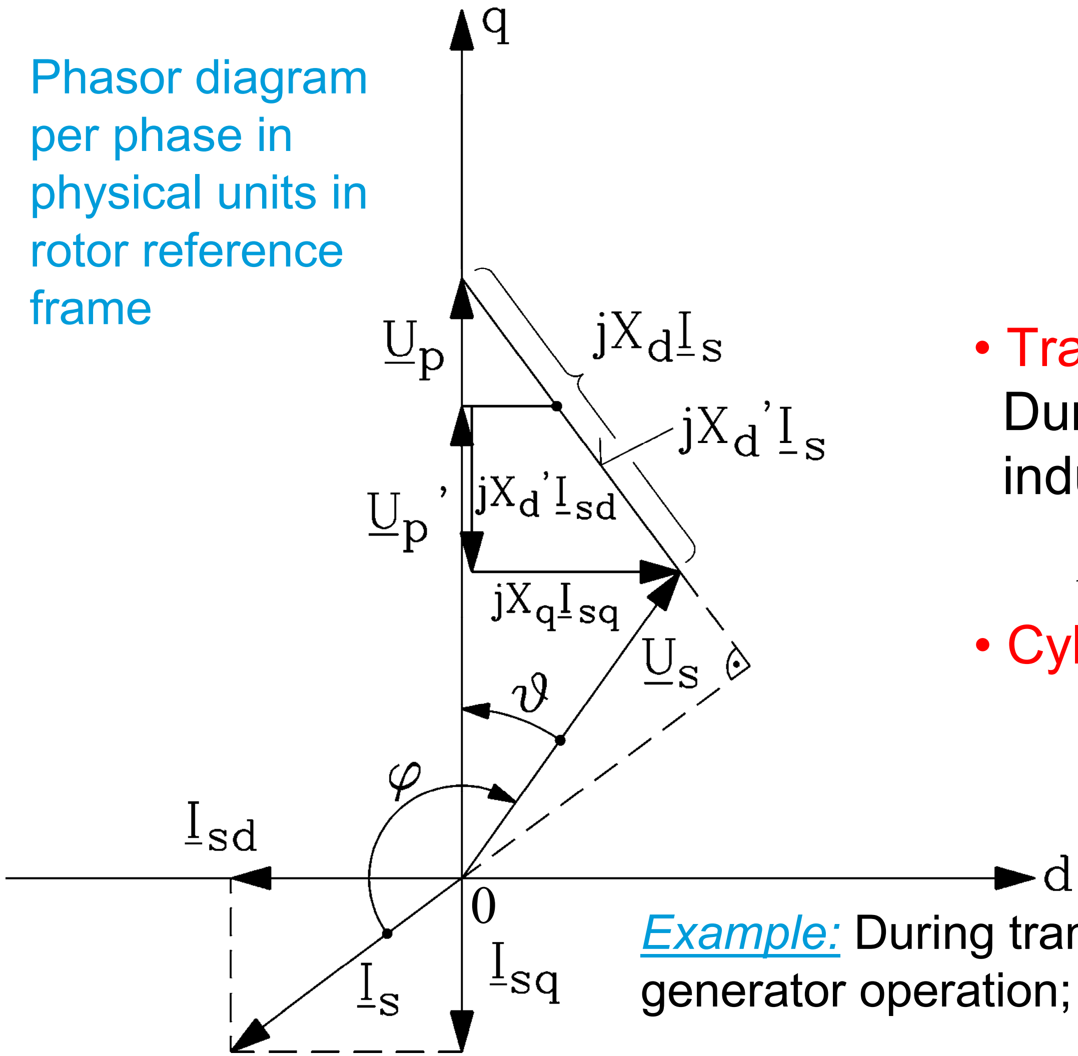
Phasor diagram per phase in physical units in rotor reference frame



$\tau < 3\tau'_d$:

$$u'_p = \frac{x_{dh}}{x_f} \cdot \omega_s \psi_f = const.$$

- Transient reactance:
  During transient state stator winding inductance is transient inductance !

$$X'_d < X_d \qquad X'_q = X_q$$

- Cylindrical rotor synchronous machine:

$$X'_q = X_q = X_d$$

Example: During transient state for over-excited generator operation; stator resistance neglected (R_s = 0)

Transient electric machine power $P_{e,dyn}$ of cylindrical rotor machine (1)

- Calculating overload power from phasor diagram at given voltage U_s, U'_p and $R_s = 0$:

$$P_{e,dyn} = m_s \cdot \mathrm{Re}\{\underline{U}_s \cdot \underline{I}_s^*\} = m_s \cdot \mathrm{Re}\{(U_d + jU_q)\cdot(I_d - jI_q)\} = m_s \cdot (U_d I_d + U_q I_q)$$

$$U_d = U_s \cdot \sin\vartheta,\quad U_q = U_s \cdot \cos\vartheta,\quad I_d = (U_q - U'_p)/X'_d,\quad I_q = -U_d / X_d$$

$$P_{e,dyn} = m_s \cdot \left[U_d \cdot (U_q - U'_p)/X'_d - U_q U_d / X_d\right]$$

$$P_{e,dyn} = -m_s \cdot \left[U_d U'_p / X'_d - U_d U_q \cdot (X'^{-1}_d - X^{-1}_d)\right]$$

$$P_{e,dyn} = -m_s \cdot \left(\frac{U_s U'_p}{X'_d}\cdot\sin\vartheta - \frac{U_s^2}{2}\cdot\left(\frac{1}{X'_d} - \frac{1}{X_d}\right)\cdot\sin(2\vartheta)\right) \qquad M_{e,dyn} = \frac{P_{e,dyn}}{\Omega_{syn}}$$

- Looks like salient pole machine power characteristic, but is cylindrical rotor characteristic in transient state !

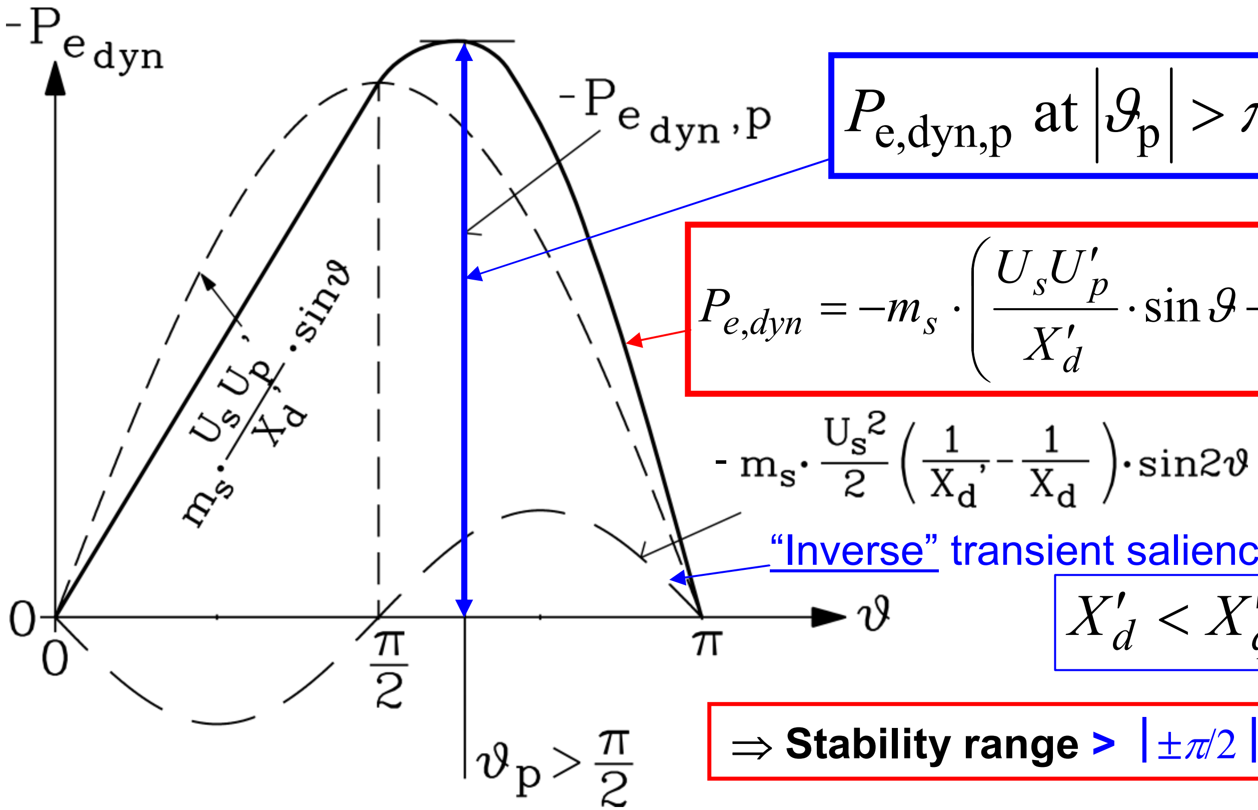
- Dynamic pull-out torque: $M_{p,dyn} = P_{e,dyn,p} / \Omega_{syn}$ $\qquad M_{p,dyn} > M_{p0}$

8. Dynamics of synchronous machines

Transient electric machine power $P_{e,dyn}$ of cylindrical rotor machine (2)

p.u.-power: $p_{e,dyn} = \frac{P_{e,dyn}}{(m_s/2)\cdot \hat{U}_N \hat{I}_N} = -\frac{u_s u'_p}{x'_d}\cdot \sin\vartheta + \frac{u_s^2}{2}\cdot\left(\frac{1}{x'_d}-\frac{1}{x_d}\right)\cdot\sin(2\vartheta)$

Generator mode:



$P_{e,dyn,p}$ at $|\vartheta_p| > \pi/2$

$$P_{e,dyn} = -m_s \cdot \left(\frac{U_s U'_p}{X'_d}\cdot \sin\vartheta - \frac{U_s^2}{2}\cdot\left(\frac{1}{X'_d}-\frac{1}{X_d}\right)\cdot\sin(2\vartheta)\right)$$

"Inverse" transient saliency between *d*- and *q*-axis:

$$X'_d < X'_q = X_d$$

⇒ **Stability range > $|\pm\pi/2|$, increased overload power**

8. Dynamics of synchronous machines

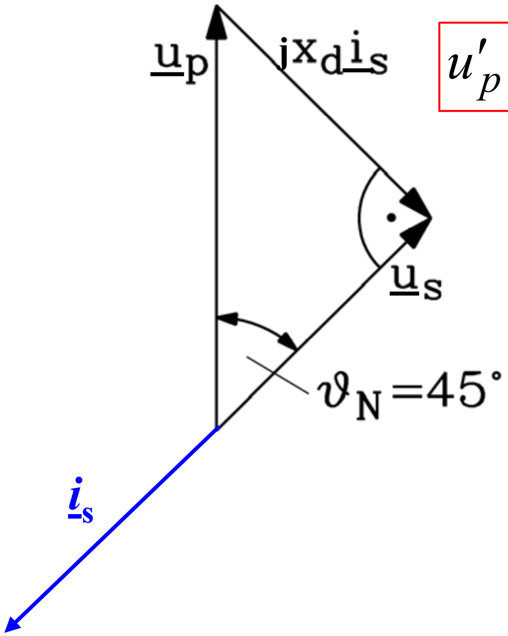
Transient pull-out power & transient pull-out torque

- Transient pull-out power / torque much bigger than at steady state, e.g.: 3.68/1.41 = 2.6!

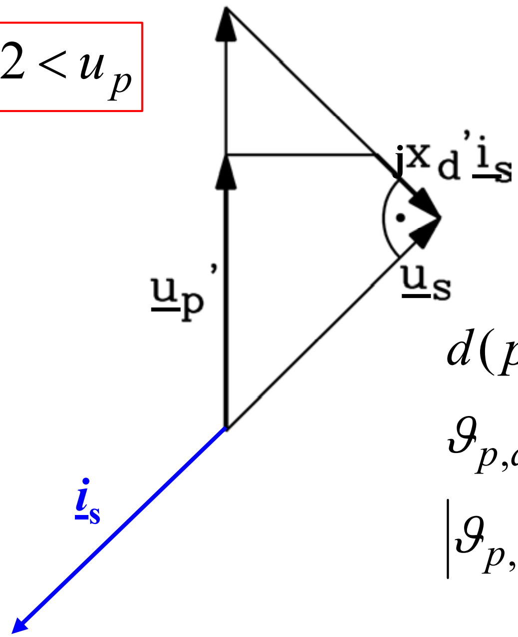
Example:

Over-excited synchronous generator with cylindrical rotor:

Data: $u_s = 1$, $i_s = 1$, $\vartheta_N = 45°$, $x_d = 1$, $x'_d = 0.3$, $r_s \approx 0$. $\cos\varphi_s = -1$



$$u'_p = 0.92 < u_p$$



$$\frac{M_{p,dyn}}{M_{p0}} = \frac{3.68}{1.41} = 2.61\,!$$

$$d(p_{e,dyn})/d\vartheta = 0:$$

$$\vartheta_{p,dyn} = \pm 116.8°$$

$$\left|\vartheta_{p,dyn}\right| = 116.8° > 90°$$

Synchronous pull-out power:

$$P_{e,p0}/(m_s U_N I_N) = -u_s u_p / x_d = -1.41$$

Increased dynamic pull-out power:

$$p_{e,dyn,p} = P_{e,dyn,p}/(m_s U_N I_N) = -3.68$$

Transient electric machine power $P_{e,dyn}$, salient pole machine

- Calculating overload power from phasor diagram at given voltage U_s, U'_p and $R_s = 0$:

$$P_{e,dyn} = m_s \cdot \mathrm{Re}\{\underline{U}_s \cdot \underline{I}_s^*\} = m_s \cdot \mathrm{Re}\{(U_d + jU_q)\cdot(I_d - jI_q)\} = m_s \cdot (U_d I_d + U_q I_q)$$

$$U_d = U_s \cdot \sin\vartheta,\quad U_q = U_s \cdot \cos\vartheta,\quad I_d = (U_q - U'_p)/X'_d,\quad I_q = -U_d / X_q$$

$$P_{e,dyn} = -m_s \cdot \left(\frac{U_s U'_p}{X'_d} \cdot \sin\vartheta - \frac{U_s^2}{2} \cdot \left(\frac{1}{X'_d} - \frac{1}{X_q} \right) \cdot \sin(2\vartheta) \right)$$

$$p_{e,dyn} = \frac{P_{e,dyn}}{(m_s/2)\cdot \hat{U}_N \hat{I}_N} = -\frac{u_s u'_p}{x'_d} \cdot \sin\vartheta + \frac{u_s^2}{2} \cdot \left(\frac{1}{x'_d} - \frac{1}{x_q} \right) \cdot \sin(2\vartheta)$$

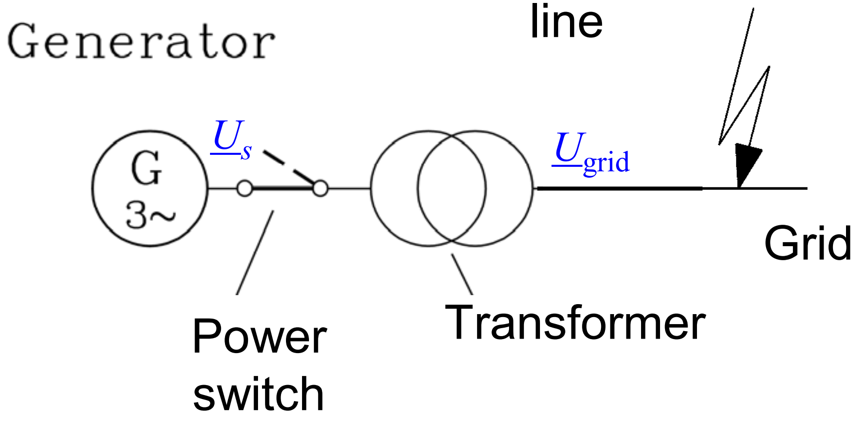
- “Inverse” transient saliency between *d*- and *q*-axis:

	Cylindrical rotor machine	Salient pole rotor machine
Synchronous state	$X_d = X_q$	$X_d > X_q$
Transient state	$X'_d < X_q = X_d$	$X'_d < X_q < X_d$

8. Dynamics of synchronous machines

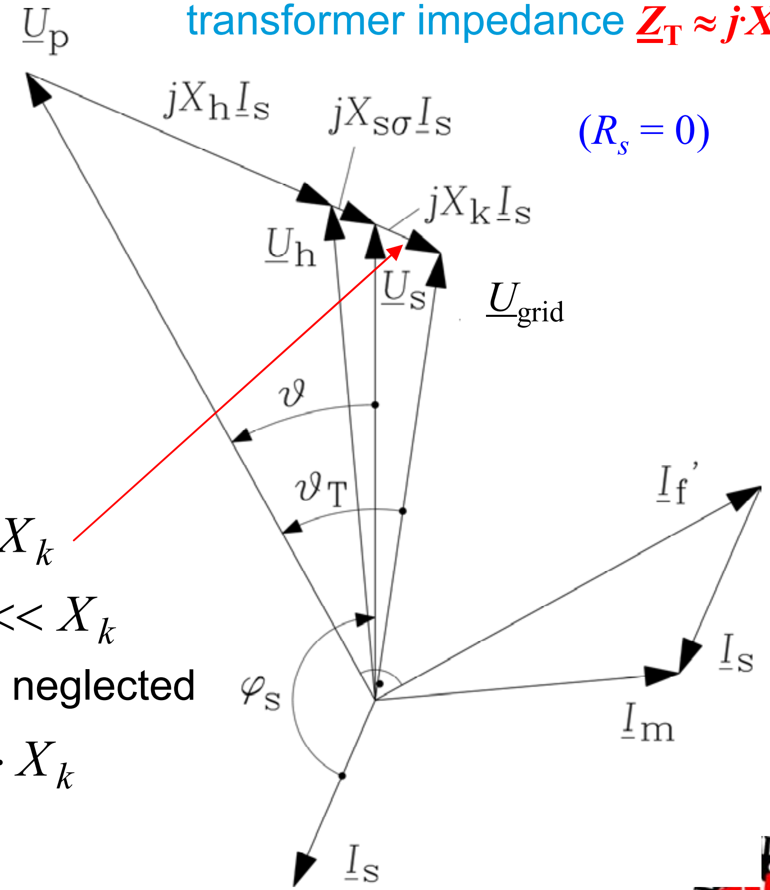
Including of transformer impedance $\underline{Z}_T \approx j \cdot X_k$

Short circuit on the transmission line



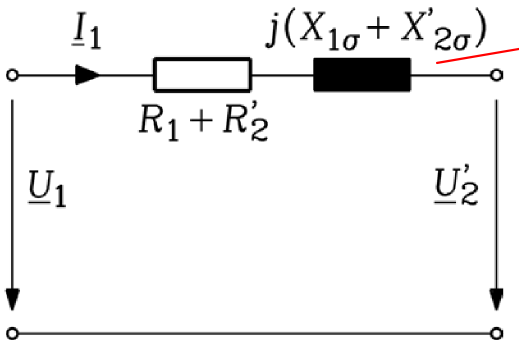
Steady state phasor diagram of synchronous machine, including transformer impedance $\underline{Z}_T \approx j \cdot X_k$

$(R_s = 0)$



Transformer equivalent circuit per phase

(magnetizing current neglected):



$$X_{1\sigma} + X'_{2\sigma} = X_k$$

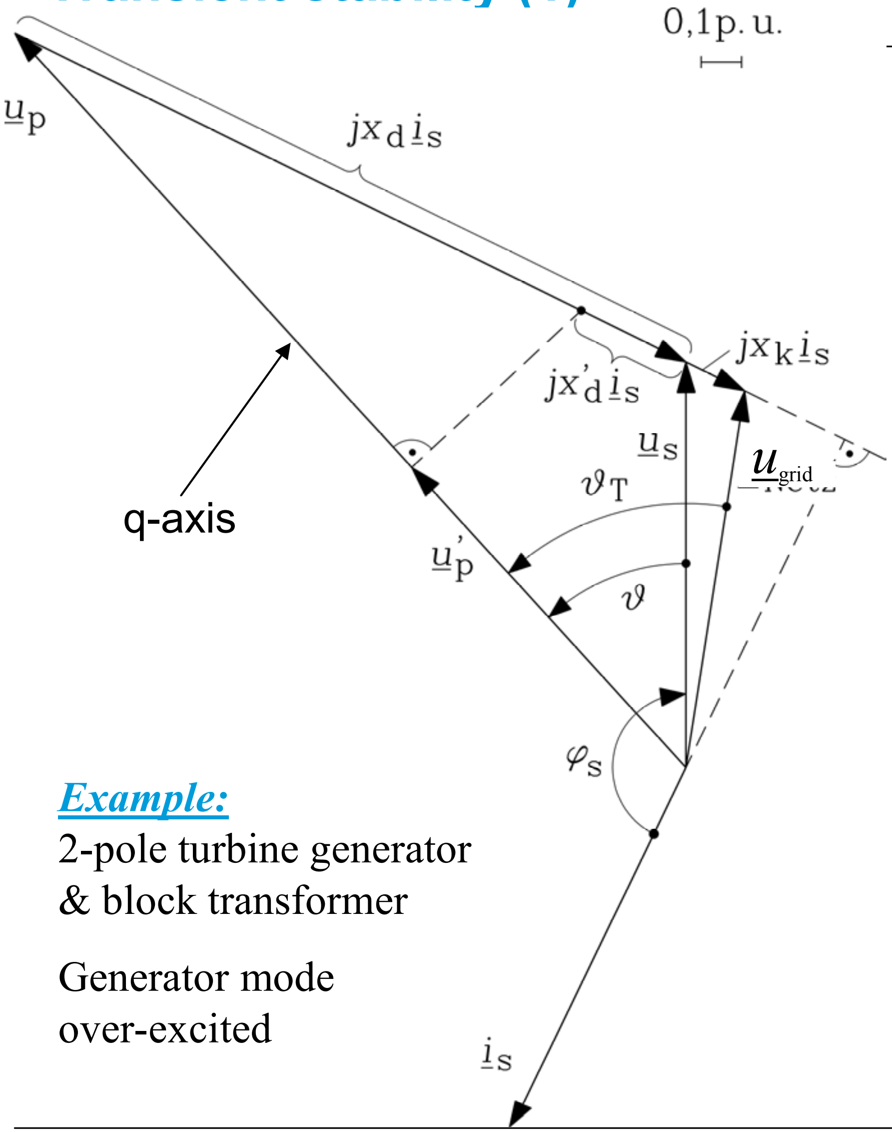
$$R_k = R_1 + R'_2 << X_k$$

Transformer resistances neglected

$$\underline{Z}_T = R_k + j \cdot X_k \approx j \cdot X_k$$

8. Dynamics of synchronous machines

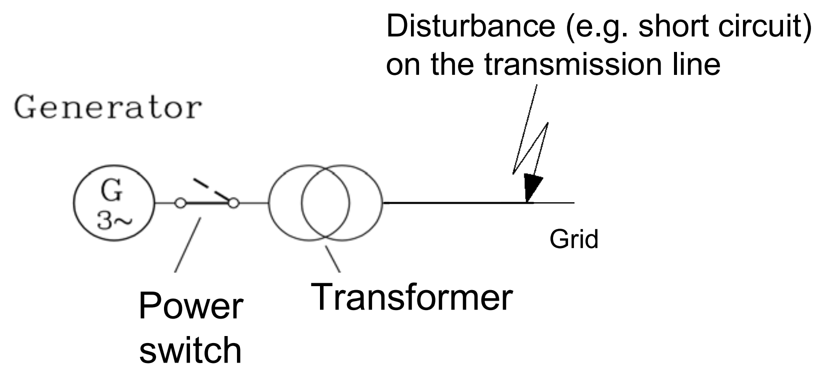
Transient stability (1)



- Synchronous machine is operating after a "transient disturbance" with the transient phasor diagram during the short time of ca. $3T'_d$
- Load angle ϑ_T: Between *q*-axis and "impressed" grid voltage $\underline{u}_{grid}$!
- Reduced back EMF $\underline{u}_p'$, but increased dynamic pull-out torque $M_{p,dyn}$!
- Resultant impedance $x'_d + x_k$
- Resistances r_s, r_k neglected!

Example:
2-pole turbine generator
& block transformer

Generator mode
over-excited



8. Dynamics of synchronous machines

Transient stability (2)

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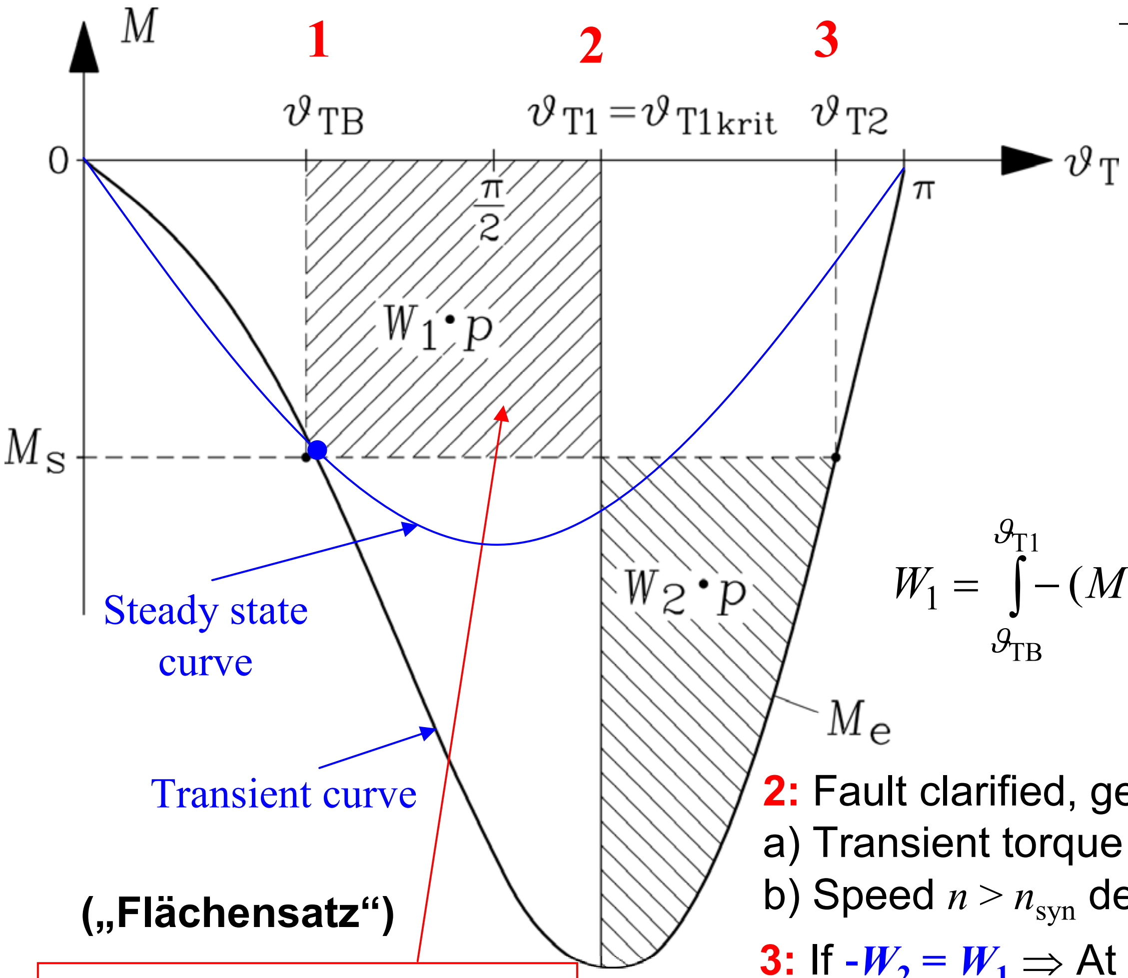

(„Flächensatz“)

Maximum allowable: $W_{1,crit}$!
Equal “areas” $|W_1 \cdot p| = |W_2 \cdot p|$

1: Steady state operation ϑ_{TB}, M_s, when short circuit occurs

a) Machine shifts fast from sub-transient to transient state

b) Switch disconnects generator $\Rightarrow i_s, M_e = 0$

c) Turbine torque M_s accelerates generator: $n > n_{syn}$, doing the work $W_1 > 0 \Rightarrow$ load angle ϑ_T increases

$$W_1 = \int_{\vartheta_{TB}}^{\vartheta_{T1}} -(M_s / p) \cdot d\vartheta_T > 0 \qquad W_2 = \int_{\vartheta_{T1}}^{\vartheta_{T2}} \frac{M_e - M_s}{p} \cdot d\vartheta_T < 0$$

2: Fault clarified, generator reconnected, i_s current flows:

a) Transient torque M_e brakes the generator set: Work $W_2 < 0$

b) Speed $n > n_{syn}$ decreases, but load angle still increases to ϑ_{T2}

3: If $-W_2 = W_1 \Rightarrow$ At ϑ_{T2} the gen-set speed reaches again synchronous speed $n = n_{syn} \Rightarrow$ Generator synchronous again =
Transient stability is fulfilled!

8. Dynamics of synchronous machines

Transient stability: Sketch of load angle ϑ_T & speed n

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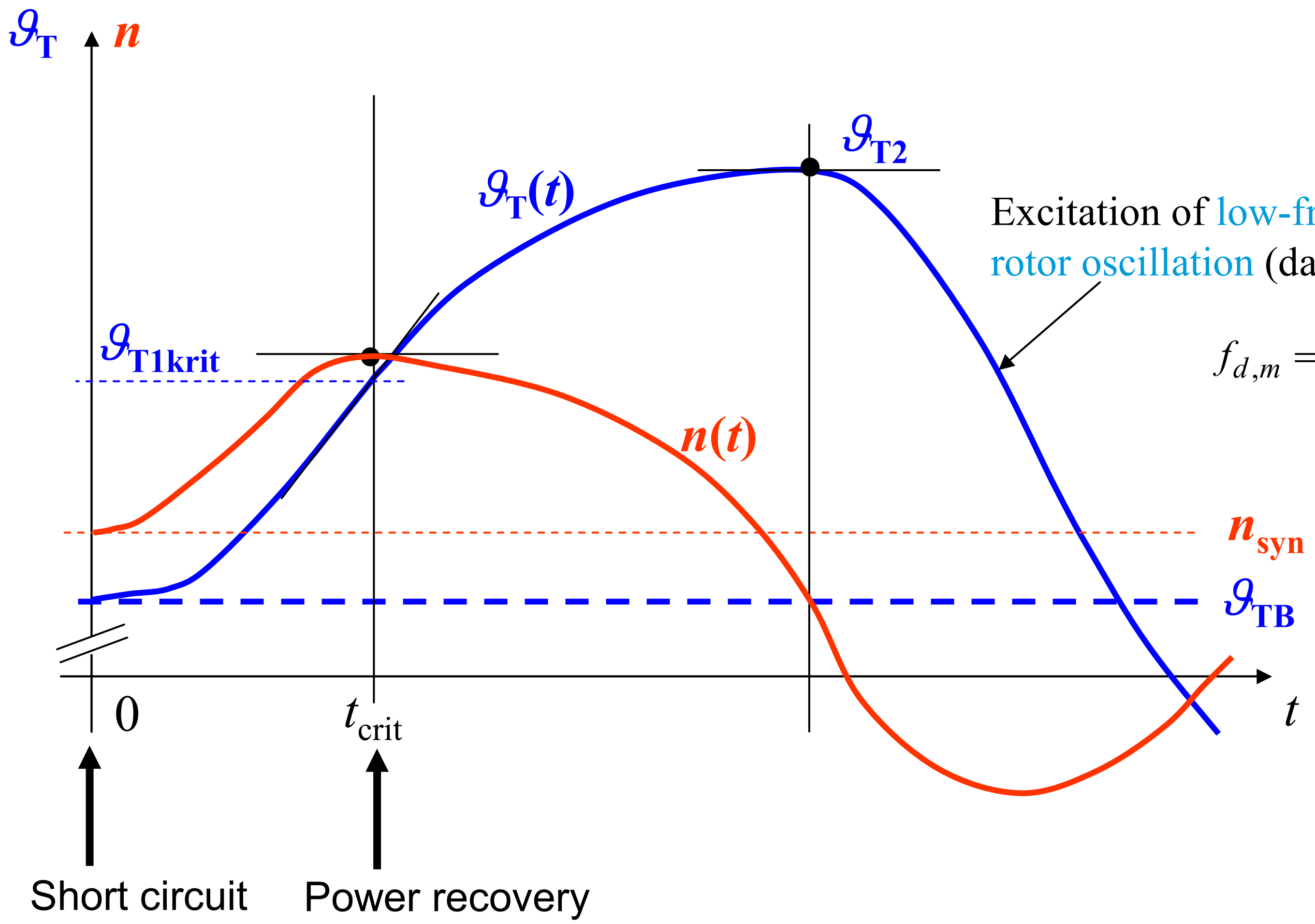
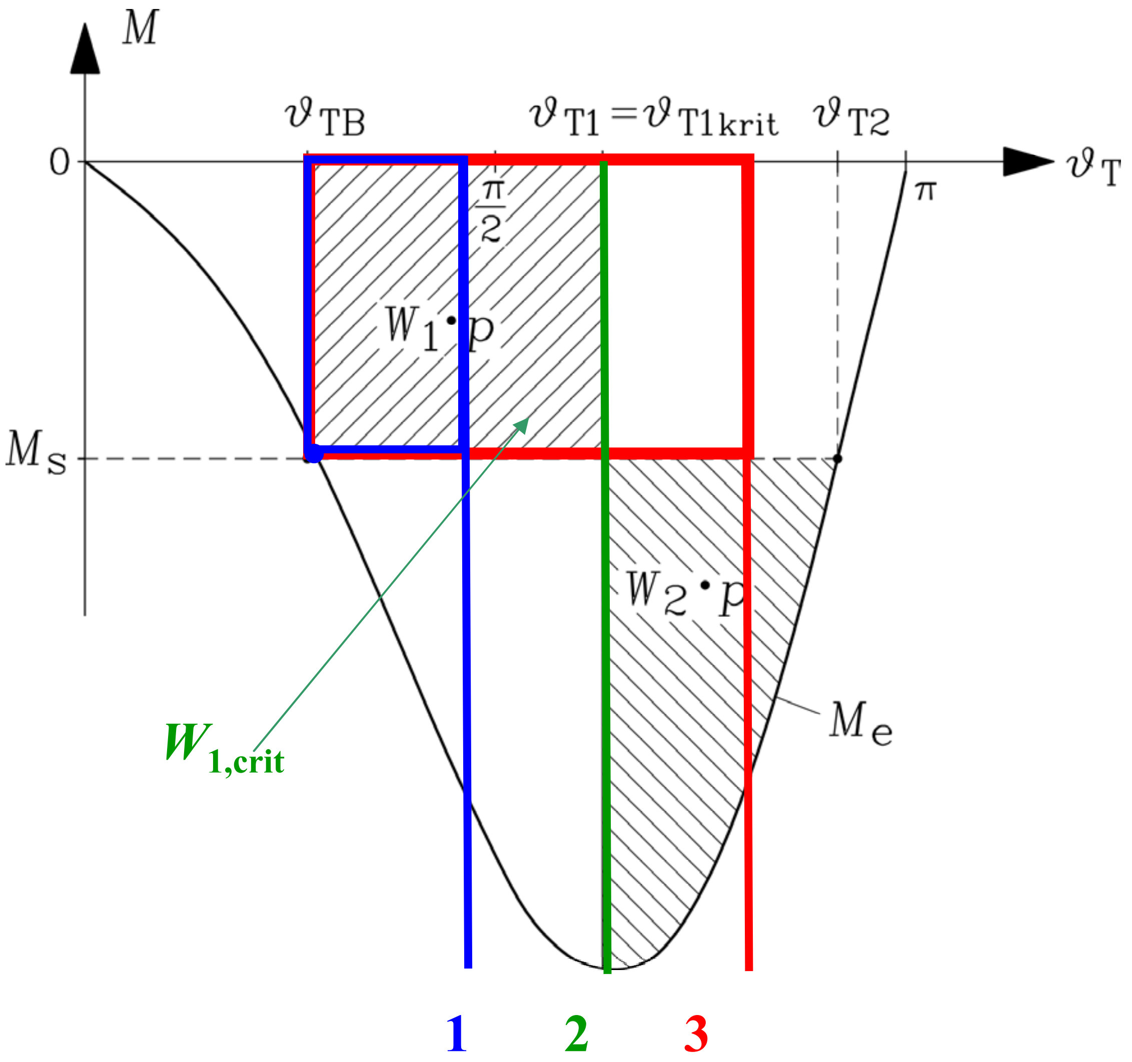


Excitation of low-frequent ($f_{d,m}$) mechanical torsional rotor oscillation (damping α by damper cage)

$$f_{d,m} = \frac{1}{2\pi} \cdot \sqrt{\frac{p \cdot |c_\vartheta|}{J} - \alpha^2} \qquad c_\vartheta = \frac{\partial M_e}{\partial \vartheta}$$

Transient stability (3)



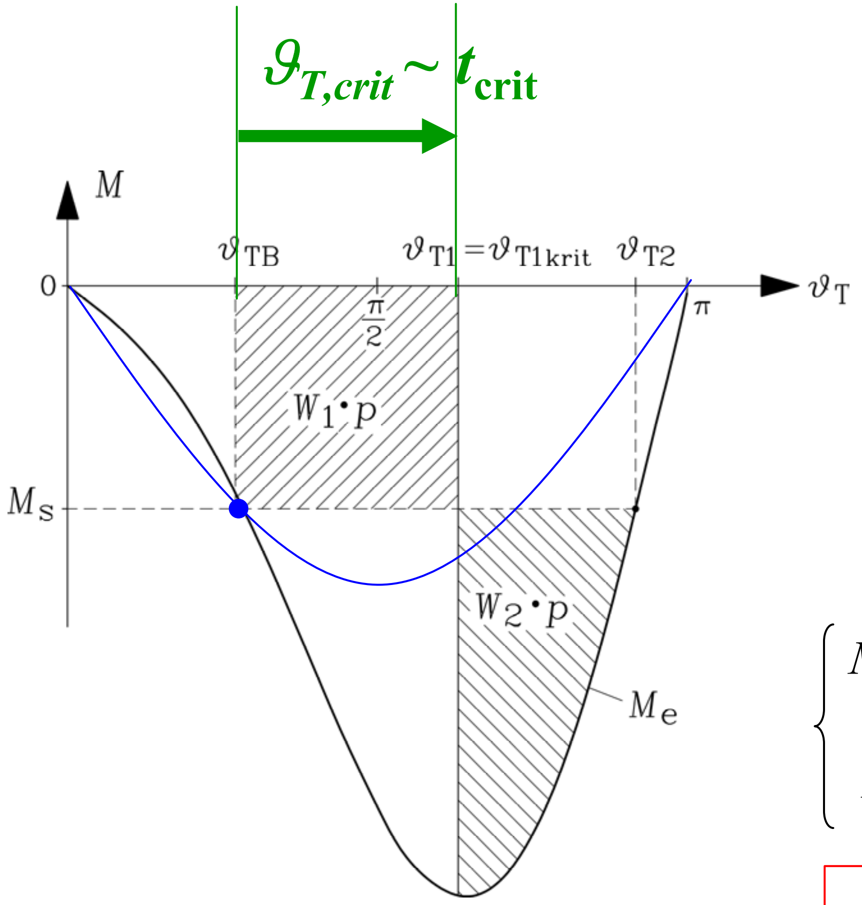
Case 1: $W_1 < W_{1,crit}$: Stable operation, because small acceleration ⇒ generator re-synchronizes.

Case 2: $W_1 = W_{1,crit}$: Critical case: Still stable operation ⇒ generator re-synchronizes

Case 3: $W_1 > W_{1,crit}$: Too big acceleration. Sufficient braking not possible within load curve range $0 \leq \vartheta_T \leq \pi$: $\vartheta_T > \pi$ = Slipping. Unstable operation ⇒ generator does not re-synchronize. It is further accelerated and must be switched off the grid.

New synchronization process needed, starting from no-load!

Critical fault clearing time t_{crit}



Case 2: $W_1 = W_{1,crit} = W_2$

Case 2: $W_1 = W_{1,crit}$: Critical case: Still stable operation, generator re-synchronizes

$$J \cdot \frac{\ddot{\gamma}_e}{p} = M_e - M_s = -M_s \quad \gamma_e : \text{el.degrees}$$

$$\ddot{\gamma}_e = \dot{\Omega}_m \cdot p = \ddot{\vartheta} \qquad \vartheta_{T1crit} = -p \cdot \frac{M_s}{J} \cdot \frac{t_{crit}^2}{2} + \vartheta_{TB}$$

$$t_{crit} = \sqrt{(\vartheta_{T1crit} - \vartheta_{TB}) \cdot \frac{2J}{-pM_s}}$$

Consumer reference frame:
Generator: $M_s < 0$

$$\begin{cases} M_s = P/(\eta \cdot \Omega_{syn}) = S_N \cos\varphi_s /(\eta \cdot \Omega_{syn}) & \Omega_{syn} = \omega_N / p \\ T_J = J \cdot \frac{\omega_N}{p \cdot M_B} = J \cdot \frac{\omega_N^2}{p^2 \cdot S_N} & \eta \approx 1 \end{cases}$$

$$t_{crit} = \sqrt{(\vartheta_{T1crit} - \vartheta_{TB}) \cdot \frac{2T_J}{\omega_N \cdot (-\cos\varphi_s)}}$$

Example: Critical fault clearing time t_{crit}

$$t_{crit} = \sqrt{(\vartheta_{T1crit} - \vartheta_{TB}) \cdot \frac{2T_J}{\omega_N \cdot (-\cos\varphi_s)}}$$

Example: 2-pole turbine generator:

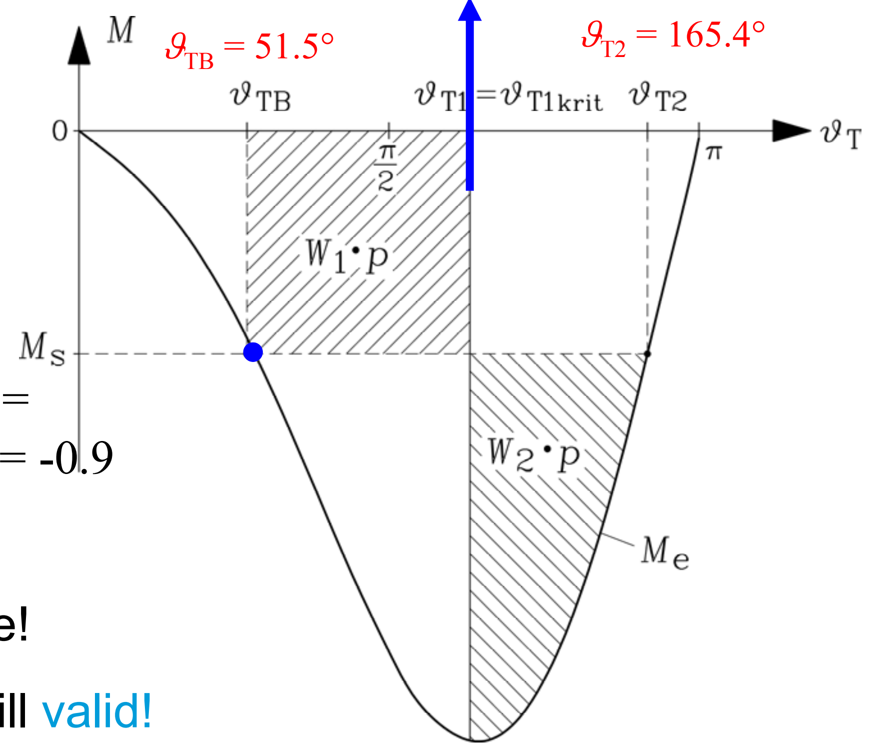
$S_N = 850$ MVA, 50 Hz ; $T_J = 5.4$ s

$\cos\varphi_s = -0.9$ (consumer reference frame)

$\vartheta_{T1crit} = 113.45° = 1.98$ rad, $\vartheta_{TB} = 51.5° = 0.899$ rad

$$t_{crit} = \sqrt{(1.98 - 0.899) \cdot \frac{2 \cdot 5.4}{2\pi 50 \cdot 0.9}} = 0.203 s$$

$M_s / M_B =$
$= \cos\varphi_s = -0.9$



- The fault must be cleared within 203 ms, otherwise the generator set will not re-synchronize!
- $t_{crit} < T'_d$, so that transient machine condition is still valid!

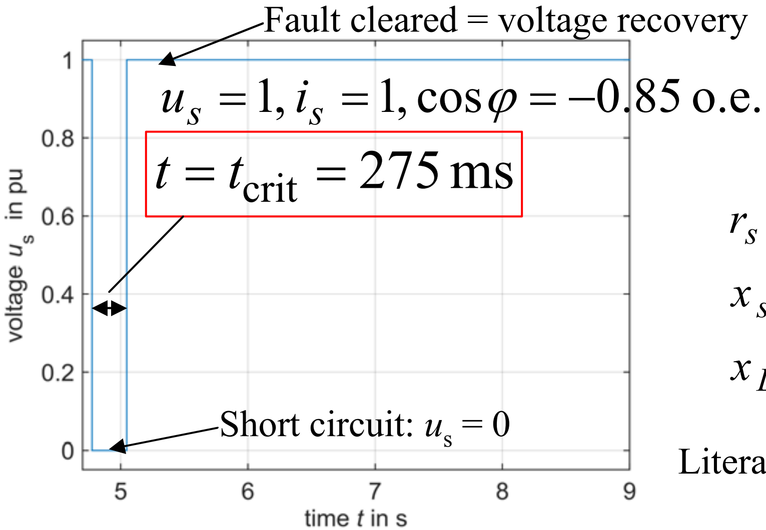
8. Dynamics of synchronous machines

Nowadays operation of generator-sets during the fault

- Due to the short "critical fault clearing time" t_{crit} the generator set is kept operating at the grid even after a severe fault (e.g. short circuit)
- The power $p \sim u_s i_s$ to the grid is in the worst-case zero (fault near generator: $u_s = 0$)
- The average air-gap short circuit torque is nearly zero: $m_{e,av} = M_e \approx 0 \Rightarrow$ Turbine accelerates generator-set due to zero (or small) braking torque $M_e = 0$.
- When the fault is cleared (and the faulty line is switched off) within $t < t_{crit}$ (transient stability), the grid voltage u_s suddenly appears again at the generator terminals via the healthy parallel lines $\Rightarrow i_s$, $M_e > 0$ = "load step"
- This "load step" causes a new transient $i_s(t)$, $M_e(t)$, which might cause torsion resonance
- Resonance torque must stay within mechanical safety limits !

8. Dynamics of synchronous machines

Example: Transient stability at sudden short circuit



2-pole turbine generator 600 MVA, 26 kV, 50 Hz, 3000/min, I_{fN} = 1800 A, U_{fN} = 146 V, no transformer; short circuit after rated generator load

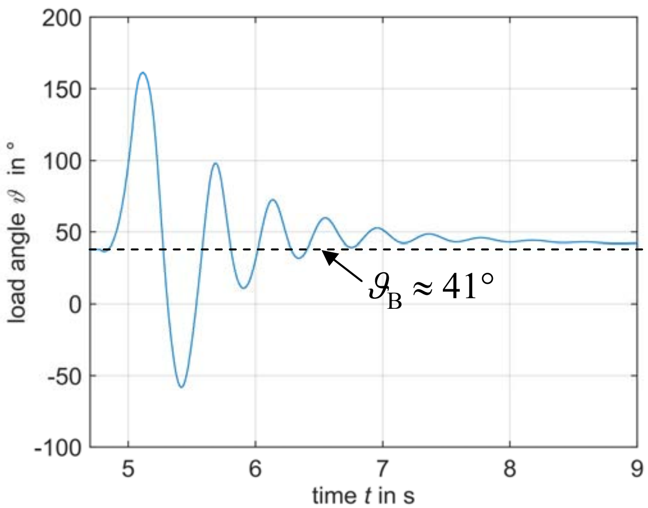
$r_s = 0.004$, $r_f = 0.001$, $r_D = 0.0187$, $r_Q = 0.0867$, $\tau_J = 1200$

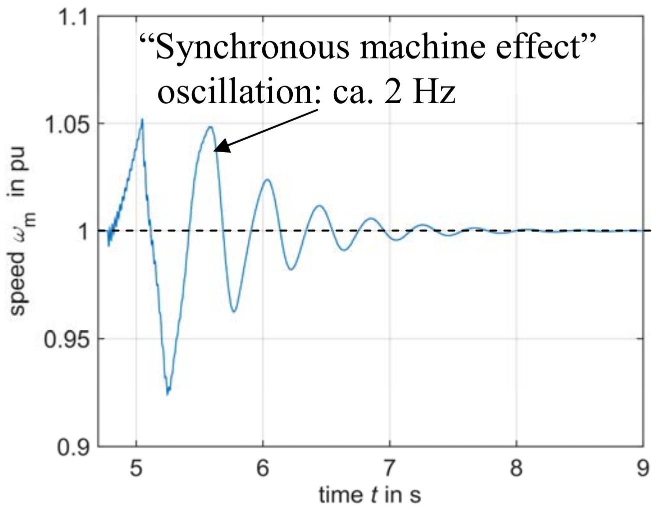
$x_{s\sigma} = 0.19$, $x_{dh} = 1.73$, $x_{qh} = 1.66$,

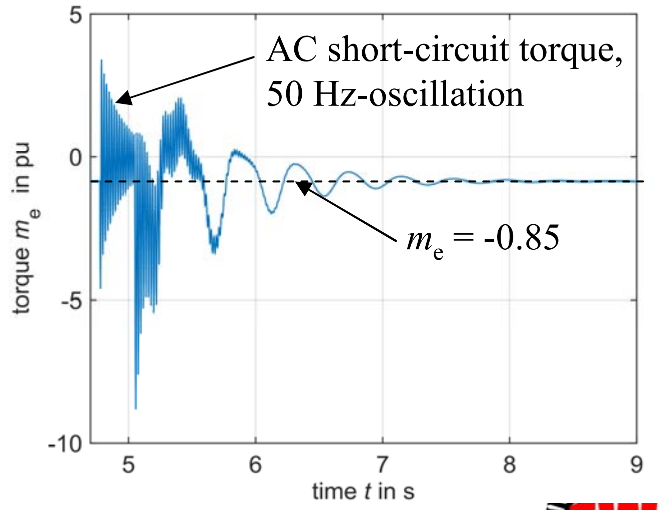
$x_{D\sigma} = 0.1313$, $x_{Q\sigma} = 0.0731$, $x_{f\sigma} = 0.1642$

(Source: ABB Birr, Switzerland (now GE))

Literature: D. Oeding, B. Oswald / El. Kraftwerke & Netze, Springer, Berlin, 2016







8. Dynamics of synchronous machines

Example: Transient instability at sudden short circuit

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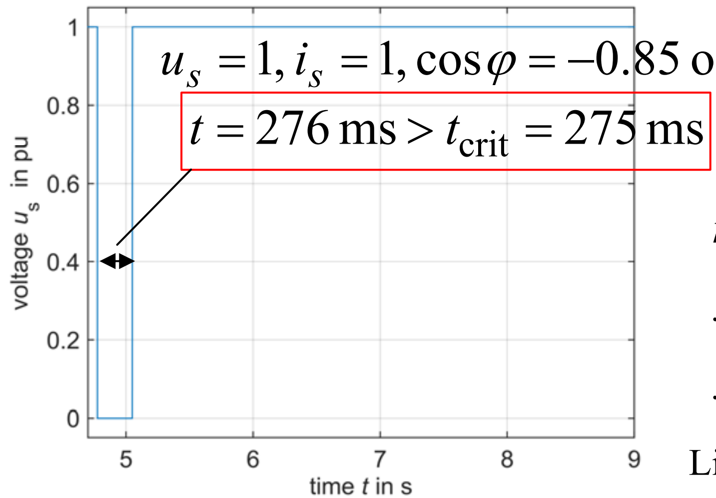


2-pole turbine generator 600 MVA, 26 kV, 50 Hz, 3000/min, $I_{\text{fN}} = 1800$ A, $U_{\text{fN}} = 146$ V:

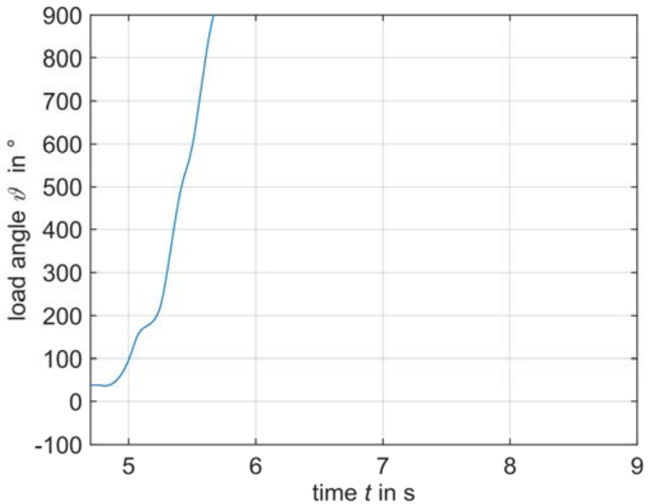
(Source: ABB Birr, Switzerland (now GE))

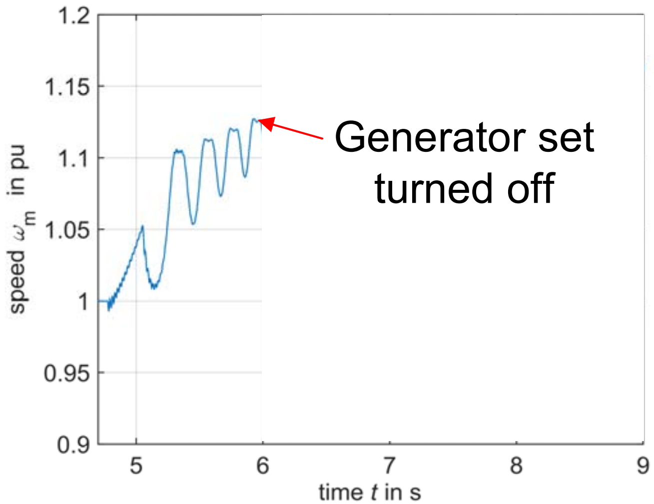
$r_s = 0.004\,,\ r_f = 0.001\,,\ r_D = 0.0187\,,\ r_Q = 0.0867\,,\ \tau_J = 1200$

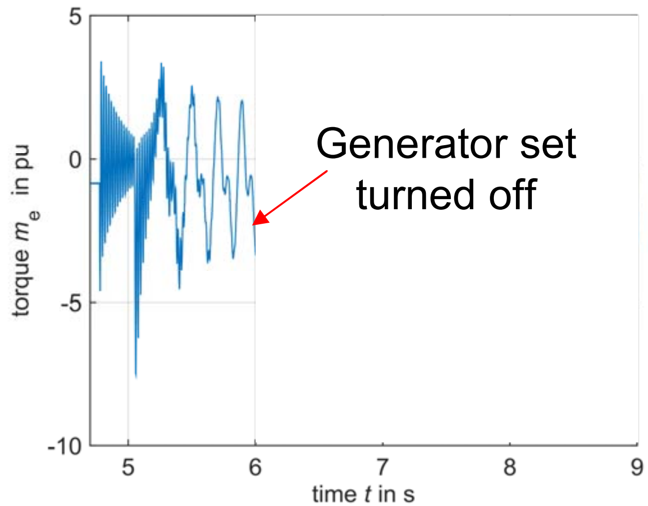
$x_{s\sigma} = 0.19\,,\ x_{dh} = 1.73\,,\ x_{qh} = 1.66\,,$

$x_{D\sigma} = 0.1313\,,\ x_{Q\sigma} = 0.0731\,,\ x_{f\sigma} = 0.1642$

Literature: D. Oeding, B. Oswald / El. Kraftwerke & Netze, Springer, Berlin, 2016





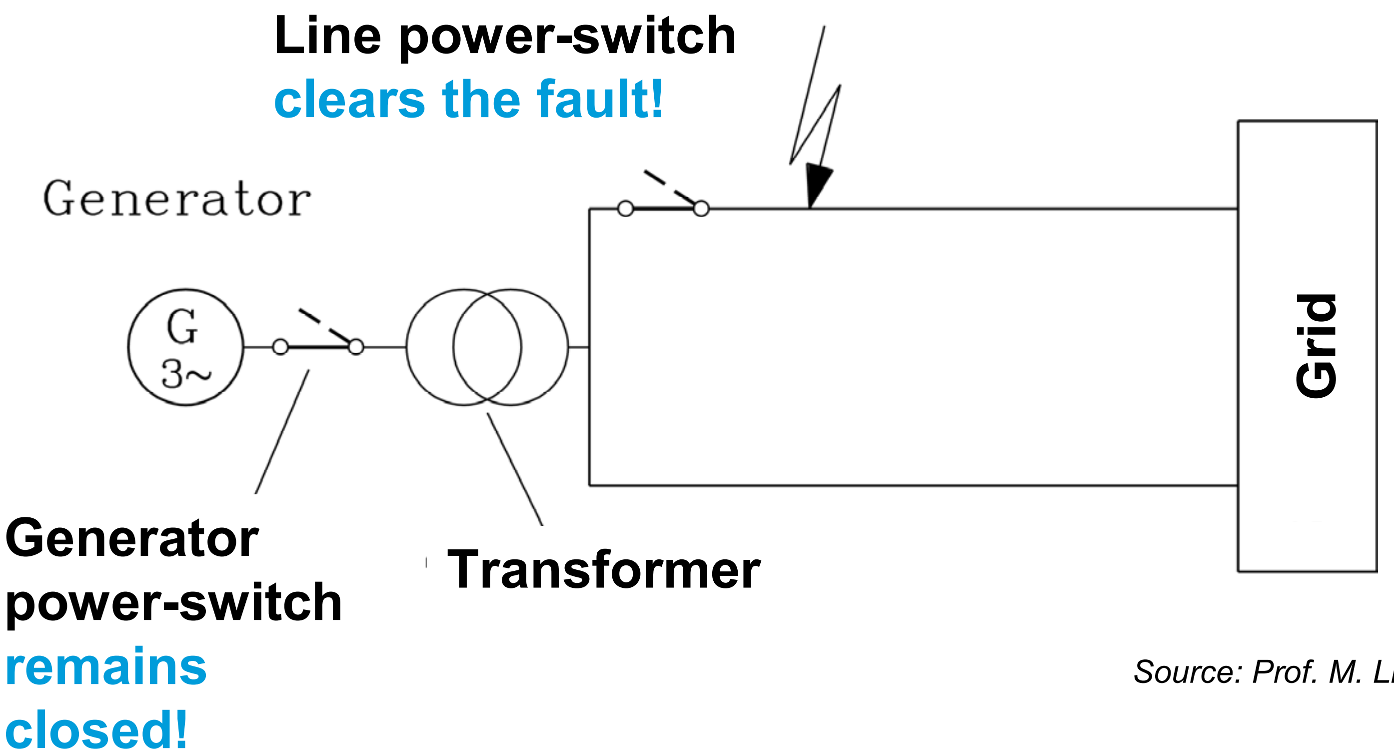


8. Dynamics of synchronous machines

Example: Transient stability

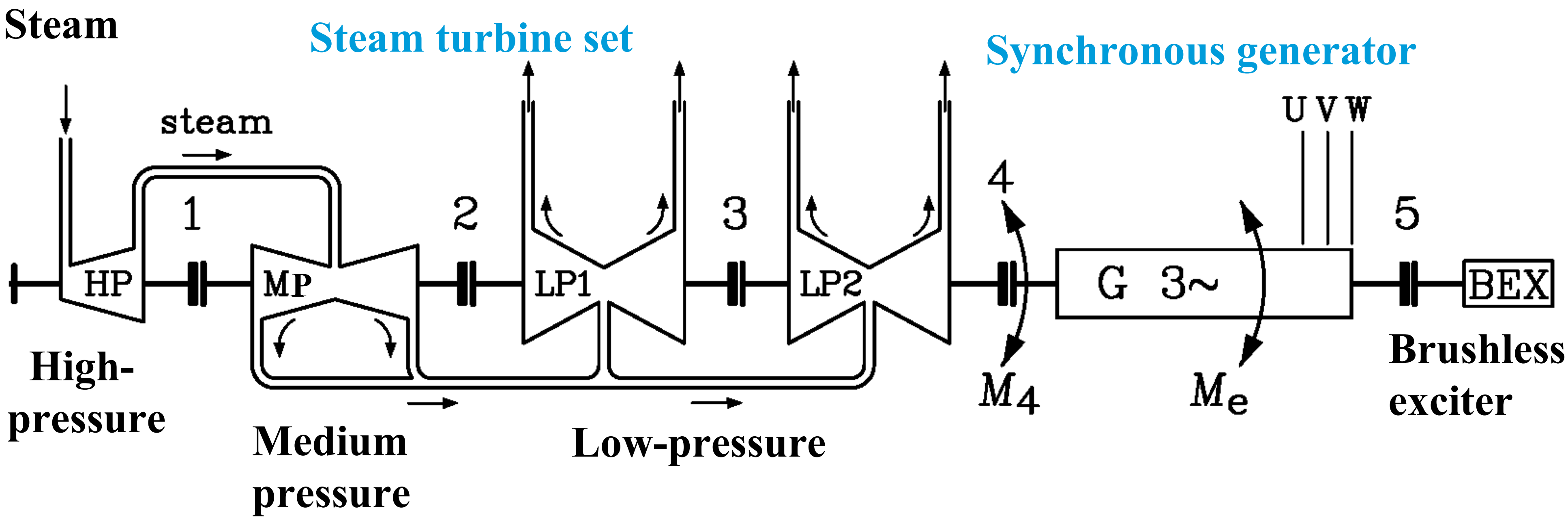
Due to sudden short circuit on a parallel line the generator operates on a short circuit and is accelerated by the turbine

Short circuit on parallel line



Source: Prof. M. Liese, TU Dresden

Numerical calculation of shaft torque during fault clearing



M_e: electromagnetic air-gap torque

M_4: shaft torque at coupling no. 4

Source: Prof. M. Liese, TU Dresden

Numerically calculated torque in the air gap at coupling no. 4

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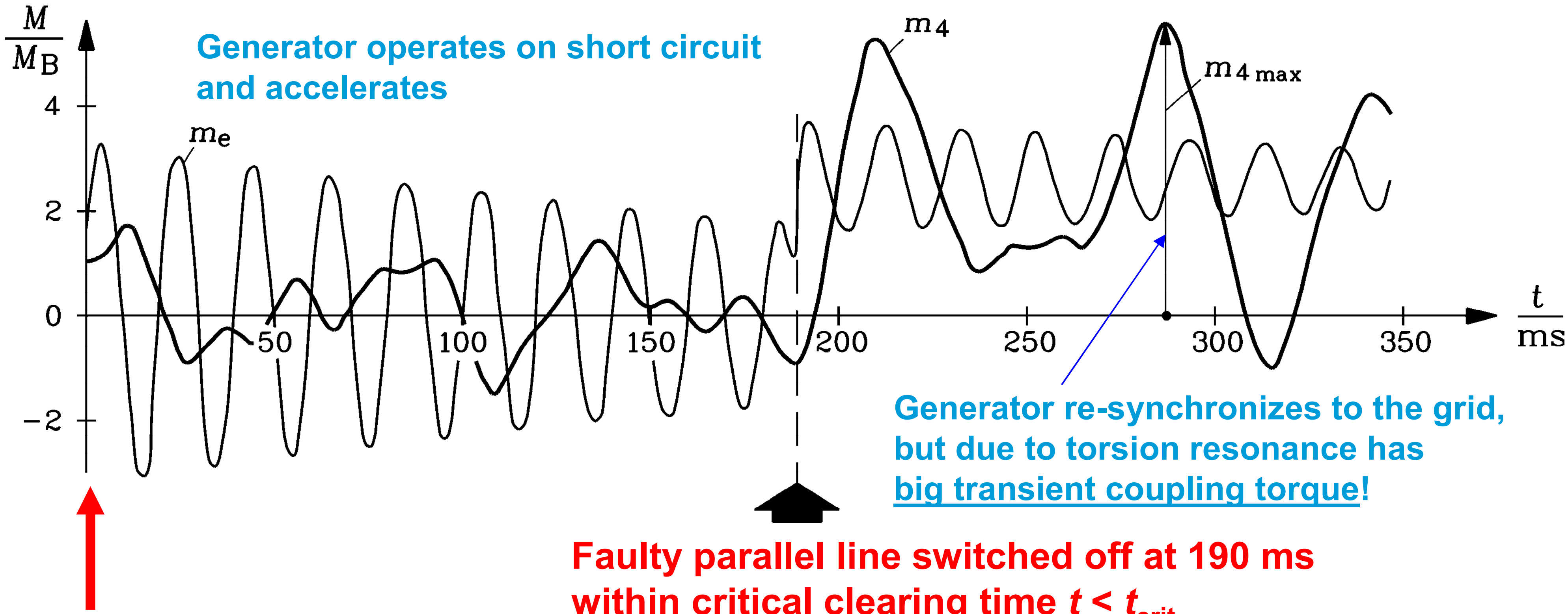
Generator operates on short circuit and accelerates



Generator re-synchronizes to the grid, but due to torsion resonance has <u>big transient coupling torque</u>!

Faulty parallel line switched off at 190 ms <u>within</u> critical clearing time $t < t_{crit}$

Short circuit on parallel line

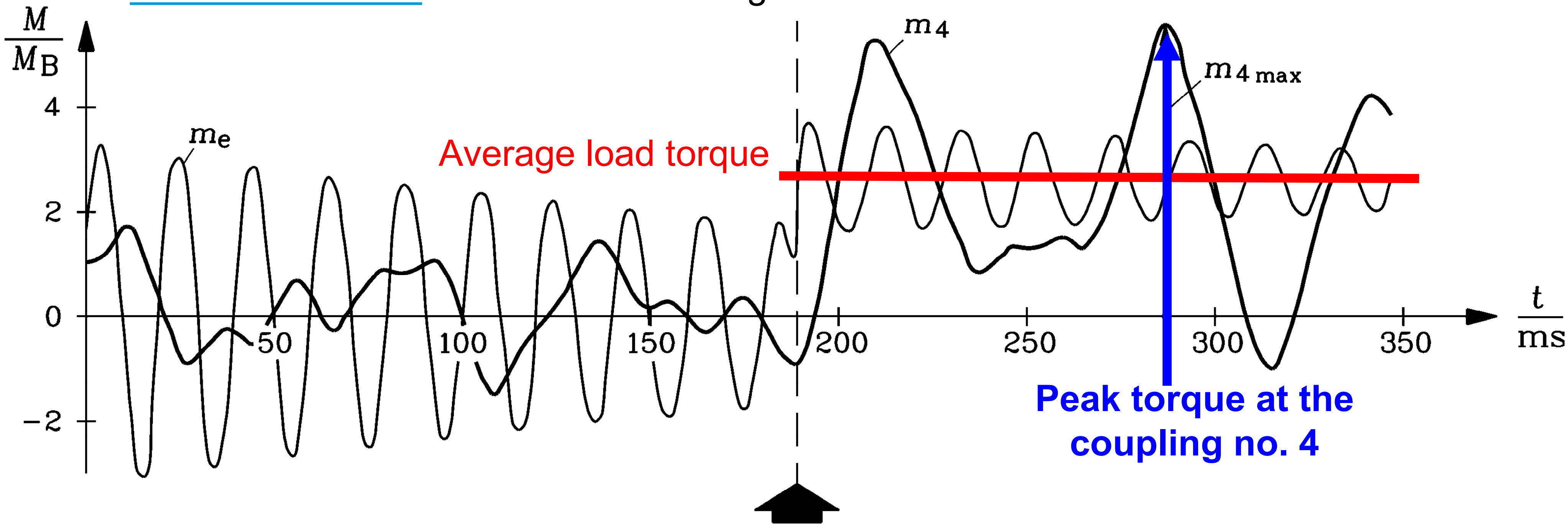
m_e: electromagnetic air-gap torque

m_4: shaft torque at coupling no. 4

Source: Prof. M. Liese, TU Dresden

8. Dynamics of synchronous machines

Peak torque at the coupling no. 4 is BIGGER than air gap torque due to torsion resonance excitation in the long shaft



Parallel line switched off within critical clearing time $t < t_{crit}$

m_e: electromagnetic air-gap torque

m_4: shaft torque at coupling no. 4

Source: Prof. M. Liese, TU Dresden

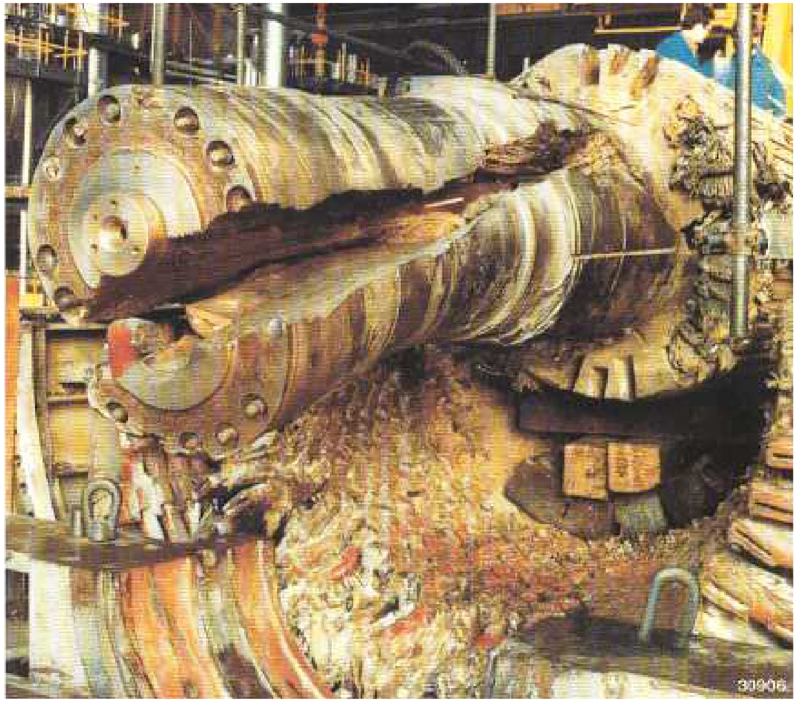
8. Dynamics of synchronous machines

Breaking of shaft at *Porcheville* (F)

- Breaking of the shaft due to too large peak torque after re-synchronization (resonance effect) !
- Due to torsion resonance the exciting air-gap torque M_e causes big shaft torques, especially at coupling no. 4, where M_4 exceeds the air-gap torque !
- In case of too weak shaft design, the turbine shaft may break $\Rightarrow$ lessons learned!

Example:
Broken shaft of the two-pole 600 MW turbine generator at the thermal power plant *Porcheville, France*, during re-synchronization of the generator after short circuit clearing.
(1977)

Source: Prof. Dr. M. Liese, TU Dresden



Summary:
Transient stability of electrically excited synchronous machines

- Transient state: Induced damper currents i_D, i_Q have already vanished, but still transient field current Δi_f component flows in addition to DC field current i_{f0}

- Transient field current Δi_f increases dynamic stability of synchronous machine

- During transient time constant increased dynamic pull-out torque $M_{p,dyn}$

- Increased transient pull-out torque $M_{p,dyn}$ helps to stabilize grid

- Critical fault clearing time t_{crit} is increased by increased transient torque $M_{e,dyn}$

- Transient stability peak torque limit $M_{p,dyn}$ much bigger than steady-state stability torque limit $M_{p,0}$

- At grid voltage recovery after fault resonant torque amplification may occur $\Rightarrow$ Careful turbine-generator shaft design necessary

What did you learn in this course ?

- How to design an electric machine electromagnetically!
- Example of squirrel cage induction machine was chosen, as it is the working horse of modern drive technology.
- Wound-rotor induction machine is included in text book !
- Thermal design has been presented !
- Dynamics of
  a) DC machine,
  b) induction machine and
  c) synchronous machine
  was discussed, using for linear or linearized equations *Laplace* transform !
- For non-linear equations MATLAB/Simulink program package was introduced !

That´s all, folks !

Thank you for your attention !

Good luck for your further studies !