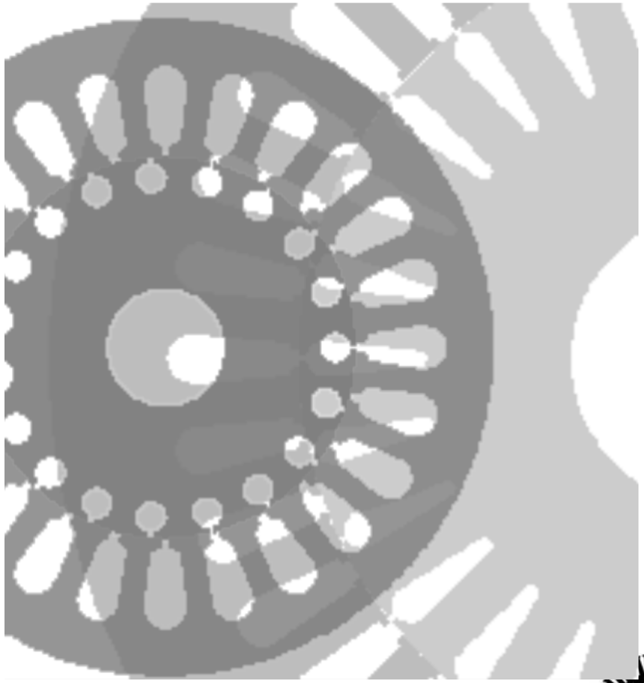
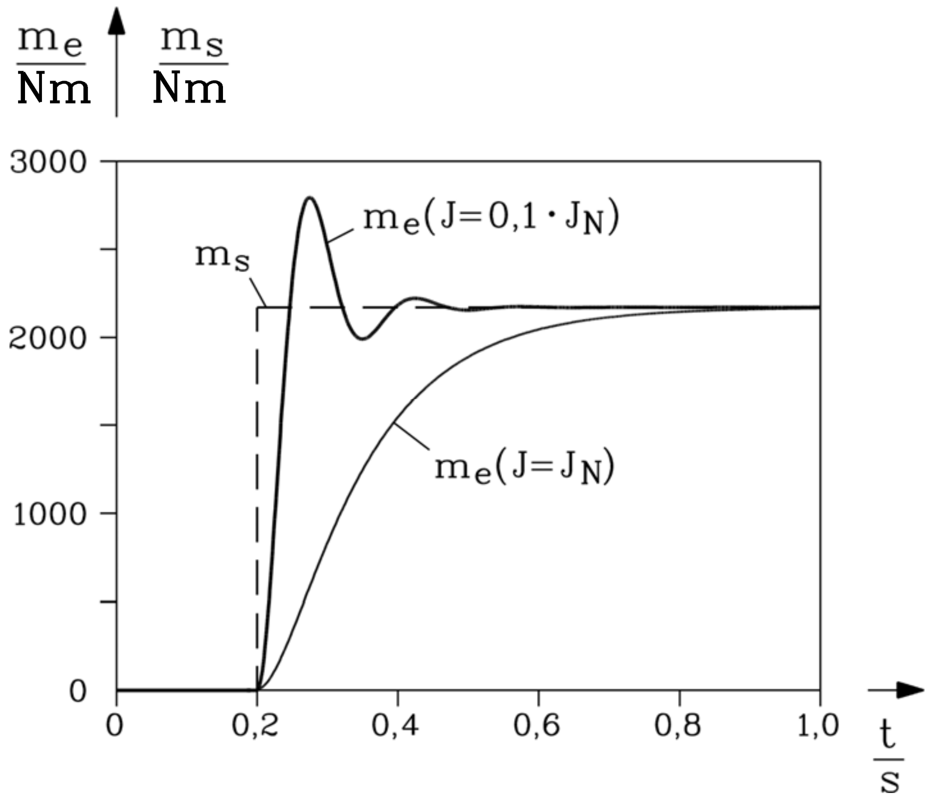
Energy Converters – CAD and System Dynamics



Source:
SPEED program

4. Dynamics of electrical machines



4. Dynamics of electric machines

4.1 Motivation: Why do we need dynamic theory of electric machines ?

4.2 Methods for calculation of transient machine operation

4. Dynamics of electrical machines

Motivation for dynamic simulations

Why do we need dynamic theory of electric machines ?

- *Controlled drives*
- *Switching of electric machines in normal operation*
- *Failures in electric machines*
- *Inverter operation*
- *Stability of operation*

For switching of motors, sudden failures, for response of machines to controller operation, for inverter operation and for investigation of stability dynamic modelling of electric machines is necessary.

4. Dynamics of electric machines

4.1 Motivation: Why do we need dynamic theory of electric machines ?

4.2 Methods for calculation of transient machine operation

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Differential equations instead of algebraic equations !

DC machinery: $U = R \cdot I \quad \rightarrow \quad u(t) = R \cdot i(t) + L \cdot di(t)/dt$

AC machinery: $\underline{U} = R \cdot \underline{I} + j\omega L \cdot \underline{I} \quad \rightarrow \quad u(t) = R \cdot i(t) + L \cdot di(t)/dt$

Mechanical system: $F_e = F_s \quad \rightarrow m \cdot dv(t)/dt = f_e(t) - f_s(t)$

Solving of "ordinary" differential equations:

- Dynamic models have only **time *t* as variable … "ordinary"!**
- **Initial conditions** are necessary: e. g. current $i(t=0) = I_0$

a) Linear differential equations

b) Non-linear differential equations

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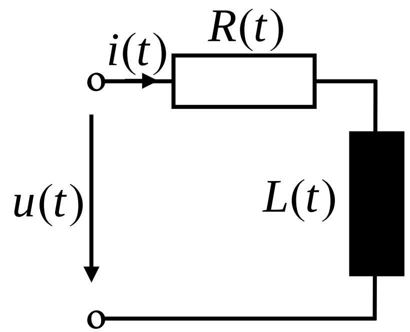
Example: Dynamic coil current $i(t)$



- Coil with time-dependent resistance $R(t)$ and inductance $L(t)$ is energized by a given voltage $u(t)$ for $t \geq 0$.
- The coil current $i(t)$ is to be calculated!

$$\psi(t) = L(t) \cdot i(t)$$

$$R(t) \cdot i(t) + d\psi(t)/dt = u(t) \Rightarrow R(t) \cdot i(t) + d(L(t) \cdot i(t))/dt = u(t)$$

$$R(t) \cdot i(t) + dL/dt \cdot i(t) + L \cdot di/dt = \underbrace{\left[R(t) + dL/dt\right]}_{R_{eq}(t)} \cdot i(t) + L(t) \cdot di/dt = u(t)$$

$$\boxed{R_{eq}(t) \cdot i(t) + L(t) \cdot di(t)/dt = u(t)}$$

Linear ordinary 1st order differential equation

Linear superposition of solutions

If $i_1(t)$ and $i_2(t)$ are solutions, then also $i_3(t) = i_1(t) + i_2(t)$ is a solution !

$$R_{eq}(t) \cdot i(t) + L(t) \cdot di(t)/dt = u(t) \neq 0$$

Example:

a) Homogenous equation: $R_{eq}(t) \cdot i(t) + L(t) \cdot di(t)/dt = 0$

Homogenous solution $i_h(t)$

b) Inhomogenous equation: $R_{eq}(t) \cdot i(t) + L(t) \cdot di(t)/dt = u(t)$

Particular solution $i_p(t)$

c) Resulting solution: $i(t) = i_h(t) + i_p(t)$

Linear differential equations with constant coefficients

Example: $R_{eq}(t)\cdot i(t)+L(t)\cdot di(t)/dt=u(t) \Rightarrow R\cdot i(t)+L\cdot di(t)/dt=u(t)$

Coefficients are constant: $R(t) = R$, $L(t) = L$

Example:

Separately excited DC machine

DC supply voltage U_f switched on: $u_f(t) = U_f$, $t \ge 0$

Calculate field current increase $i_f(t)$

$W_m = L_f i_f^2 / 2$

Energy does not "jump",

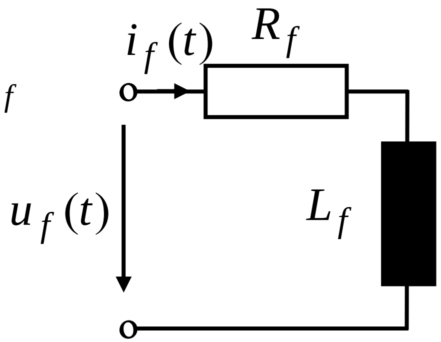
(otherwise: infinite power,

$P = dW_m/dt = "\Delta W_m/0" \to \infty$)

$W_m(0-) = W_m(0+)$

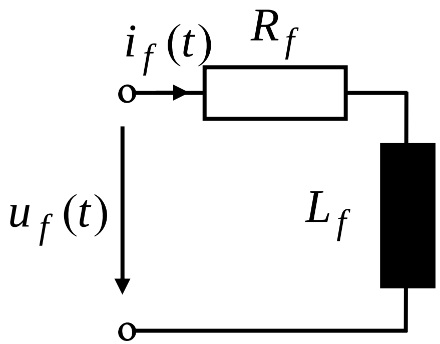
- Differential equation: $R_f\cdot i_f(t)+L_f\cdot di_f(t)/dt=u_f(t)=U_f$
- Initial condition: $i_f(0)=0$.
- Homogenous solution: $i_h(t)=C\cdot e^{\lambda t}\to R_f+L_f\cdot\lambda=0\Rightarrow\lambda=-R_f/L_f$
- Particular solution: $i_p(t)=K\to R_f\cdot K+L_f\cdot dK/dt=U_f \quad\to\quad K=U_f/R_f$
- Initial condition determines constant C:

$$i_f(0)=i_h(0)+i_p(0)=C\cdot e^0+K=0:\quad C=-K$$



Solution: $i_f(t)=i_h(t)+i_p(t)=\dfrac{U_f}{R_f}\cdot(1-e^{-\frac{t}{T_f}}),\quad t\ge 0,$, time constant: $T_f=L_f/R_f$.

Example: Separately excited DC machine

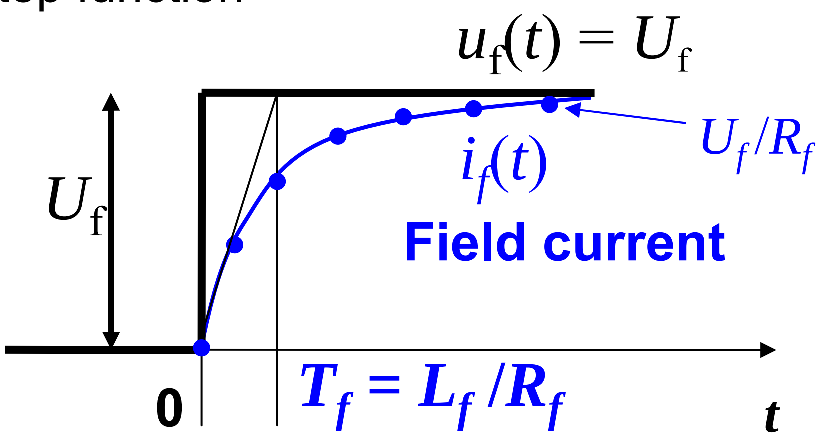


DC supply voltage U_f switched on: $u_f(t) = U_f$, $t \geq 0$

Calculate field current increase $i_f(t)$, $t \geq 0$

$$u_f(t) = U_f \cdot \varepsilon(t) \qquad \varepsilon(t) = \begin{cases} 0, t < 0 \\ 1, t \geq 0 \end{cases}$$ *Heaviside* step function

Field current: $$i_f(t) = \frac{U_f}{R_f} \cdot \left(1 - e^{-t/T_f}\right)$$



After infinitely long time (in reality after ca. $3T_f$) the field current is a DC current: $i_f = I_f = U_f/R_f$.

Laplace transform: Definition

$$F(\underline{s}) = L\{f(t)\} = \int_{t=0}^{\infty} f(t) \cdot e^{-\underline{s} \cdot t} \cdot dt$$

$\underline{s}$: Complex *Laplace* variable („*Laplace* operator“) [s^{-1}]

t: Time [s]

- Linear transformation of an arbitrary time function *f*(*t*), which is zero for $t \leq 0$.

- Used for solving linear differential equations.

4. Dynamics of electrical machines

Laplace transform table (1)

$f(t)$, $t > 0$ and zero, $t \le 0$	$F(\underline{s})$
K	$K/\underline{s}$
t	$1/\underline{s}^2$
t^n , $n = 1, 2, 3, ...$	$n/\underline{s}^{n+1}$
$e^{b \cdot t}$	$\frac{1}{\underline{s} - b}$
$\sin(b \cdot t)$	$\frac{b}{\underline{s}^2 + b^2}$
$\cos(b \cdot t)$	$\frac{\underline{s}}{\underline{s}^2 + b^2}$
$\sinh(b \cdot t)$	$\frac{b}{\underline{s}^2 - b^2}$
$\cosh(b \cdot t)$	$\frac{\underline{s}}{\underline{s}^2 - b^2}$

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Laplace transform table (2)

Linearity	$f_1(t) + f_2(t)$	$F_1(\underline{s}) + F_2(\underline{s})$
	$k \cdot f(t)$	$k \cdot F(\underline{s})$
Similarity	$f(t/b)$	$b \cdot F(b \cdot \underline{s})$
	$f(t \cdot c)$	$\frac{1}{c} \cdot F(\underline{s}/c)$
Shifting	$f(t-\tau)$	$e^{-\underline{s}\cdot\tau} \cdot F(\underline{s})$
	$f(t) \cdot e^{-b \cdot t}$	$F(\underline{s}+b)$
Derivation	$df/dt = f'$	$\underline{s} \cdot F(\underline{s}) - f(0)$
	$d^n f / dt^n = f^{(n)}$, n = 1, 2, ...	$\underline{s}^n \cdot F(\underline{s}) - \underline{s}^{n-1} \cdot f(0) - \underline{s}^{n-2} \cdot f'(0) - - \ldots - f^{(n-1)}(0)$

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Laplace transform table (3)

Integration	$\int_0^t \int_0^t \ldots f(t) \cdot dt \cdot dt \cdot \ldots$, *n* times	$\frac{1}{\underline{s}^n} \cdot F(\underline{s})$
Convolution	$f_1(t) * f_2(t) = \int_{-\infty}^{\infty} f_1(t-\tau) \cdot f_2(\tau) \cdot d\tau$	$F_1(\underline{s}) \cdot F_2(\underline{s})$
Limits	$\lim_{t \to 0+} f(t)$	$\lim_{\underline{s} \to \infty} \underline{s} \cdot F(\underline{s})$
	$\lim_{t \to \infty} f(t)$	$\lim_{\underline{s} \to 0} \underline{s} \cdot F(\underline{s})$

Proof

Note: $$f_1(t) * f_2(t) = \int_{-\infty}^{\infty} f_1(t-\tau) \cdot f_2(\tau) \cdot d\tau = \int_{-\infty}^{\infty} f_1(\tau) \cdot f_2(t-\tau) \cdot d\tau$$

Proof: $$f_1(t) * f_2(t) = \int_{-\infty}^{\infty} f_1(t-\tau) \cdot f_2(\tau) \cdot d\tau : \quad t - \tau = x, -dx = d\tau$$

$$-\infty < \tau < \infty \Leftrightarrow \infty > x > -\infty$$

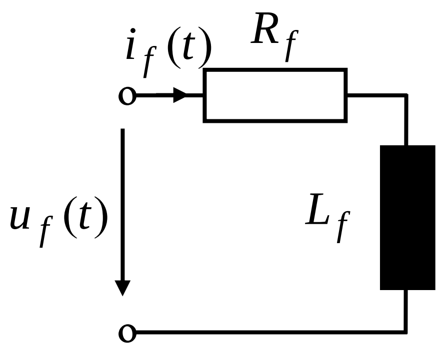
$$\int_{\tau=-\infty}^{\tau=\infty} f_1(t-\tau) \cdot f_2(\tau) \cdot d\tau = \int_{x=\infty}^{x=-\infty} f_1(x) \cdot f_2(t-x) \cdot (-dx) =$$

$$= \int_{x=-\infty}^{x=\infty} f_1(x) \cdot f_2(t-x) \cdot dx$$

Example:

Switching on at $t = 0$ DC excitation voltage $U_f : u_f(t) = U_f\,,\ t \ge 0$

Calculate current increase $i_f(t)$:



- **Differential equation:** $R_f \cdot i_f(t) + L_f \cdot di_f(t)/dt = u_f(t) = U_f \cdot \varepsilon(t)$
- **Initial condition:** $i_f(0) = 0$.

- *Laplace* transform: $L(i_f(t)) = I(\underline{s})$

$$L\left(R_f \cdot i_f(t) + L_f \cdot di_f(t)/dt\right) = R_f \cdot I(\underline{s}) + L_f \cdot \left(\underline{s} \cdot I(\underline{s}) - i_f(0)\right) = L(U_f) = U_f / \underline{s}$$

- Solution of algebraic equation:

$$R_f I(\underline{s}) + L_f \cdot \underline{s} I(\underline{s}) = \frac{U_f}{\underline{s}} \quad \rightarrow \quad I(\underline{s}) = \frac{U_f}{\underline{s}} \cdot \frac{1}{R_f + \underline{s} L_f} = \frac{U_f}{R_f} \cdot \left(\frac{1}{\underline{s}} - \frac{T_f}{1 + \underline{s} T_f}\right)$$

with $T_f = L_f / R_f$. Initial condition is already implemented in algebraic equation !

- **Inverse transformation, using Tables, yields with** $1/\underline{s} \leftrightarrow 1$ **and** $\dfrac{1}{1 + \underline{s} T_f} \leftrightarrow \dfrac{e^{-t/T_f}}{T_f}$

the solution:

$$\underline{\underline{L^{-1}\left(\underline{I}(\underline{s})\right) = i_f(t) = \frac{U_f}{R_f} \cdot \left(1 - e^{-\frac{t}{T_f}}\right),\ t \ge 0}}$$

4. Dynamics of electrical machines

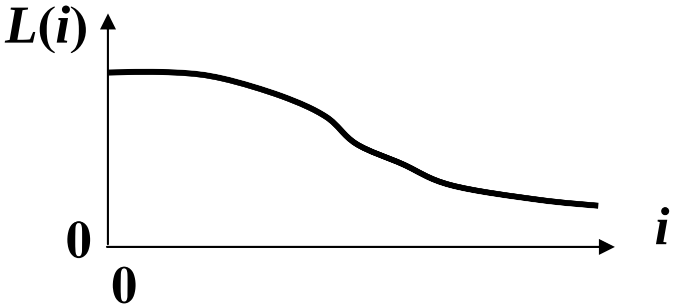
Solving of non-linear differential equations

- Superposition of two solutions **does not yield** another solution of that equation !
- Soving usually must be done **numerically** in step-by-step integration
  with finite step length, starting from *t* = 0 with the value for initial condition.
- Integration method: *Euler´s algorithm, preferred:* method of *Runge-Kutta*.
- Optimum step length exists !
- Commercial software:
  - *MATLAB/Simulink®,*
  - *DYMOLA/Modelica®,*
  - SIMPLORER, ...

Example:

Iron saturation decreases inductance

$$R \cdot i(t) + d(L(i) \cdot i(t))/dt = u(t)$$



Example: Saturation-dependent flux linkage $\psi(i)$

Example:
Non-linear differential equation with non-linear dependence $\psi(i)$

Coil with resistance R and non-inductance $L(i)$ is energized by a given voltage $u(t)$ for $t \geq 0$.

The coil current $i(t)$ is to be calculated!

$$\psi(t) = L(i(t)) \cdot i(t)$$

$$R \cdot i(t) + d\psi(t)/dt = u(t) \Rightarrow R \cdot i(t) + d(L(i) \cdot i(t))/dt = u(t)$$

$$R \cdot i(t) + \frac{dL(i)}{di} \cdot \frac{di}{dt} \cdot i(t) + L(i) \cdot \frac{di}{dt} = R \cdot i(t) + \underbrace{\left[\frac{dL(i)}{di} \cdot i(t) + L(i) \right]}_{L_{eq}(i)} \cdot \frac{di}{dt} = u(t)$$

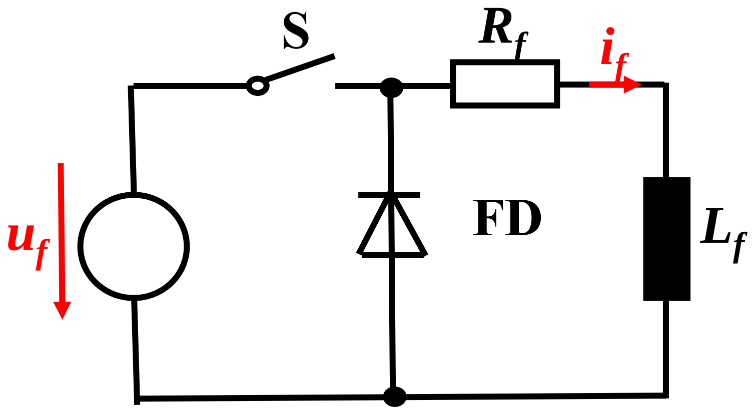
$$\boxed{R \cdot i(t) + L_{\mathrm{eq}}(i) \cdot di(t)/dt = u(t)}$$

Example for *Euler´s* method

Example:

Solving of differential equation with *Euler´s* method

Switching off at t = 0 a DC current i_f via a switch S and a free-wheeling diode FD



Initial condition:

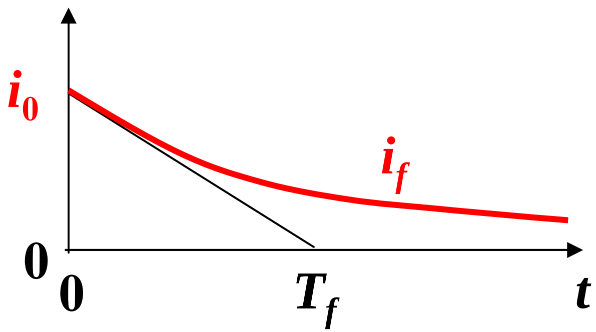
Magnetic energy does not "jump": $W_m(0-) = W_m(0+)$

$$W_m = L_f i_f^2(t=0-)/2 = L_f i_f^2(t=0+)/2$$

$i_f(0) = i_0$ Ideal FD: $u_{FD} = 0 \Leftrightarrow i_{FD} \geq 0$

Differential equation:

$$R_f \cdot i_f(t) + d(L_f(i_f) \cdot i_f(t))/dt = 0$$

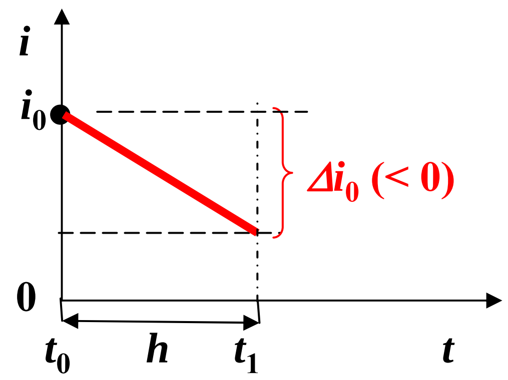


If $L_f = \text{const.}: i_f(t) = i_0 \cdot e^{-t/T_f}, T_f = L_f / R_f$

Solving of differential equation with *Euler´s* method

Example:

- NON-LINEAR differential equation $R \cdot i(t) + L_{eq}(i) \cdot (di(t)/dt) = 0$
- Solved numerically with ***Euler´s* method**:
- $\frac{di}{dt} = -\frac{R \cdot i(t)}{L_{eq}(i)}$, $i' = f(i,t)$, initial condition $i(t_0) = i_0$.
- Integration step length $\Delta t = h$: $\Delta i_0 = f(i_0, t_0) \cdot h$.
- Point by point solution: $t_1 = t_0 + h$: $i_1 = i(t_0 + h) \approx i_0 + \Delta i_0$.
- Next point $t = t_0 + h + h = t_1 + h$: $\Delta i_1 = f(i_1, t_1) \cdot h$
- $i_2 = i(t_1 + h) \approx i_1 + \Delta i_1$ and so on.



General rule is $i_{n+1} = i_n + h \cdot f(i_n, t_n)$ with *n* = 1, 2, 3, ... to be calculated in recursive way.

Thus the values i_n at t_n are taken instead of the exact (but unknown) function $i(t)$.

4. Dynamics of electrical machines

Runge-Kutta´s method versus *Euler´s* method

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Example:

Comparison of *Euler* and *Runge-Kutta* method for $-R = L = 1$: "negative damping"

$di/dt = i(t), \quad i(0) = 1$

- Exact solution is known $i(t) = e^t \; , \quad T = -1$

Source:
H.-J. Dirschmid, Springer-Verlag

t	$i(t)\ (h = \Delta t = 0.2)$	$i(t)\ (h = \Delta t = 0.2)$	$i(t)$
	Euler	***Runge-Kutta***	***exact solution***
0	1.0	1.0	1.0
0.2	1.2	1.2214	1.2214027
0.4	1.44	1.49182	1.4918247
0.6	1.728	1.82211	1.8221188
0.8	2.0736	2.22552	2.2255409
1.0	2.48832	2.71825	2.7182818
1.2	2.98598	3.32007	3.3201169
...	...	...	...

The deviation from the exact solution is much smaller with *Runge-Kutta* in comparison with *Euler*´s method.

Summary:
Methods for calculation of transient machine operation

- Linear differential equations allow superposition of solutions
- Linear diff. equ. with constant coefficients: Homogeneous and particular solution
- *Laplace* transform used for solving linear differential equations
- Non-linear ordinary differential equations solved by time-stepping
- RUNGE-KUTTA time stepping numerical solution method widely used

4. Dynamics of electrical machines

Tutorial

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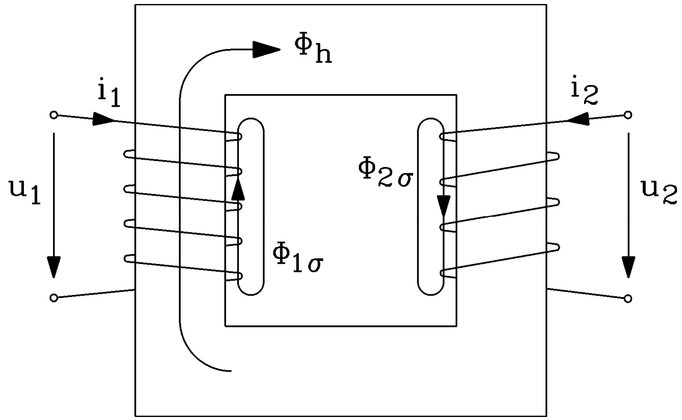
Example: Transformer in-rush current (1)

- Switching the primary winding (N_1 turns) of a single-phase transformer to the grid voltage u_1, secondary winding is open.
- Worst case: Zero crossing of voltage u_1 at switching $t = 0$



$$\psi_1 = N_1 \cdot (\Phi_{1\sigma} + \Phi_h) = (L_{1\sigma} + L_h) \cdot i_1 \quad i_1(0) = 0$$

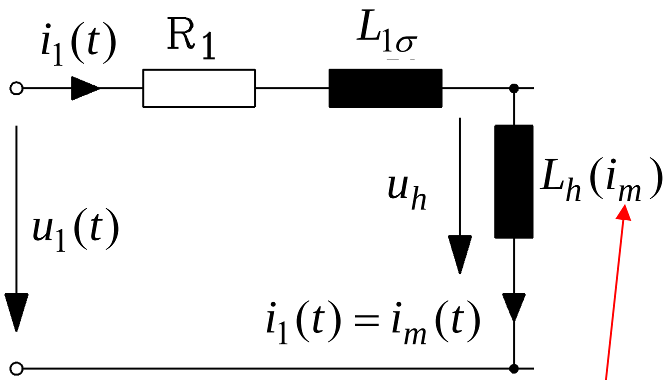
$$u_1(t) = \hat{U} \cdot \sin(\omega \cdot t) = R_1 i_1(t) + d\psi_1(t)/dt \quad t \geq 0$$

$$u_1(t) = \hat{U} \cdot \sin(\omega \cdot t) \approx d\psi_1(t)/dt$$

$$\psi_1(t) \approx \int_0^t \hat{U} \cdot \sin(\omega \cdot t) \cdot dt = -\frac{\hat{U} \cdot \cos(\omega \cdot t)}{\omega} + C$$

$$\psi_1(0) = 0 \Rightarrow C = \frac{\hat{U}}{\omega} \qquad \boxed{\psi_1(t) = \frac{\hat{U}}{\omega} \cdot (1 - \cos(\omega t))}$$

DC flux component — AC flux component



Non-linear main inductance: Decreases strongly with current due to iron core saturation!

4. Dynamics of electrical machines

Example: Transformer in-rush current (2)

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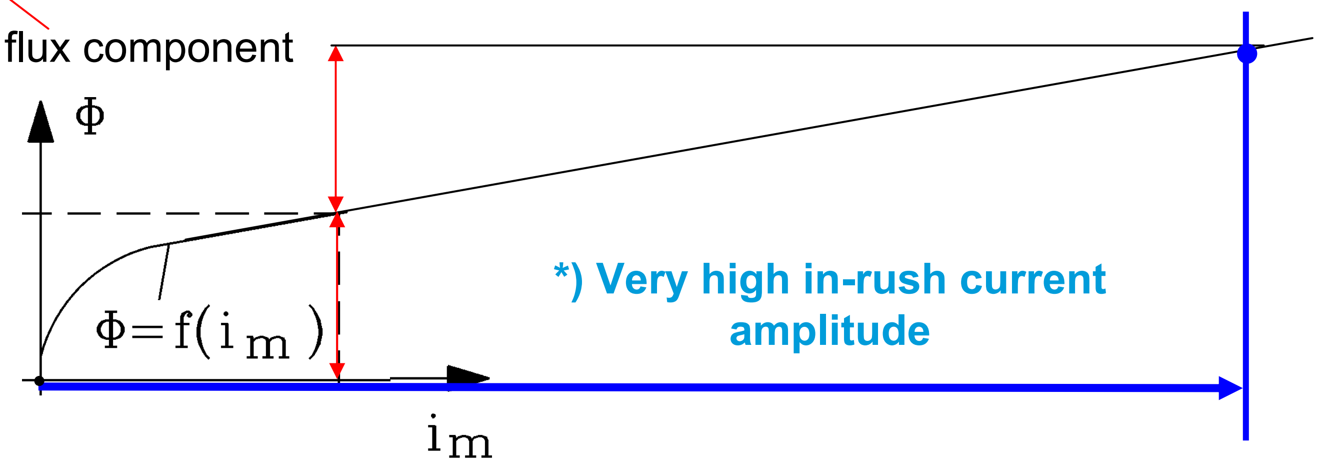
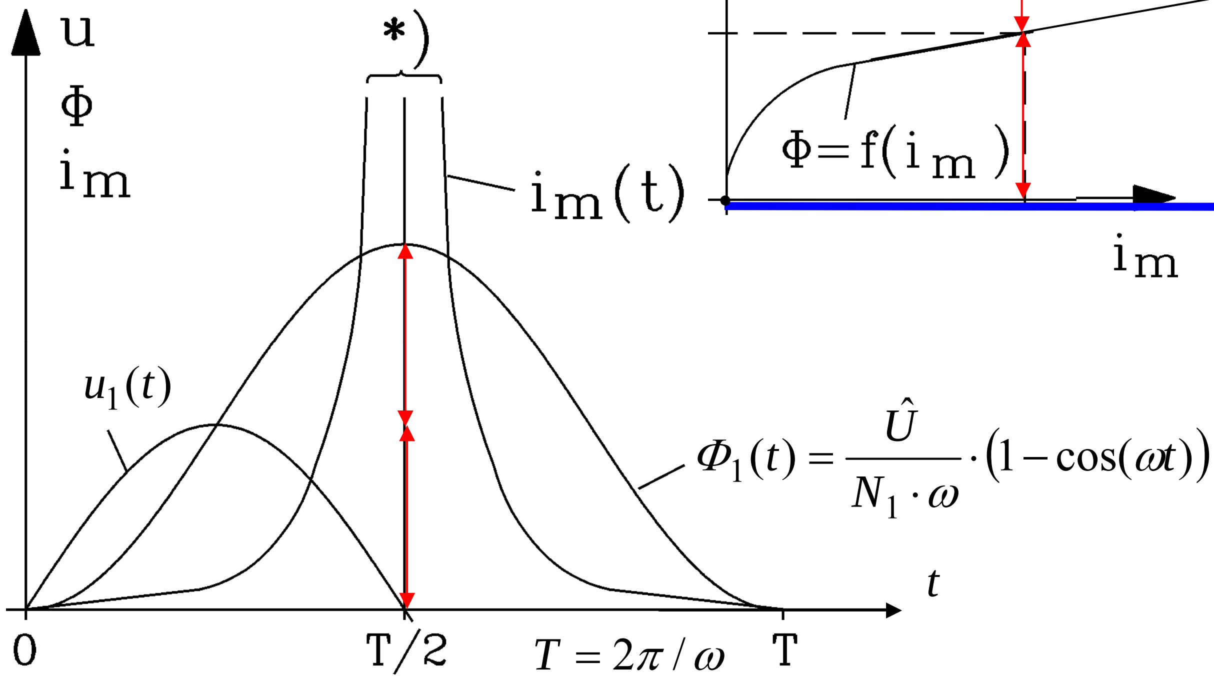
$$\psi_1(t) = N_1 \cdot \Phi_1(t) = \frac{\hat{U}}{\omega} \cdot \left(1 - \cos(\omega t)\right)$$

DC flux component AC flux component

$$i_1(t) = i_m(t)$$

- R_1 = 0: No damping of DC flux component





DC and AC flux component add after half period *T*/2: = double rated flux occurs = very high saturation = very low main inductance L_h = very high magnetizing and no-load primary current = **IN-RUSH CURRENT!**

Up to 15-times rated current!

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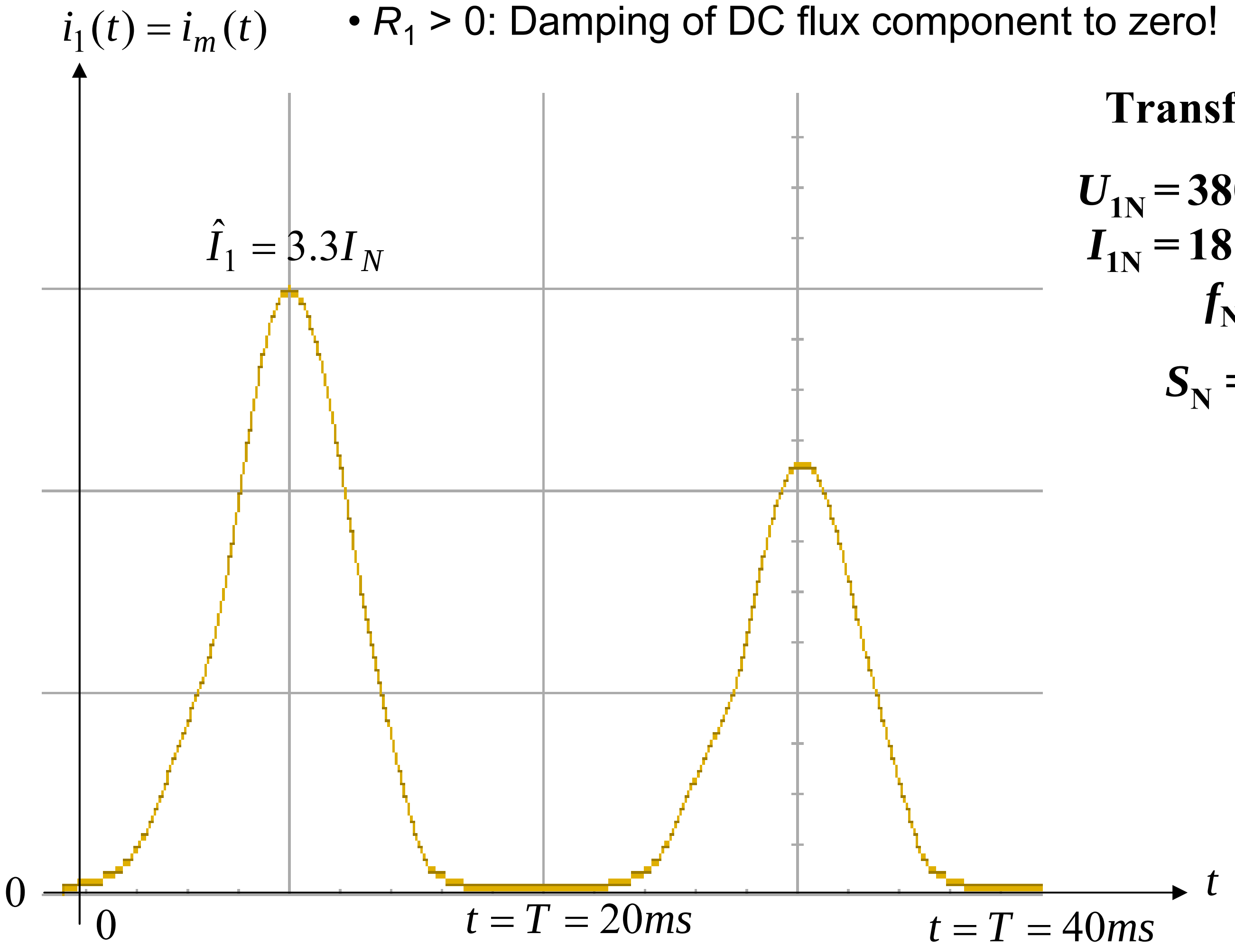
Measured in-rush current of a small transformer

- $R_1 > 0$: Damping of DC flux component to zero!



Transformer rating:

$U_{1N} = 380\ \text{V Y} / 220\ \text{V}\ \Delta$

$I_{1N} = 18\ \text{A Y} / 31.1\ \text{A}\ \Delta$

$f_N = 50\ \text{Hz}$

$S_N = 11.8\ \text{kVA}$

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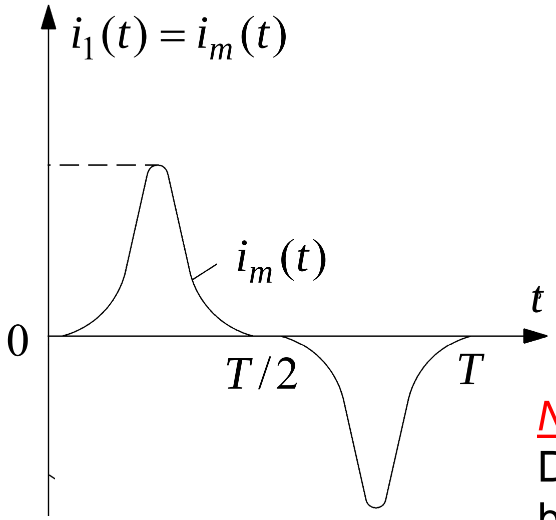
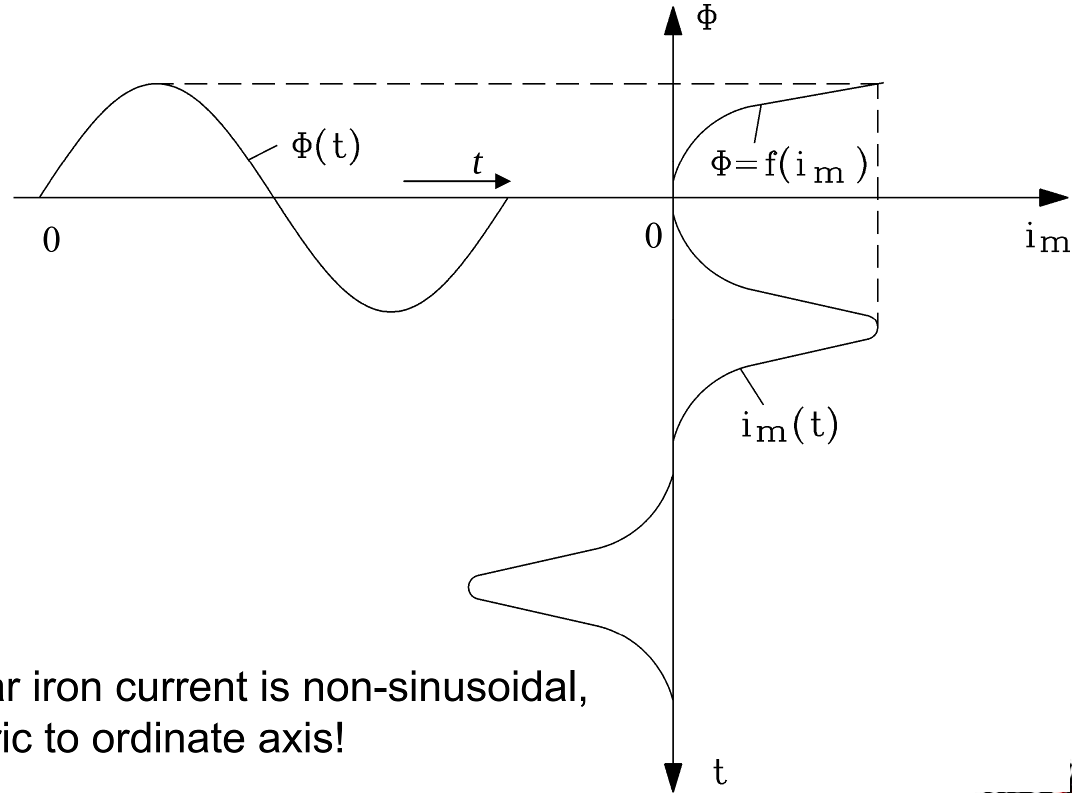
Steady-state transformer primary no-load current

- Due to I^2R losses the DC current component decays!
- The DC flux component vanishes, only the sinusoidal AC flux component remains.
- The AC no-load current is very small, but non-sinusoidal due to the big, but non-linear $L_h(i_m)$.

$$\Phi_1(t) = -\frac{\hat{U}}{N_1 \cdot \omega} \cdot \cos(\omega t)$$





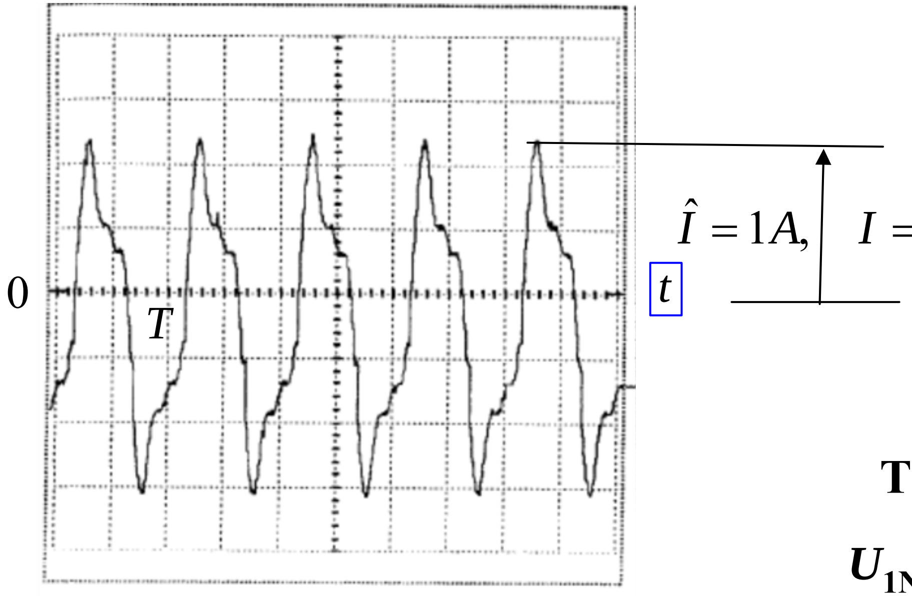
Note:
Due to non-linear iron current is non-sinusoidal, but still symmetric to ordinate axis!

Tutorial

Measured steady-state transformer primary no-load current

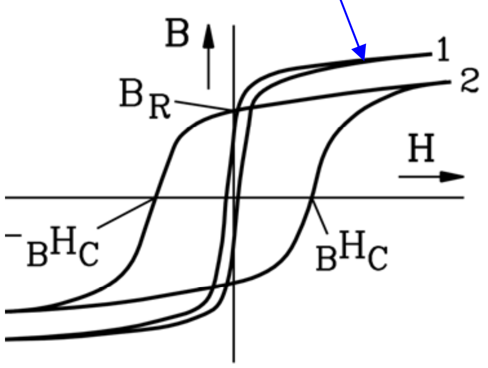
- Due to iron hysteresis loop the no-load current is not ordinate-symmetric, but still abscissa-symmetric!

$i_1(t) = i_m(t)$



$\hat{I} = 1A, \quad I = 0.6A = 0.03 \cdot I_N$

iron hysteresis loop



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