Energy Converters – CAD and System Dynamics

1. Basic design rules for electrical machines
2. Design of Induction Machines
3. Heat transfer and cooling of electrical machines
4. Dynamics of electrical machines
5. Dynamics of DC machines
6. Space vector theory
7. Dynamics of induction machines
8. Dynamics of synchronous machines

Source: SPEED program

2. Design of Induction Machines

Source:

Breuer Motors,

Germany

2. Design of induction machines

2.1 Main dimensions and basic electromagnetic quantities of induction machines

2.2 Scaling effect in electric machines

2.3 Stator winding low and high voltage technology

2.4 Stator winding design

2.5 Rotor cage design

2.6 Wound rotor design

2.7 Design of main flux path of magnetic circuit

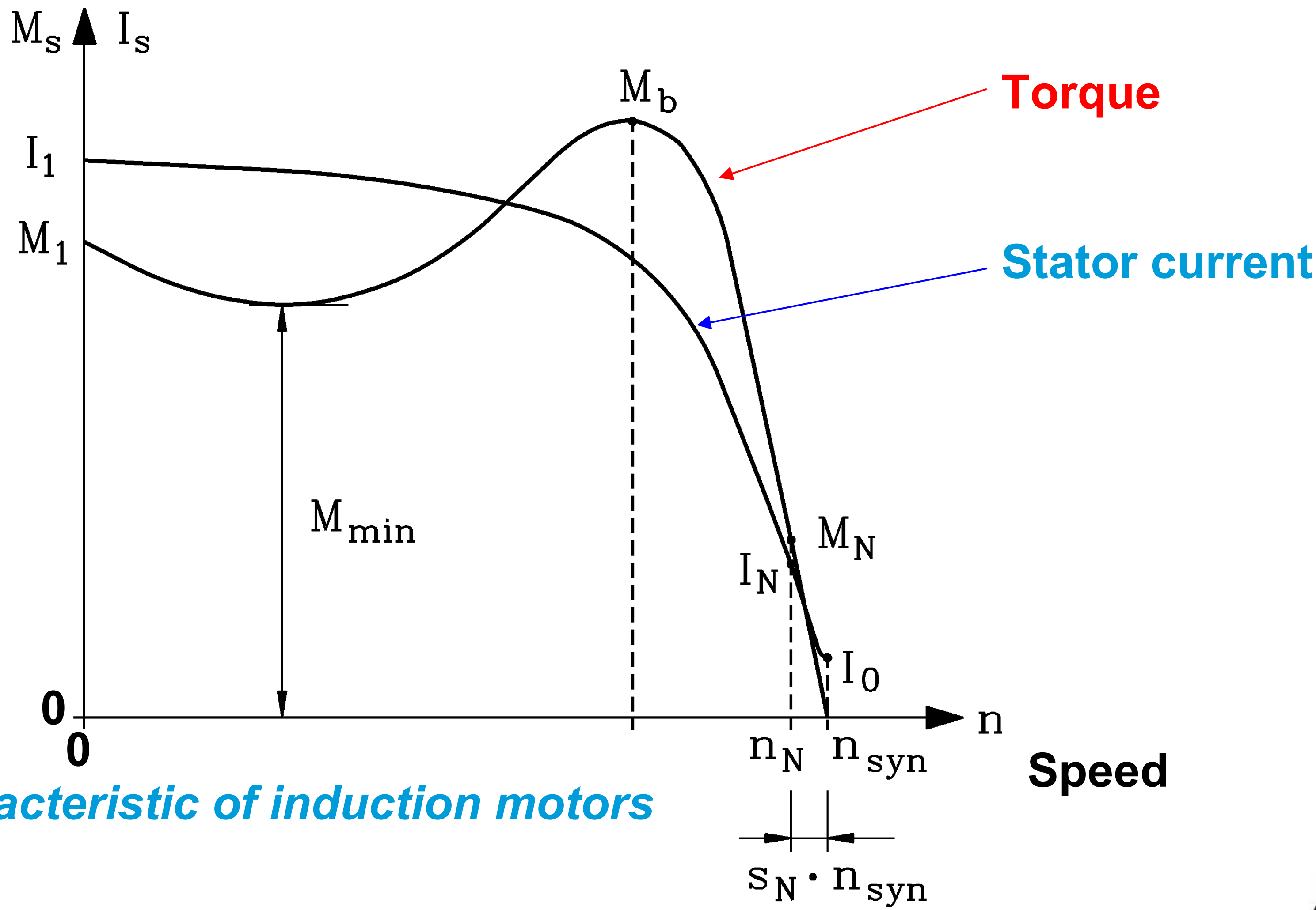
2.8 Stray flux and inductance

2.9 Influence of saturation on inductance

2.10 Masses and losses

2. Design of Induction Machines

Demands for design



Output characteristic of induction motors vs. speed

2. Design of Induction Machines

Typical demands acc. to Int. Standard IEC 60034-1:

Rated operation:	*Tolerances for measured values:*
Measured rated slip s_N	$\pm$20% of calculated rated slip $s_{N,calc}$
Measured power factor $\cos\varphi_N$	$-(1-\cos\varphi_{N,calc})/6$
Measured efficiency η_N	$-0.15\cdot(1-\eta_{N,calc})$ for $P_N \le 50\ \mathrm{kW}$
	$-0.10\cdot(1-\eta_{N,calc})$ for $P_N > 50\ \mathrm{kW}$
Overload capability:	
Measured breakdown torque M_b:	
Demand: for 15 s: > $1.6M_N$	-10% of calculated breakdown torque $M_{b,calc}$
Starting parameters (at $s = 1$):	
Measured starting torque M_1	-15% ... +25% of calculated starting torque $M_{1,calc}$
Measured starting current I_1	+20% of calculated starting current $I_{1,calc}$
Measured minimum torque M_{min}	
("saddle" torque):	Demands: $M_{min} > 0.5M_1$

2. Design of Induction Machines

Example: Efficiency tolerance

Calculated efficiency $\eta_N = 0.9$
Efficiency tolerance: $Tol = -0.10 \cdot (1 - \eta_{N,calc})$ for $P_N > 50\ \mathrm{kW}$

$$Tol = -0.1 \cdot (1 - 0.9) = -0.01$$

Minimum admissible measured efficiency:

$$\eta_{meas,\min} = \eta_N + Tol = 0.9 - 0.01 = 0.89$$

Main dimensions of induction machines

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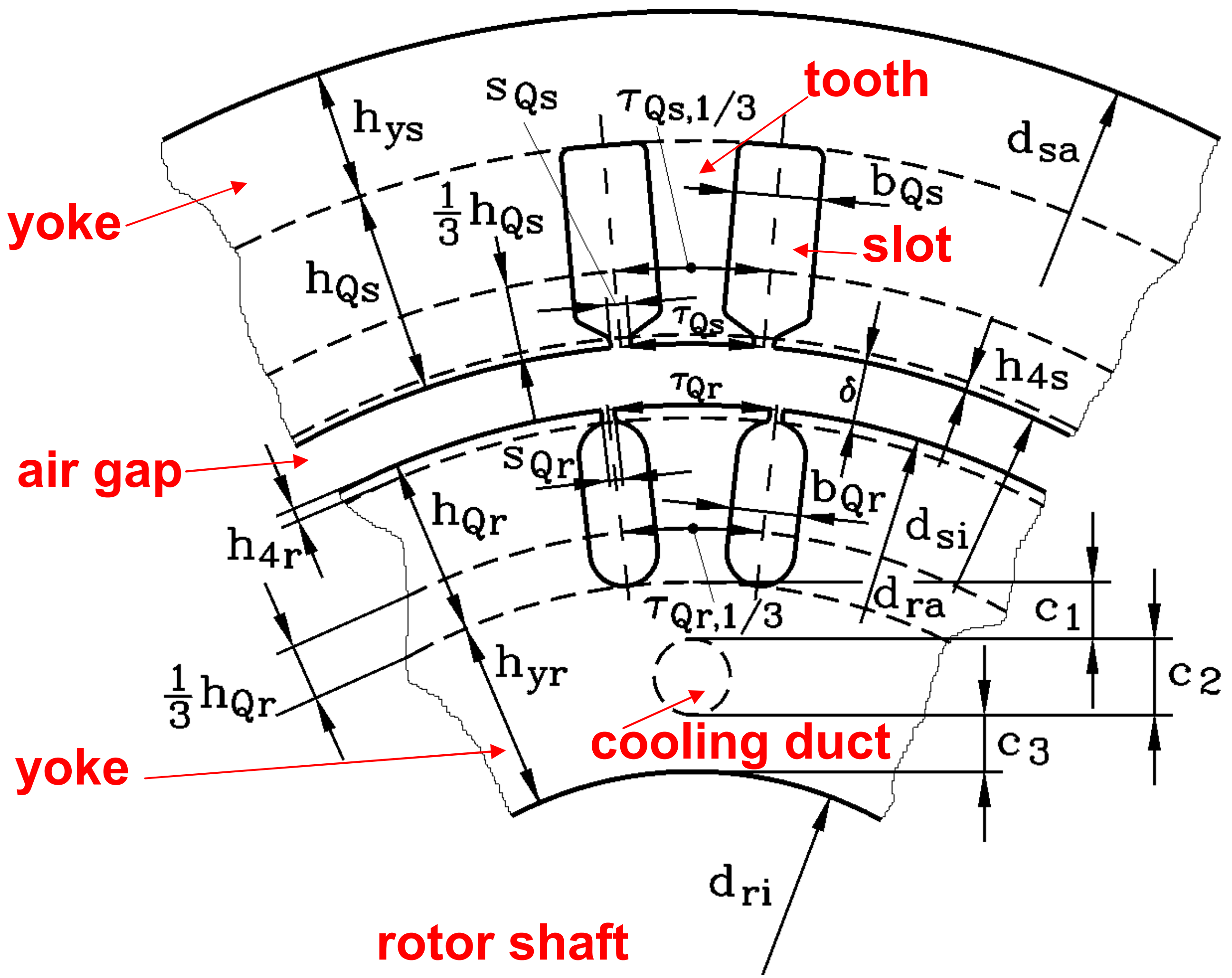
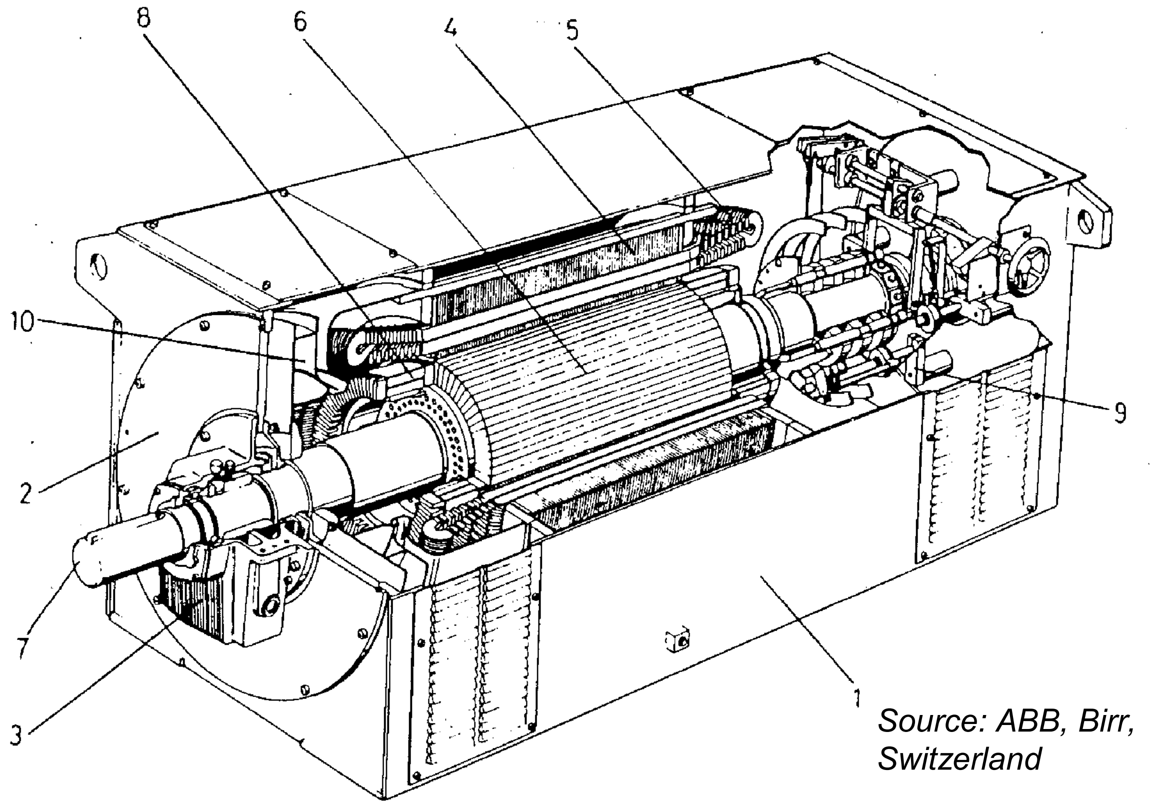


These dimensions have to be defined during design process to get the wanted machine output characteristic !

2. Design of Induction Machines



Source: ABB, Birr, Switzerland

1: Stator housing

2: End shield

3: Oil cooler for sleeve bearings

4: Stator iron stack

5: Overhang of two layer form wound stator winding

6: Rotor iron stack **with axial ducts in rotor iron back**

7: Shaft end

8: Overhang of rotor two layer winding

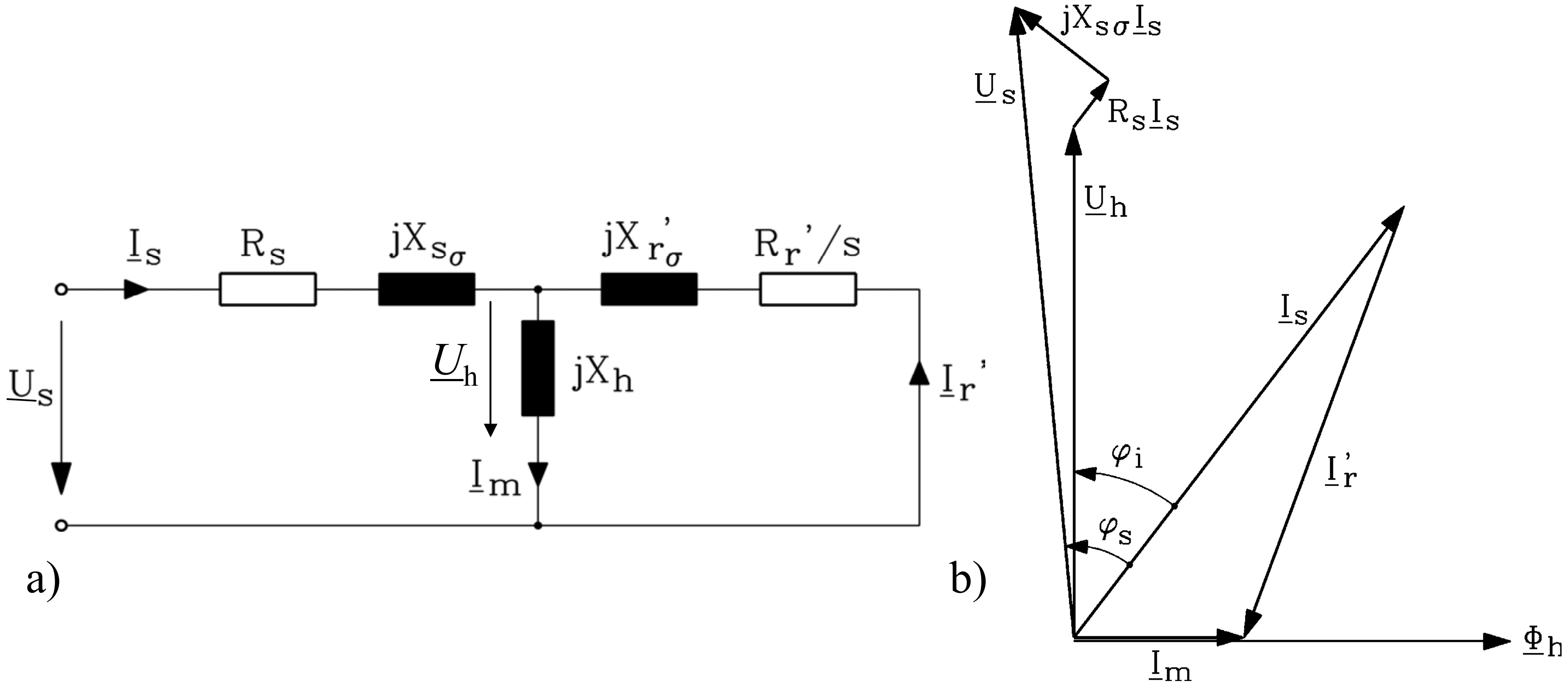
9: Three-phase rotor slip rings with brush contacts and hand wheel to lift brushes and to short circuit the rotor

10: Shaft mounted radial fan for generating cooling air flow, which passes through openings in stator housing

Design calculation yields equivalent circuit parameters

a) Equivalent circuit per phase for fundamental air gap flux density distribution, sinusoidal currents and voltages (here: iron losses $P_{Fe,s}$ neglected)

b) Corresponding phasor diagram for motor operation



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Summary:
Main dimensions and basic electromagnetic quantities of induction machines

- Design requirements by standards (IEC 60034) and by customers
- Electromechanical design implies magnetic, thermal and mechanical issues
- Design must be „translated“ into equivalent circuit for performance prediction

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2. Design of induction machines

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2. Design of Induction Machines

Scaling effect of power

- **Velocity of rotor** at synchronous speed at stator bore $v_{syn} = 2 f_s \tau_p$ is proportional to **velocity of air flow in air gap** of open ventilated machines with fans mounted on shaft.
- **Better cooling at bigger pole pitch**, so higher current loading possible: $A_s \sim \tau_p$.
- **Flux** $\Phi_h = \frac{2}{\pi} \cdot \tau_p \cdot l_{Fe} \cdot \hat{B}_{\delta,1}$ is excited by coils. For **minimum copper losses** at given flux, surface $\tau_p \cdot l_{Fe}$ needs **minimum length** per turn of $2 \cdot (\tau_p + l_{Fe})$, yielding $\tau_p = l_{Fe}$.
- **Result:** $A_s \sim \tau_p$, $l_{Fe} \sim \tau_p$ and $d_{si}\pi = 2p\tau_p \Rightarrow d_{si} \sim \tau_p \cdot p$
- **Rated apparent power** S_N with ***Esson´s equation:*** $S_N = 3U_{sN} I_{sN}$, $S_\delta = 3U_h I_s$

$$S_N \approx S_\delta = \frac{\pi^2}{\sqrt{2}} \cdot k_{w1} \cdot A_s \cdot \hat{B}_{\delta 1} \cdot d_{si}^2 \cdot l_{Fe} \cdot \frac{f_s}{p} = (2 \cdot \sqrt{2} \cdot k_{w1} \cdot f_s \cdot \hat{B}_{\delta 1}) \cdot p \cdot A_s \cdot \tau_p^2 \cdot l_{Fe} = K \cdot p \cdot \tau_p^4,$$

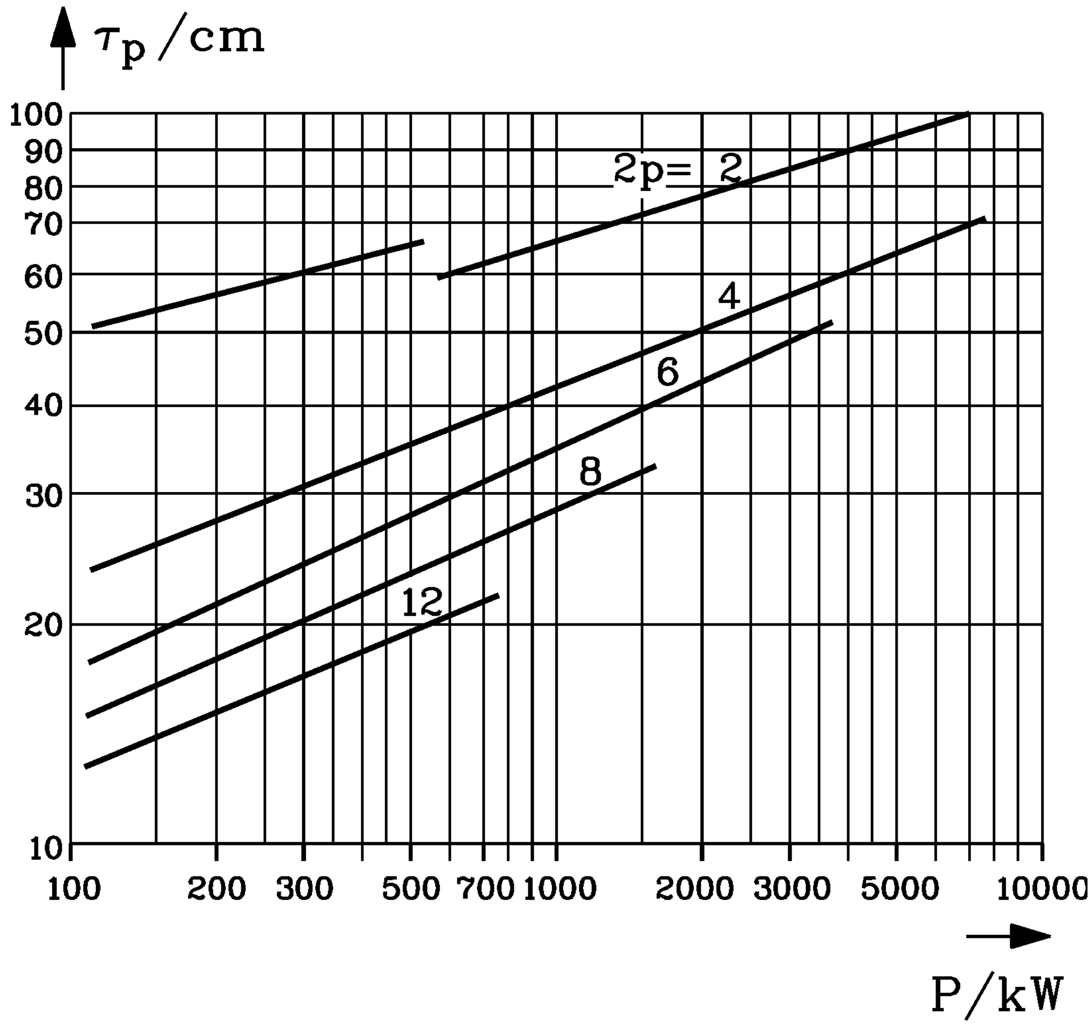
$$S_N \sim P_N \sim p \cdot \tau_p^4 \sim p \cdot A_s^4 \sim p \cdot l_{Fe}^4$$

Increase of pole pitch, iron length and current loading with $\sqrt[4]{P_N / p}$ **!**

$$\tau_p \sim \sqrt[4]{P_N / p}, \quad l_{Fe} \sim \sqrt[4]{P_N / p}, \quad A_s \sim \sqrt[4]{P_N / p}$$

2. Design of Induction Machines

Main dimensions may be chosen from curves, where parameters of already built machines are summarized



Behind these curves ESSON´s utilization number is "hidden" for

- a given induction machine type
- an open ventilation cooling system
- a winding temperature limit 125°C

$$\tau_p \sim \sqrt[4]{P_N / p}$$

Increase of pole pitch of induction machines for pole numbers 2, 4, 6, 8, 12

$$y = \lg(\tau_p) \sim \lg(\sqrt[4]{P_N / p})$$

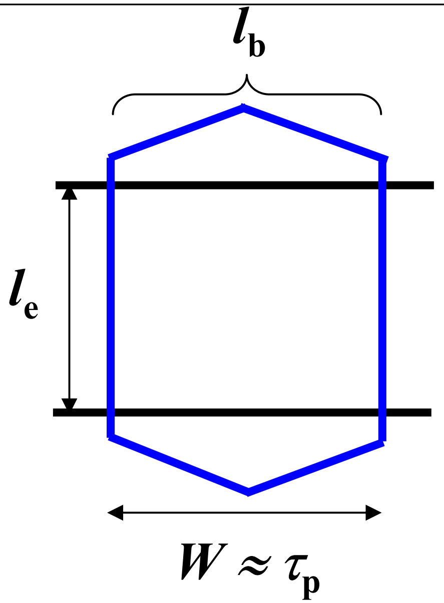
$$y = \lg(\tau_p) \sim \lg(P_N)/4 - \lg(p)/4$$

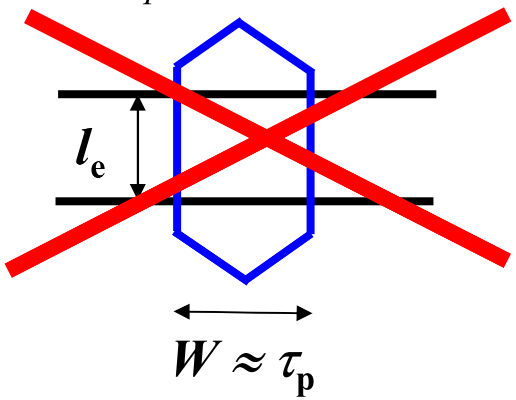
$$\boxed{y \sim x/4 - \lg(p)/4}$$

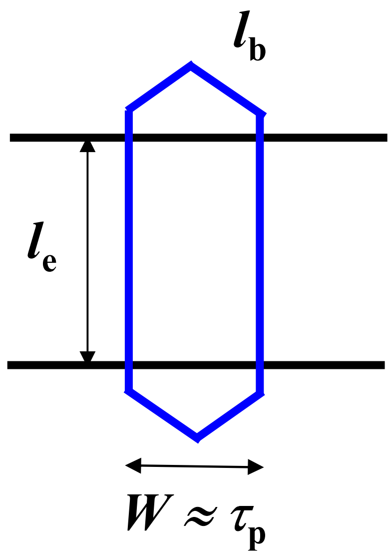
Diagram is a straight line in double logarithmic scaling

Iron stack length vs. pole pitch

Pole pitch τ_p shrinks with increasing pole count $2p$ $\tau_p = d_{si}\pi/(2p)$







$2p = 4$

$l_e \approx \tau_p$

At low pole count $2p = 4$ the condition $l_e \approx \tau_p$ for minimum copper losses at given pole area $l_e \cdot \tau_p$ is used

$2p > 4$

$l_e \approx \tau_p$

At higher pole count $2p > 4$ for the condition $l_e \approx \tau_p$ the induction machines become axially very short, with a dominating ratio $l_b/l_e > 1$

$2p > 4$

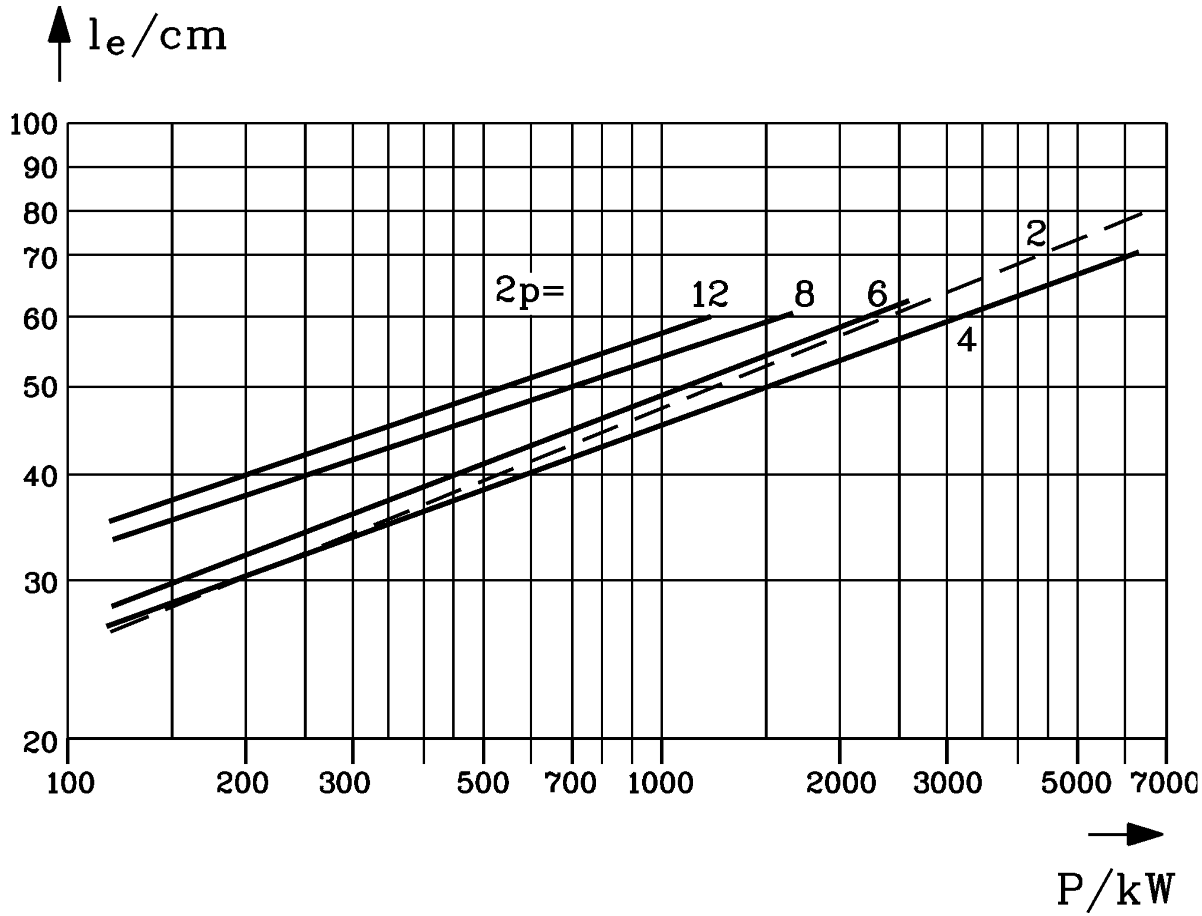
$l_e > \tau_p$

Therefore:

At higher pole count $2p > 4$ the iron length is increased $l_e > \tau_p$.

(Equivalent) iron stack length

Increase of (equivalent) iron stack length l_e of induction machines for pole count $2p$ = 2, 4, 6, 8, 12



l_e: Equivalent stack length (due to radial ventilation ducts)

$2p$ = 4: $l_e \sim \tau_p \sim \sqrt[4]{P_N}$

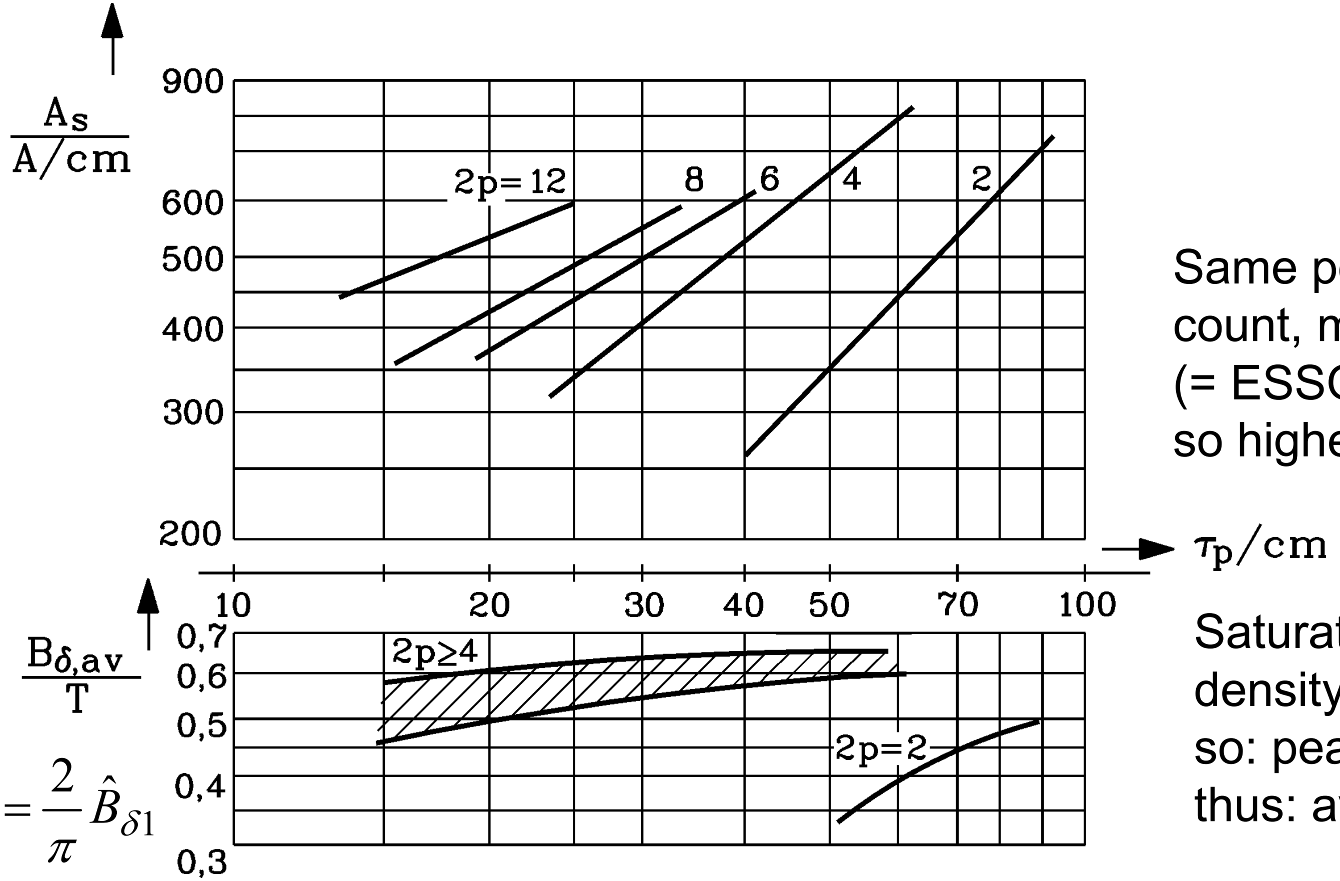
$2p$ > 4: $l_e \sim k(p) \cdot \sqrt[4]{P_N}$

$k(p) > 1$

Current loading and air gap flux density

Stator current loading A_s & average air gap flux density $B_{\delta,av}$:

For pole count $2p$ = 2, 4, 6, 8, 12; open ventilated air cooled machines, Thermal Class B (temperature rise 80 K over 40 °C ambient);



$$B_{\delta,av} = \frac{2}{\pi}\hat{B}_{\delta 1}$$

$$A_s \sim \tau_p$$

Same pole pitch, but higher pole count, means bigger machine (= ESSON-number C ↑), so higher A_s is possible.

Saturation limits tooth flux density to 2 T, so: peak air gap $\hat{B}_\delta$ to 1 T, thus: average $B_{\delta,av}$ to 0.66 T

2. Design of Induction Machines

Comparison of 2- and 4-pole machine cross section

For the same bore diameter d_{si} the pole pitch of the 2-pole machine is twice of the 4-pole machine.

Hence for the same air gap flux density the flux Φ per pole is twice. Therefore for the same yoke flux density B_y we need twice big yoke height h_{ys} for 2-pole machines, yielding a huge stator iron mass.

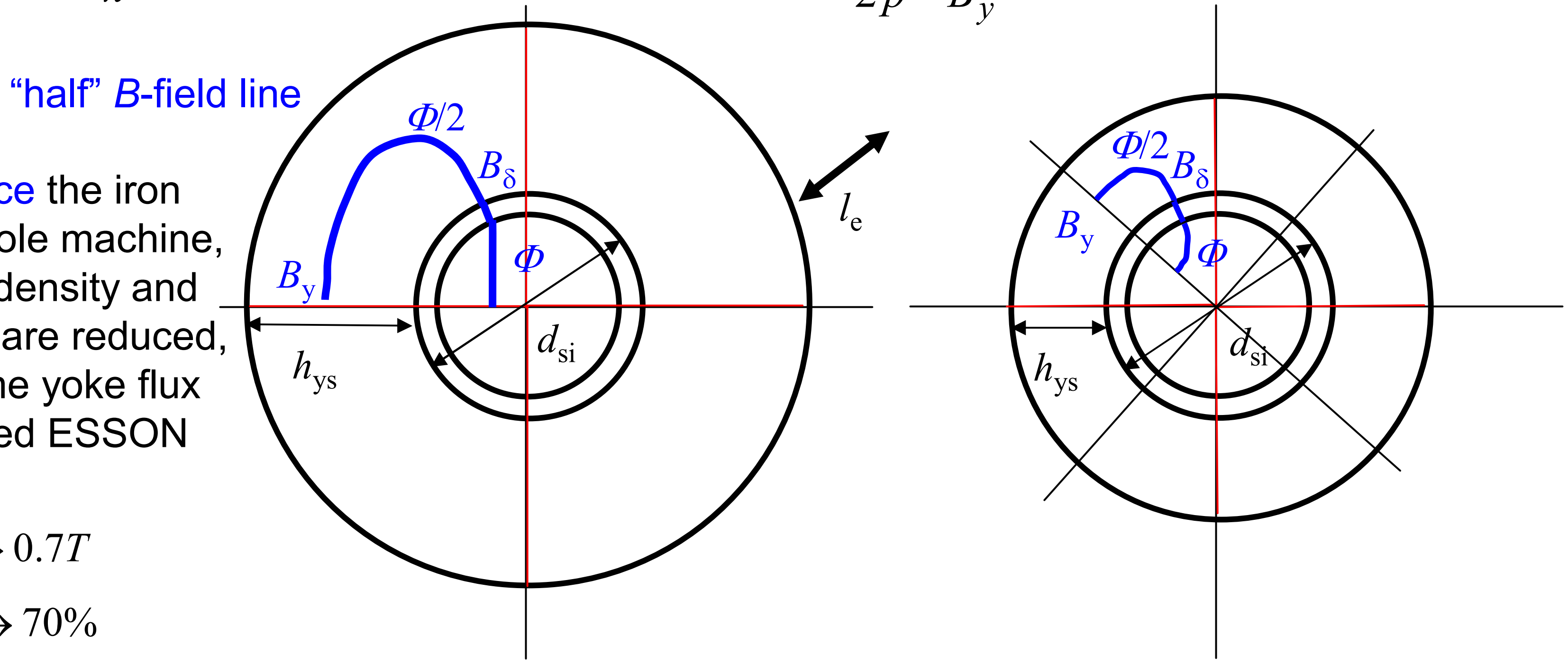
$$\tau_p = d_{si}\pi/(2p) \qquad \Phi = \frac{2}{\pi}\tau_p l_e \hat{B}_{\delta 1} \qquad \Phi/2 = h_{ys} l_e B_y \quad \rightarrow \quad h_{ys} = \frac{d_{si}}{2p}\cdot\frac{\hat{B}_{\delta 1}}{B_y}$$

In order to reduce the iron mass of the 2-pole machine, the air gap flux density and the yoke height are reduced, yielding the same yoke flux density = reduced ESSON number!

$$\hat{B}_{\delta 1} = 1.0T \rightarrow 0.7T$$

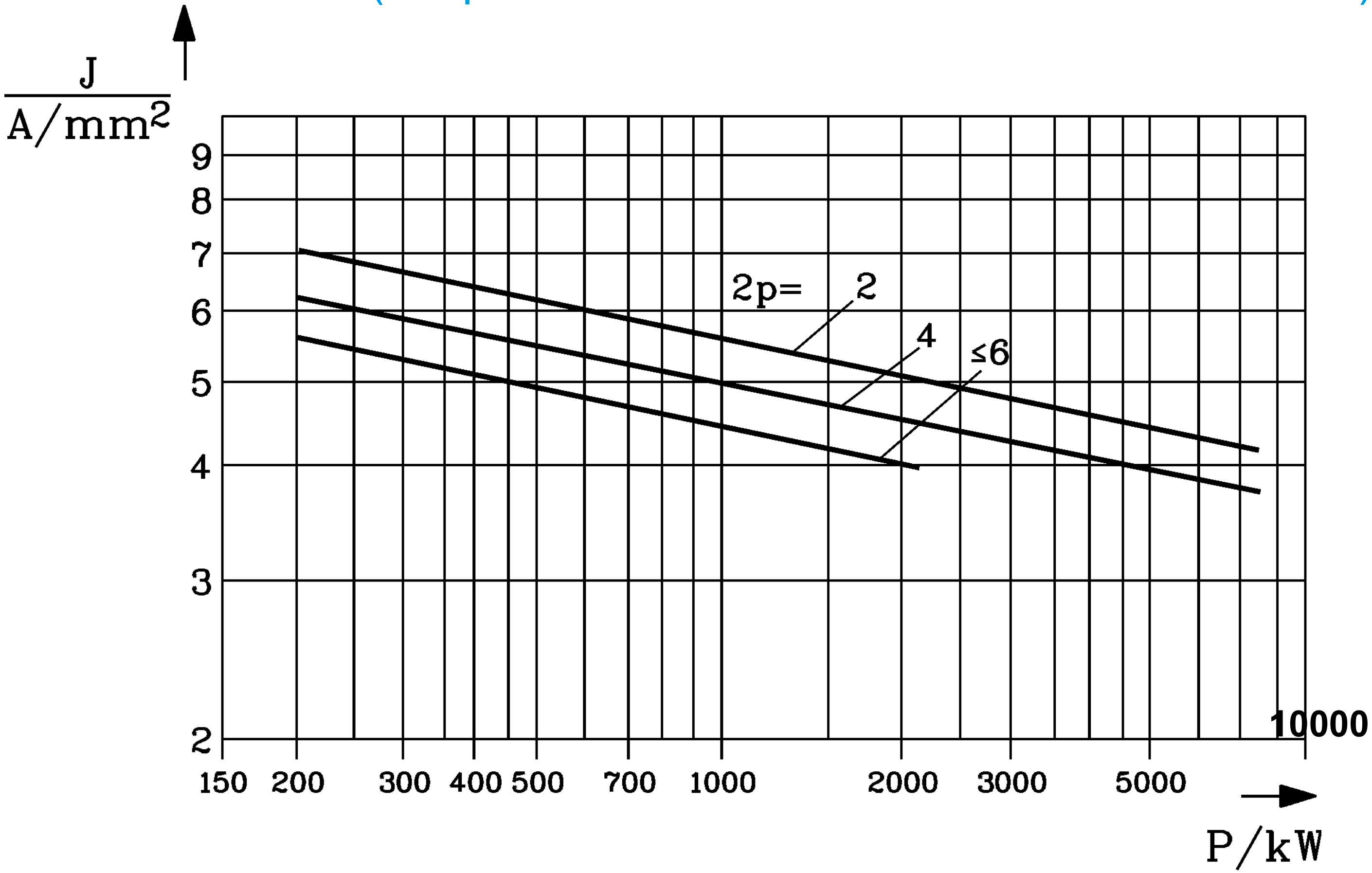
$$h_{ys} = 100\% \rightarrow 70\%$$

$$C \sim A_s \hat{B}_{\delta 1} = 100\% \rightarrow 70\%$$



2-pole machine **4-pole machine**

Decrease of current density J of 6 kV induction machines
for pole count $2p$ = 2, 4 and $\geq$ 6; open ventilated air cooled machines,
Thermal Class B (temperature rise $\Delta\vartheta$ = 80 K over 40 °C ambient)



$$P \sim L^4, J \sim 1/\sqrt{L}$$

$$J \sim 1/\sqrt[8]{P}$$

$$\lg J \sim \lg P^{-1/8}$$

$$\lg J \sim -0.125 \cdot \lg P$$

2. Design of Induction Machines

Pole count scaling effect of current density *J* vs. power *P*

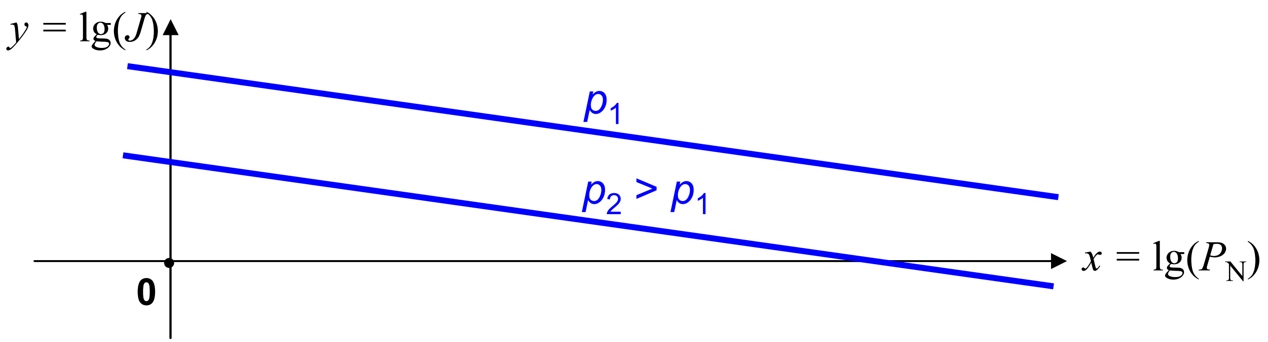
$$A_s \sim \tau_p = d_{si} \cdot \pi /(2p) \qquad d_{si} \approx L \qquad A_s \sim L / p$$

$$l_{Fe} \approx L$$

$$S_N \approx S_\delta = \frac{\pi^2}{\sqrt{2}} \cdot k_{w1} \cdot A_s \cdot \hat{B}_{\delta 1} \cdot d_{si}^2 \cdot l_{Fe} \cdot \frac{f_s}{p} = \left(\frac{\pi^2}{\sqrt{2}} \cdot k_{w1} \cdot \hat{B}_{\delta 1} \cdot f_s \right) \cdot \frac{A_s \cdot d_{si}^2 \cdot l_{Fe}}{p} = \overline{K} \cdot \frac{L^4}{p^2} \approx P_N$$

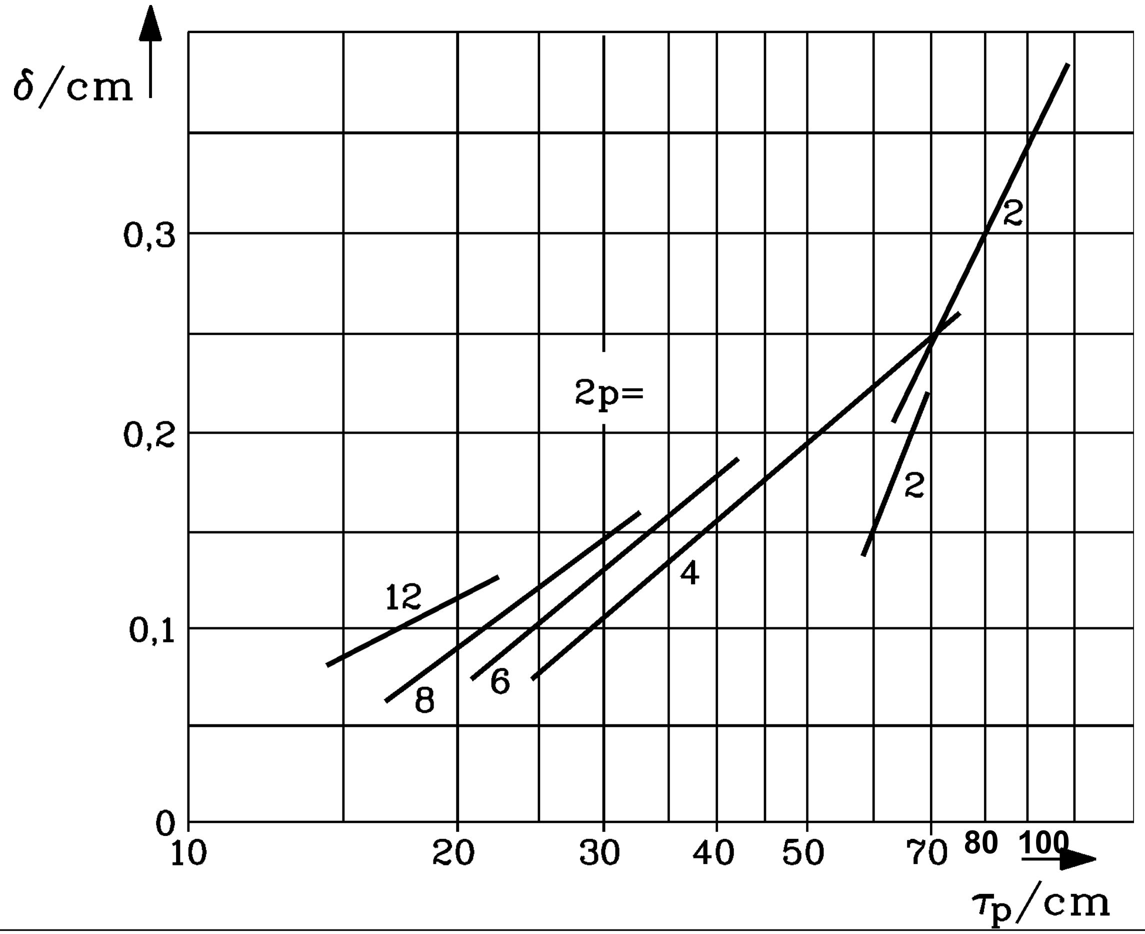
$$L = \sqrt[4]{P_N \cdot p^2 / \overline{K}} \qquad J = \frac{1}{\sqrt{L}} = \frac{1}{\sqrt[8]{P_N \cdot p^2 / \overline{K}}}$$

$$y = \lg(J) = \frac{\lg(\overline{K})}{8} - \frac{\lg(P_N)}{8} - \frac{\lg(p)}{4} \qquad \boxed{y = \lg(J) = -\frac{x}{8} + \frac{\lg(\overline{K})}{8} - \frac{\lg(p)}{4}}$$



Air gap width

Increase of air gap width δ with increasing induction machine size (pole pitch τ_p)
for pole count $2p$ = 2, 4, 6, 8 and 12



For the same pole pitch τ_p the higher pole count $2p$ gives a bigger diameter d_{si}:

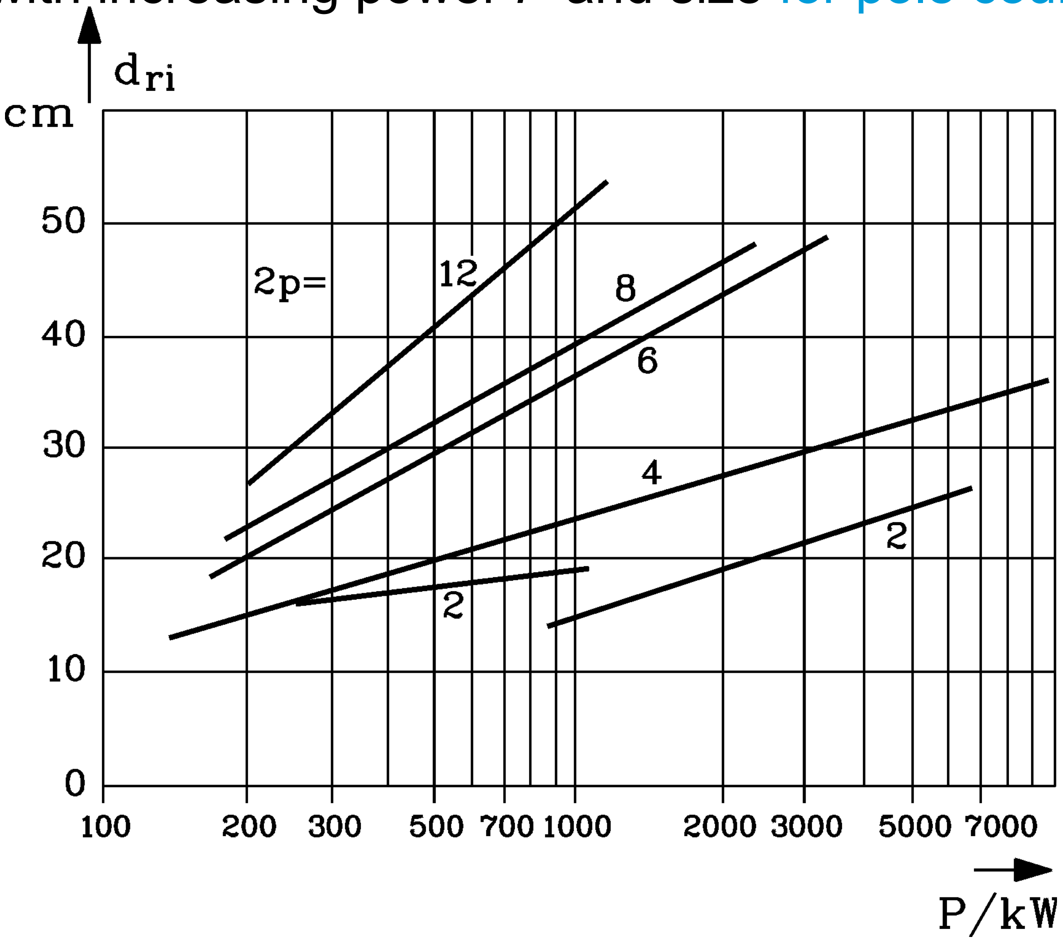
→ need for bigger air gap δ to avoid touching of rotor, caused by shaft bending

$$\frac{\delta}{d_{si}} \approx \text{const.} \Rightarrow \tau_p = \frac{d_{si} \cdot \pi}{2p}$$

$$\delta \sim d_{si} \sim \tau_p \cdot p$$

Shaft diameter

Increase of shaft diameter d_{sh} (= inner rotor stack diameter d_{ri}) with increasing power P and size for pole count $2p$ = 2, 4, 6, 8 and 12



Bigger power P at higher pole count $2p$ means

- lower speed n, so higher torque M, so
- bigger shaft diameter d_{ri} is required

$$\lg(d_{ri}) \sim \lg(P_N)/3 + \lg(p)/3$$

2. Design of Induction Machines

Torsion stress requires a certain shaft diameter d_{ri}

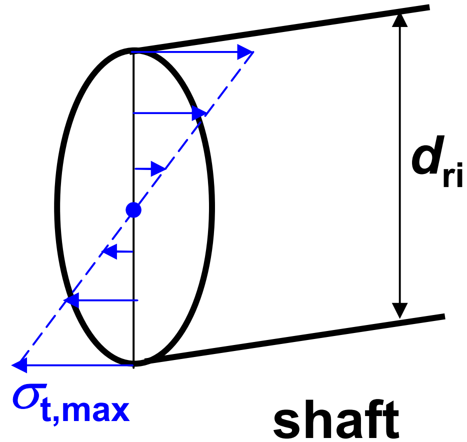
Torsion stress σ_t at torque *M*:

$$\sigma_{t,\max} = \frac{M \cdot (d_{ri}/2)}{I_p} \qquad I_p = \frac{\pi}{2} \cdot \left(\frac{d_{ri}}{2}\right)^4$$

$$M = \frac{P_N}{2\pi \cdot n} = \frac{P_N}{2\pi \cdot (f_s / p)} \sim P_N \cdot p$$

$$\sigma_{t,\max} \sim \frac{P_N \cdot p}{d_{ri}^3} \qquad d_{ri} \sim \sqrt[3]{P_N \cdot p}$$

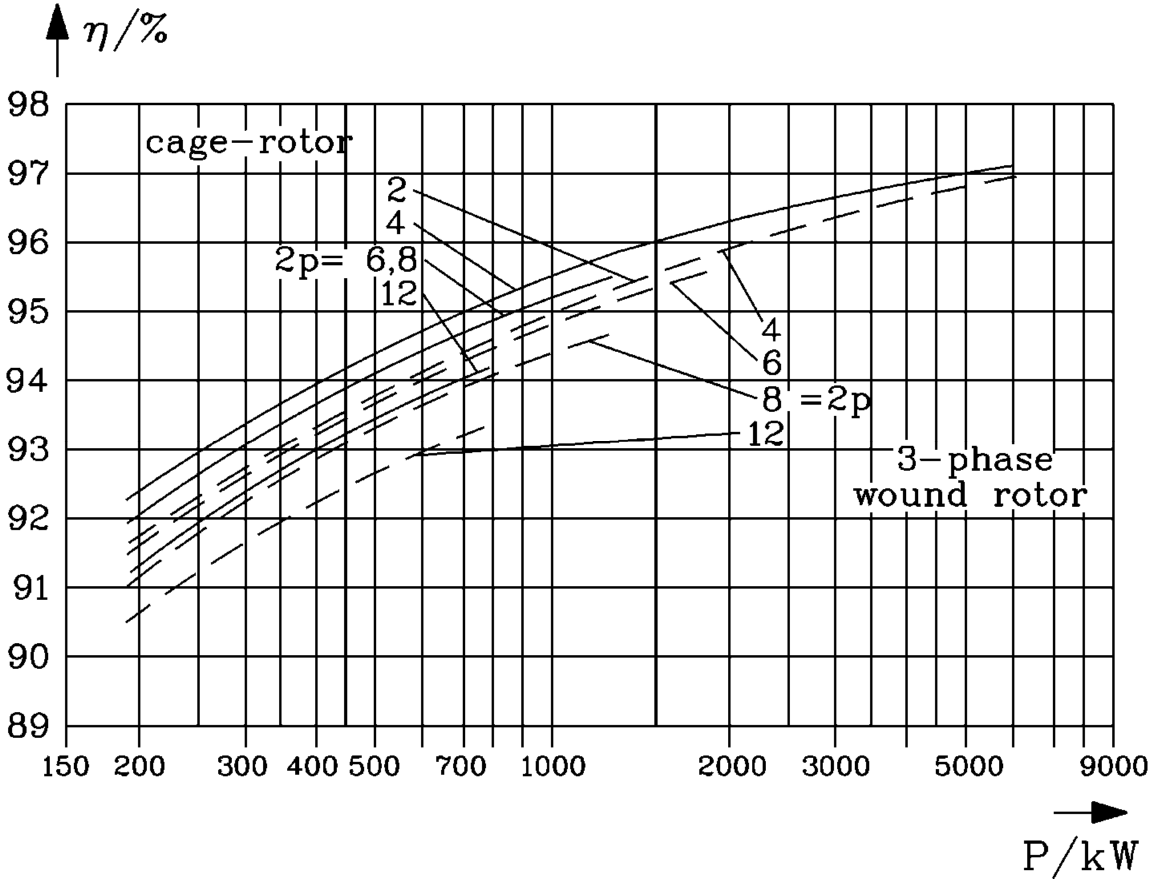
$$\lg(d_{ri}) \sim \lg(P_N)/3 + \lg(p)/3$$



2. Design of Induction Machines

Efficiency

Increase of motor efficiency η with increasing induction machine power P
at pole count $2p$ = 2, 4, 6, 8 and 12 for cage and wound rotor, 6 kV, 50 Hz, Thermal Class B ($\Delta\vartheta$ = 80 K), open ventilated machines



Power rise with lengths:

$$P \sim C \cdot L^3 \sim A_s \hat{B}_{\delta 1} L^3 \sim L^4$$

$$P \sim L^4$$

At constant loss density $p_d = P_d / V$ losses rise with volume:

$$P_d = p_d V \sim L^3$$

So efficiency increases with bigger machines ($P = P_{out}$).

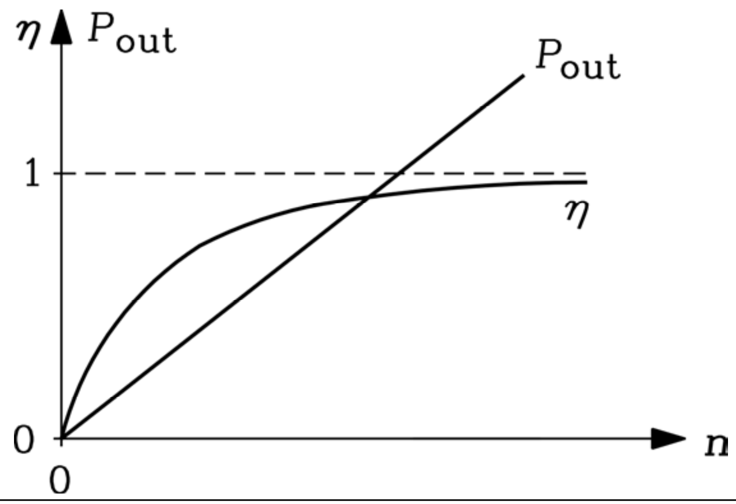
$$\eta = \frac{P}{P + P_d} = \frac{1}{1 + P_d / P} = \frac{1}{1 + k / L}$$

k: constant

Efficiency increases with speed at given torque

- **At a given torque *M* efficiency η increases at dominant I^2R-losses with speed n.**
- **Torque** $M_e = P_\delta/(2\pi \cdot n_{syn}) \sim 3U_h I_s/(f_s/p) \sim 3 \cdot f_s \cdot \Phi_h \cdot I_s/(f_s/p) \sim p \cdot \Phi \cdot I_s =$ **"Flux x Current„**
- **Efficiency** $\eta = \frac{P_{out}}{P_{in}} = \frac{P_{out}}{P_{out} + P_d} = \frac{2\pi \cdot n \cdot M}{2\pi \cdot n \cdot M + 3 \cdot R \cdot I^2} \sim \frac{n \cdot p \cdot \Phi \cdot I}{n \cdot p \cdot \Phi \cdot I + k' \cdot R \cdot I^2}$

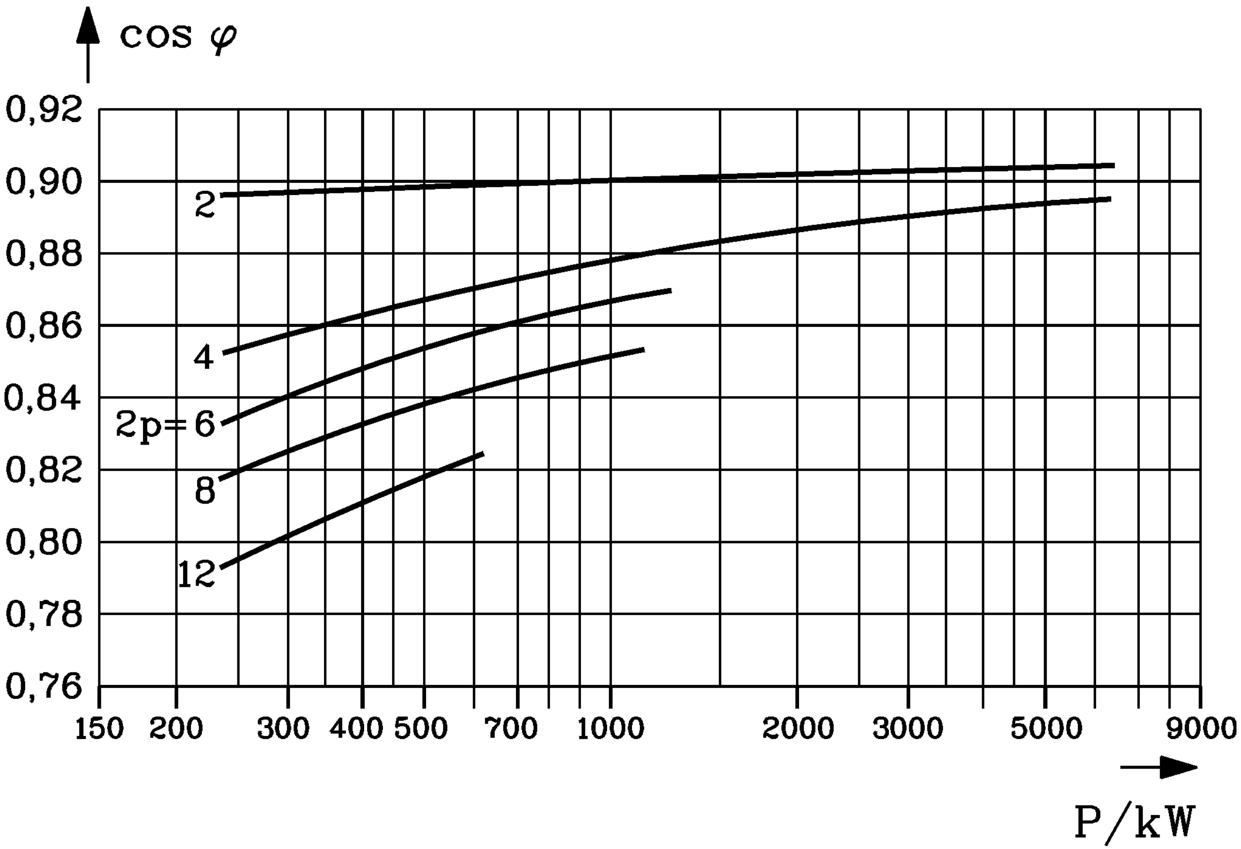
$$\eta \sim \frac{n}{n + \bar{k} \cdot I} \qquad I = I_{sN} = \text{const.}$$



2. Design of Induction Machines

Power factor

Increase of power factor $\cos\varphi$ with increasing induction machine power P
at pole count $2p$ = 2, 4, 6, 8 and 12 for cage and wound rotors, 6 kV, 50 Hz,
Thermal Class B ($\Delta\vartheta$ = 80 K), open ventilated machines



a) In bigger machines the ratio "stray flux vs. main flux" is smaller due to higher slot numbers Q, yielding higher power factor $\cos\varphi$.

b) High pole count machines have

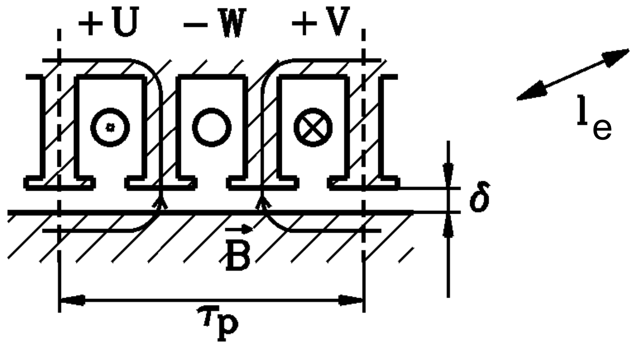
(1) low slot number q per pole & phase, so higher harmonic leakage,
(2) smaller ratio τ_p/δ, so lower magnetizing reactance,

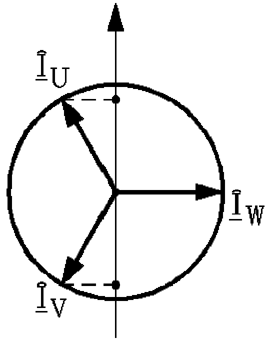
yielding lower power factor $\cos\varphi$.

2. Design of Induction Machines

Induction machines with high pole count have poor cosφ

Example:
m = 3, q = 1





- Magnetizing current $I_m \sim U_s/(2\pi L_s) \sim \delta/\tau_p$
  Inductance per phase $L_s = L_{s\sigma} + L_h$ $\qquad L_h \sim l_e \cdot \tau_p/\delta$
- (a) High pole count $2p \Rightarrow$ pole pitch $\tau_p = d_{si}\pi/(2p)$ small
  (b) Mechanical lower limit for air gap $\delta_{min} \Rightarrow \tau_p/\delta_{min}$ gets too small for big 2*p*!

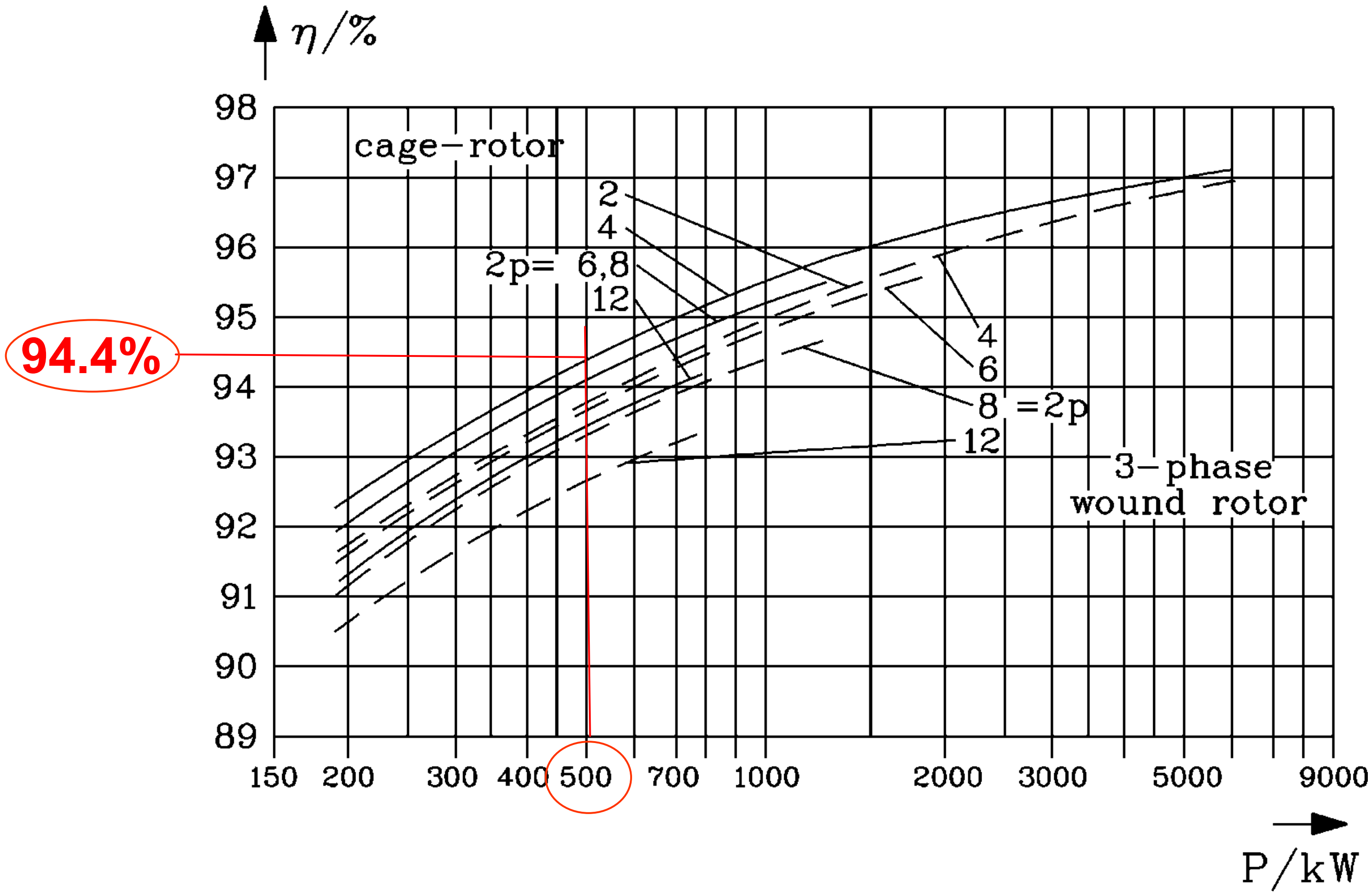
Example: Machine for very low speed 28/min:
I: High pole count induction machine not feasible for low speed, because cosφ is too low
II: 4-pole induction machine frame size 400 mm with gear i = 50 is feasible

	P/ kW	n/min^{-1}	f/ Hz	$2p$	d_{si}/ m	τ_p/mm	δ/mm	δ/d_{si} /%	τ_p/δ	l/ m	cosφ	I_m/I_N
I	750	28	22.4	96	5	164	5	0.1	32.8	0.35	0.6	0.8
II	640	1514	50	4	0.45	353	2	0.4	176.5	0.66	0.91	0.27

Design example:

Motor, P_N = 500 kW, 6 kV, 50 Hz, 4 poles

2*p* = 4, 500 kW, cage machine: Estimation of motor efficiency η



2. Design of Induction Machines

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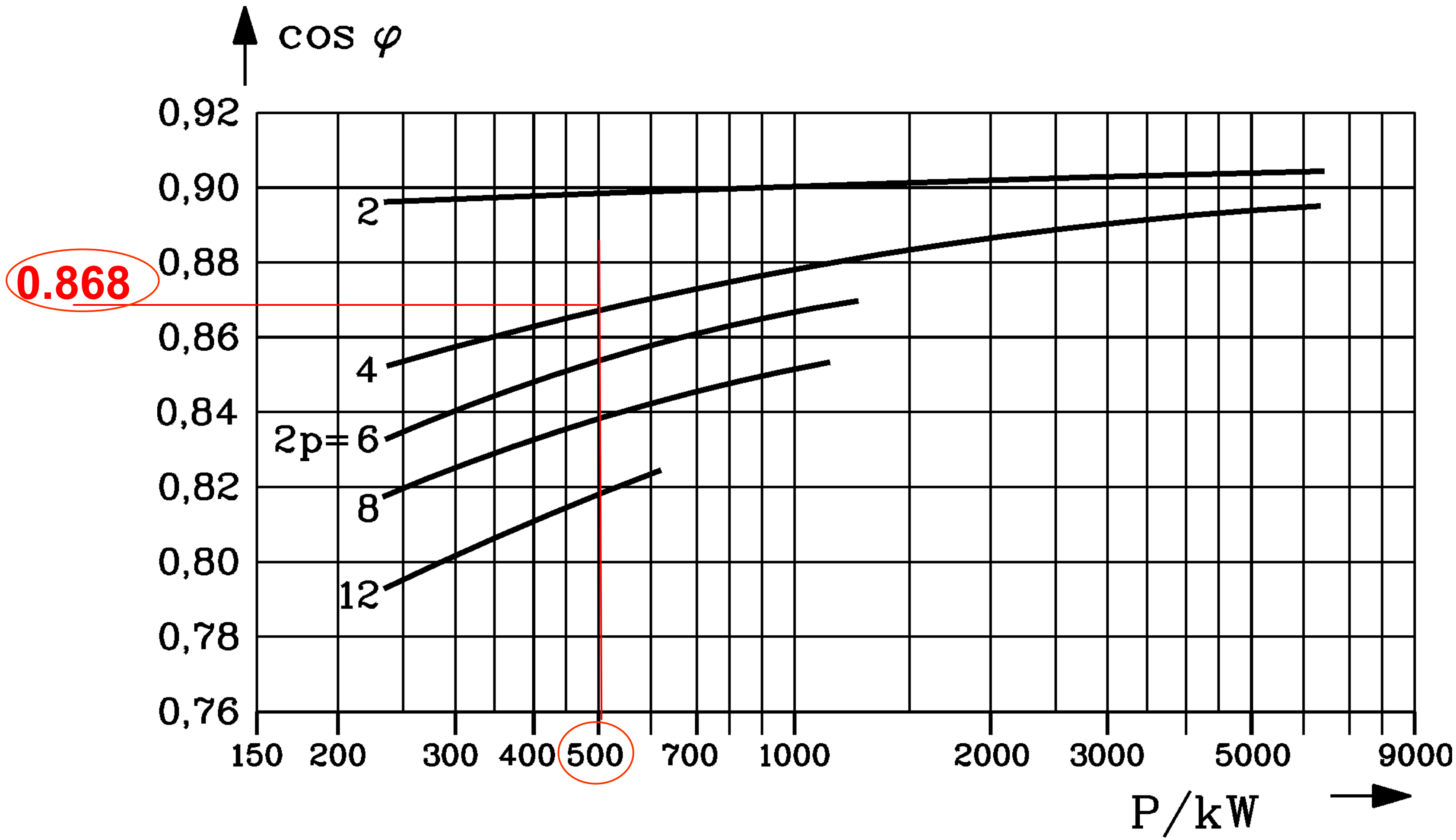
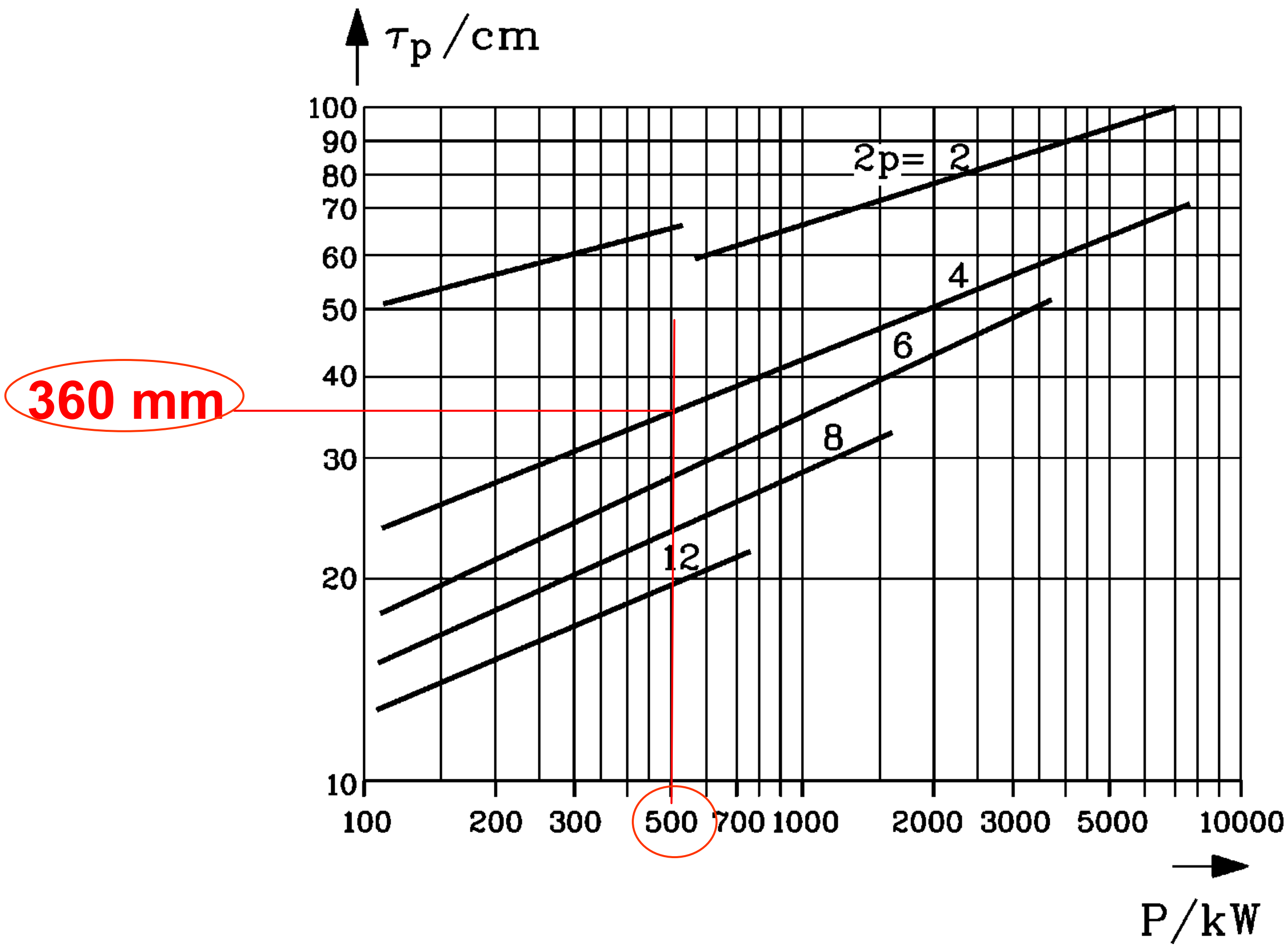
500 kW, induction machine: Estimation of power factor $\cos\varphi$

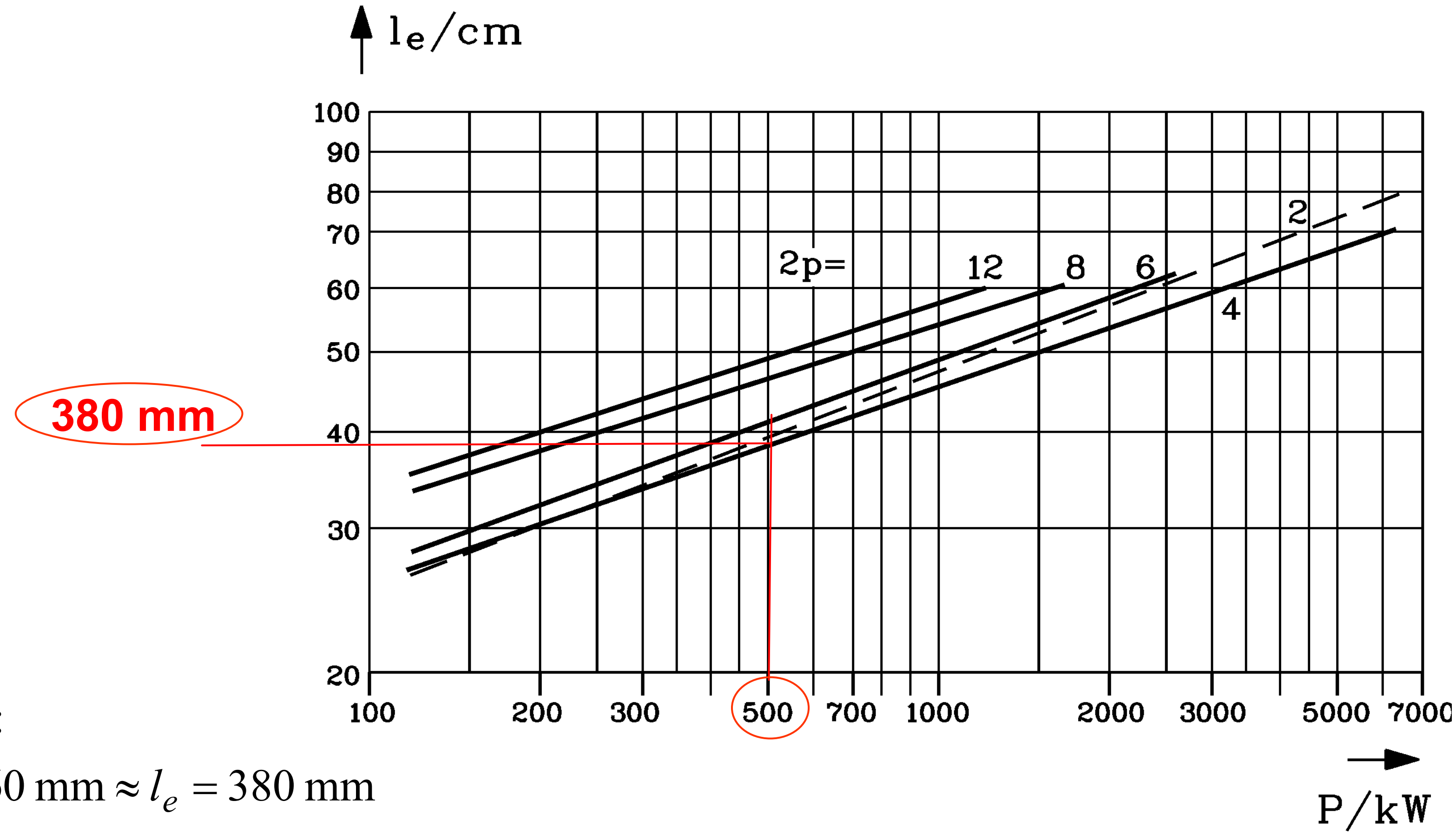


2. Design of Induction Machines

2p = 4, 500 kW, induction machine: Estimation of pole pitch τ_p



2p = 4, 500 kW, cage machine: Estimation of (equivalent) stack length l_e

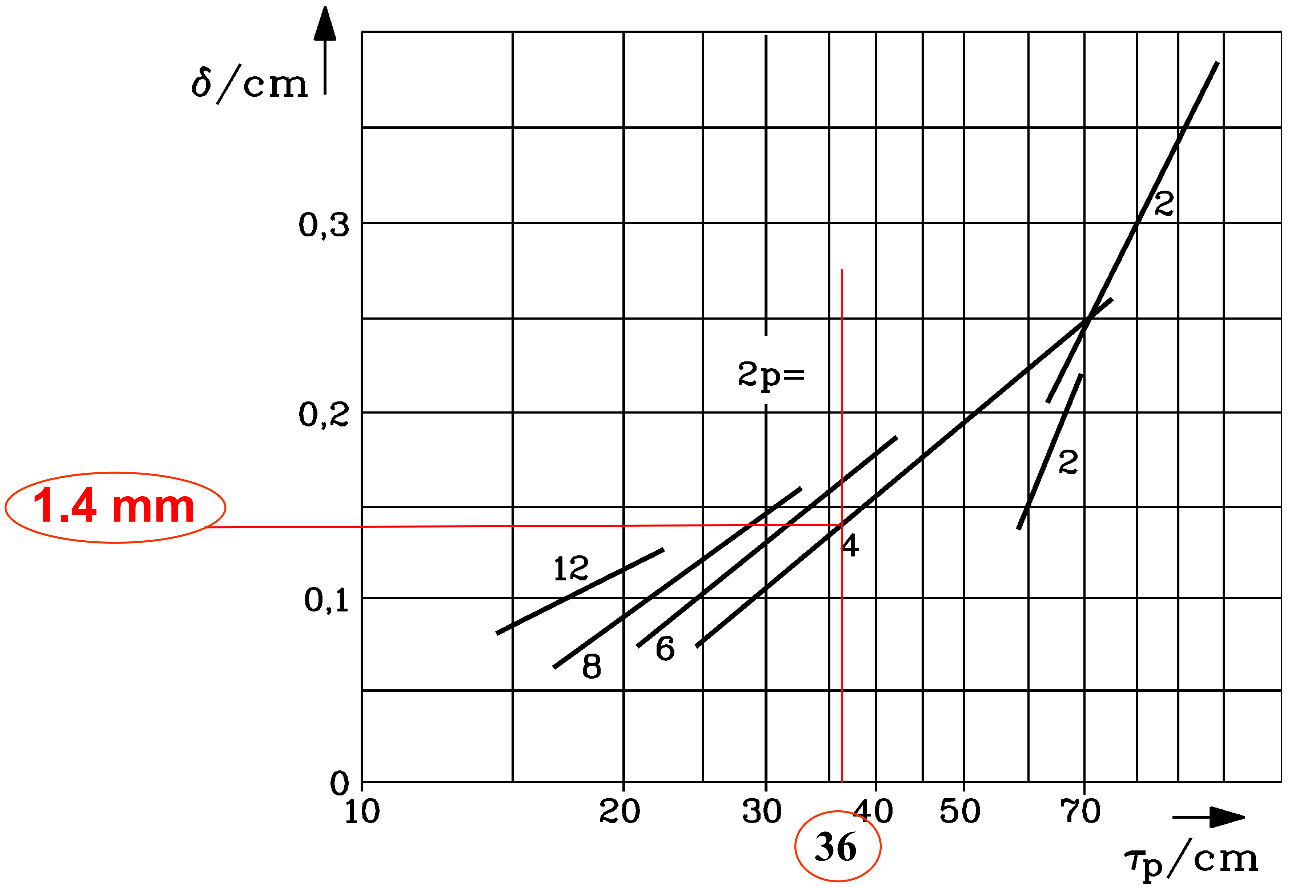


$2p = 4:$

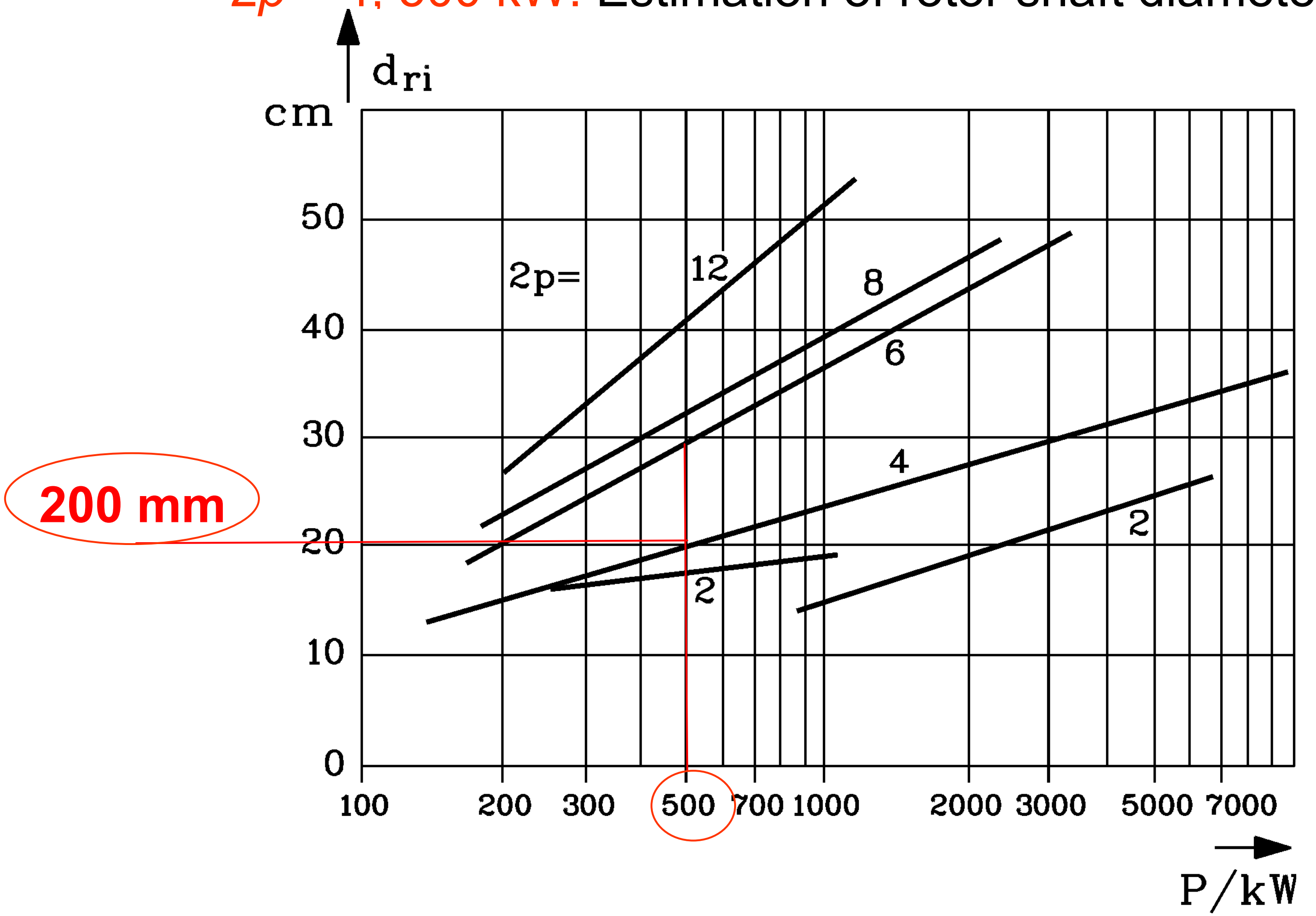
$\tau_p = 360 \text{ mm} \approx l_e = 380 \text{ mm}$

2. Design of Induction Machines

Pole pitch τ_p = 360 mm, $2p$ = 4, 500 kW: Estimation of air gap δ



2*p* = 4, 500 kW: Estimation of rotor shaft diameter d_{ri}



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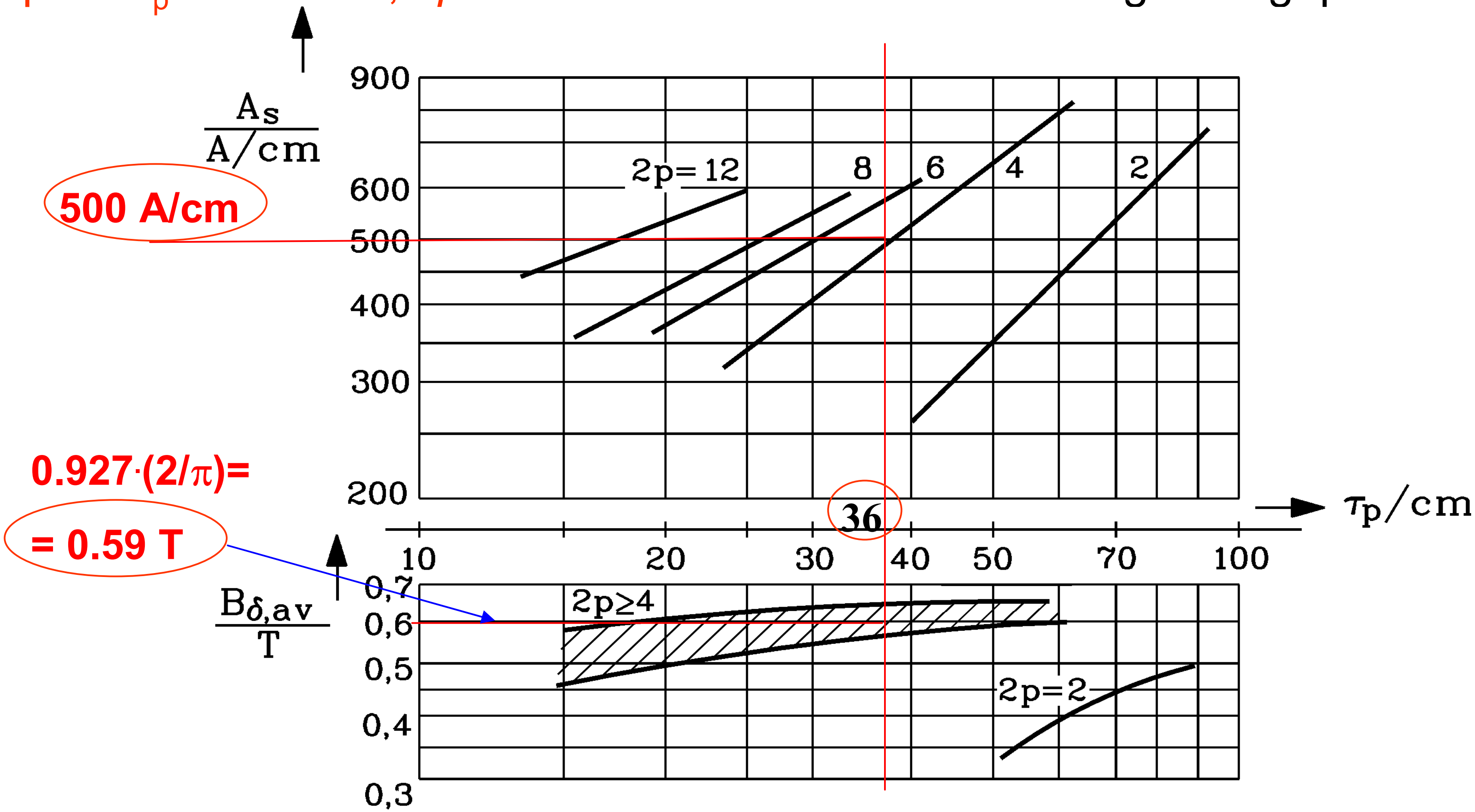
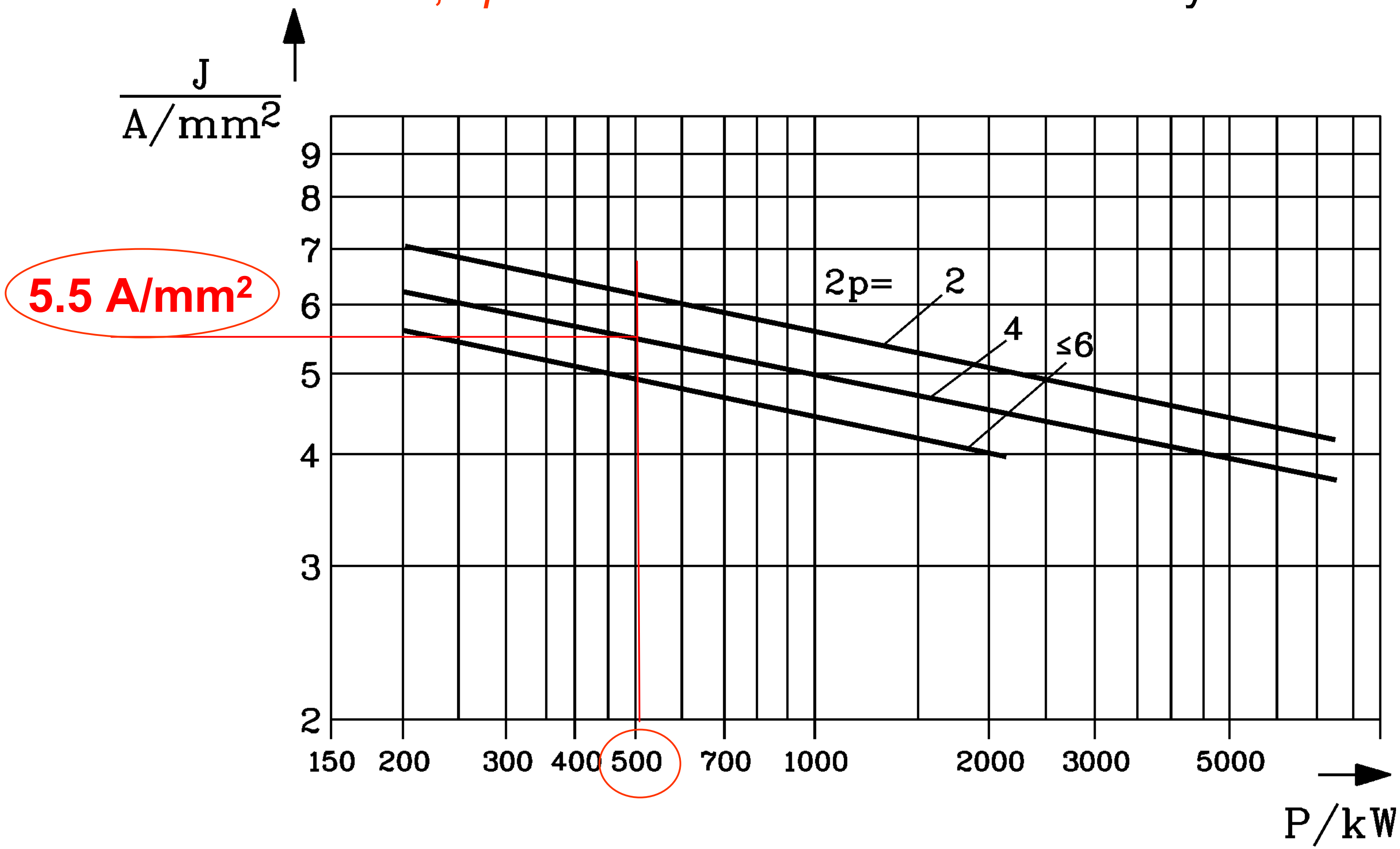
Pole pitch τ_p = 360 mm, 2*p* = 4: Estimation of current loading & air gap flux density



2. Design of Induction Machines

500 kW, $2p$ = 4: Estimation of current density J



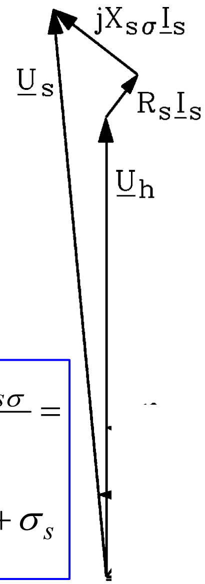
2. Design of Induction Machines
Design example

Motor, P_N = 500 kW, 6 kV, 50 Hz, 4 poles:

Estimated values: η_N = 94.4 %, $\cos\varphi_N$ = 0.868

Motor output power: 500 kW, apparent power: $S_N = \dfrac{P_N}{\eta_N \cdot \cos\varphi_N} = 610\ \text{kVA}$

Motor current: $I_{sN} = \dfrac{S_{sN}}{\sqrt{3}\cdot U_{sN}} = \dfrac{610}{\sqrt{3}\cdot 6} = 59\ \text{A}$,



$$\frac{U_s}{U_h} \approx \frac{X_h + X_{s\sigma}}{X_h} = 1 + \frac{X_{s\sigma}}{X_h} = 1 + \sigma_s$$

Synchronous speed: $n_{syn} = f_s / p = 1500$ /min

Pole pitch: 360 mm, stack length: 380 mm

air gap: 1.4 mm, shaft diameter: 200 mm

Current loading: 500 A/cm, current density: 5.5 A/mm², $A_s \cdot J_s = 2750$ (A/cm)(A/mm²)

Stator bore diameter: $d_{si} = 2p\tau_p / \pi = 458\ \text{mm}$.

Internal apparent power: ($\sigma_s \cong 0.08/2 = 0.04$): $S_\delta = S/(1+\sigma_s) = 610/1.04 = 587$ kVA

Electromagnetic utilization: $C = S_\delta / (d_{si}^2 \cdot l_e \cdot n_{syn}) = 4.9\ \text{kVA·min/m}^3$

Flux density (k_{w1} = 0.91 estimated): $C = \dfrac{\pi^2}{\sqrt{2}} \cdot k_{w1} \cdot A_s \cdot \hat{B}_{\delta 1} \quad \Rightarrow \quad \hat{B}_{\delta 1} = \underline{\underline{0.927}}$ T

Summary:
Scaling effect in electric machines

- "Typical length" L used as scale
- Scaling laws for
  power P, air gap flux density B_δ, current loading A, current density J
- Assumptions for
  given cooling system & type of construction must be kept in mind
- Pole count influence on active iron mass ~ $d_{sa}^2 \cdot l_e$, efficiency η, power factor $\cos\varphi$
- Scaling laws verified with results of built machines

2. Design of induction machines

2.1 Main dimensions and basic electromagnetic quantities of induction machines

2.2 Scaling effect in electric machines

2.3 Stator winding low and high voltage technology

2.4 Stator winding design

2.5 Rotor cage design

2.6 Wound rotor design

2.7 Design of main flux path of magnetic circuit

2.8 Stray flux and inductance

2.9 Influence of saturation on inductance

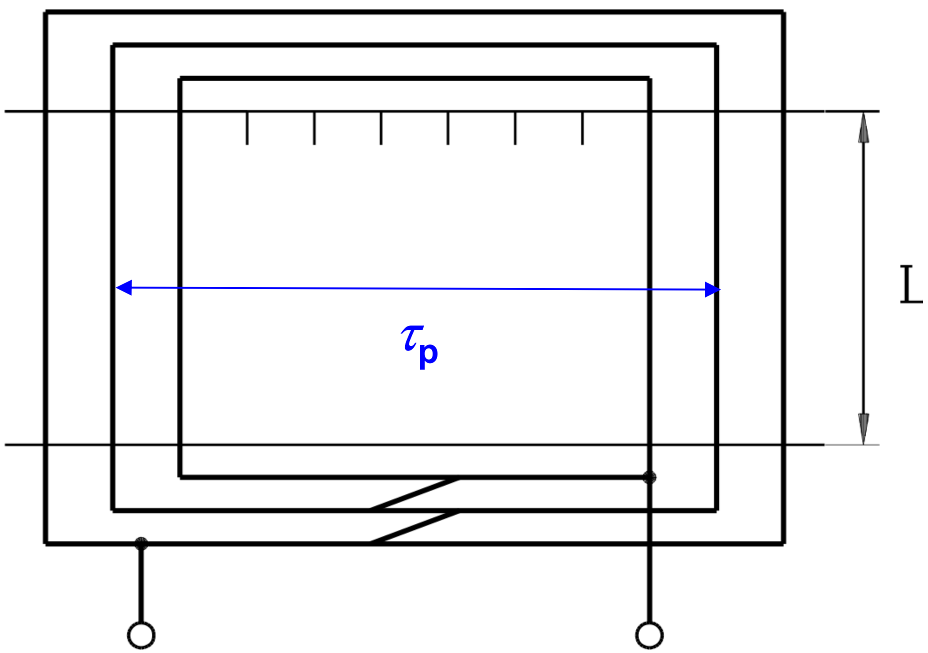
2.10 Masses and losses

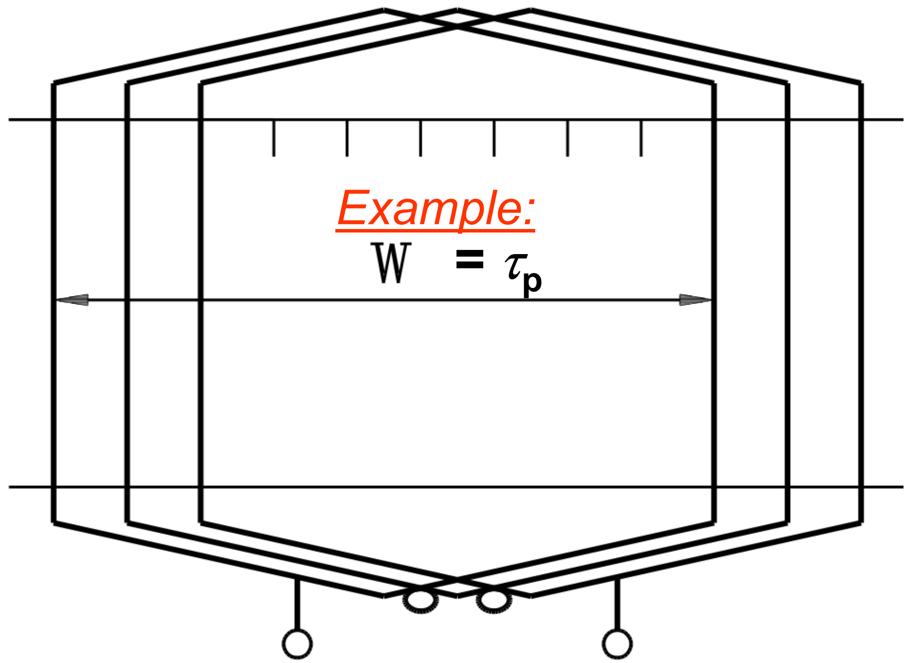
2. Design of Induction Machines

Coil groups per pole and phase

Example: q = 3 per group

L: Total axial iron stack length including radial ventilation ducts





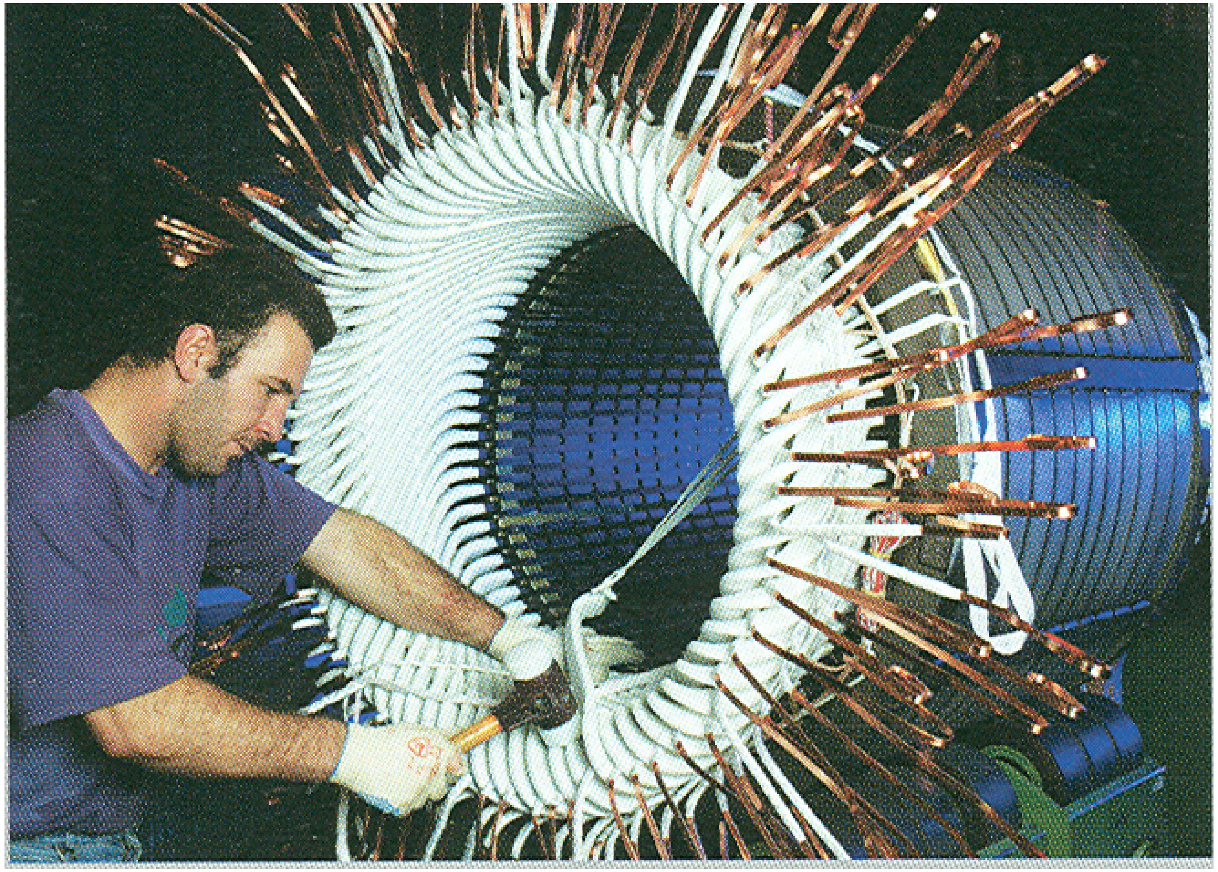
concentric coils

for single layer winding

coils with identical span

for double layer winding

Two-layer four-pole three-phase stator winding manufacturing (72 slots) for a doubly-fed induction wind generator

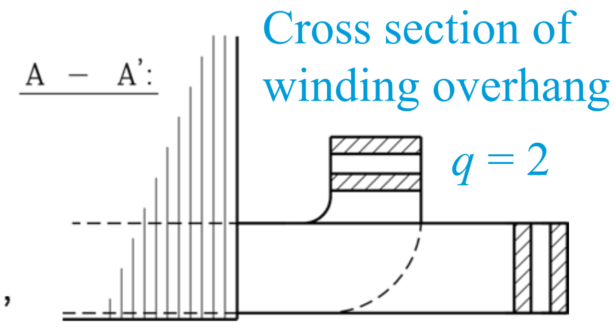


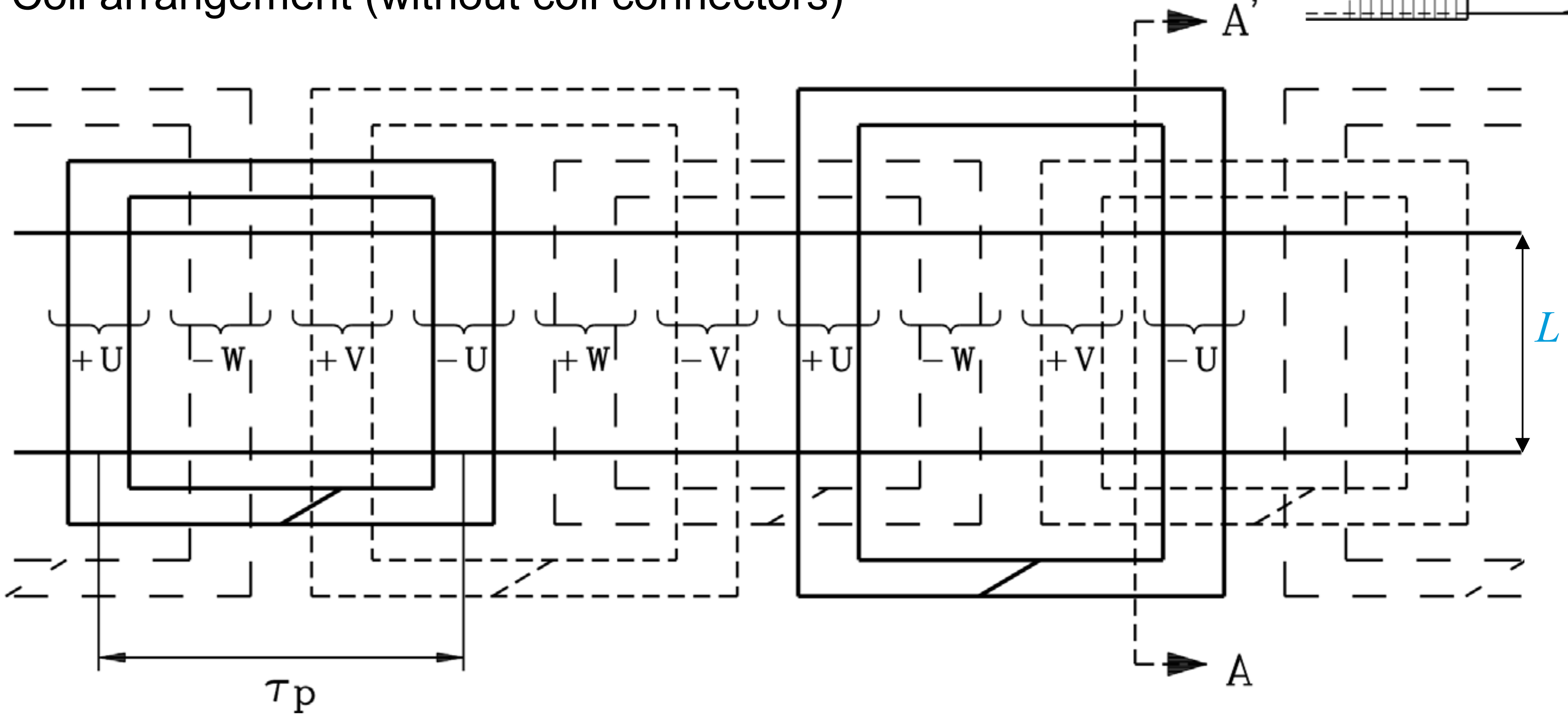
Source: Winergy Germany

2. Design of Induction Machines
Single layer winding

Two different sizes of concentric coil groups possible for $2p = 4, 8, \ldots$

Example: $q = 2$ coils per pole and phase,
Coil arrangement (without coil connectors)

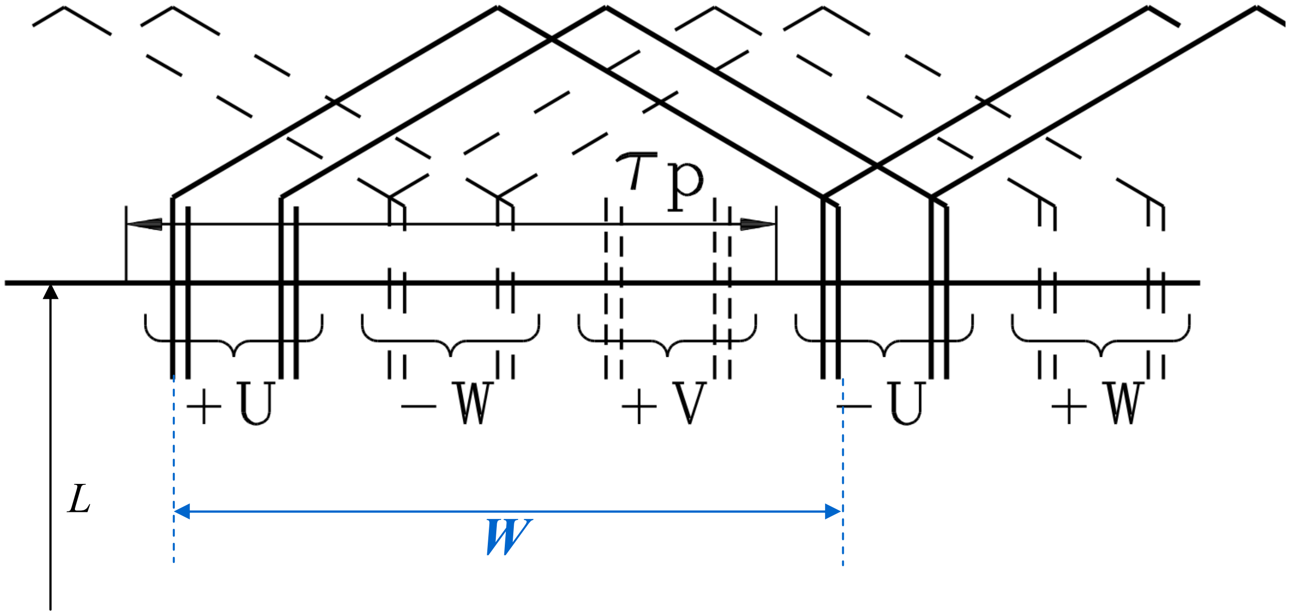


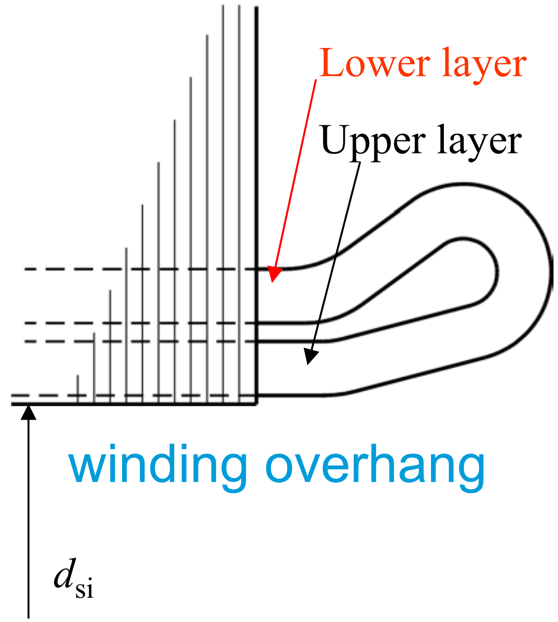


2. Design of Induction Machines
Two layer winding

Identical sizes for all coils

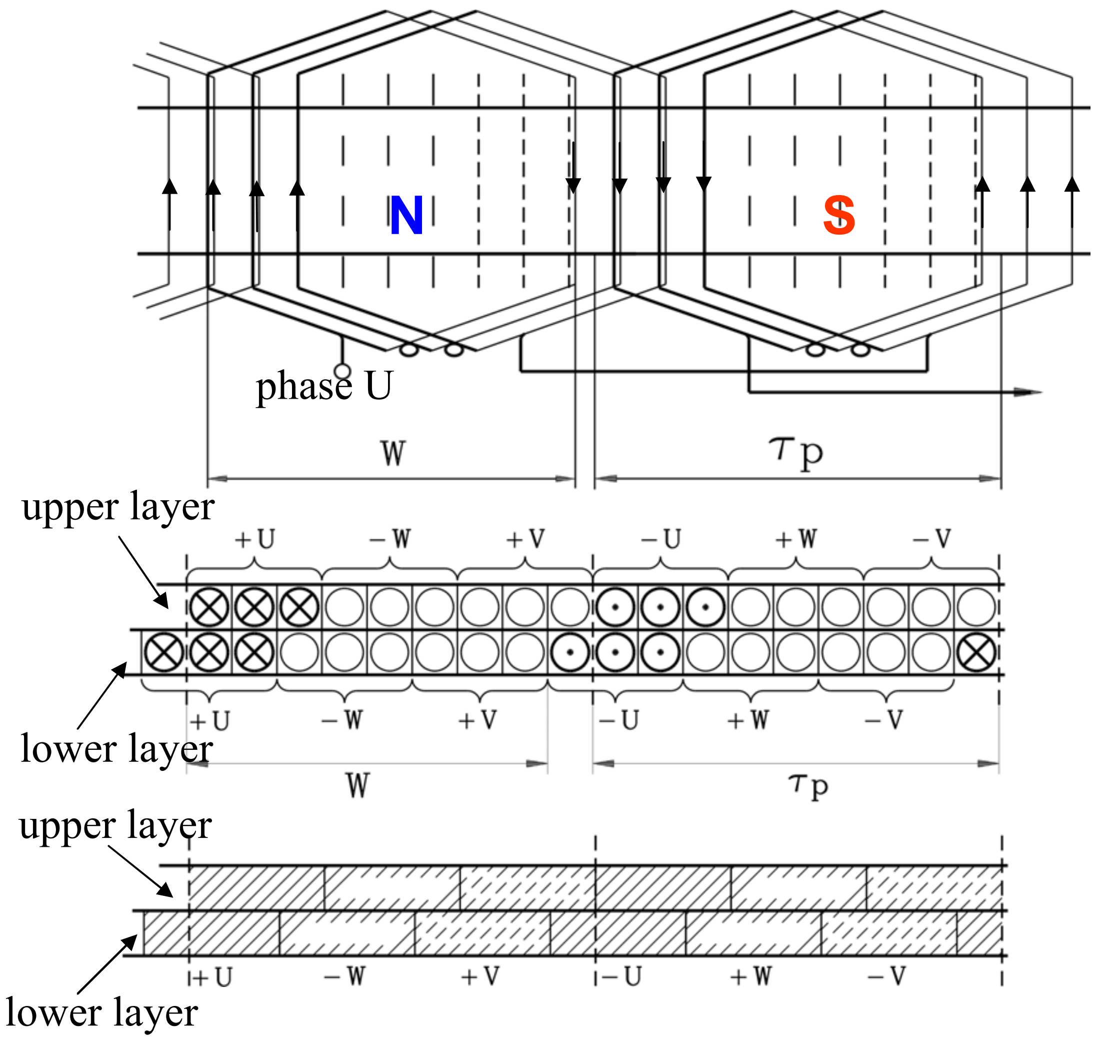
Example: q = 2 coils per pole and phase
Coil arrangement with full pitched coils: $W = \tau_p$





2. Design of Induction Machines

Pitched two layer lap-wound winding $W < \tau_p$



Arrangement of pitched coils ($W < \tau_p$) per phase and connection of coil groups for N- and S-pole

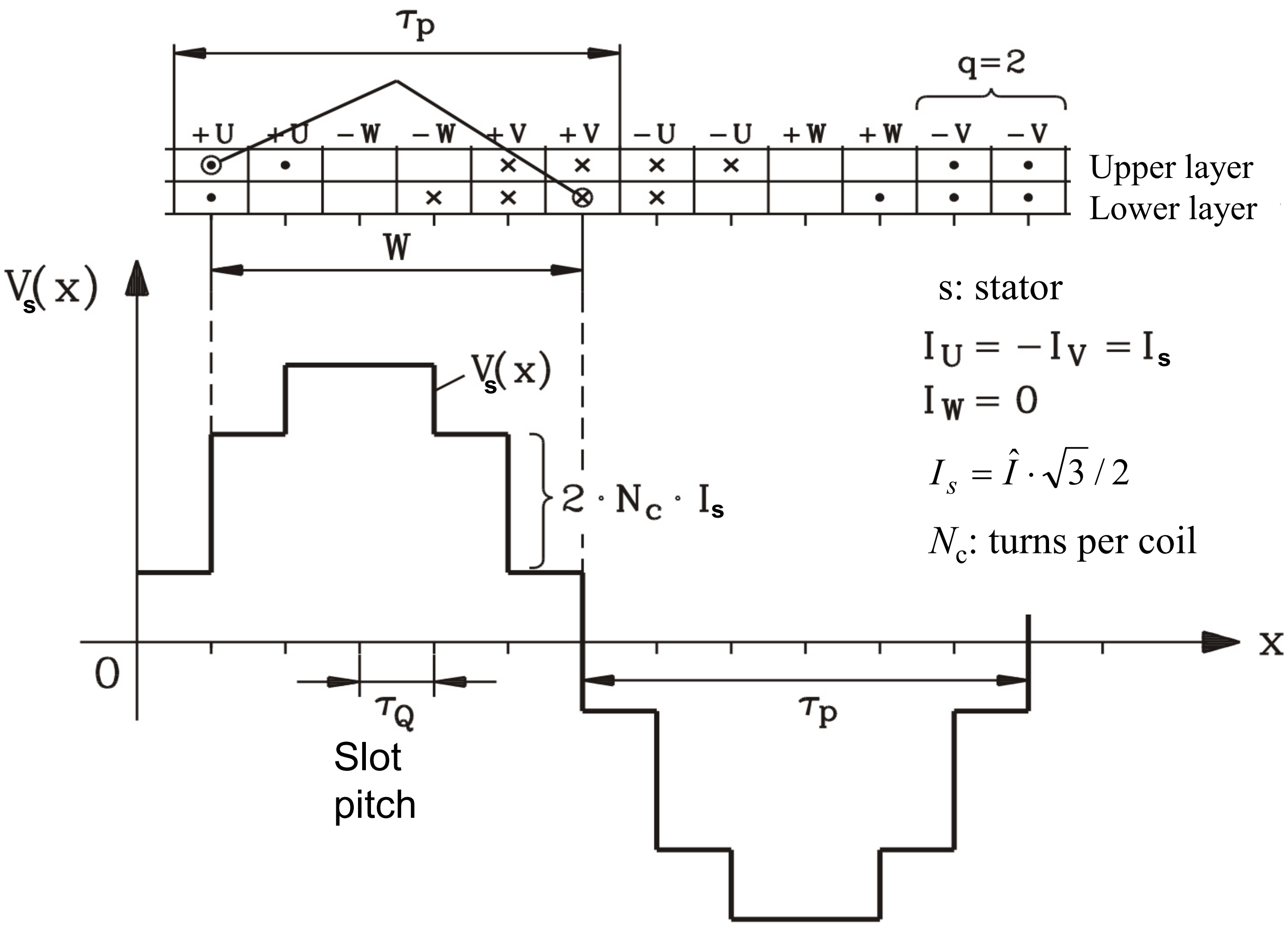
Example:
q = 3 coils per pole and phase,
pitch $W/\tau_p = 8/9$

Cross section of coil arrangement in slots, showing all three phases U, V, W in upper and lower layer, being symbolized by phase belts

Phase belt without depicting single slots

2. Design of Induction Machines

Momentary plot of MMF distribution

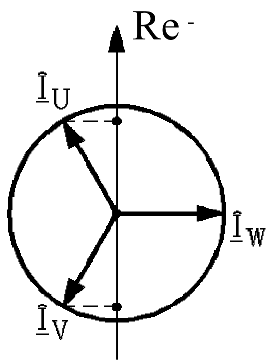


MMF distribution $V_s(x)$ (magneto-motive force) distribution of a two-layer winding

Example:

q = 2 slots per pole and phase

Coil pitch 5/6



Winding factor $k_{ws\nu}$

$$k_{ws\nu} = k_{ds\nu} \cdot k_{ps\nu}$$

m: phase count (e.g. 3)
ν: ordinal number

$$k_{ds\nu} = \frac{\sin\left(\frac{\nu \cdot \pi}{2 \cdot m}\right)}{q \cdot \sin\left(\frac{\nu \cdot \pi}{2 \cdot q \cdot m}\right)}$$ **Distribution factor**

$$k_{ps\nu} = \sin\left(\nu \frac{\pi}{2} \cdot \frac{W}{\tau_p}\right)$$ **Pitching factor**

- Reduction of harmonic induced voltage e.g. for q = 3, m = 3 (distribution factor):
  for $\nu = -5: k_{ds,-5} = 0.218$, $\nu = 7: k_{ds,7} = -0.177$.
  Also fundamental flux linkage is reduced a little bit: $\nu = 1: k_{ds,1} = 0.960$.

- The flux linkage with stator winding of harmonic flux density waves
  with ν pole pairs is reduced for W/τ_p = 8/9 e.g. (pitching factor):
  For $\nu = -5: k_{ps,-5} = 0.64$, $\nu = 7: k_{ps,7} = -0.34$.

Note that also fundamental flux linkage is reduced a little bit: $\nu = 1: k_{ps,1} = 0.984$

2. Design of Induction Machines

High voltage form wound stator coil with several turns N_c for two-layer winding

Winding overhang

coil side, inserted in slot.
Black:
Semi-conducting anti-corona screen

Red: Anti-humidity varnish

coil terminals

Source:

Andritz Hydro, Austria

2. Design of Induction Machines

Inserting a two-layer winding into stator slots

Radial ventilation duct

Tooth

Slot

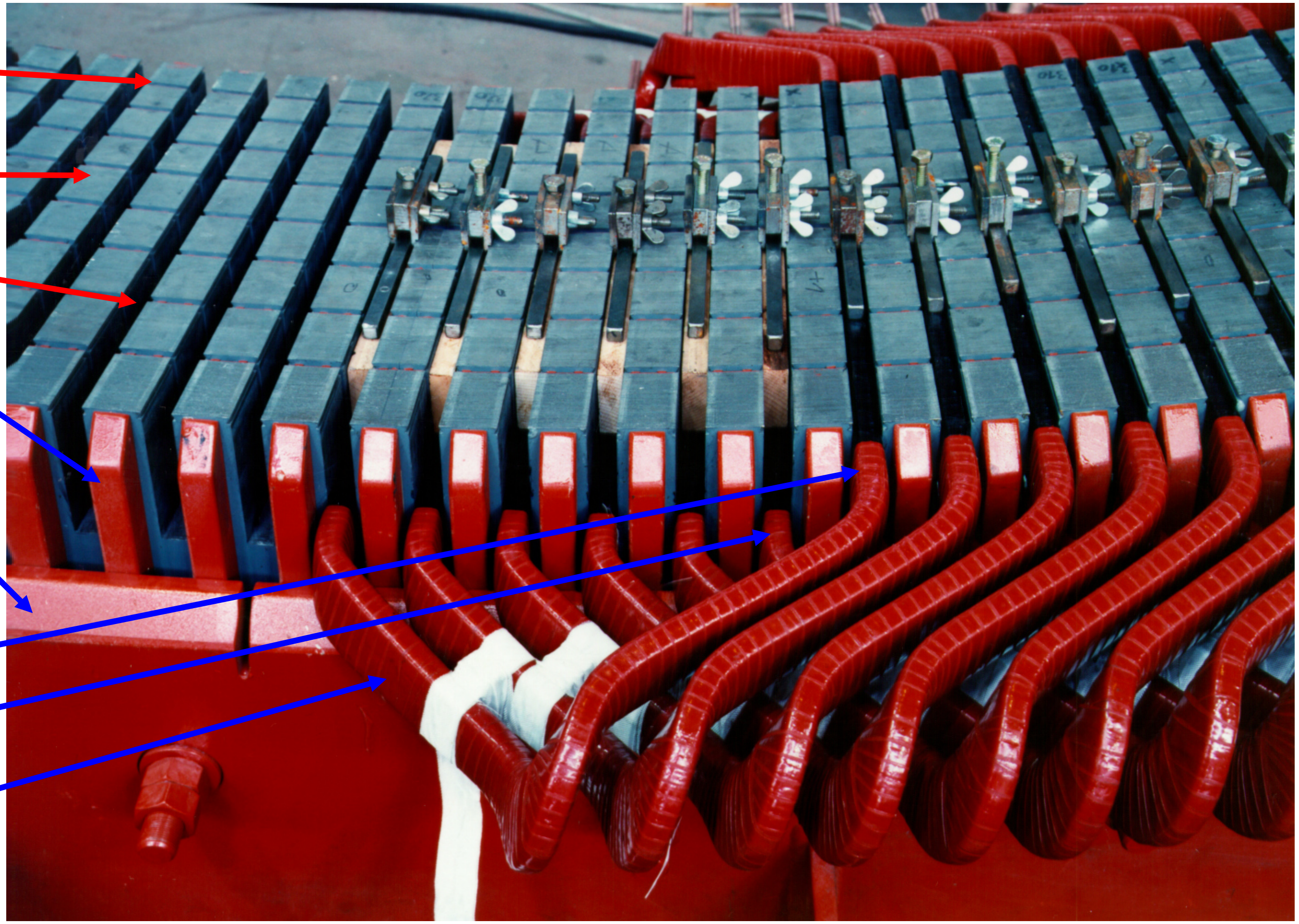
Massive iron clamping finger

Press plate

upper layer

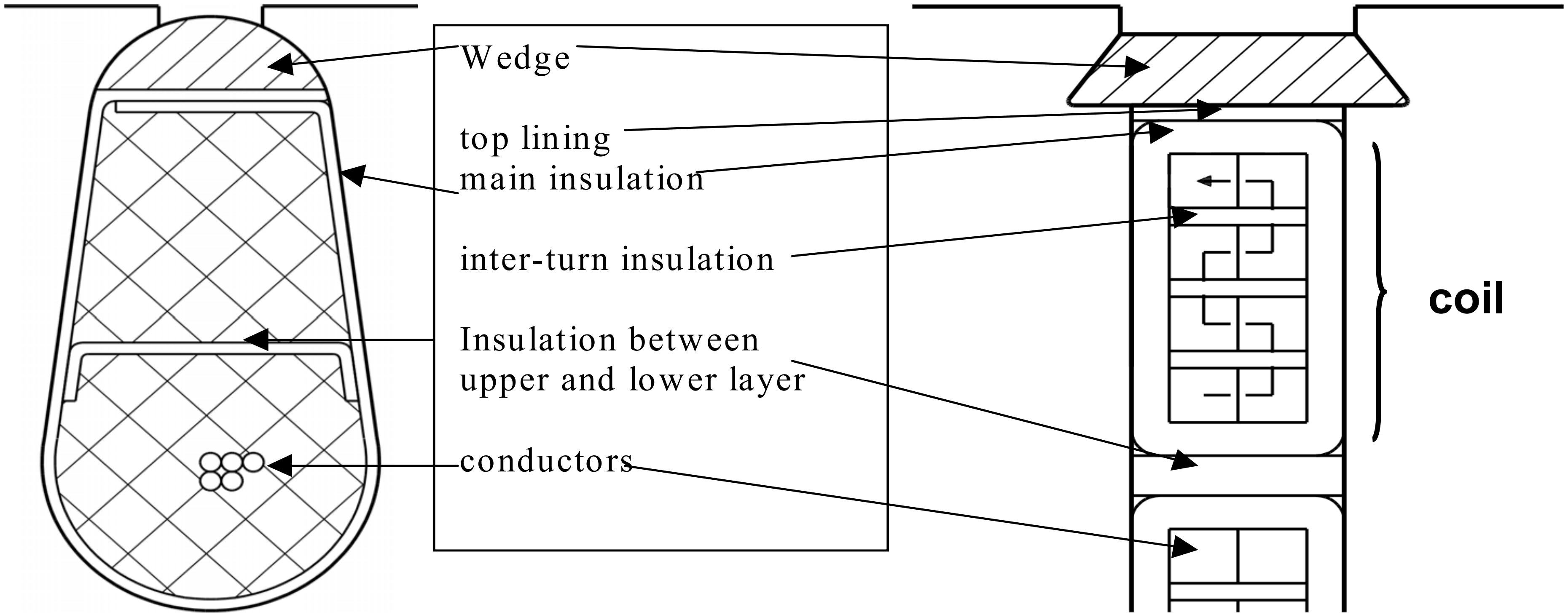
lower layer

winding overhang



Source: Andritz Hydro, Austria

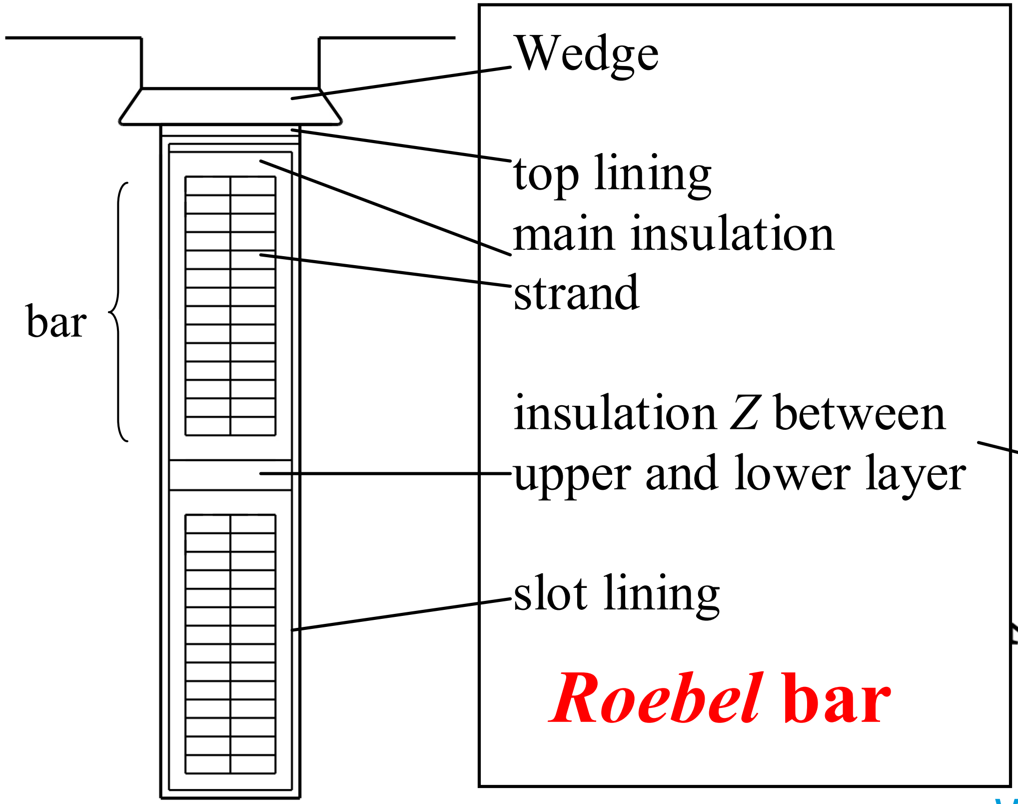
Two-layer winding: Slot and coil arrangement, Winding insulation



Oval semi-closed slot for round wire low voltage

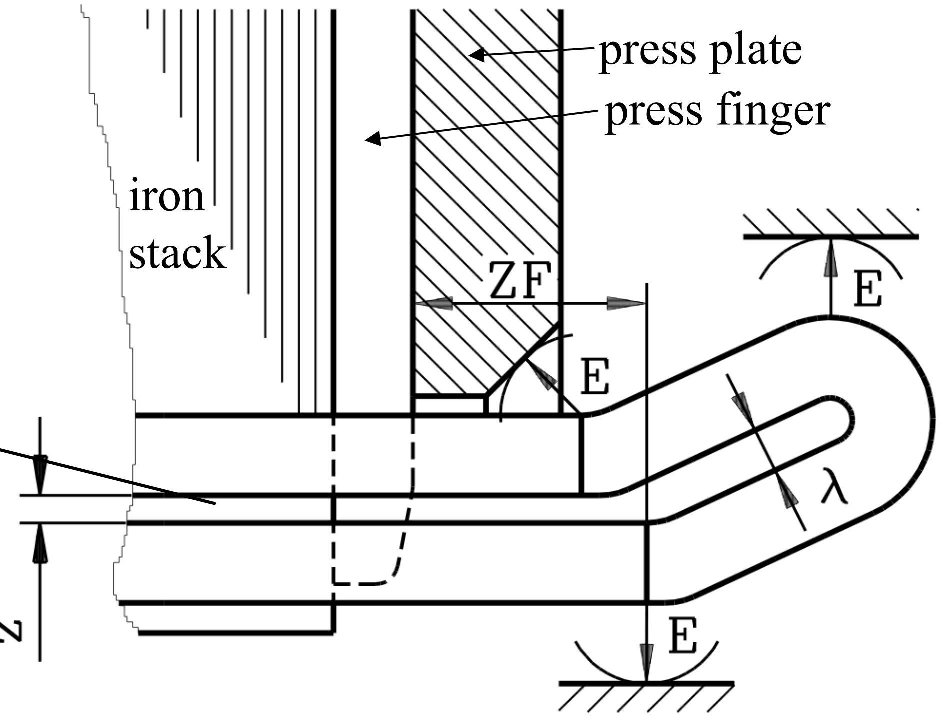
Rectangular slot for form wound high voltage coil arrangement with $N_c = 8$ turns per coil

Stator winding details



Cross section in slot
with 24 strands per bar

$N_c = 1$



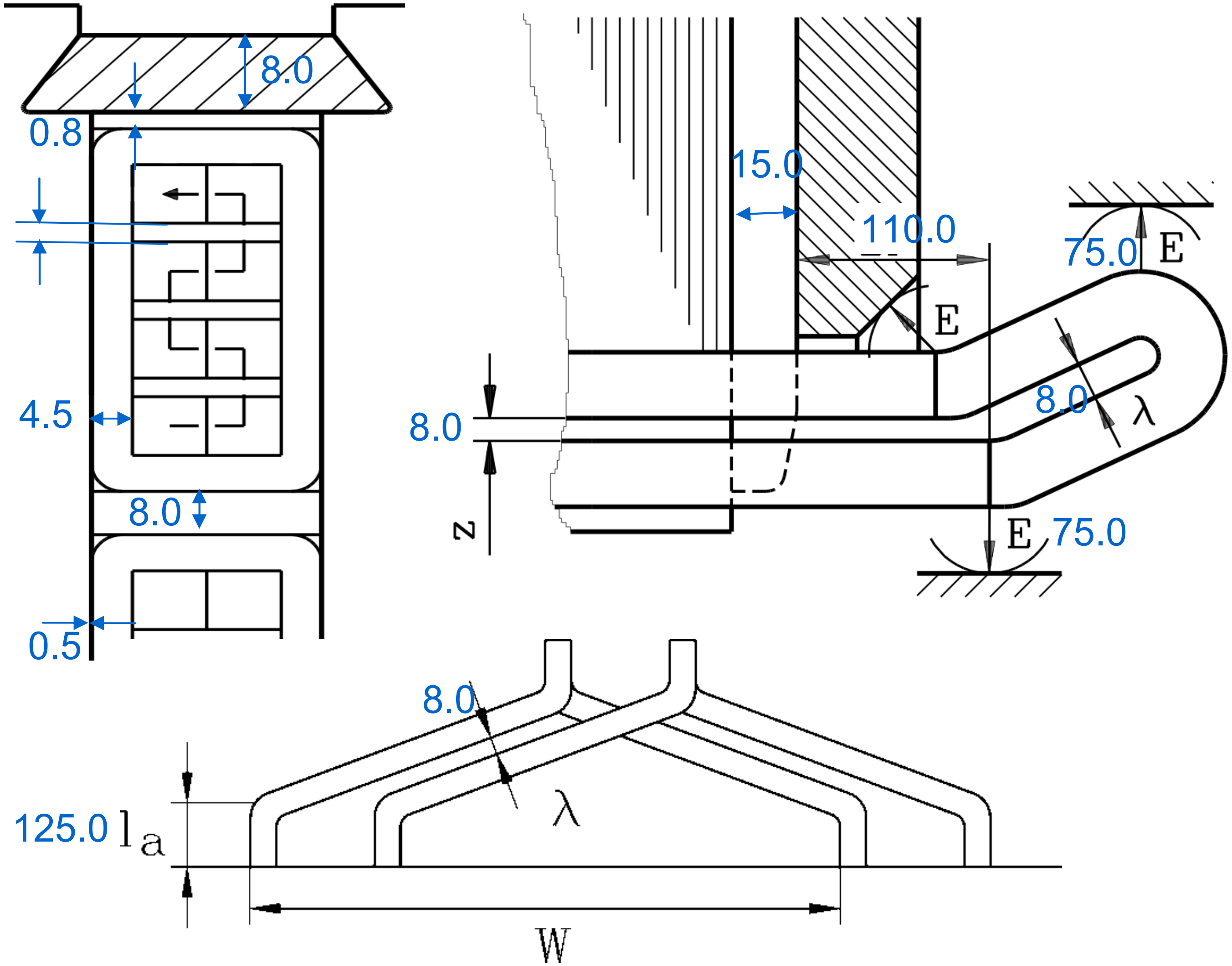
- Winding overhang with clearing *E* to end shields (on earth potential).
- Clearing *ZF* of end of main insulation from pressing construction of iron stack end

2. Design of Induction Machines

High voltage insulation for stator winding 3.3 k V ... 16.5 kV

Rated voltage (line-to-line, r.m.s.)	kV	3.3	6.6	11.0	13.8	16.5
Slot main insulation, thickness d	mm	1.5	2.2	3.2	3.8	4.5
Inter-layer insulation, thickness Z	mm	3	4	6	7	8
Top lining thickness	mm	0.3	0.4	0.6	0.7	0.8
Winding overhang main insulation, thickness d_s	mm	2	2.5	3.5	4	4
Clearing of slot main insulation ZF	mm	25	45	80	100	110
Clearing between coils in winding overhang λ	mm	3	4	6	7	8
Clearing from winding overhang to earth potential E	mm	25	35	55	65	75
Total conductor insulation thickness (both sides), glass fibre, inter-turn voltage < 80 V	mm	0.3	0.4	0.4	0.4	0.5
Slot lining thickness in vertical direction and slot play	mm	1	2	2	2	2

2. Design of Induction Machines
High voltage insulation for stator winding



Example:
High voltage insulation for stator winding at U_N = 16.5 kV
(Dimensions in mm)

Drawings NOT to scale!

Summary:
Stator winding low and high voltage technology

- Low voltage winding up to $U_N \leq 1000$ V, AC, r.m.s.:
  usually single-layer winding
- Round wire winding and enamel insulation for low voltage
- High voltage: $U_N > 1000$ V, AC, r.m.s.
- Special high voltage insulation system, open slots, rectangular conductors
- High voltage: Usually double-layer winding
- Large machines: Prefabricated, fully insulated coils
- Smaller machines: Complete resin impregnation of stator with inserted coils

2. Design of induction machines

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2.10 Masses and losses

2. Design of Induction Machines

Low voltage winding with round copper wire

- **Low voltage** winding is usually manufactured of round wire:
  Slot fill factor k_f for slot design is used.

$$k_f = \frac{A_{Cu}}{A_Q}$$

Copper cross section

Slot cross section

	single layer winding	double layer winding
• Slot fill factor k_f	≤ 0.45	≤ 0.42

- **Round wire diameter d_{Cu} < 1.1 mm for easy bending!**

Example:

$A_Q = 194$ mm², single layer winding with round copper, $N_c = 13$, $a_s = 1$.
Each turn per coil consists of $a_i = 8$ parallel wires with diameter
$d_{Cu} = 1.0$ mm: $A_L = a_i \cdot (d_{Cu}^2 \cdot \pi / 4) = 8 \cdot (1^2 \cdot \pi / 4) = 6.28\text{mm}^2$

$\Rightarrow$ Slot fill factor: $A_{Cu} = N_c \cdot A_L = 13 \cdot 6.28 = 81.7\text{mm}^2, k_f = 81.7 / 194 = \underline{\underline{0.421}}$

Cleaning of the two-layer stator winding of surplus resin after complete stator resin impregnation

Source: Winergy Germany

Stator two-layer winding design

- Chosen number of slots: $q_s = 5$, $Q_s = 2p \cdot m_s \cdot q_s = 4 \cdot 3 \cdot 5 = 60$, leads to 15 slots per pole, $d_{si} = 458$ mm
- Slot pitch: $\tau_{Qs} = d_{si}\pi / Q_s = 24.0$ mm
- Coil pitching is possible in steps of one slot pitch: W/τ_p = 14/15, 13/15 etc., chosen pitch W/τ_p = 12/15 leads to $k_{ps,1} = 0.951$.
  The influence of 5th space harmonic is completely eliminated: $k_{ps,5} = 0$.
- Distribution factor: $k_{ds,1} = 0.957$, stator winding factor: $k_{ws,1} = 0.957 \cdot 0.951 = 0.910$
- With chosen air gap flux density 0.9 T main flux per pole of fundamental $\nu = 1$ is

$$\Phi_h = \frac{2}{\pi} \cdot \tau_p \cdot l_{Fe} \cdot \hat{B}_{\delta,1} = \frac{2}{\pi} \cdot 0.36 \cdot 0.38 \cdot 0.9 = 78.4 mWb$$

- Choice of number of turns per phase N_s:

Estimated induced voltage per phase: $U_h = \frac{U_N / \sqrt{3}}{1+\sigma_s} = \frac{6000/\sqrt{3}}{1.04} = 3330$ V

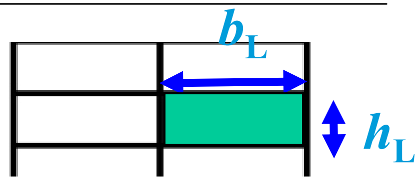
$$U_h = \sqrt{2}\pi f_s \cdot N_s k_{ws1} \cdot \Phi_h \quad \Rightarrow \quad N_s = 210.17$$

$$N_c = N_s \cdot a_s / (2pq_s) = 210.17 \cdot 1/(2 \cdot 2 \cdot 5) = 10.5 \qquad a_s = 1$$

- **Final values:** $N_c = 10 \quad N_s = 200 \quad U_h = 3330 V \quad \hat{B}_{\delta,1} = 0.946 T$

2. Design of Induction Machines

Selection of available profile copper wire



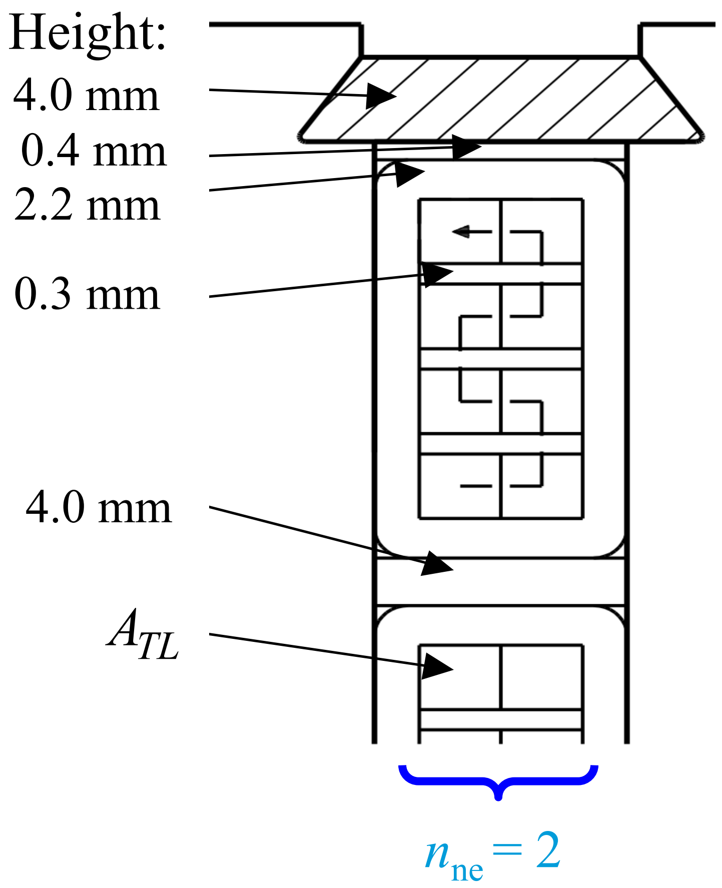
Breadth b_L (mm)	(mm) Conductor height h_L								
	1.8	*2*	*2.24*	*2.5*	*2.8*	*3.15*	*3.55*	*4*	*4.5*
5	**8.637**	**9.637**	**10.84**	**11.95**	**13.45**	**15.20**	**17.22**	**-**	**-**
5.6	**9.717**	**10.84**	**12.18**	**13.45**	**15.13**	**17.09**	**19.33**	**21.54**	**-**
6.3	**10.98**	**12.24**	**13.75**	**15.20**	**17.09**	**19.30**	**21.82**	**24.34**	**27.49**
7.1	**12.42**	**13.84**	**15.54**	**17.20**	**19.33**	**21.82**	**24.66**	**27.54**	**31.09**

- Dimensions (without enamel coating) and cross section
- Edges of wire rounded by 0.5 mm .... 1.0 mm radius
  Note: 12.42 mm² < 1.8 x 7.1 = 12.78 mm² due to rounded edges
- $h_L < b_L$ to minimize eddy currents, induced by the AC slot stray flux

2. Design of Induction Machines
Profile copper winding

Example: 6.6 kV, $a_i = 1$, $N_c = 8$

- Number of adjacent turns: n_{ne}
- Number of strands per turn: a_i
- Cross section area of strand: A_{TL}



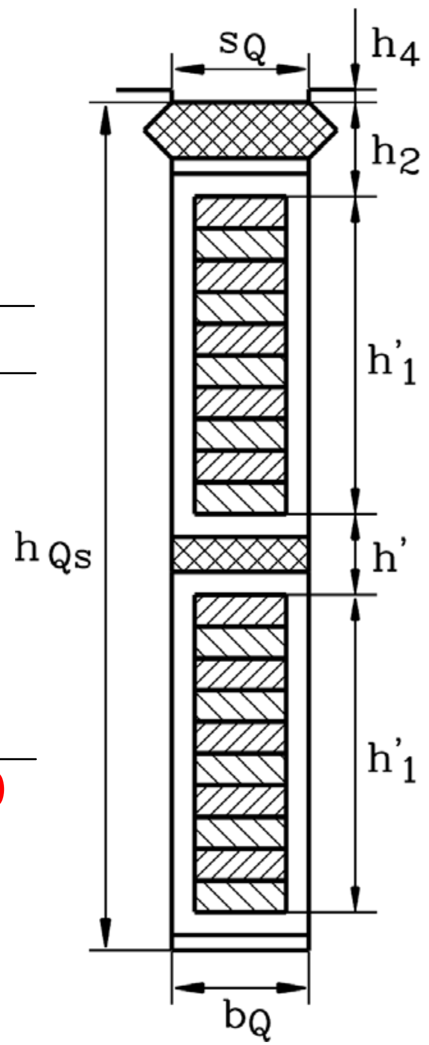
- Current per phase 59 A, voltage per phase 3460 V, U_N = 6.0 kV
- Current density limit 5.5 A/mm², N_s = 200, N_c = 10, $\tau_{Qs} = 24.0$ mm
- Parallel paths: a_s = 1, a_i = 1.
- Conductor cross section: $A_{TL} = I_s/(J_s \cdot a_s \cdot a_i) = 59/(5.5 \cdot 1 \cdot 1) = 10.73$ mm²
- Chosen slot breadth: $b_{Qs} = 12.5$ mm (= $0.52 \cdot \tau_{Qs}$)
- Chosen conductor dimensions (Table 2.4-2): $b_L = 7.1$ mm, $h_L = 1.8$ mm
- Inter-turn voltage: $U_s / N_s = 3460/200 = 17.3$ V < 80 V:
- Table 2.3-3: Conductor insulation thickness d_{ic} = 0.4 mm (both sides)
- Additional inter-turn insulation d_i = 0.3 mm

$$N_c \cdot (h_L + d_{ic}) + (N_c - 1) \cdot d_i = 10 \cdot (1.8 + 0.4) + 9 \cdot 0.3 = 24.7 \text{ mm}$$

$$n_{ne} \cdot (b_L + d_{ic}) = 1 \cdot (7.1 + 0.4) = 7.5 \text{ mm}$$

Slot design - height

Slot height design:		mm	
Number of insulated turns per coil one above the other = 10		24.7	
Main insulation		4.4	
Insulated coil side	24.7 + 4.4 = 29.1		29.1
Two coils per slot		58.2	
Inter-layer insulation	$Z = 4.0$	4.0	
Slot lining (thickness 0.15 mm)		0.45	
Wedge $h_{wedge} = 4.0$, $h_4 = 0.5$ mm		4.5	
Top and bottom lining		0.8	
Vertical play		1.05	
Slot height $h_{Qs} + h_4$			**69.0**



$h_4 = 0.5$ mm

$s_Q = 12.5$ mm

$h_2 = 4 + 0.4 + 2.2 = 6.6$ mm

$h'_1 = 24.7$ mm

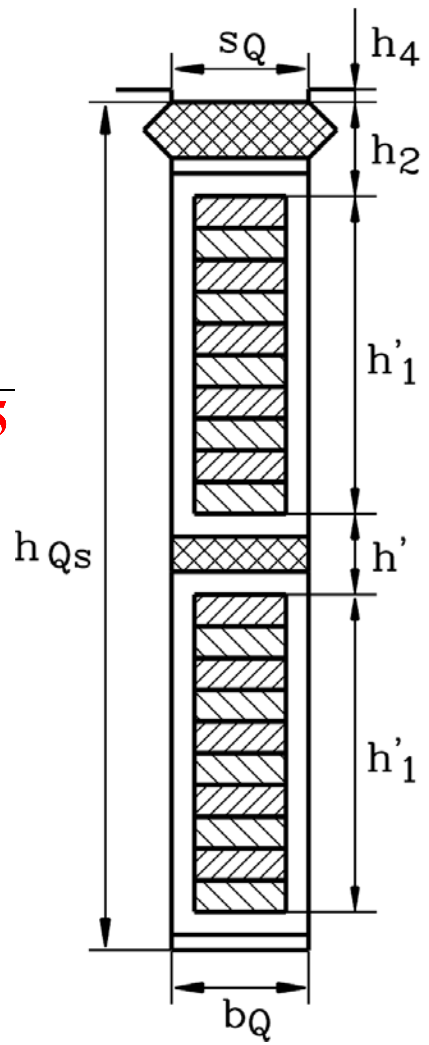
$h' = 2.2 + 4 + 2.2 = 8.4$ mm

$h'_1 = 24.7$ mm

$b_Q = 12.5$ mm

Slot design - width

Slot width design:	mm	
Number of adjacent insulated turns $n_{ne} = 1$	7.5	
Main insulation	4.4	
Slot lining (thickness 0.15 mm)	0.3	
Play	0.3	
Slot width b_Q		**12.5**



$h_4 = 0.5$ mm

$s_Q = 12.5$ mm

$h_2 = 4+0.4+2.2=$
$= 6.6$ mm

$h'_1 = 24.7$ mm

$h' = 2.2+4+2.2=$
$= 8.4$ mm

$h'_1 = 24.7$ mm

$b_Q = 12.5$ mm

Slot fill factor

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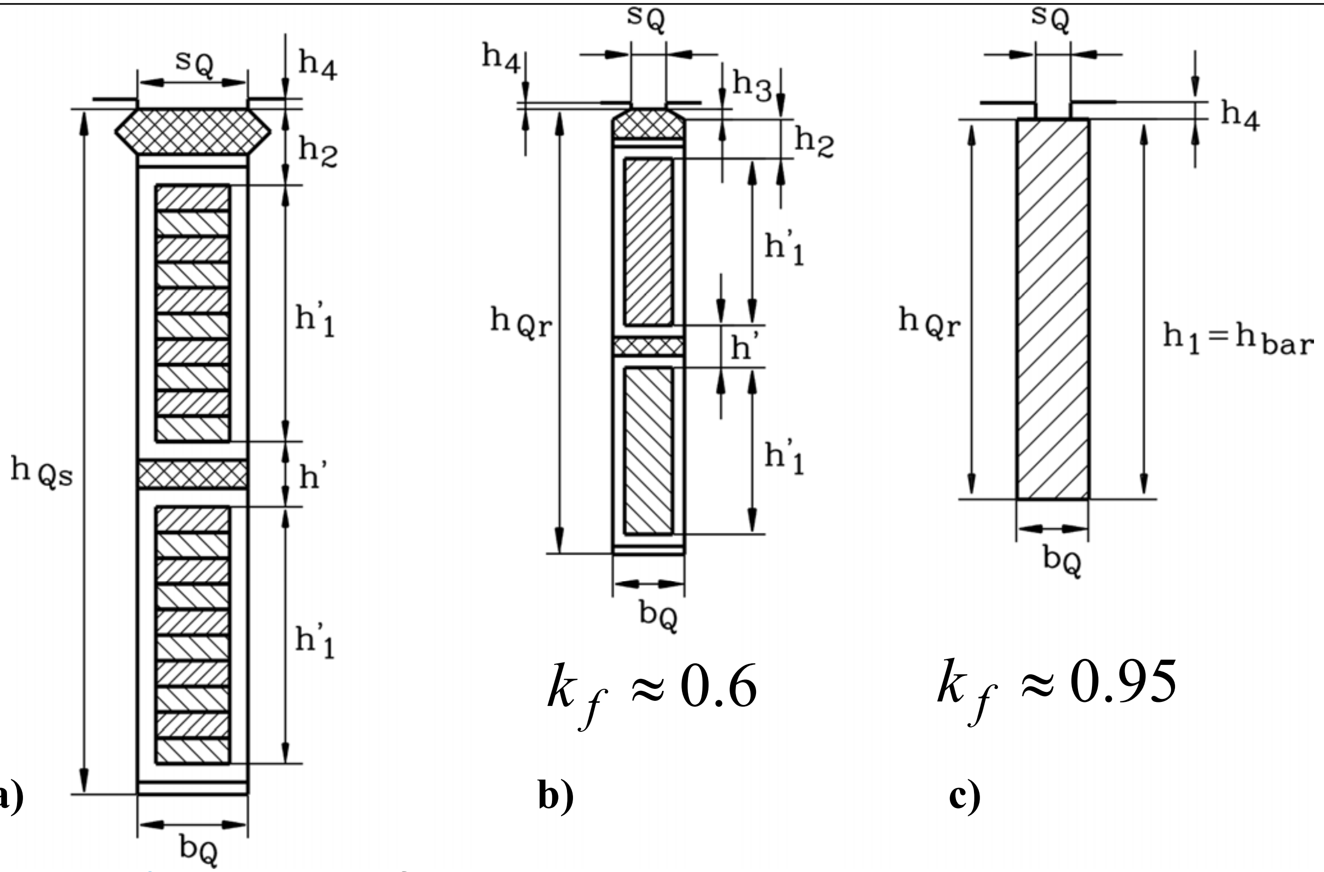
Slot fill factor:

$$k_f = \frac{A_{Cu}}{A_Q}$$

$$k_f = \frac{2 \cdot 10 \cdot 12.42}{69 \cdot 12.5} = 0.3$$

Low slot fill factor due to high voltage winding!



$k_f \approx 0.6$ $\quad$ $k_f \approx 0.95$

a) b) c)

a) Two coils (two-layer winding) in one slot for 6 kV, 500 kW induction machine

b) Wound rotor induction machine: Rotor slot design for low voltage winding < 1kV

c) Cage rotor: Deep rotor bar in slot: Very low voltage, no insulation!

Check of winding design

- Limit of height of conductor h_L (or strand) to avoid eddy current losses (current displacement): "reduced conductor height" $\xi < 0.3 \ldots 0.35$

- Conductor cross section concerning current density J_s

- Coil main insulation thickness with respect to rated voltage U_N

- Conductor insulation with respect to inter-turn voltage $U_{turn} \approx U_N / N_s < 80$ V

- Sufficient tooth width to avoid increased iron saturation: < 2.2 T

- Resulting thermal utilization $A_s \cdot J_s$

Checking of eddy currents at 20°C

Checking of eddy currents at 20°C: $\xi = h_L / d_E$

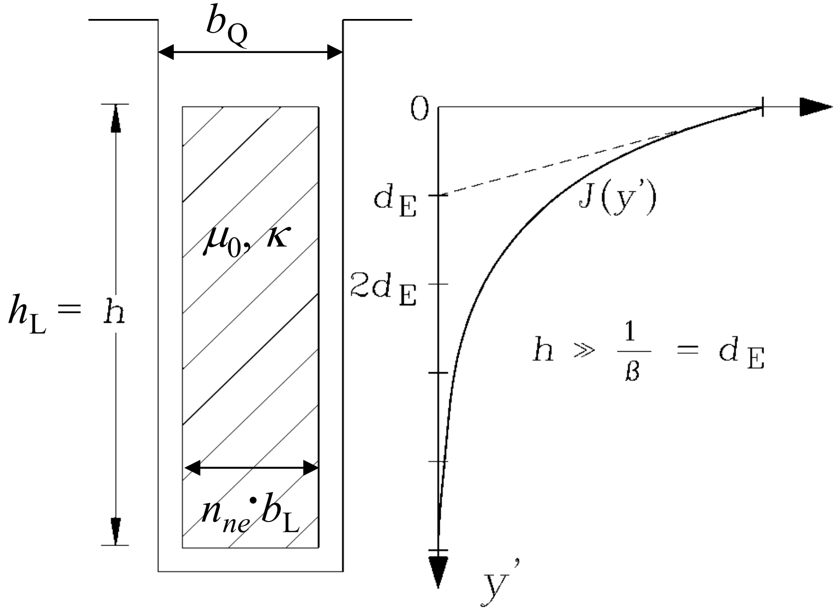
ξ = conductor height / penetration depth of eddy currents

$$d_E = \sqrt{\frac{2 \cdot b_Q}{\mu_0 \cdot \omega_s \cdot \kappa \cdot n_{ne} \cdot b_L}}$$

$$d_E = \sqrt{\frac{2 \cdot 12.5}{4\pi \cdot 10^{-7} \cdot 2\pi \cdot 50 \cdot 57 \cdot 10^6 \cdot 1 \cdot 7.1}} = 12.5\text{mm}$$

$$\xi = 0.0018 / 0.0125 = \underline{\underline{0.144}} < 0.35$$

Example: $n_{ne} = 1$



Check of current density and figures of merit

- Checking of current density and thermal utilization:

$$J_s = I_s/(a \cdot a_i \cdot A_{TL}) = 59/(1 \cdot 1 \cdot 12.42) = \underline{\underline{4.75}}\ \mathrm{A/mm^2} < 5.5\ \mathrm{A/mm^2}$$

$$A_s = \frac{2m_s N_s I_s}{2p\tau_p} = \frac{2 \cdot 3 \cdot 200 \cdot 59}{4 \cdot 36} = 492\ \mathrm{A/cm}$$

$$A_s \cdot J_s = 492 \cdot 4.75 = \underline{\underline{2337}}\ \mathrm{(A/cm)(A/mm^2)}:$$

Result fits to limits for open ventilated machine with 80 K temperature rise.

- Electromagnetic utilization:

$$C = \frac{\pi^2}{\sqrt{2}} \cdot k_{ws} \cdot A_s \cdot \hat{B}_{\delta 1} = \frac{\pi^2}{\sqrt{2}} \cdot 0.91 \cdot 49200 \cdot 0.946 = 295585\ \mathrm{VAs/m^3} = \underline{4.93}\ \mathrm{kVA \cdot min/m^3}$$

Summary:
Stator winding design

- Increased insulation thickness *d* with increased rated high voltage U_N
- Choice of winding type and number of slots per pole and phase *q*
- Choice of current density *J* and current loading *A*
- Detailed slot design for HV winding
- Slot fill factor k_f for LV winding, depending on manufacturing abilities

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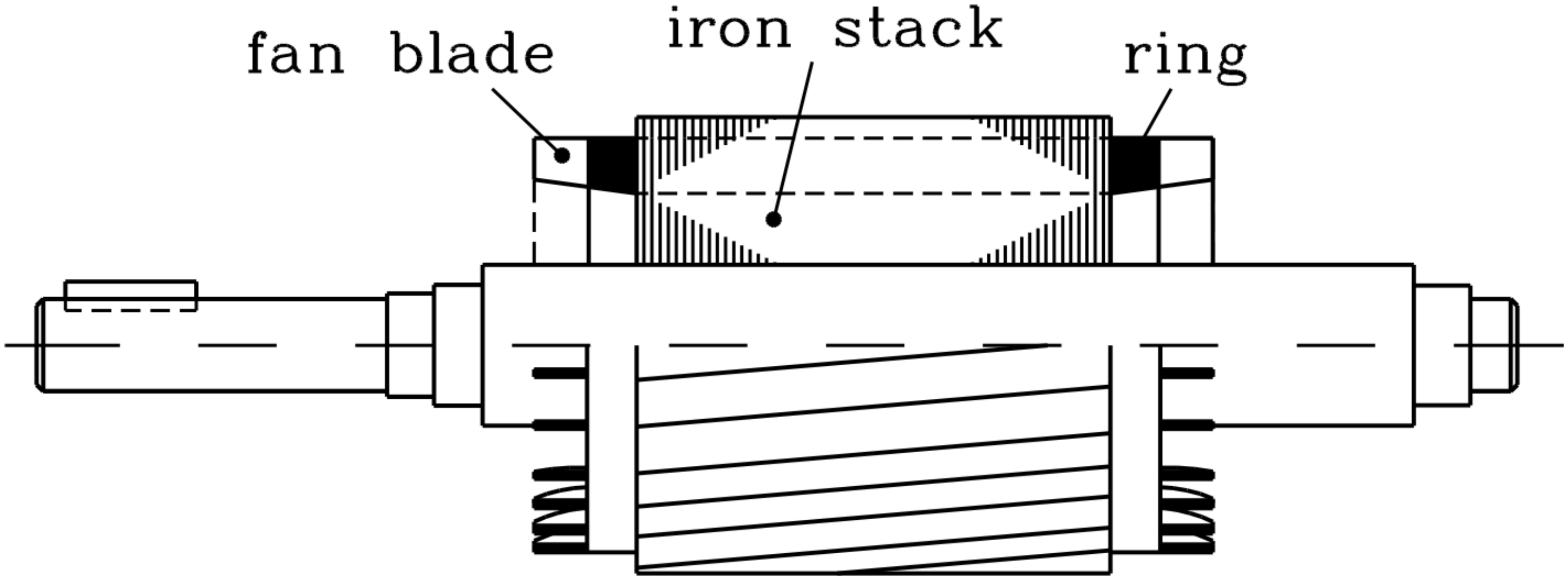
Die-cast aluminium cage rotor with skewed rotor bars and small fan blades at the rings

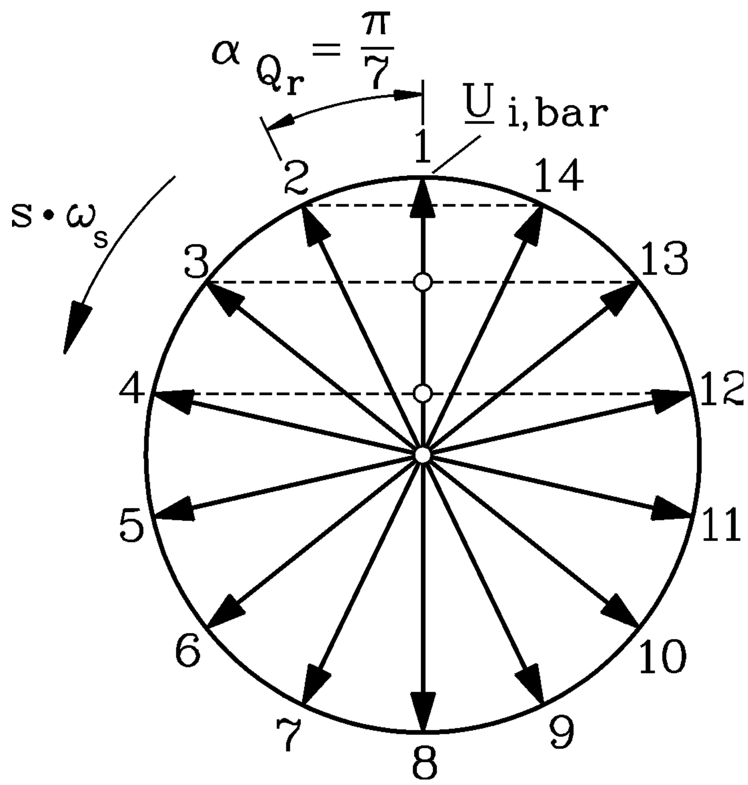


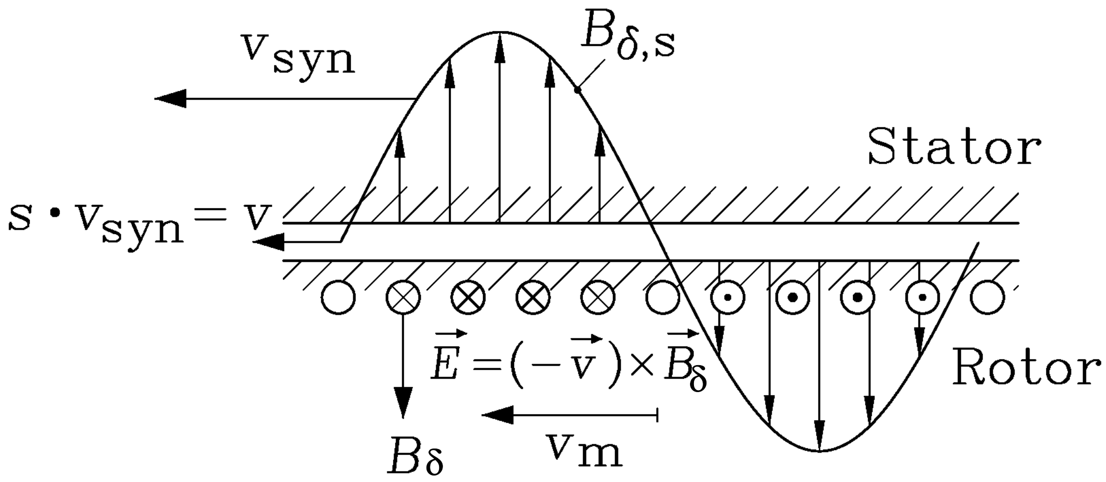
Source: H. Kleinrath, Studientext

2. Design of Induction Machines

Induced phase shifted bar voltages

Example: Q_r/p = 14 bars per pole pair   *Example:* Q_r/p = 10 bars per pole pair





$$U_i \sim |\vec{v} \times \vec{B}_\delta| \longrightarrow U_{i,bar} = s \cdot v_{syn} \cdot l \cdot \hat{B}_{\delta 1} / \sqrt{2}$$

Use of non-insulated bars due to low bar voltage!

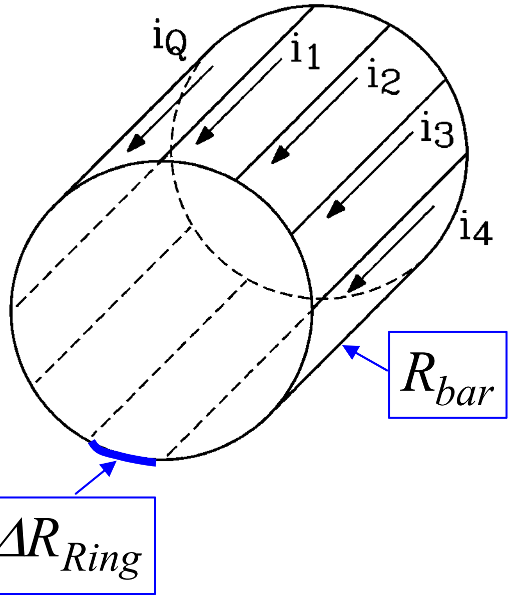
Example: $2p = 4,\ \tau_p = 36\ \text{cm},\ f_s = 50\ \text{Hz},\ l = 1\ \text{m},\ \hat{B}_{\delta 1} = 1\text{T} : v_{syn} = 2 f_s \tau_p = 36\ \text{m/s}$

$s = 1 : U_{i,bar} = s \cdot v_{syn} \cdot l \cdot \hat{B}_{\delta 1} / \sqrt{2} = 1 \cdot 36 \cdot 1 \cdot 1 \cdot 0.71 = 25.5\ \text{V};\ s_N = 1\% : U_{i,bar} = 0.26\ \text{V}$

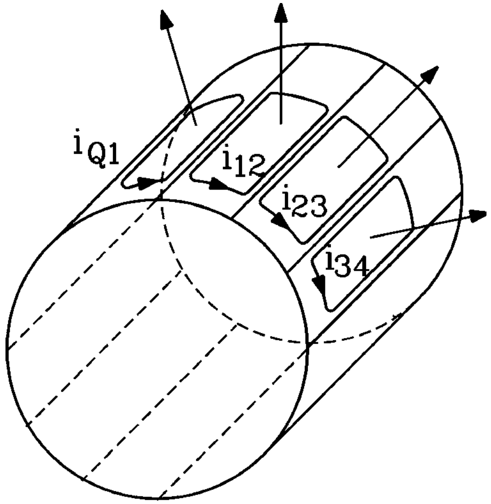
2. Design of Induction Machines

Squirrel cage: Bar and ring currents

a) Bar currents

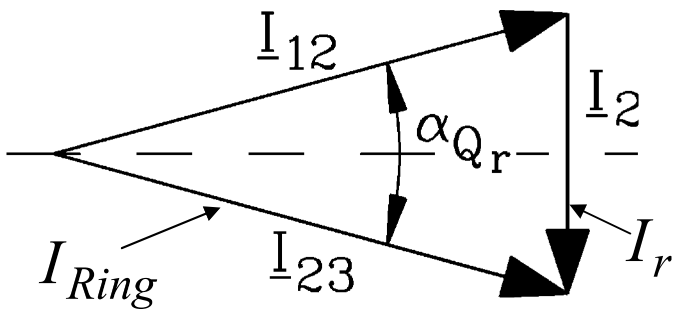


b) Ring currents



$$\alpha_{Qr} = 2\pi p / Q_r$$

$$i_{23} = i_{12} + i_2$$



$$I_{Ring} = I_r / (2 \cdot \sin(p\pi / Q_r))$$

$$P_{Cu,r} = Q_r \cdot R_{bar} \cdot I_r^2 + 2 \cdot Q_r \cdot \Delta R_{Ring} \cdot I_{Ring}^2 = Q_r \cdot (R_{bar} + \Delta R_{Ring}^*) \cdot I_r^2 = Q_r \cdot R_r \cdot I_r^2$$

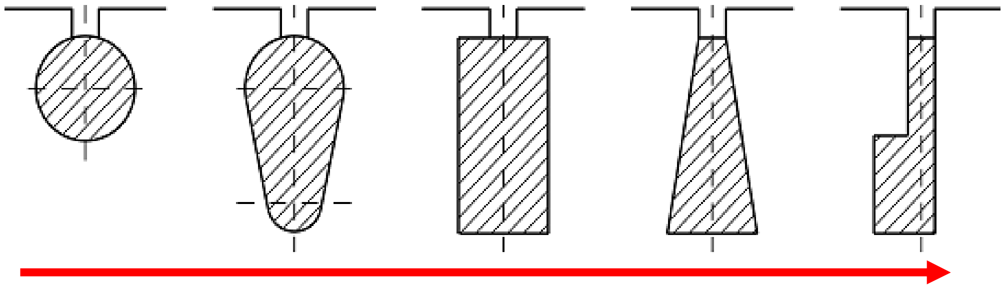
$$\Delta R_{Ring}^* = \Delta R_{Ring} \cdot \frac{1}{2 \cdot \sin^2(\pi \cdot p / Q_r)}$$

Equivalent bar **series resistance**

2. Design of Induction Machines

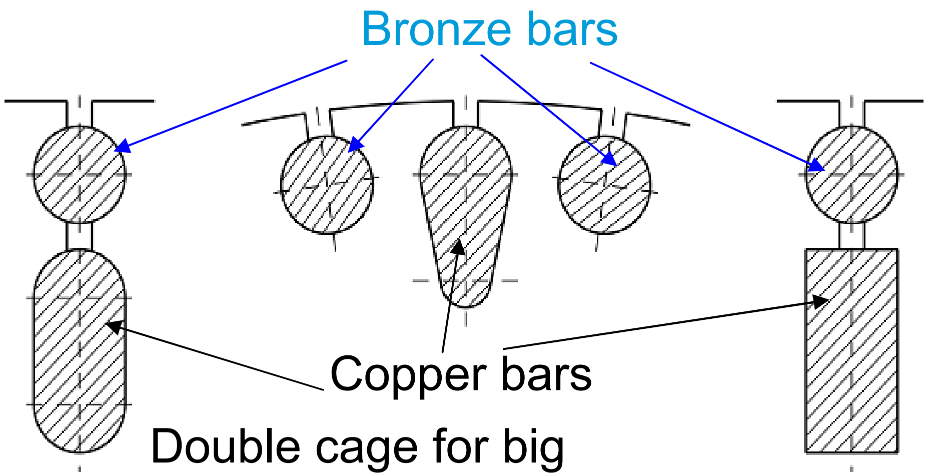
Shape of rotor bars in rotor slots

Copper bars:



Increasing current displacement effect!

Source: T. Bohn, Energietechnik, TÜV Rheinland



Double cage for big starting torque M_1

At big slip $|s| > |s_b|$:

"Current displacement":

Resulting rotor bar current i_r (inclusive eddy currents) flows mainly in upper half of bar !

So it is using only part of rotor bar cross section, which leads to increase of effective rotor bar resistance $R_{r,AC}$ (AC resistance).

$$R_{r,AC} > R_{r,DC}$$

$$P_{Cu,r} = Q_r \cdot R_r \cdot I_r^2 = s \cdot P_\delta \Big|_{s=1} = P_\delta = \frac{\omega_s}{p} \cdot M_1$$

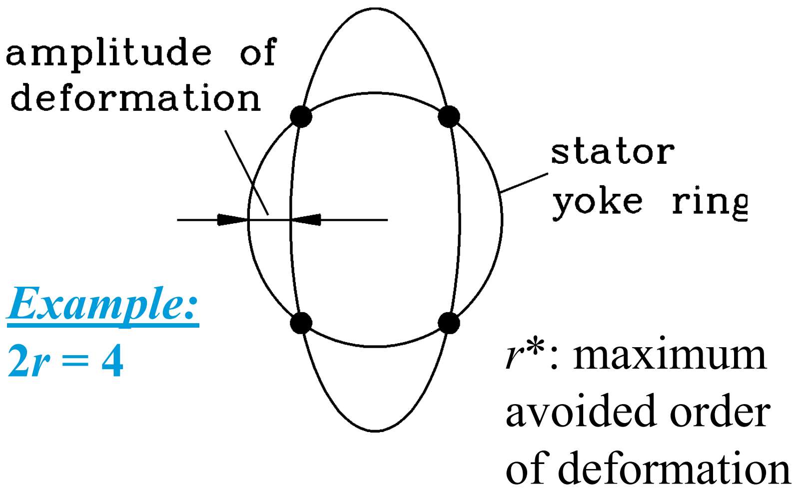
$$M_1 \sim R_r(s=1) = R_{r,AC}(s=1)$$

2. Design of Induction Machines
Choice of rotor slot numbers

1. No cogging $Q_s \neq Q_r$

2. For skewed cage: $0.8Q_s \leq Q_r < Q_s$

Minimum inter-bar currents



3. Minimization of acoustic noise:

3a) $|Q_s - Q_r| \neq 0, 1, 2, \ldots r^*, 2p, 2p \pm 1, 2p \pm 2, \ldots, 2p \pm r^*$

3b) Avoid (Q_s, Q_r) pairs with high number of common dividers!

4. No pulsating rotor bending force: Choose Q_r **even**

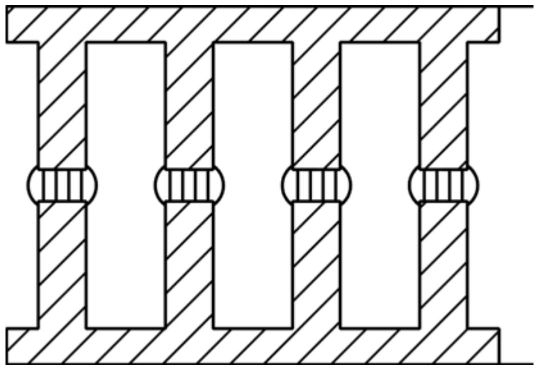
to avoid radial force waves with $2r = 2$ nodes !

5. Minimize flux pulsation in teeth = reduce additional losses:

Q_r/Q_s not below 0.8 and not above 1.2, otherwise slot frequent flux pulsation is too big

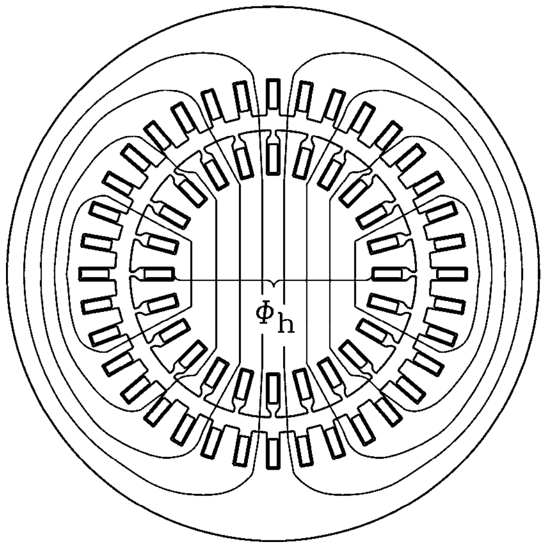
Cogging torque & bending vibrations

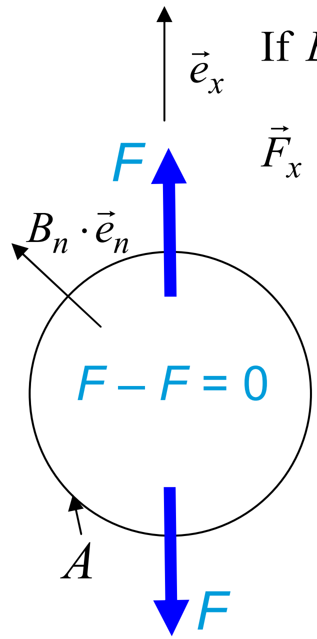
(from lecture: "Motor development for electrical drive systems")



$Q_s = Q_r$:

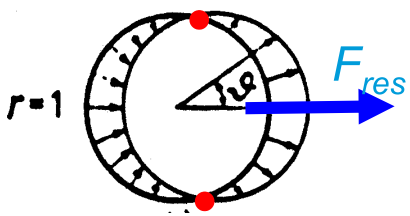
Big cogging force: FORBIDDEN!



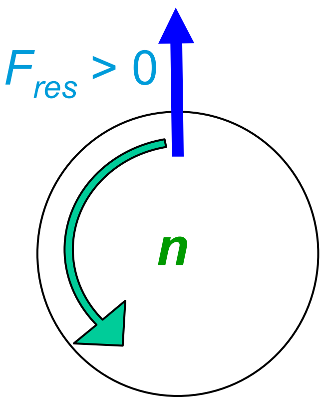


If $B_t << B_n$:

$$\vec{F}_x \approx \vec{e}_x \cdot \oint_A \frac{B_n^2}{2\mu_0} \cdot (\vec{e}_n \cdot \vec{e}_x) \cdot dA$$



$2r = 2$



Even rotor slot number:

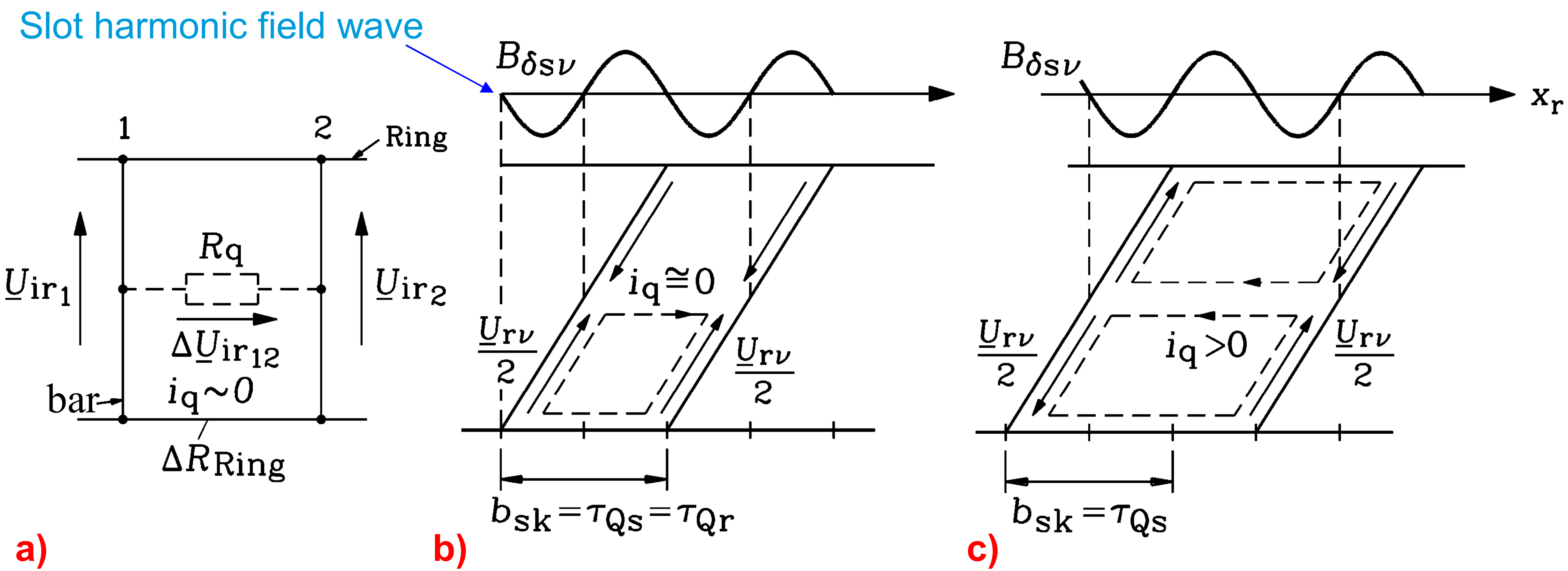
Yields with even stator slot number $Q_s = 2p \cdot m_s q_s$ a symmetrical upper and lower flux density with symmetrical upper and lower radial force.

Odd rotor slot number:

Single-sided magnetic rotating pull F_{res} ⇒ excites bending shaft vibrations with frequency $f = n$

Losses due to inter-bar currents

(from lecture: "Motor development for electrical drive systems")



a) Unskewed cage: No inter-bar current, because $\Delta R_{Ring} << R_q$

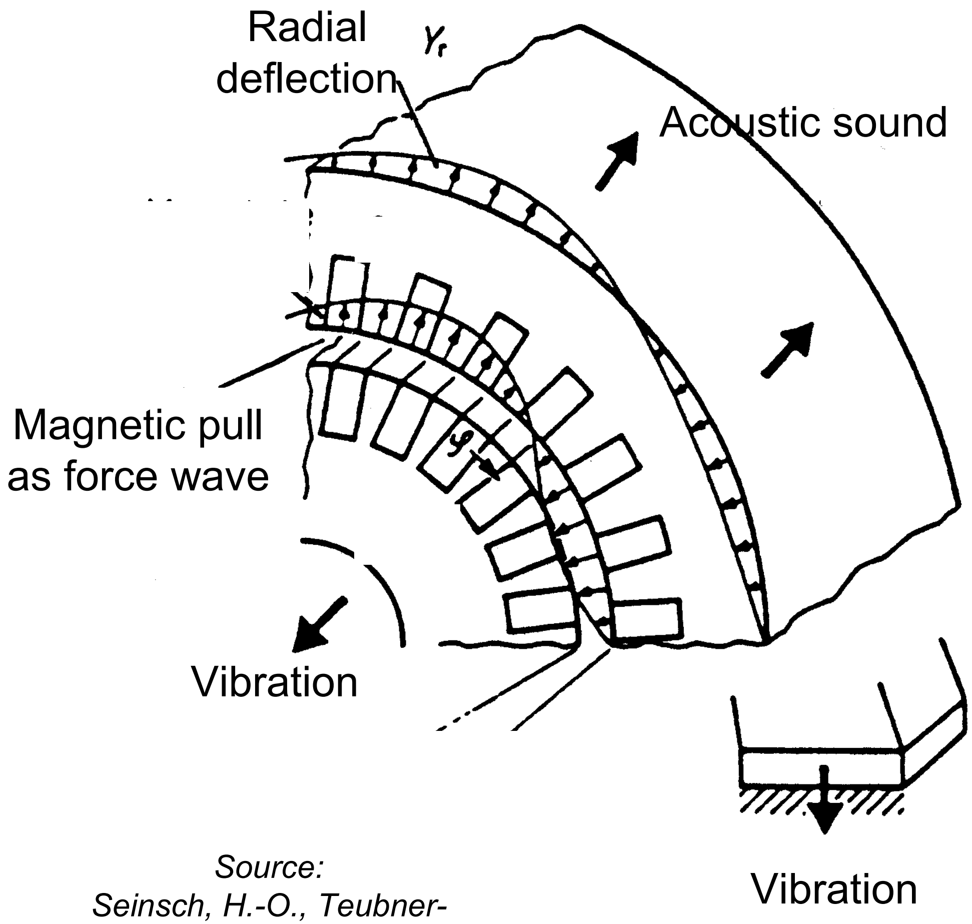
b) Skewed cage: Slot numbers equal $Q_r = Q_s$: No inter-bar current

c) $Q_r = Q_s/1.5 = 0.67 \cdot Q_s < 0.8 \cdot Q_s$: BIG harmonic inter-bar current flows, as harmonic voltages add up.

2. Design of Induction Machines

Electromagnetic acoustic noise

(from lecture: "Motor development for electrical drive systems")

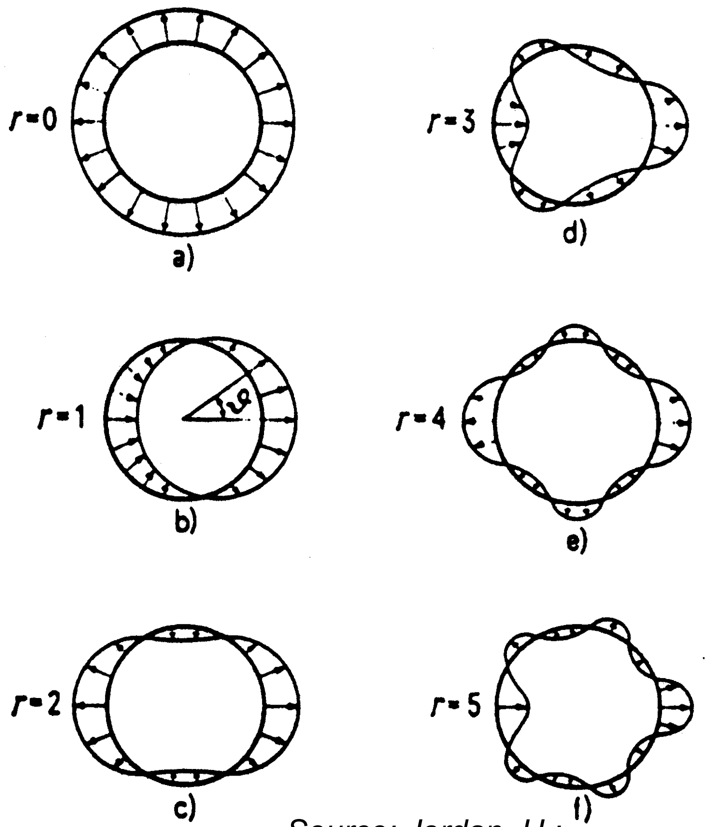


Source:
Seinsch, H.-O., Teubner-Verlag, Stuttgart

- The stator iron may be regarded as a steel ring, whereas the rotor is a steel cylinder.
- Therefore the stator is less stiff than the rotor and is bent by the radial force waves.
- As the iron surface is shaken with this frequency f_{Ton}, the surrounding air is compressed and de-compressed with this frequency f_{Ton}.
- So acoustic sound waves are generated with that tonal frequency f_{Ton} to be heard by e.g. human beings.

Deformation of the stator yoke – Acoustic noise

(from lecture: "Motor development for electrical drive systems")



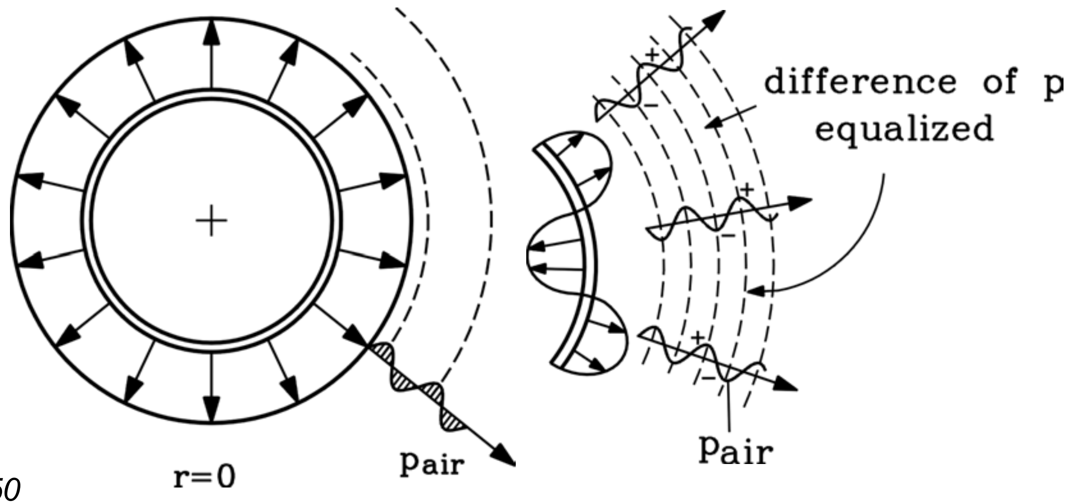
Source: Jordan, H.; Der geräuscharme Elektromotor, Verlag Girardet, Essen, 1950

- $2r = 0$:

Stator surface oscillates in phase along the stator circumference, so a far reaching sound pressure wave p_{air} is generated.

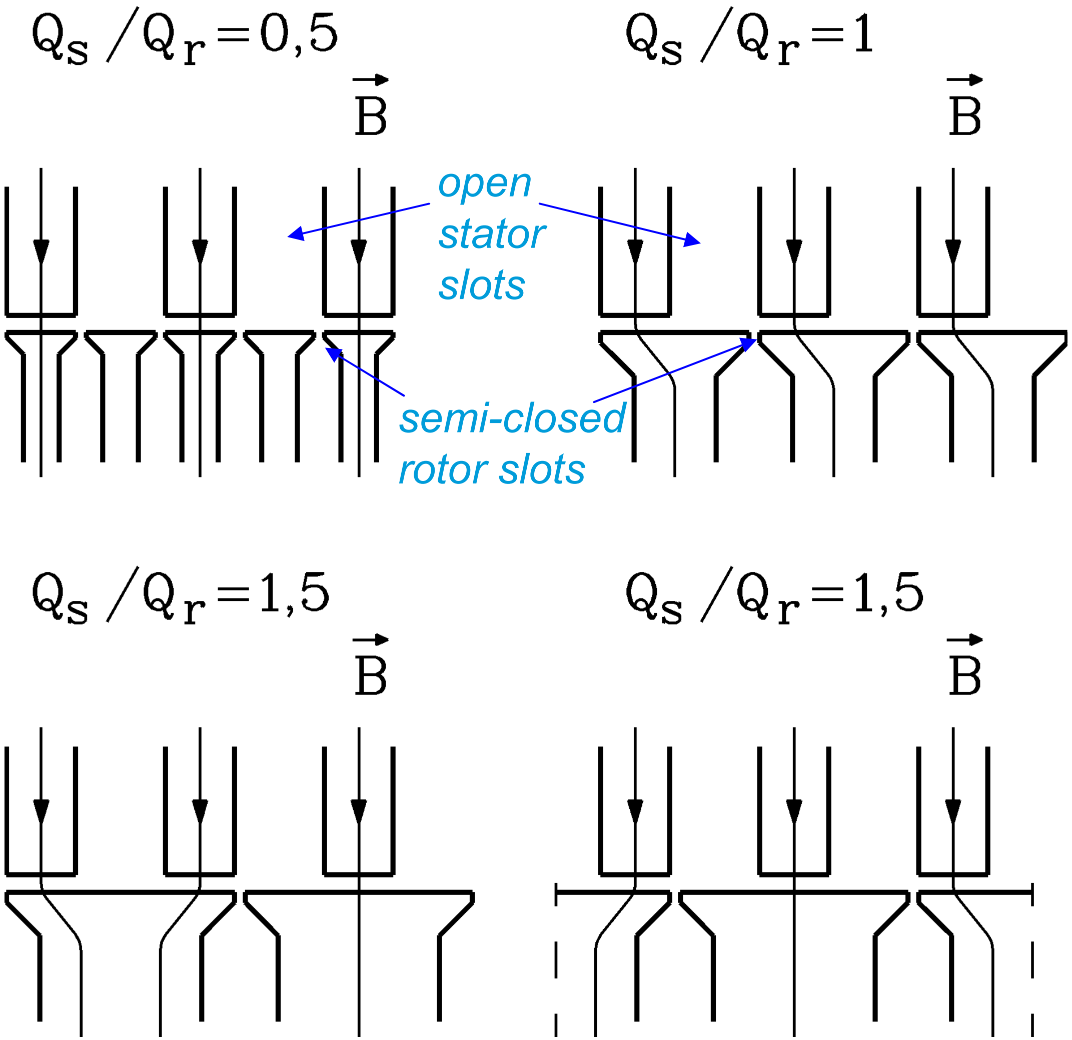
- $2r > 0$:

With increased node number r the sound pressure p_{air} is equalized strongly along the circumference = no far reaching sound.



Stator is approximated for the "far sound pressure field" as a vibrating sphere.

Rotor tooth flux pulsation depends on ratio Q_s/Q_r



Q_s/Q_r = 0.5:

Flux pulsation between 0 and 200%
(Average: 100%)

Q_s/Q_r = 1 (forbidden !):

No flux pulsation: 0%

Q_s/Q_r = 1.5:

Flux pulsation between 66% and 133%

Choice of rotor slot count

Example:

Choice of rotor slot numbers:

$2p = 4$, $Q_s = 60$, unskewed rotor:

Rule 1: $Q_r \neq 60$

Rule 5: $Q_r = (0.8 \ldots 1.2) \cdot Q_s = 48 \ldots 72$

Rule 4: Take only even numbers!

(44) (46) 48 50 ~~52~~ ~~54~~ ~~56~~ ~~58~~ ~~62~~ ~~64~~ ~~66~~ ~~68~~ ~~70~~ ~~72~~ (74) (76)

(70, 72: if skewed)

Rule 3: Choose e.g. $r^* = 4$:

$$|Q_s - Q_r| \neq 0, 1, 2, \ldots r^*, 2p, 2p \pm 1, 2p \pm 2, \ldots, 2p \pm r^* = 0, 1, 2, 3, 4, 4, 3, 5, 2, 6, 1, 7, 0, 8$$

Rule 2: For **skewed** rotor only slot numbers $Q_r < Q_s$ remain:

e.g. $Q_r = 50$ is chosen.

2. Design of Induction Machines
Typical slot numbers of 3-phase cage induction motors

- Typical stator slot numbers Q_s: q_s = integer ("integer slot winding")

$2p$ // q_s	1	2	3	4	5	6	7	8	9	10
2			18	24		36		48	54	60
4		24	36	48	60					
6	18	36	54	72						
8	24	48	72							

- Typical stator/rotor slot numbers for skewed line-start cage motors $Q_s > Q_r$:

(The rules 1 … 5 are not always obeyed strictly!)

$2p$	Q_s / Q_r					
2	24/22 36/28 48/36 48/40			60/48		
4	36/28 48/36 48/44			60/44 60/50		
6	36/30 54/48			72/54		
8	48/44					

Estimation of rotor bar current

Current transfer ratio:

$$\ddot{u}_I = \frac{k_{ws} m_s N_s}{k_{wr} m_r N_r} = \frac{2 k_{ws} m_s N_s}{Q_r}$$

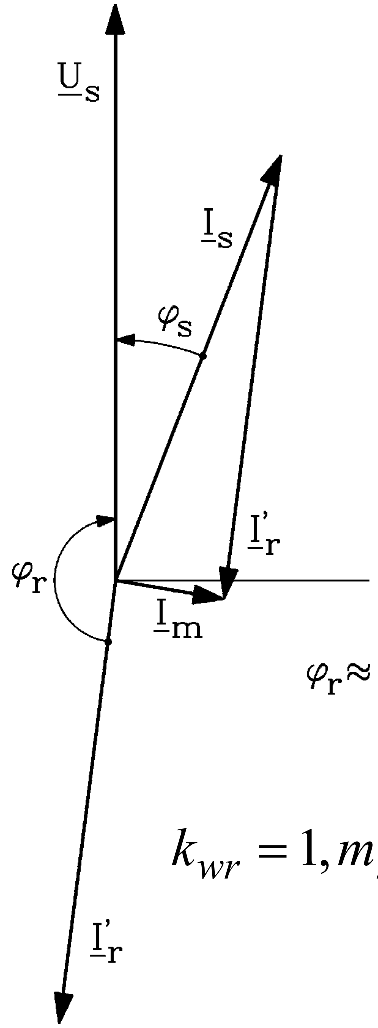
Rotor bar current:

$$I'_r = I_r / \ddot{u}_I \approx I_s \cdot \cos\varphi_s$$

$$I'_r = I_r / \ddot{u}_I \approx I_s \cdot \cos\varphi_s = 59 \cdot 0.87 = 51.33\text{A}$$

$$\ddot{u}_I = \frac{2 k_{ws} m_s N_s}{Q_r} = \frac{2 \cdot 0.91 \cdot 3 \cdot 200}{50} = 21.84$$

$$I_r = \ddot{u}_I \cdot I'_r = 21.84 \cdot 51.33 = \underline{\underline{1121}}\ \text{A}$$



$\varphi_r \approx 180°,\ |\cos\varphi_r| \approx 1$

$$k_{wr} = 1, m_r = Q_r, N_r = 1/2$$

2. Design of Induction Machines
Choice for rotor slot shape

$$d_{ra} = d_{si} - 2\delta = 458 - 2 \cdot 1.4 = 455.6 \text{ mm}$$

$$\tau_{Qr} = d_{ra}\pi / Q_r = 455.6\pi / 50 = 28.6 \text{ mm}$$

$$b_{Qr} \le \tau_{Qr} / 2 = 14.3 \text{ mm}$$

Deep bar rotor to increase starting torque:

„Deep“ bar = big ratio $h_{\mathrm{Cur}}/b_{\mathrm{Cur}} \ge 8$ (here: 8)

Choice:

$h_{\mathrm{Cur}} = 40$ mm

$b_{\mathrm{Cur}} = 5$ mm

cross section: $A_{\mathrm{Cur}} = 200 \text{ mm}^2$

$$h_{Qr} \cong h_{Cur}$$

$$b_{Qr} \cong b_{Cur}$$

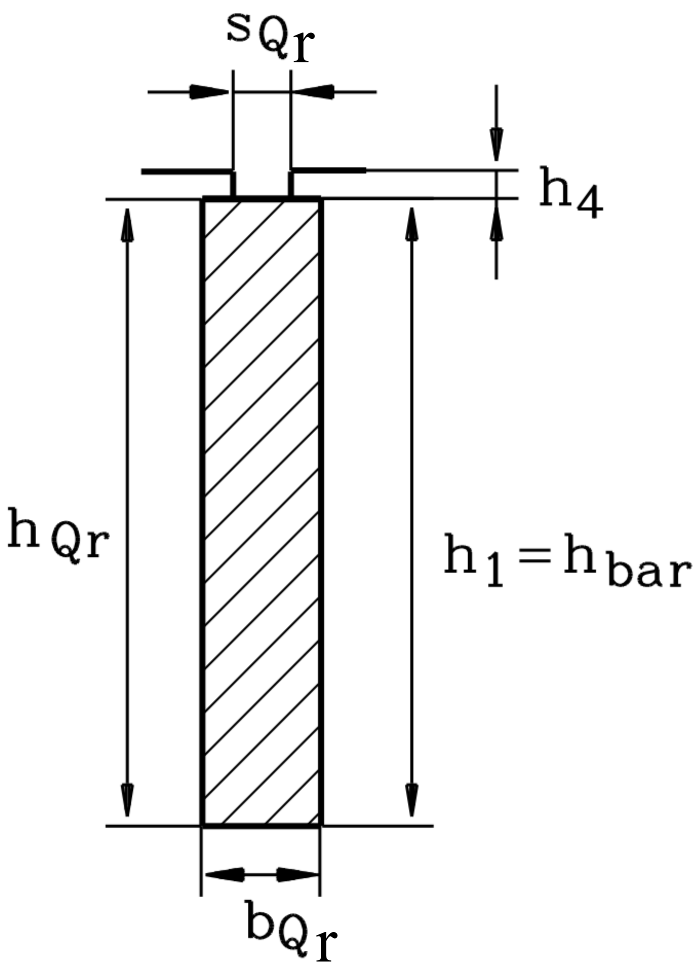
Semi-closed rotor slot to hold the bar against the centrifugal force

- Choice: $s_{\mathrm{Qr}} = 2.5$ mm

$h_1 = h_{\mathrm{Cur}} = 40$ mm, $b_1 = b_{\mathrm{Cur}} = 5$ mm (+ 0.1 mm play)

$h_4 = 3.4$ mm, $s_{\mathrm{Qr}} = 2.5$ mm

$h_{\mathrm{Qr}} = 40.1 + 3.4 = \underline{43.5}$ mm, $b_{\mathrm{Qr}} = \underline{5.1}$ mm

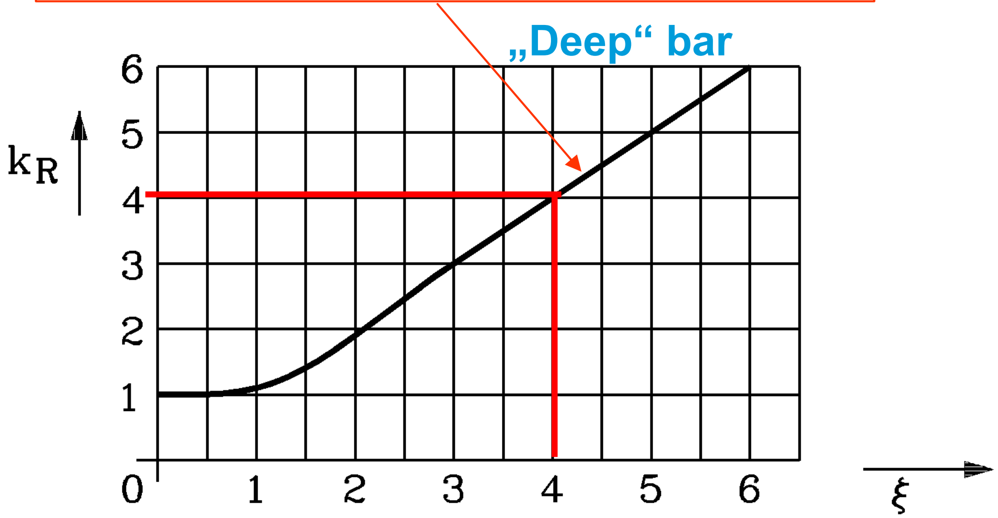


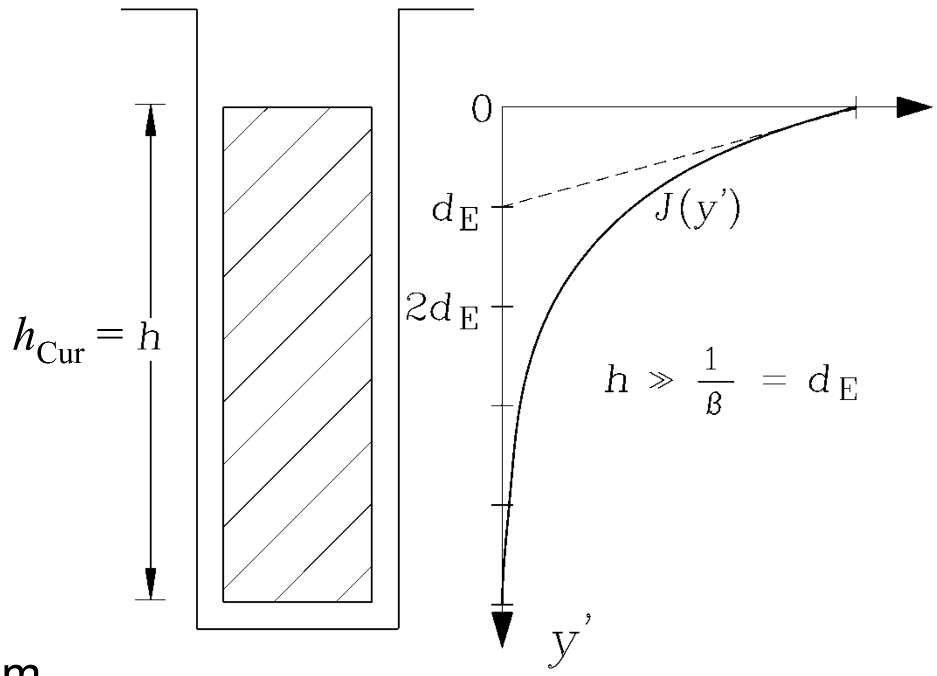
2. Design of Induction Machines

Choice of „reduced conductor height" $h_{Cur}/d_E = \xi$

$$R_{AC} = k_R \cdot R_{DC} \approx \xi \cdot R_{DC} \qquad \xi > 2$$

$$b_{Qr} \approx b_{Cu,r} : d_E\big|_{s=1} = 1/\sqrt{\mu_0 \cdot \pi \cdot f_s \cdot \kappa(\vartheta)}$$



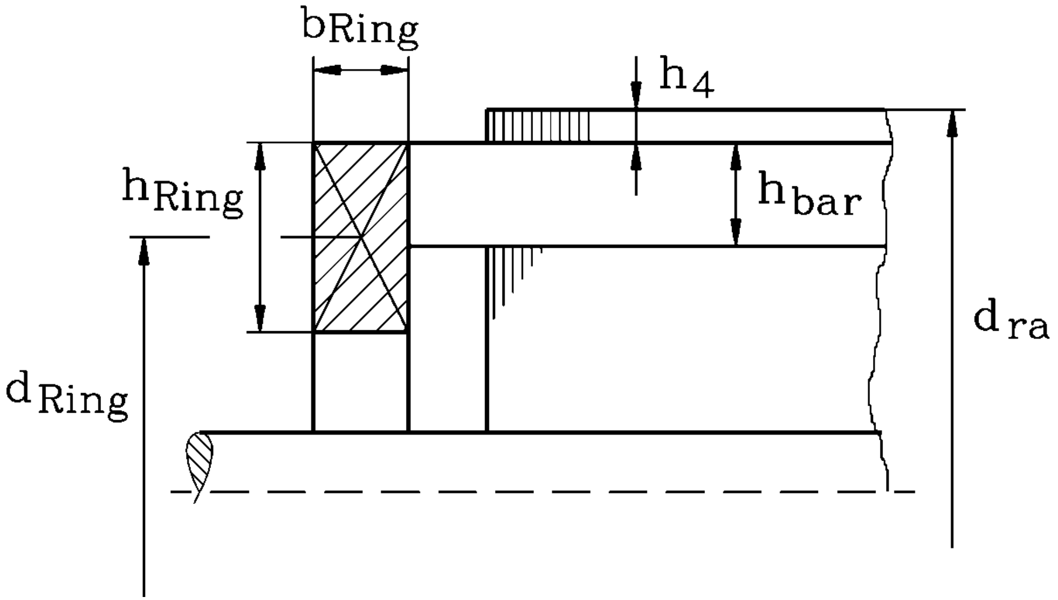


Example:

50 Hz, copper bar: d_E = ca. 10 mm, h_{Cur} = 40 mm

$\xi = h_{Cur}/d_E = 4$: Expected bar resistance increase: Factor $k_R = 4$ at stand still $s = 1$

$$d_E(s=1) = 1/\sqrt{\pi \cdot f_s \cdot \mu_{Cu} \cdot \kappa_{Cu}(75°C)} = 1/\sqrt{\pi \cdot 50 \cdot 4\pi \cdot 10^{-7} \cdot 50 \cdot 10^6} = 0.0101\text{m} = 1.01\text{cm}$$



4 times bigger ring current than bar current: Bigger ring cross section: e. g.:

Choice:

$h_{\mathrm{Ring}} = 80$ mm, $b_{\mathrm{Ring}} = 10$ mm,

cross section: $A_{\mathrm{Ring}} = 800\ \mathrm{mm}^2$

Rotor bar current density: $J_r = I_r / A_{Cur} = 1121/200 = 5.6\ \mathrm{A/mm}^2 := J_{Ring}$

Rotor ring current: $I_{Ring} = I_r /(2 \cdot \sin(p\pi / Q_r)) = 1121/(2 \cdot \sin(2\pi / 50)) = \underline{\underline{4472}}\ \mathrm{A}$

Necessary ring cross section: $A_{Ring} = I_{Ring} / J_{Ring} = 4472 / 5.6 = 800\ \mathrm{mm}^2$

Summary:
Rotor cage design

- Choice of number of slots per pole pair Q_r/p
- Rotor slot count must differ from stator slot count $Q_r \neq Q_s$
- Bar shapes for increased starting torque M_1 via current displacement k_R
- Skewing of rotor cage typically by one stator slot-pitch
- Die-cast cage (mainly aluminum, sometimes copper) for small machines
- Brazed copper cage for large machines > (200 … 500) kW
- Ring cross section area much bigger than bar cross section area

2. Design of induction machines

2.1 Main dimensions and basic electromagnetic quantities of induction machines

2.2 Scaling effect in electric machines

2.3 Stator winding low and high voltage technology

2.4 Stator winding design

2.5 Rotor cage design

2.6 Wound rotor design

2.7 Design of main flux path of magnetic circuit

2.8 Stray flux and inductance

2.9 Influence of saturation on inductance

2.10 Masses and losses

2. Design of Induction Machines

Wound rotor – doubly-fed induction wind generator

Doubly-fed induction generator for wind power generation

4 poles

2000 kW, 1800/min,
50 Hz,

slip range: -/+ 20% ⇒ speed range: 1500 ±300 = 1200 … 1800/min

Source:

Winergy, Germany

Details:
See lecture:
Large Generators &
High power drives

Wound rotor winding of slip-ring induction machines

- **Wound rotor winding** is usually a three-phase two-layer distributed winding.
- Due to the slip ring system usually a low voltage winding ($U_{r,LL}$ < 1000 V) is chosen by a big voltage transformer ratio $\ddot{u}_U$, so a low N_r.
- **Choice**: $N_{cr} = 1$, $Q_s \neq Q_r$ (reduced cogging torque, unskewed machine)

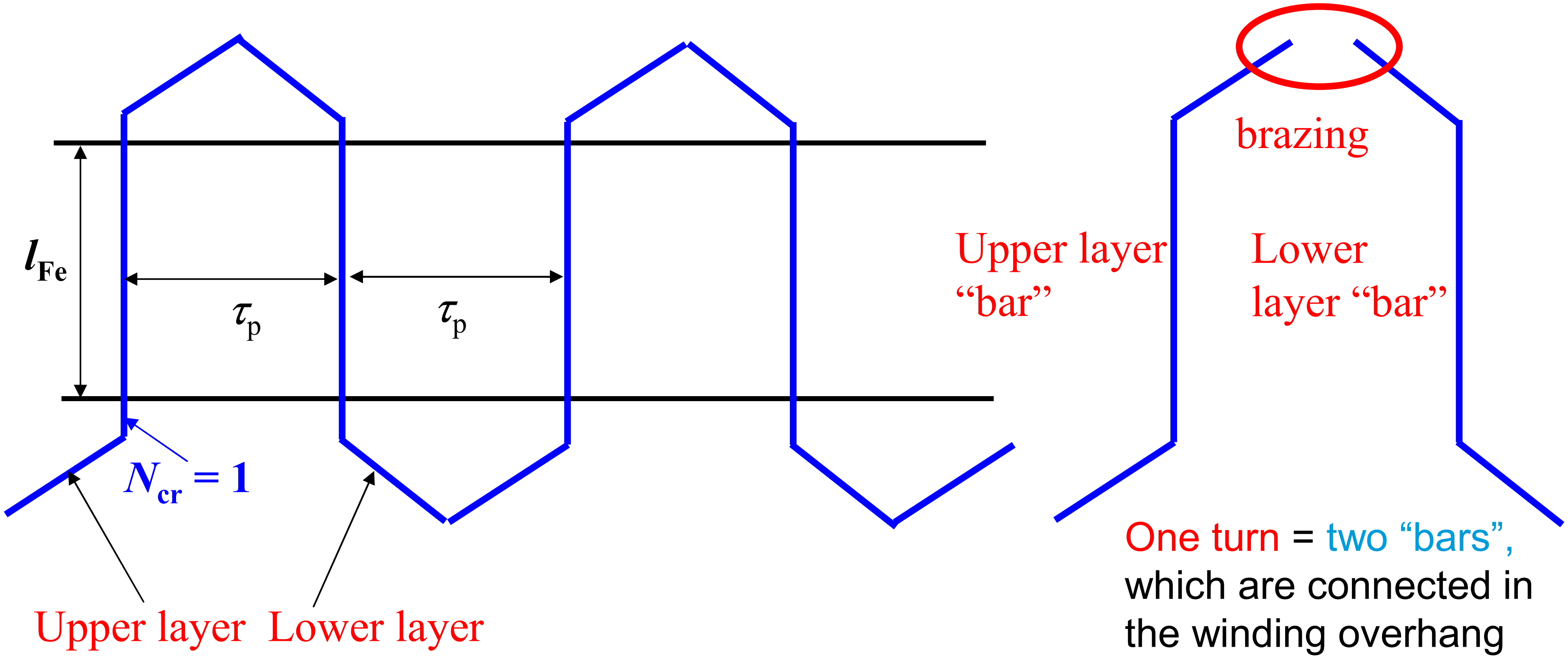
$$\ddot{u}_U = \frac{N_s k_{ws}}{N_r k_{wr}} \qquad N_r = 2p \cdot q_r \cdot N_{cr} / a_r \qquad Q_r = 2p \cdot q_r \cdot m_r \qquad Q_s = 2p \cdot q_s \cdot m_s$$

Usually: $m_s = m_r = 3$, so: $q_s \neq q_r$, e.g. $5 \neq 6$

- **Advantage of low voltage rotor winding:**
  - Thin slot insulation allowed = high copper fill factor = low rotor winding resistance R_r = low rotor copper losses
  - $N_{cr} = 1$: One turn per coil = Rotor winding is manufactured as wave winding = 50% reduction in winding overhang conductors = = further reduction of rotor winding resistance R_r

2. Design of Induction Machines

Rotor wave winding



One turn = two "bars", which are connected in the winding overhang

"Bars" are inserted into the semi-closed or closed rotor slots and are afterwards connected by brazing to form the phase winding!

2. Design of Induction Machines
Wound three-phase rotor winding manufacturing

Source:

Winergy

Germany

Rotor especially for big doubly-fed induction wind generators with rotor slip rings

Calculation example:
see text book

2. Design of Induction Machines

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$Q_s = 60,\ q_s = 5$

$$b_{Cus} \times h_{Cus} = 7.1 \times 1.8\,\text{mm}^2 \approx$$

$$\approx 12.8\,\text{mm}^2$$

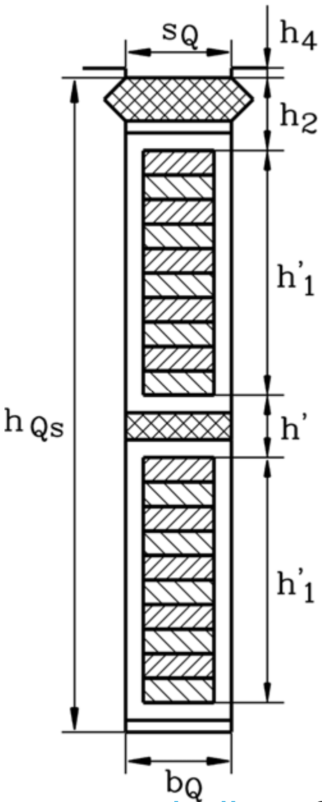
$$N_{cs} = 10$$

Total copper cross section area:

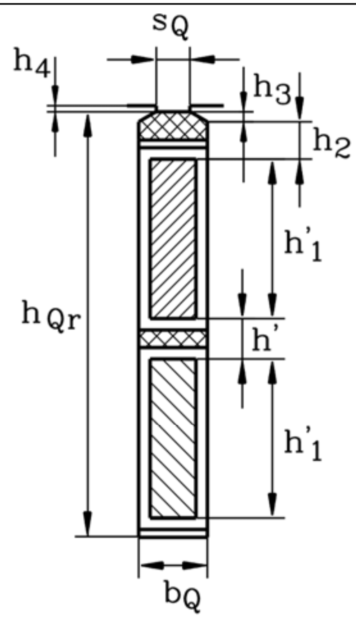
$$60 \cdot 2 \cdot 10 \cdot 12.8\,\text{mm}^2 =$$

$$= 15360\,\text{mm}^2$$

$$J_s = 4.8\,\text{A/mm}^2$$



a)



b)

$Q_r = 72,\ q_r = 6$

$$b_{Cur} \times h_{Cur} = 4.5 \times 18.0\,\text{mm}^2 \approx$$

$$\approx 81\,\text{mm}^2$$

$$N_{cr} = 1$$

Total copper cross section area:

$$72 \cdot 2 \cdot 1 \cdot 81\,\text{mm}^2 =$$

$$= 11664\,\text{mm}^2$$

$$J_r = 5.2\,\text{A/mm}^2$$

$$k_f = \frac{A_{Cu}}{A_Q}$$

a) Two-layer high-voltage stator winding, 6 kV, N_{cs} = 10, lap winding, open slot, low fill factor $k_{fs} \approx 0.3$, big slot cross section necessary for given $A_s \cdot J_s$

b) Two-layer low voltage rotor winding < 1 kV, N_{cr} = 1, wave winding, semi-closed slot, high fill factor $k_{fr} \approx 0.6$, small slot cross section necessary for given $A_r \cdot J_r$

Summary:
Wound rotor design

- Design of three-phase rotor winding similar to stator winding
- Usually two-layer winding for low voltage < 1 kV
- Wave winding with 50% reduction of winding overhang possible, if one turn per coil $N_{cr} = 1$
- Rotor slot count must differ from stator slot count $Q_r \neq Q_s$

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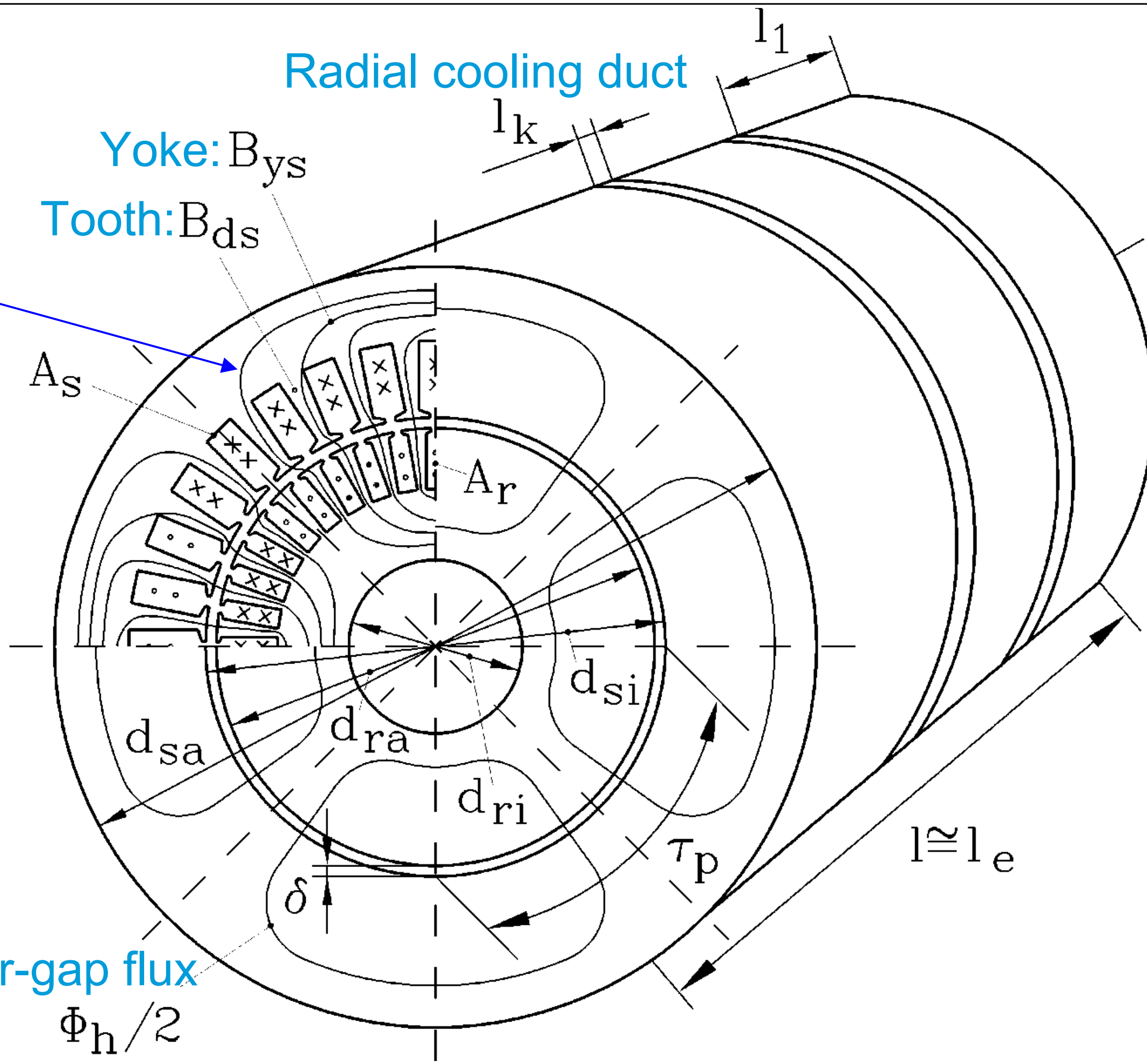
2. Design of Induction Machines

Design of main flux path of magnetic circuit



Yoke flux = ca. 50% of air-gap flux

Example:

$2p = 4$, unskewed

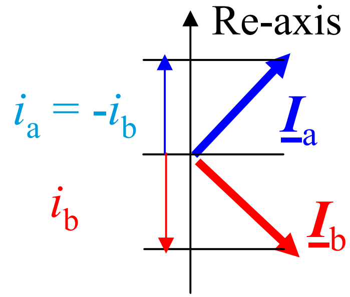
$Q_s = 32$, $Q_r = 36 > Q_s$,

$m_s = 2$, $q_s = 4$

$$Q_s = 2p \cdot m_s \cdot q_s$$

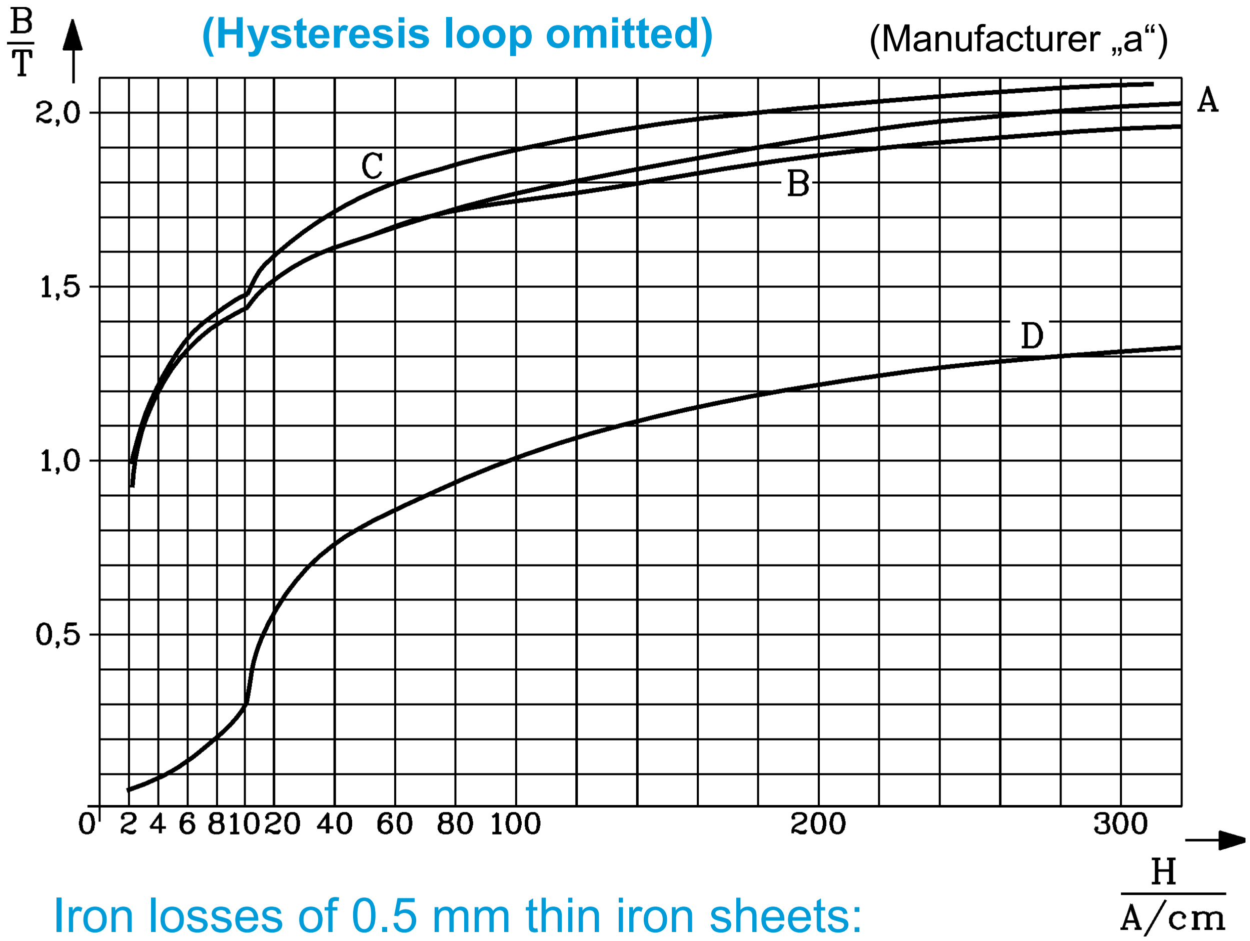
$$Q_s = 4 \cdot 2 \cdot 4 = 32$$

Example shown for two-phase stator with phases a and b at the moment: $i_a = -i_b$



2. Design of Induction Machines

Magnetization curve of silicon iron and pure iron sheets



Iron losses = *FOUCAULT´*s eddy current losses + hysteresis losses

A, B: Iron sheets with Si to increase resistance to reduce eddy current losses (*Foucault* losses)

C: Iron sheets without any Si ("pure" iron)

D: Cast iron

v_{10} = Iron losses at 1.0 T, 50 Hz per kg

Iron losses of 0.5 mm thin iron sheets:

A: sheet type III	1% Si	v_{10} = 2.3 W/kg	Hyst.: 1.6 W/kg	Foucault: 0.7 W/kg
B: sheet type IV	3.5% Si	v_{10} = 1.7 W/kg	Hyst.: 1.3 W/kg	Foucault: 0.4 W/kg

2. Design of Induction Machines

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B / T	*H* / A/cm									
	„0	„1	„2	„3	„4	„5	„6	„7	„8	„9
0.5	0.7	0.81	0.83	0.85	0.87	0.89	0.91	0.93	0.95	0.97
0.6	0.99	1.01	1.03	1.05	1.07	1.09	1.11	1.13	1.15	1.17
0.7	1.19	1.21	1.24	1.26	1.28	1.32	1.34	1.36	1.38	1.43
0.8	1.46	1.49	1.53	1.56	1.59	1.64	1.68	1.72	1.75	1.79
0.9	1.83	1.88	1.93	1.98	2.03	2.08	2.13	2.18	2.25	2.3
1.0	2.35	2.4	2.5	2.6	2.7	2.8	2.9	3.0	3.1	3.2
1.1	3.3	3.4	3.5	3.6	3.8	3.9	4.1	4.2	4.4	4.5
1.2	4.7	4.9	5.1	5.2	5.4	5.6	5.8	6.1	6.3	6.6
1.3	6.8	7.1	7.4	7.7	8.1	8.5	9.0	9.5	10.0	10.6
1.4	11.3	12.1	13.0	14.2	15.5	16.8	18.0	19.3	20.5	21.9
1.5	23.3	24.8	26.4	28.1	29.8	31.5	33.4	35.5	37.7	40.0
1.6	42	45	48	51	55	59	62	66	70	74
1.7	78	83	87	92	97	101	106	112	117	123
1.8	129	135	141	148	155	162	170	178	187	196
1.9	206	216	226	236	247	259	272	287	303	320
2.0	339	361	383	408	435	465	495	530	567	605
2.1	650	700	750	810	870	930	1000	1070	1140	1220
2.2	1300	1380	1460	1540	1620	1700	1780	1860	1940	2020
2.3	2100	2180	2260	2340	2420	2500	2580	2660	2740	2820

Magnetization characteristic *B*(*H*):

Iron sheet type III

(Manufacturer „b“)

B, *H*-values differ somewhat from manufacturer “a”!

$\mu(H) = B(H) / H$

Non-linear function: Time-stepping solution needed!

Isotropic sheets:

$$\vec{B}(H) = \mu_0 \vec{H} + \vec{J}(H)$$

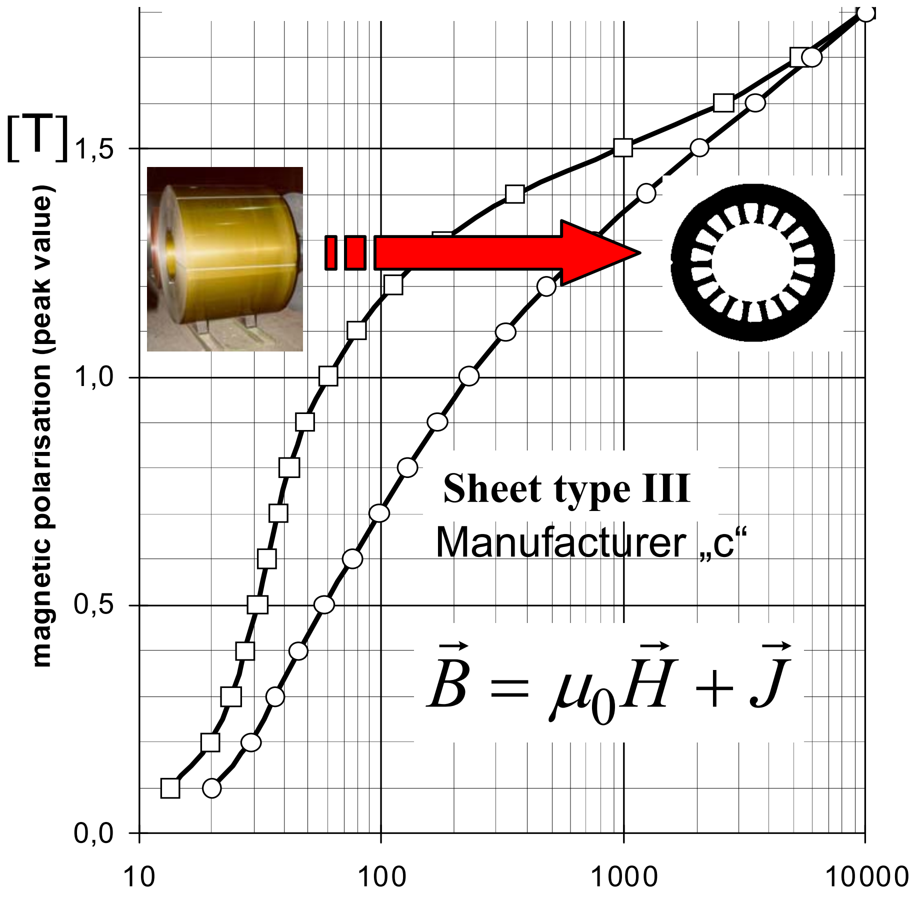
$$\vec{B} \uparrow\uparrow \vec{H} \uparrow\uparrow \vec{J}$$

$$\vec{B}(H) = \mu(H) \cdot \vec{H}$$

2. Design of Induction Machines

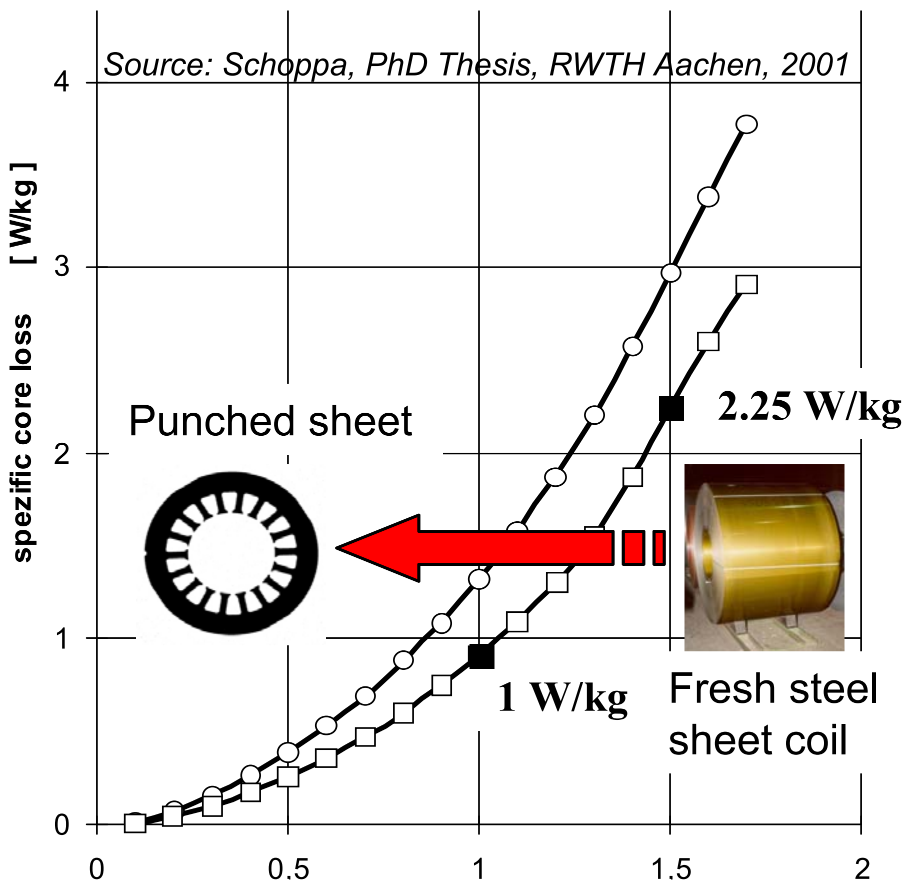
Influence of punching on iron sheet parameters

Magnetic polarization J [T]



Magnetic field strength H [A/m]

Losses per mass p at 50 Hz [W/kg]

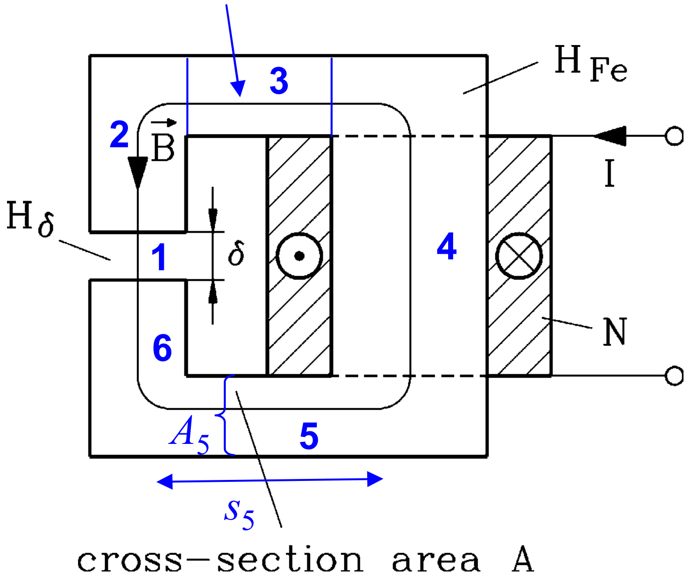


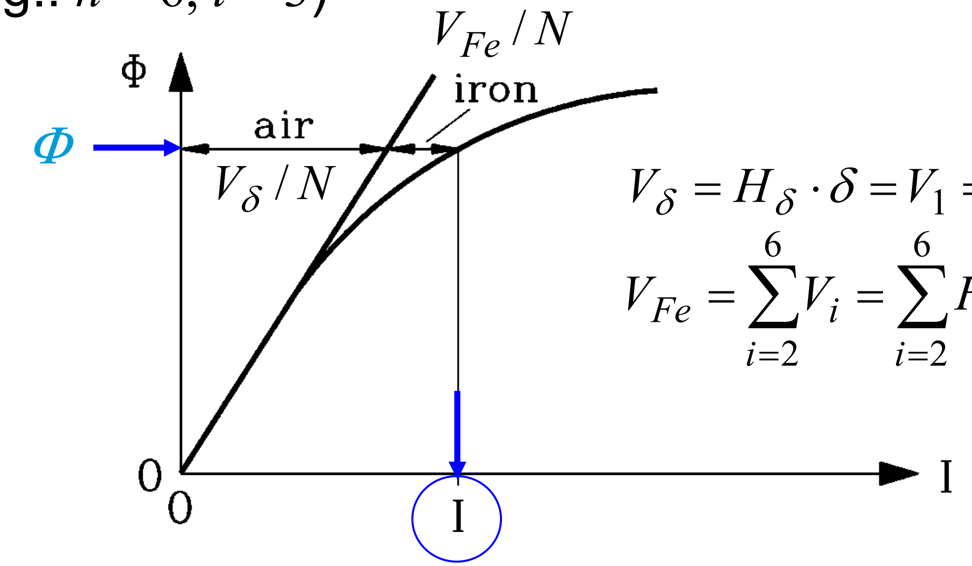
Magnetic polarization J [T]

2. Design of Induction Machines

Calculation of non-linear magnetic circuit

i*-th section of flux path *C (e.g.: $n = 6,\ i = 3$)





$$V_\delta = H_\delta \cdot \delta = V_1 = H_1 \cdot s_1$$

$$V_{Fe} = \sum_{i=2}^{6} V_i = \sum_{i=2}^{6} H_i s_i$$

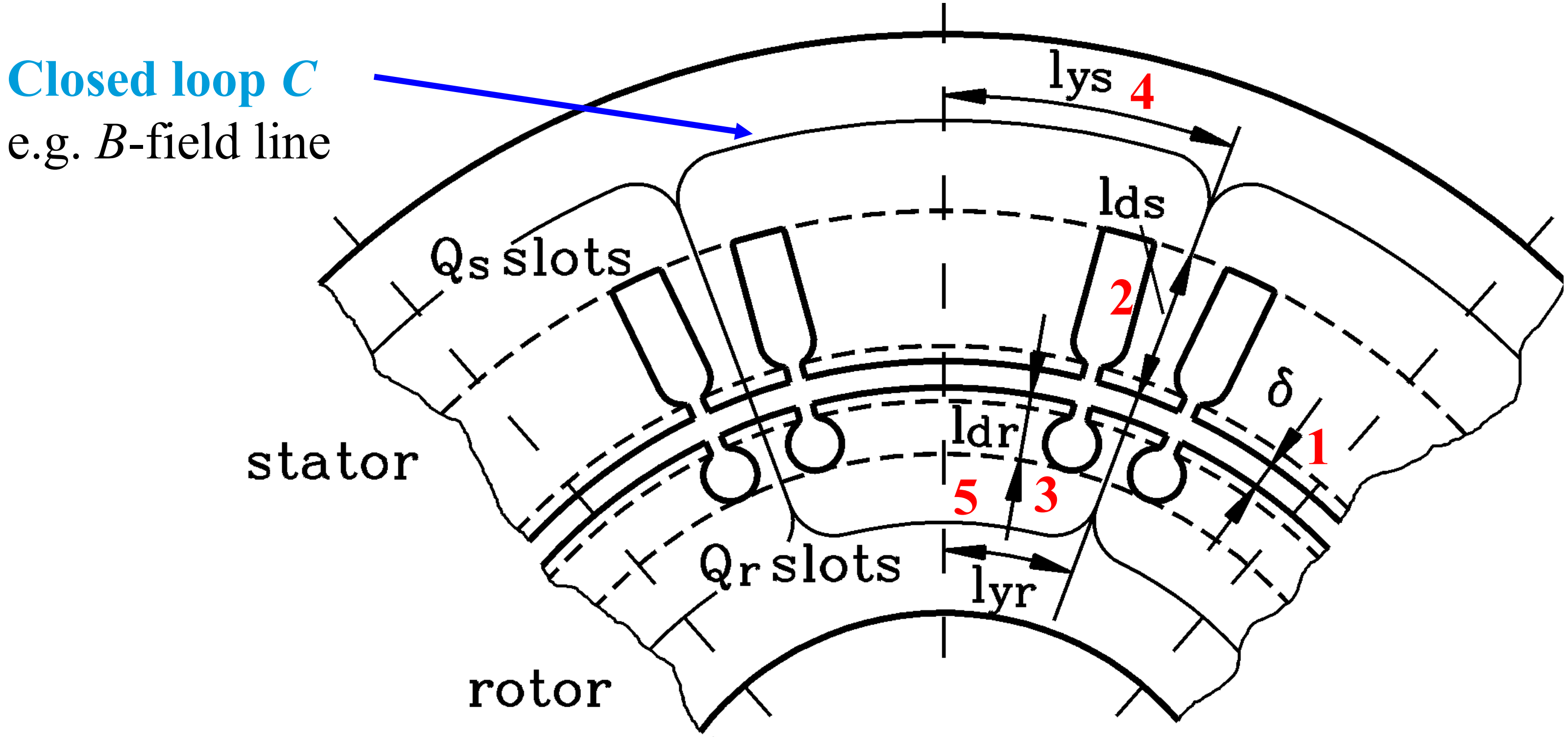
Ampere´s law:
$$\oint_C \vec{H} \bullet d\vec{s} = N \cdot I \approx \sum_i H_i \cdot s_i \quad \Rightarrow \quad \sum_{i=1}^{n} H_i \cdot s_i = \sum_{i=1}^{n} V_i = V = N \cdot I$$

Start with a certain flux Φ: $B_i = \Phi / A_i$

For i-th iron section H_i is taken by $H_i(B_i)$, for air: $H_i = B_i / \mu_0$

2. Design of Induction Machines

Real geometry of induction machine



„Half“ magnetic circuit:

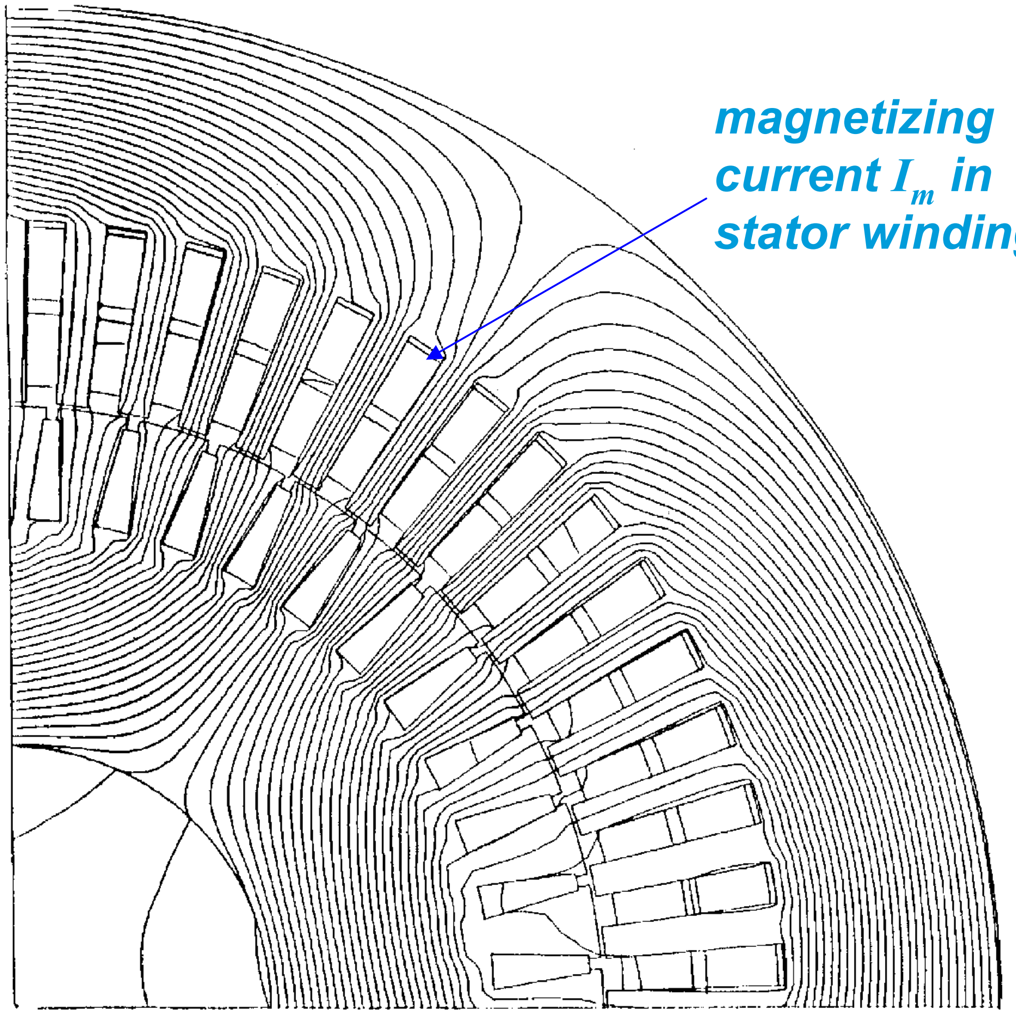
Closed loop *C* for *Ampere´s* law: Sections $i = 1, \dots, 5$:

$i = 1$: Air gap δ, which is influenced by slot openings s_Q

$i = 2, 3$: Stator and rotor teeth with length l_d (tooth section)

$i = 4, 5$: Stator and rotor yoke with half yoke length l_y (yoke section)

Numerical calculation of magnetic field with Method of Finite-Differences (see: Seminary CAD 0+3)

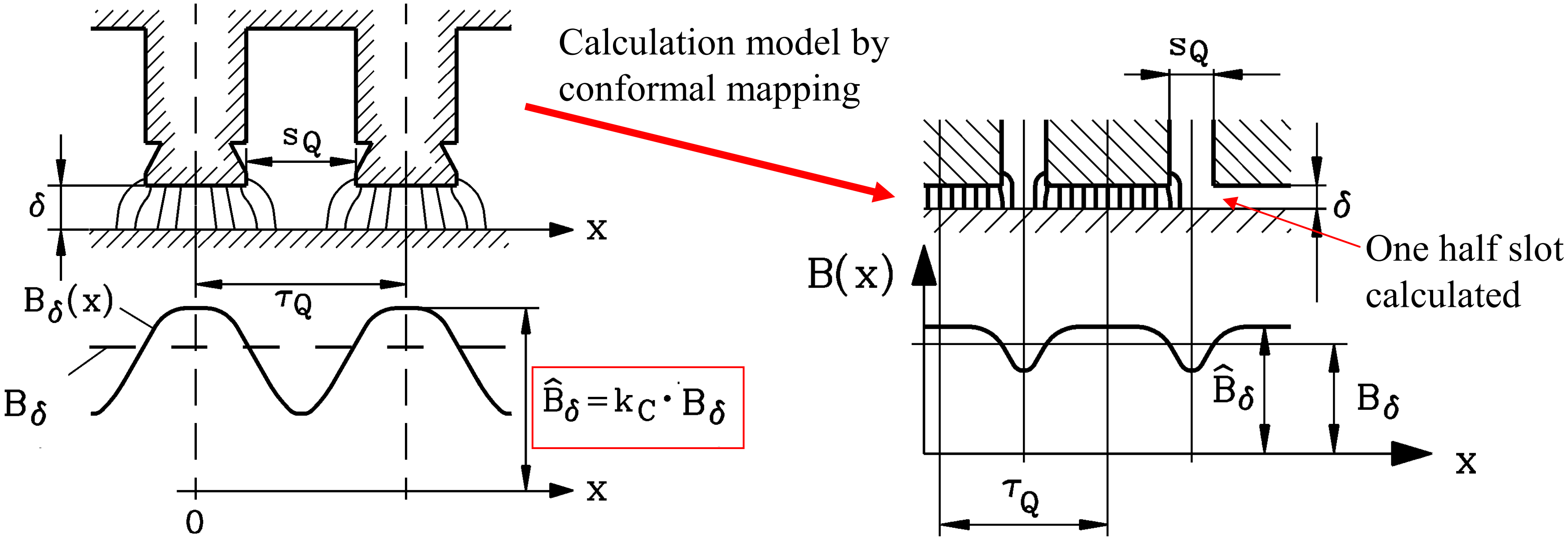


- Cross section of a three-phase 4-pole high voltage cage induction machine with wedge-type rotor slots (Q_s / Q_r = 60 / 44)

- Numerically calculated no-load flux density B at rated voltage & no-load (s = 0, rotor current zero)

In this lecture:
Analytical step by step calculation of magnetizing current I_m is done by calculating m.m.f. V_i of n sections in iron and air gap

Magnetization of air gap: Single sided slotting



Due to the slot openings the air gap flux density shows a considerable ripple. **Average flux density B_δ per slot pitch** τ_Q with respect to peak value $\hat{B}_\delta$ under the tooth tips is given by $1/k_C$ (***Carter´s coefficient k_C > 1***).

$$V_\delta = \hat{H}_\delta \cdot \delta = \hat{B}_\delta \cdot \delta / \mu_0 = B_\delta \cdot k_C \cdot \delta / \mu_0 = B_\delta \cdot \delta_e / \mu_0$$

$$k_C = \hat{B}_\delta / B_\delta = \frac{\tau_Q}{\tau_Q - \zeta(h) \cdot \delta}$$

$$V_\delta = \frac{B_\delta}{\mu_0} \cdot \delta_e$$

Equivalent air gap:

$$\delta_e = k_C \cdot \delta$$

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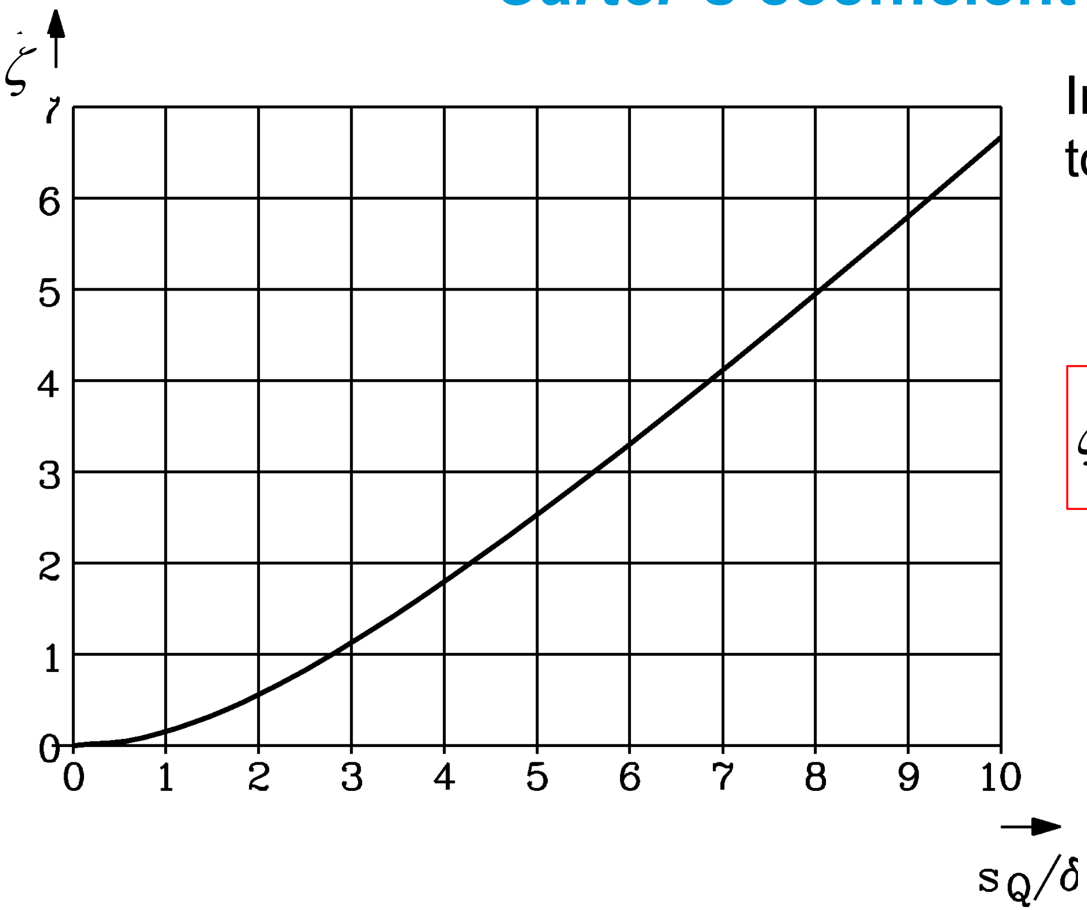
With increasing slot opening s_Q and decreasing air gap width δ *Carter´s* coefficient k_C increases !



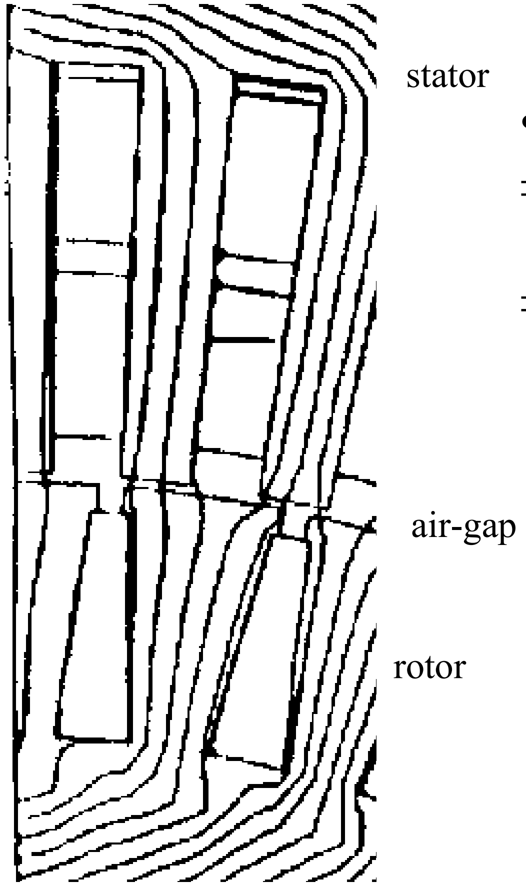
Influence of ratio $h = s_Q/\delta$ on factor ζ to determine *Carter´s* coefficient

$$\zeta(h) = \frac{2}{\pi} \cdot \left[h \cdot \arctan(h/2) - \ln\left(1 + (h/2)^2\right) \right]$$

$$\zeta(h) \approx \frac{h^2}{h+5}$$

$$h = s_Q / \delta$$

2. Design of Induction Machines
Double-sided slotting



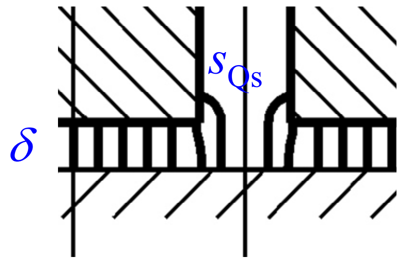
$$k_C \approx k_{Cs} \cdot k_{Cr}$$

- CARTER coefficient for double-sided slotting:

⇒ Approximately calculated by multiplication from single-sided slotting, because:

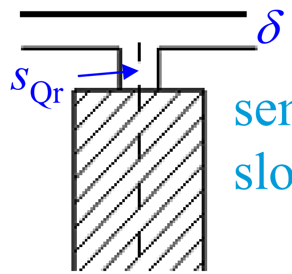
⇒ Influence of the small rotor slot openings s_{Qr} is small (due to semi-closed slots)

Slotted stator



k_{Cs}

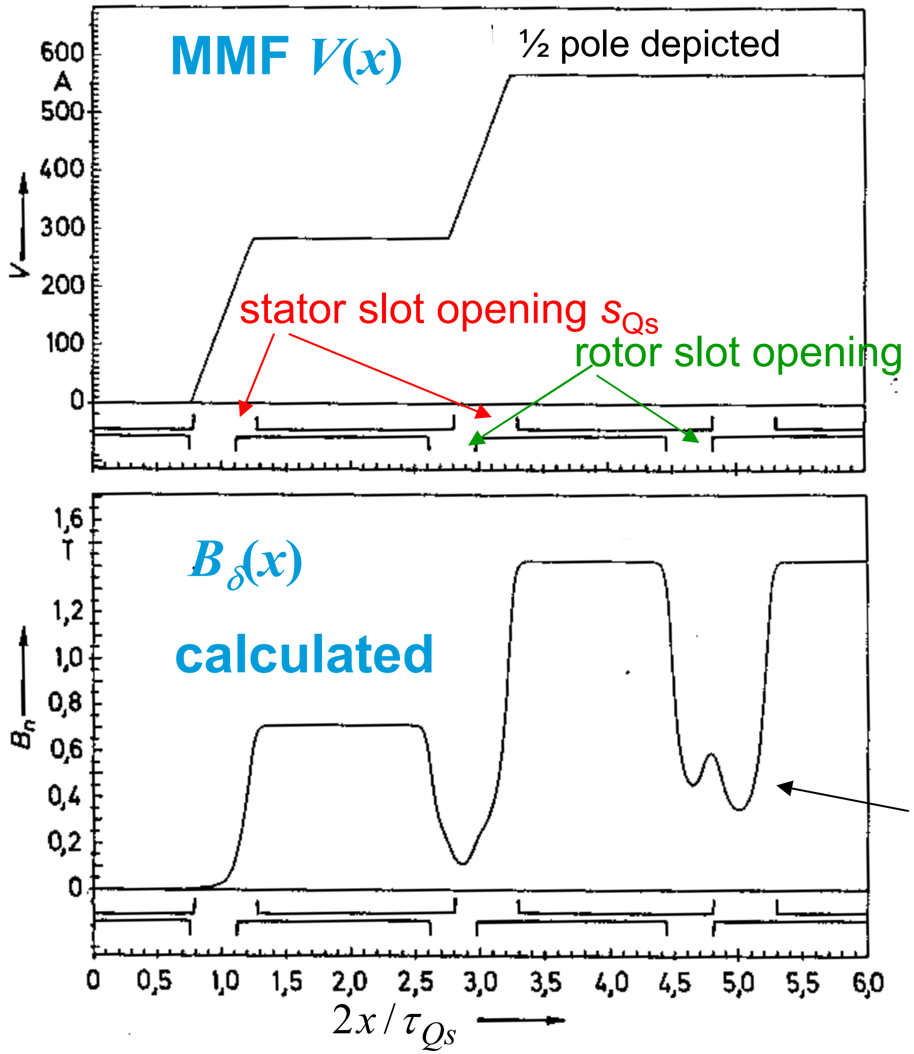
Slotted rotor



k_{Cr}

2. Design of Induction Machines

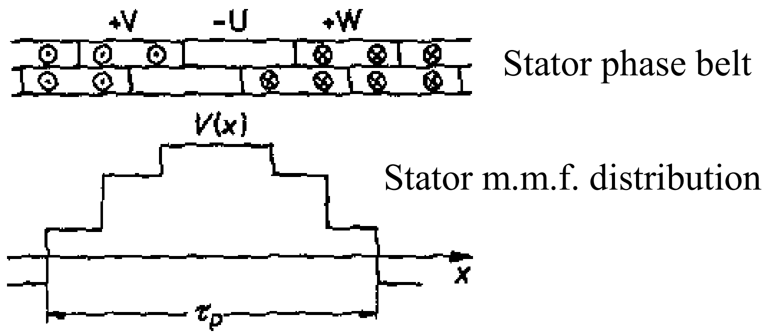
Influence of slotting on air-gap flux density



Example: $q_s = 2$, $W/\tau_p = 5/6$, $i_V = -i_W$, $i_U = 0$

Cage rotor: $Q_r/p = 13$ (rotor current zero: $i_r = 0$)

No iron saturation considered

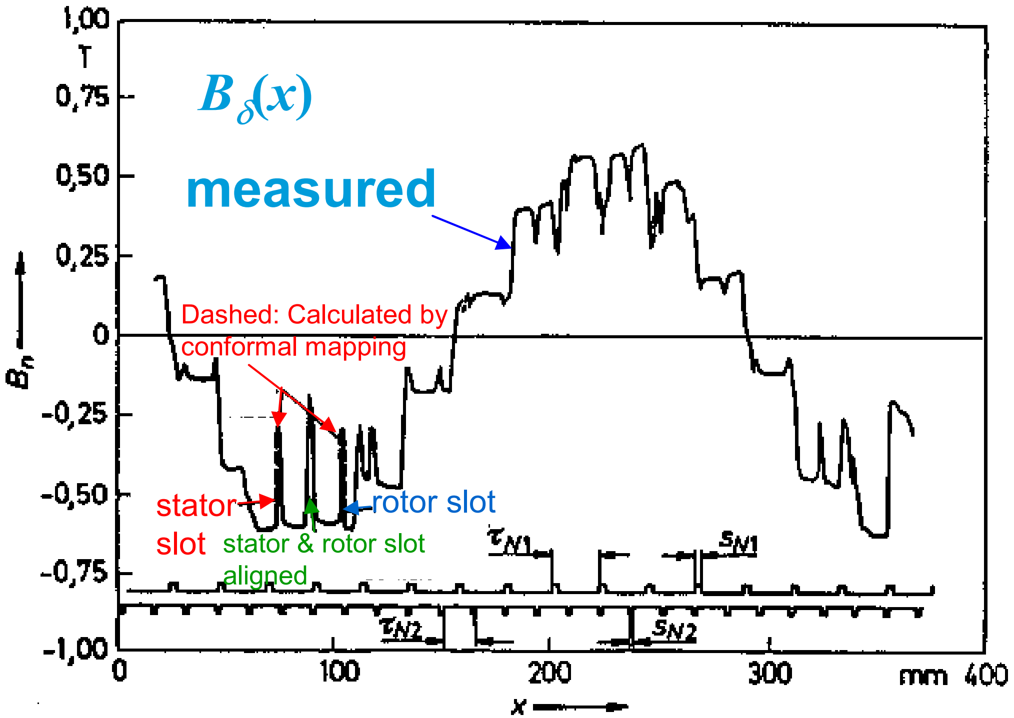


Rough consideration of slotting by multiplication of stator and rotor air gap permeance function, calculated by conformal mapping

Source: Binder, A.: Archiv f. ET, 1990

2. Design of Induction Machines

Measured unsaturated air-gap flux density with slotting influence



Semi-closed slots:

δ = 0.5 mm, s_{Qs} = 3.0 mm, s_{Qr} = 1.8 mm

Source: Binder, A.: Archiv f. ET, 1990

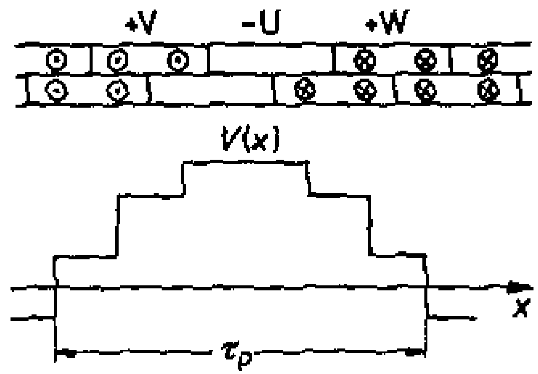
Example:

8-pole slip-ring induction motor, 30 kW, 50 Hz

q_s/q_r = 2/3, Q_s/Q_r = 48/72, non-skewed,

low voltage round-wire windings

$i_V = -i_W$ fed via DC current at rotor stand still (rotor current is zero $i_r = 0$)



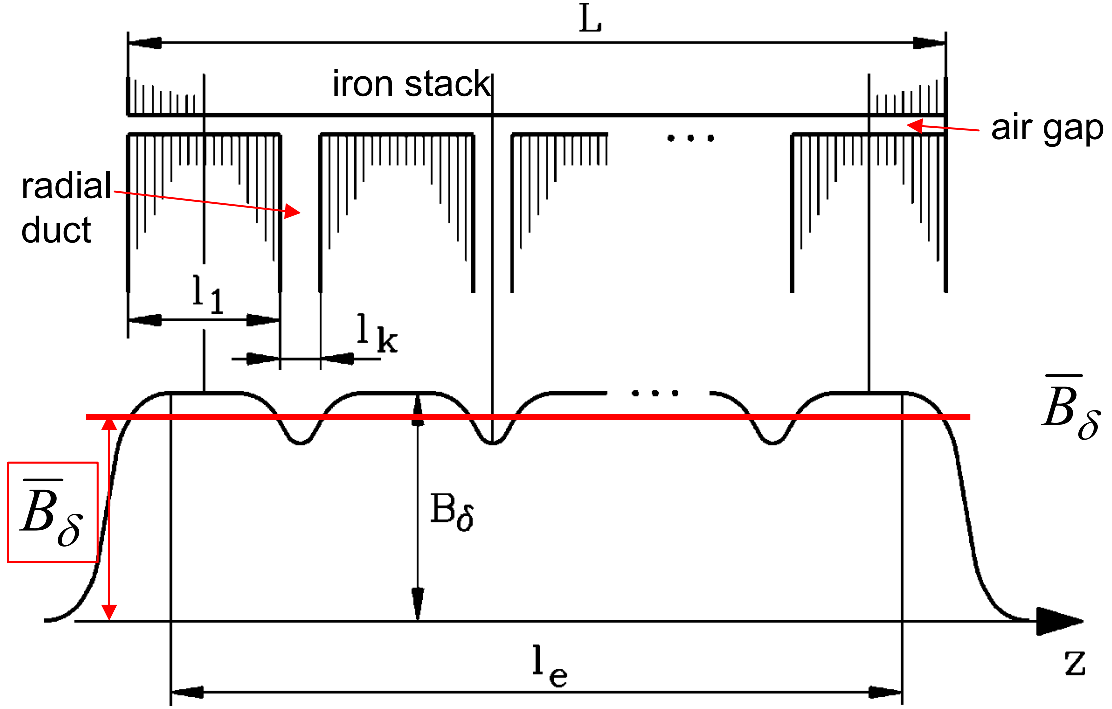
Equivalent iron length l_e

Radial ventilation ducts in iron stack lead to ripple of air gap flux density in axial direction z, which is described by magnetic equivalent iron length l_e:

z_1 sections of iron packets with length l_1, z_1-1 ducts with width l_k:

$$L = l_{Fe} + (z_1 - 1)\cdot l_k \qquad l_{Fe} = z_1 \cdot l_1$$

Taking $l_k + l_1$ instead of τ_Q, l_k instead of s_Q: $k'_C = B_\delta / \overline{B}_\delta = \dfrac{l_1 + l_k}{l_1 + l_k - \zeta(h')\cdot\delta}$ $\qquad h' = l_k / \delta$



Flux per pole: Φ

$$\Phi / \tau_p \sim \overline{B}_\delta \cdot L = B_\delta \cdot l_e > B_\delta \cdot l_{Fe}$$

$$\overline{B}_\delta \cdot L = (B_\delta / k'_C)\cdot L = B_\delta \cdot L / k'_C = B_\delta \cdot l_e$$

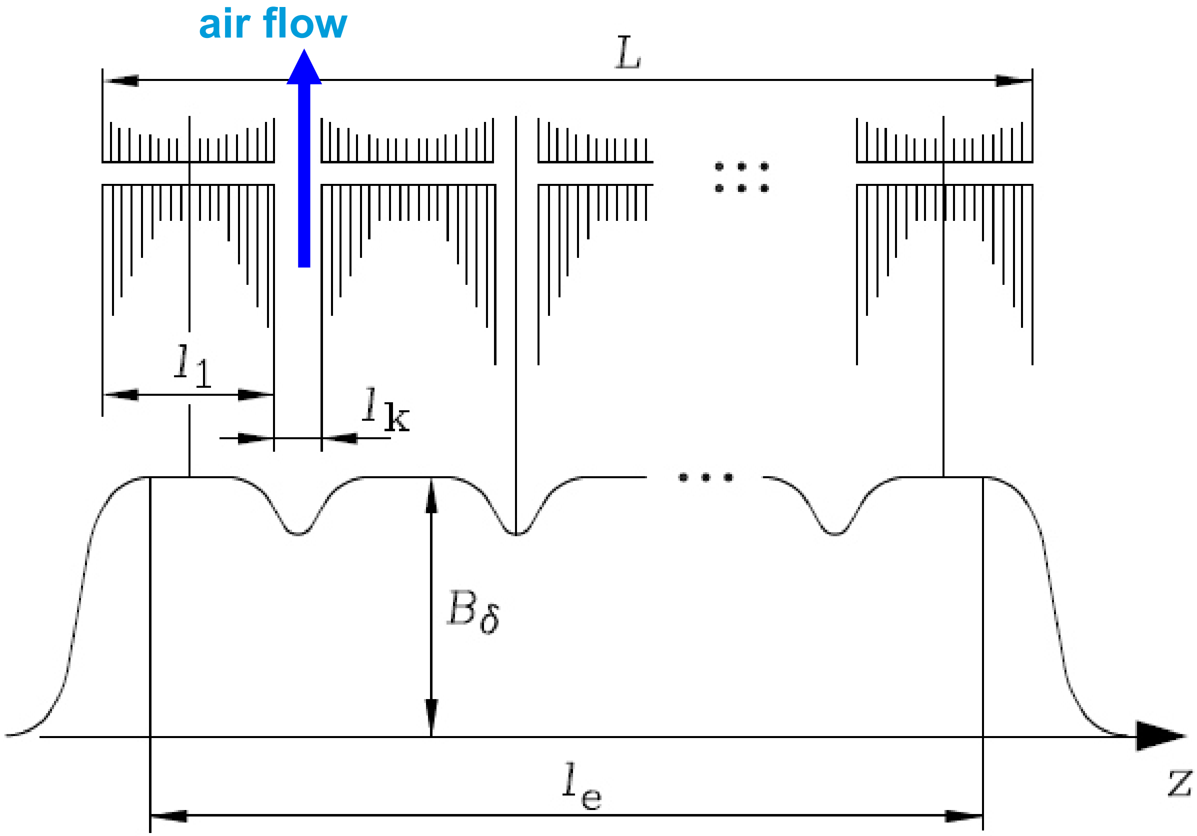
Equivalent iron length l_e:

$$l_e = L / k'_C$$

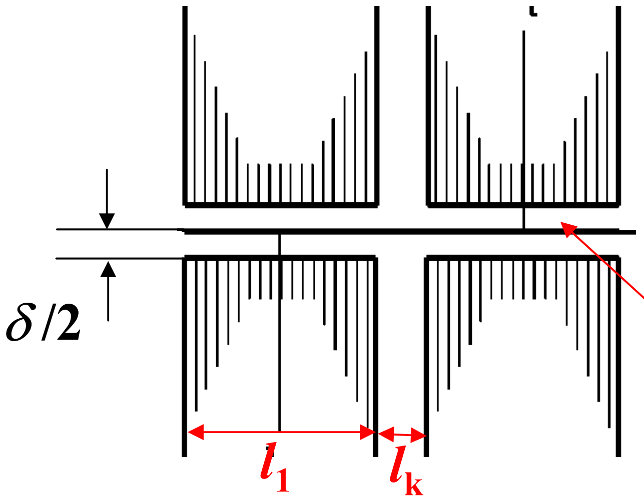
2. Design of Induction Machines

Radial ventilation ducts in stator AND rotor (1)

In stacked stator and rotor configurations radial ventilation ducts in stator and rotor shall allow a direct radial air-flow at low flow resistance!



Radial ventilation ducts in stator AND rotor (2)



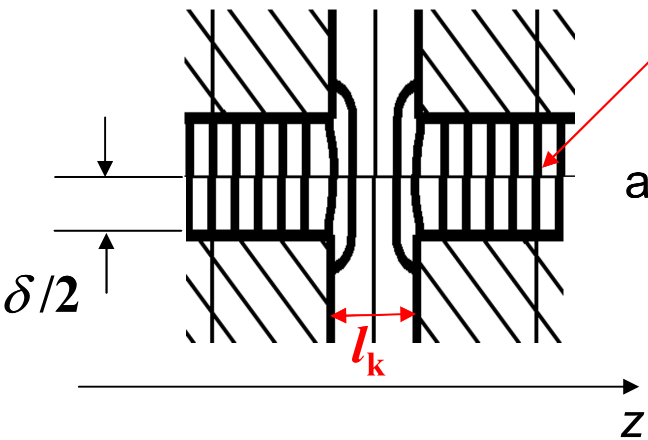
Radial ventilation ducts in stator and rotor:

Symmetric no-load air gap field !

For calculation of l_e use instead of δ only half air gap $\delta/2$!

Symmetry line of field

$$l_e = L / k'_C$$

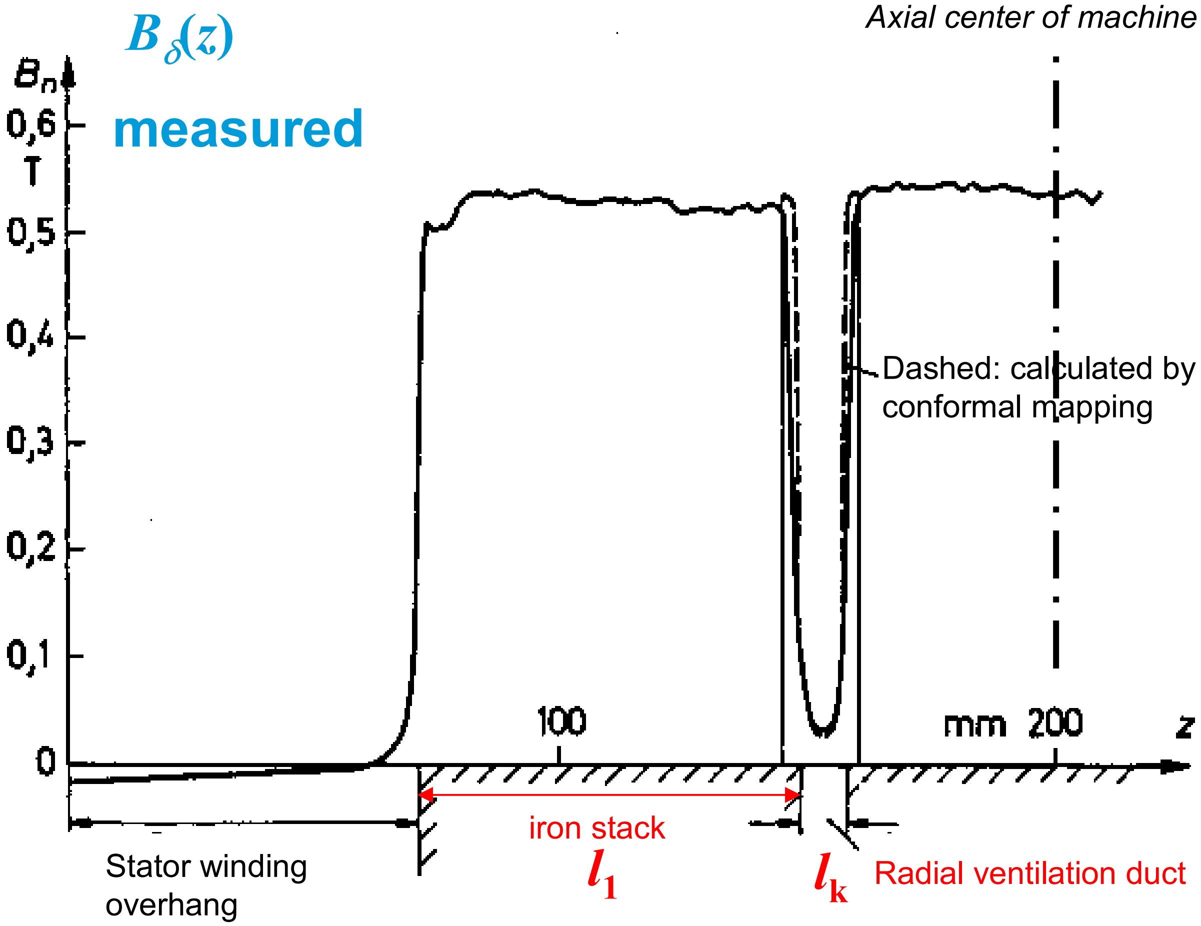


air-gap field

$$k'_C = \frac{l_1 + l_k}{l_1 + l_k - \zeta(h') \cdot (\delta/2)} \qquad h' = \frac{l_k}{\delta/2}$$

$$\delta \rightarrow \delta/2$$

Measured unsaturated air-gap flux density with cooling duct influence



Source: Binder, A.: Archiv f. ET, 1990

<u>*Example:*</u>

8-pole slip-ring induction motor, 30 kW, 50 Hz

2 radial ventilation ducts

$l_{Fe} = z_1 \cdot l_1 = 73.3 \cdot 3 = 220$ mm,

$L = l_{Fe} + (z_1 - 1) \cdot l_k = 220 + 2 \cdot 10$ mm,

$L = 240$ mm, $l_k = 10$ mm,

$z_1 = 3$ packets, $l_1 = 73.3$ mm

2. Design of Induction Machines

Example: Influence of slot openings & radial ducts

500 kW, cage induction machine, $Q_s/Q_r = 60/50$, $B_\delta = 0.858$ T,
$d_{si} = 458$ mm, air gap $\delta = 1.4$ mm, $b_{Qs} = s_{Qs} = 12.5$ mm, $s_{Qr} = 2.5$ mm,
iron stack of stator and rotor: $z_1 = 9$ sections: $l_1 = 42$ mm, 8 radial ducts with width $l_k = 10$ mm

	τ_Q / mm	s_Q / mm	$h = s_Q/\delta$	ζ	k_C
stator	24.0	12.5	8.93	5.72	1.50
rotor	28.8	2.5	1.79	0.47	1.023

Resulting: $k_C = 1.50 \cdot 1.023 = 1.54$

$$\delta_e = k_C \cdot \delta = 1.54 \cdot 1.4 = 2.156 \text{ mm}$$

$$k'_C = \frac{l_1 + l_k}{l_1 + l_k - \zeta(h') \cdot (\delta/2)} = 1.166$$

$$V_\delta = \frac{B_\delta}{\mu_0} \cdot \delta_e = \frac{0.858}{4\pi \cdot 10^{-7}} \cdot 0.002156 = \underline{\underline{1472}} \text{ A}$$

Iron stack length: $l_{Fe} = 9 \cdot l_1 = 9 \cdot 42 = 378$ mm
Total active length: $L = 9 \cdot 42 + 8 \cdot 10 = 378 + 80 = 458$ mm
Equivalent iron length: $l_e = L / k'_C = 458 / 1.166 = \underline{\underline{392}}$ mm

Example: Equivalent iron length

- Iron stack length < Equivalent iron length < Total active length

$$l_{Fe} < l_e < L$$

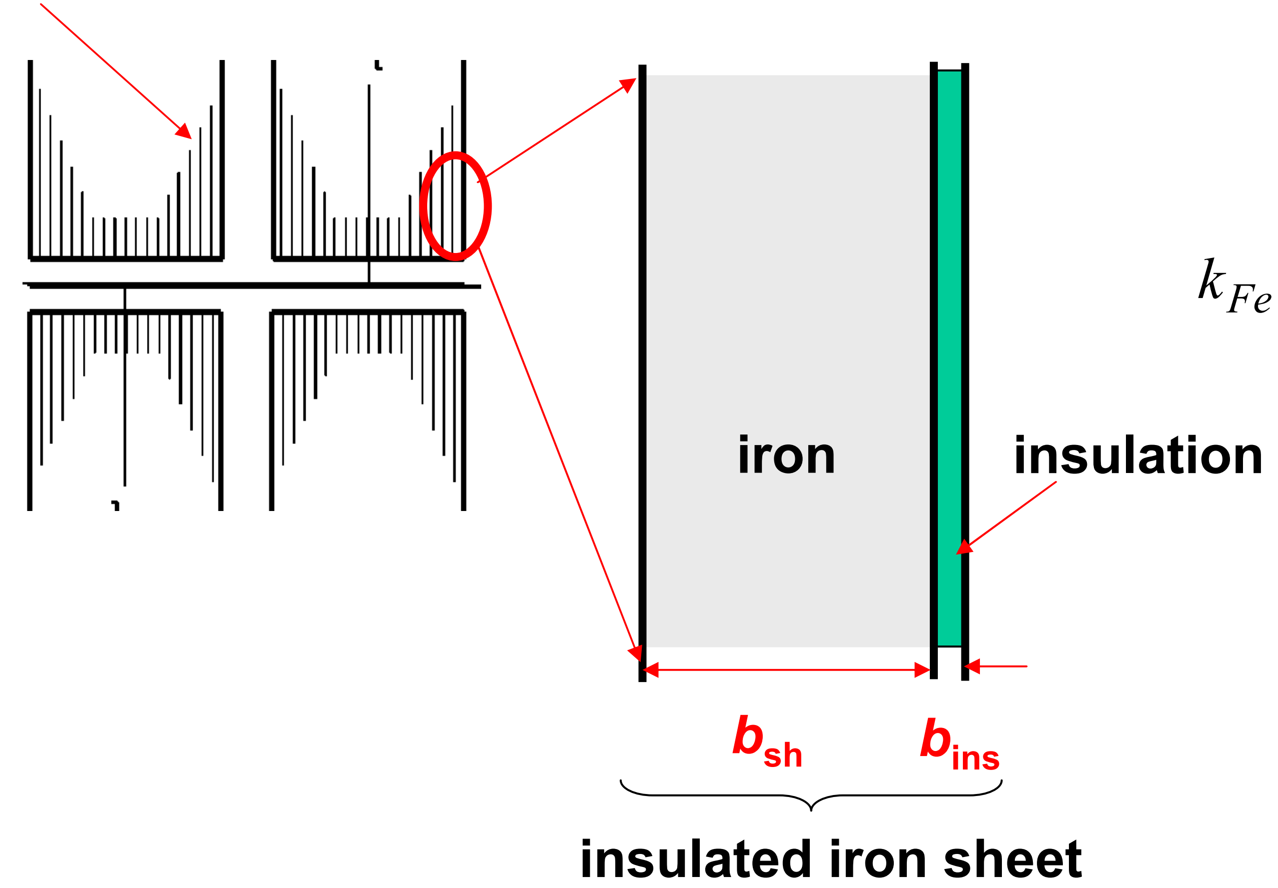
$$378\ \text{mm} < 392\ \text{mm} < 458\ \text{mm}$$

$$100\ \% < 104\ \% < 121\ \%$$

- **If you want to neglect the equivalent iron length effect, then take l_{Fe}, NOT L for the machine calculation!**

Iron fill factor k_{Fe}

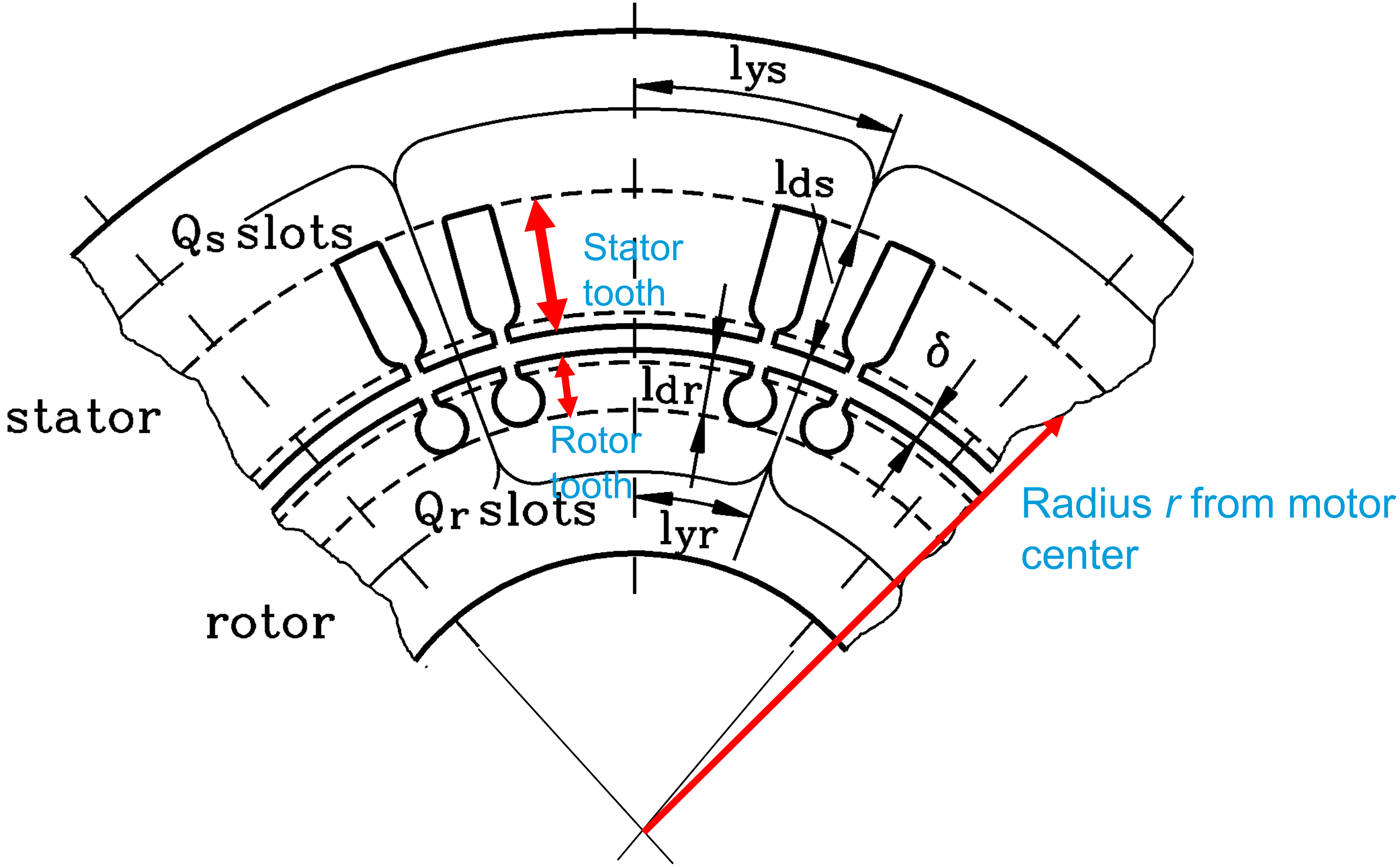
k_{Fe} < 1: Iron fill factor due to insulation of iron sheets



- Usually single-sided insulated iron sheets
- Axial iron fill factor:

$$k_{Fe} = \frac{b_{sh}}{b_{sh} + b_{ins}} \approx 0.95 \ldots 0.97$$

2. Design of Induction Machines
Magnetization of teeth



2. Design of Induction Machines

Apparent tooth flux density B'_d = NO slot flux

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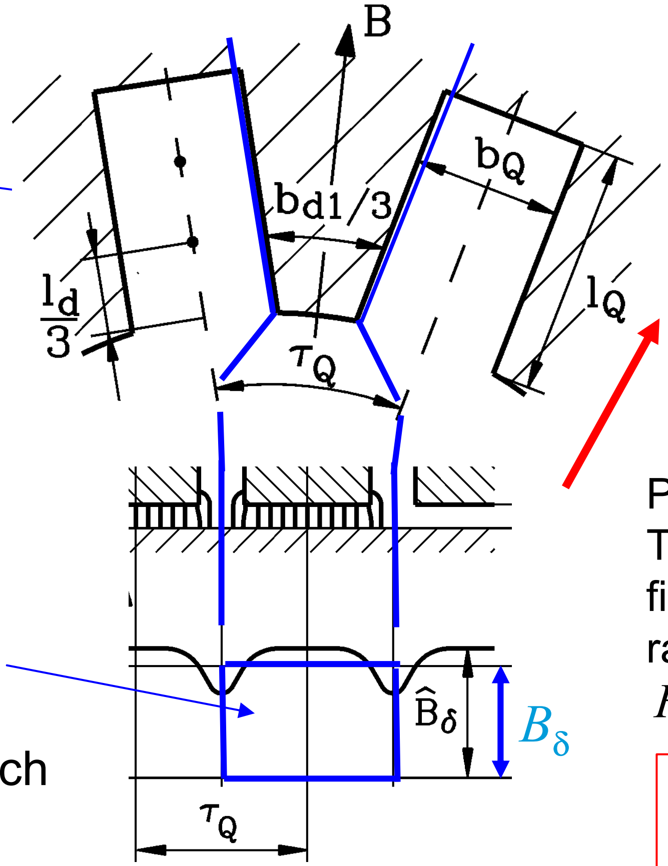
Example: Stator tooth

$$B'_d(r) = \frac{\Phi_{\delta Q}}{k_{Fe} \cdot b_d(r) \cdot l_{Fe}}$$

Flux continuity and
neglecting the slot flux

$$\Phi_{\delta Q} = B_\delta \cdot \tau_Q \cdot l_e$$

Air gap flux per stator slot pitch



Tooth length = Slot length:

$$l_Q = l_d \; (= l_{ds})$$

stator

Radius *r* from motor center

Parallel sided slots:
Tooth flux density B_d and field strength H_d depend on radius *r*

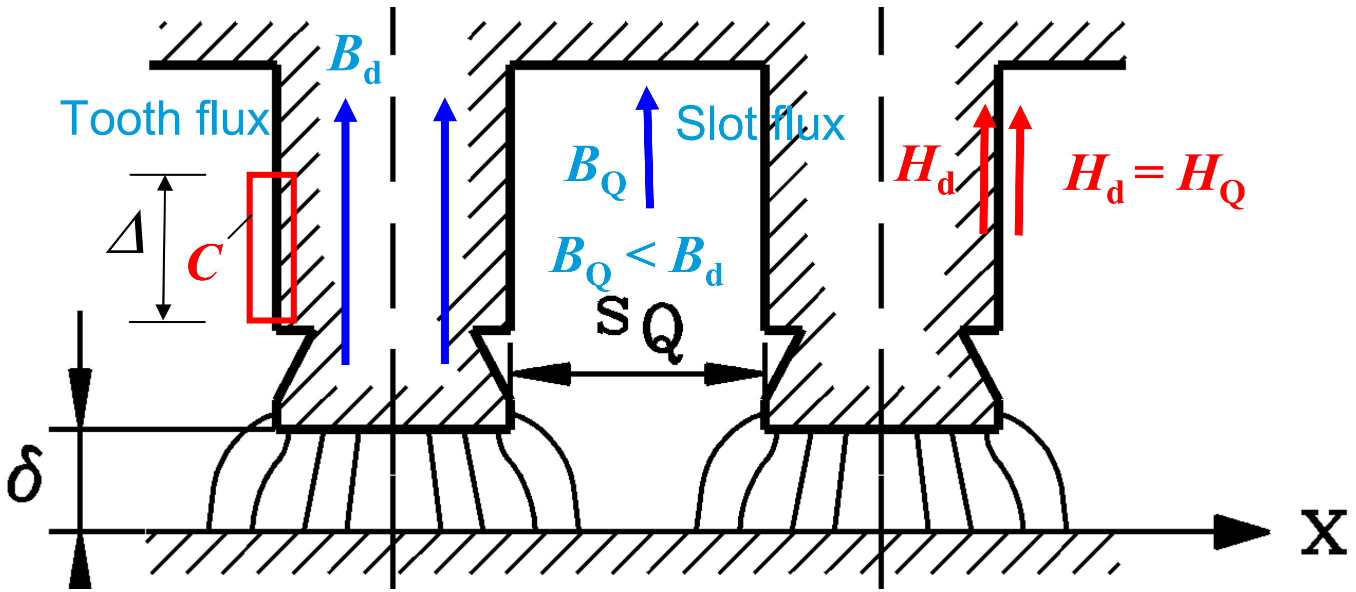
$$H_d(r) = B_d(r) / \mu_{Fe}(r)$$

"Tooth m.m.f. integration":

$$V_{ds} = \int_{d_{si}/2}^{d_{si}/2 + l_{ds}} H_{ds}(r) \cdot dr$$

2. Design of Induction Machines

Parallel-sided tooth and slot flux



$$\oint_C \vec{H} \cdot d\vec{s} = H_d \cdot \Delta - H_Q \cdot \Delta = \Theta = 0 \quad \Rightarrow \quad H_d = H_Q \qquad B_d = \mu_{Fe} H_d >> B_Q = \mu_0 H_Q = \mu_0 H_d$$

Tooth flux: $\Phi_d = B_d(r) \cdot k_{Fe} b_d(r) l_{Fe}$

Slot flux: $\Phi_Q = \mu_0 H_d(r) \cdot l_{Fe} \cdot \left[b_Q(r) + (1 - k_{Fe}) b_d(r) \right]$

Determination of field strength H_d in the tooth (1)

Air gap flux per stator slot pitch: $\Phi_{\delta Q} = B_\delta \cdot \tau_Q \cdot l_e$

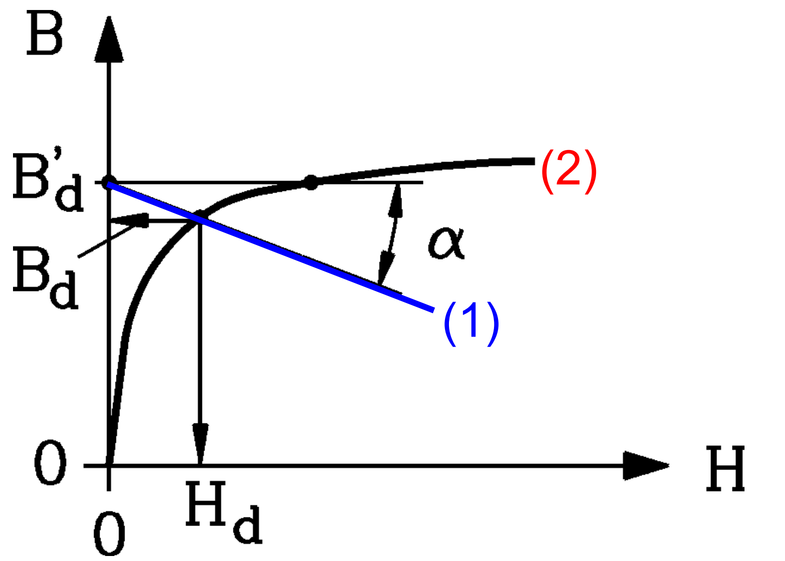
Apparent tooth flux density: $\Phi_{\delta Q} = B'_d(r) \cdot k_{Fe} \cdot b_d(r) \cdot l_{Fe}$

Flux passes mainly in teeth, but also in slots:

$$\Phi_{\delta Q} = \Phi_d + \Phi_Q = B_d(r) \cdot k_{Fe} b_d(r) l_{Fe} + \mu_0 H_d(r) \cdot l_{Fe} \cdot \left[b_Q(r) + (1 - k_{Fe}) b_d(r) \right]$$

Relationship (1) between $B_d(H_d)$: both are unknown!

$$B_d(r) = B'_d(r) - \underbrace{\frac{\mu_0}{k_{Fe}} \cdot \left(\frac{b_Q(r)}{b_d(r)} + 1 - k_{Fe} \right)}_{\mathrm{tg}\alpha} \cdot H_d(r)$$



Iron magnetization characteristic (2):

$B_{Fe}(H_{Fe}) = B_d(H_d)$

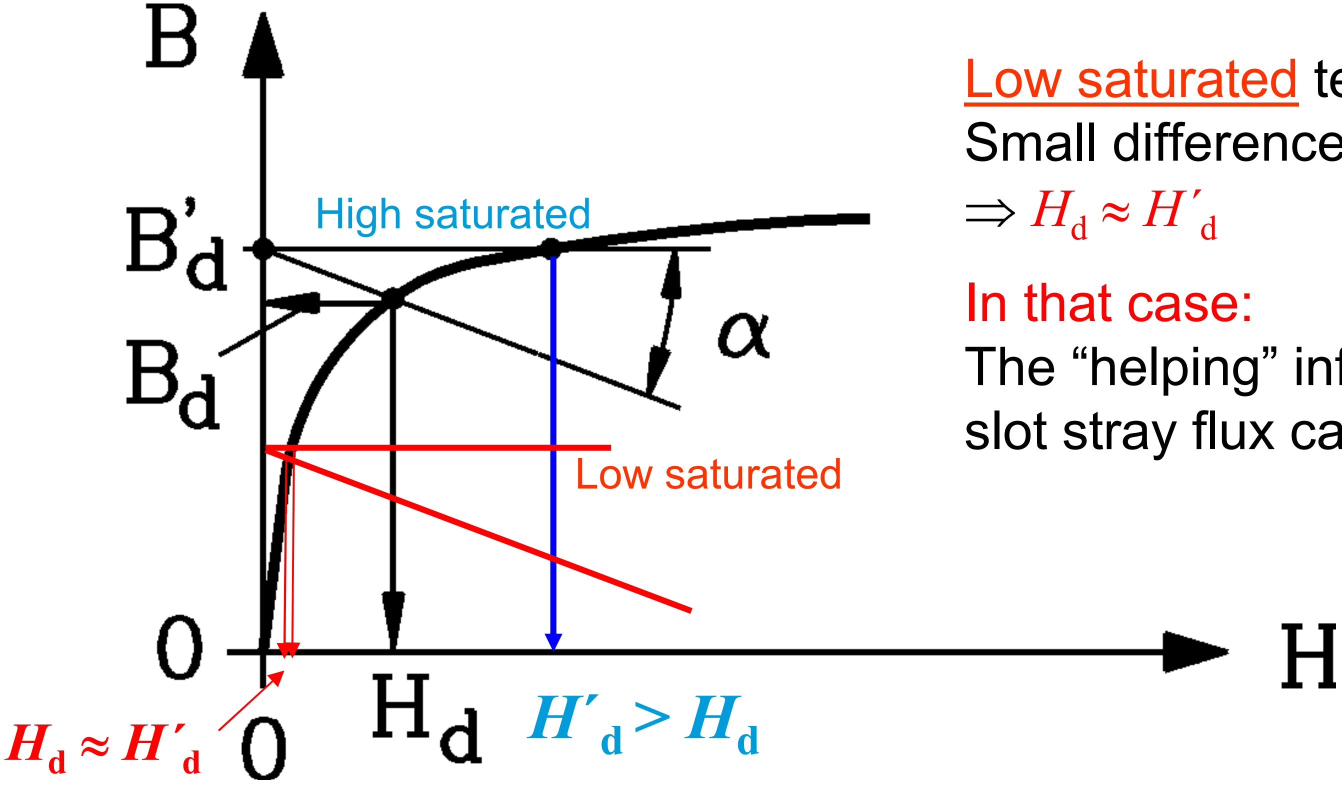
Combination of (1) and (2) yields $B_d(H_d)$!

2. Design of Induction Machines

Determination of field strength H_d in the tooth (2)

Neglecting the parallel slot flux in high saturated teeth $\Rightarrow$

$\Rightarrow$ too high tooth flux density B'_d $\Rightarrow$ too high field strength $H'_d > H_d$



Low saturated teeth:

Small difference between H_d and H'_d

$\Rightarrow H_d \approx H'_d$

In that case:

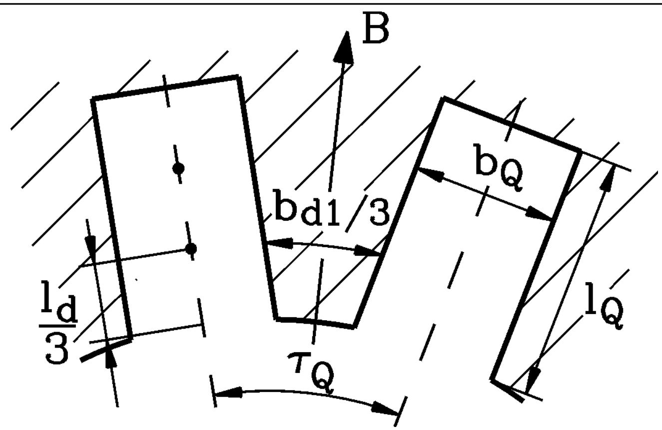
The "helping" influence of the parallel slot stray flux can be neglected!

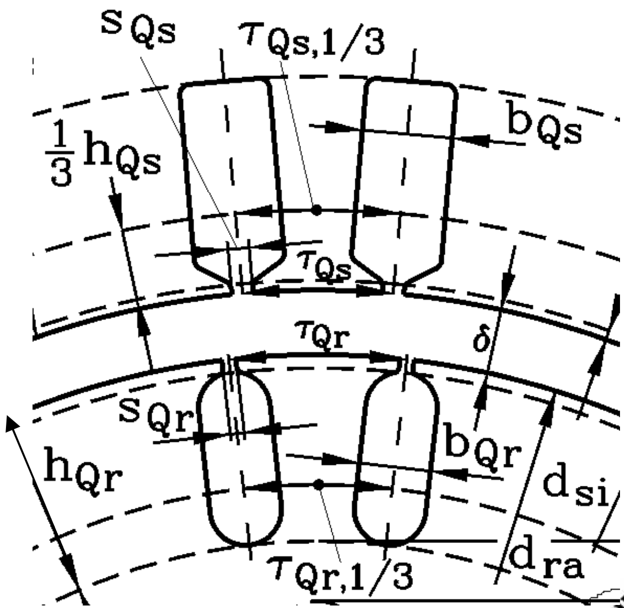
2. Design of Induction Machines

Simplified method to calculate the magnetization of teeth

(1) Parallel slot flux is neglected ⇒
"Apparent" tooth flux density B'_d is taken as real tooth flux density B_d!

(2) Parallel sided slots:
"Apparent" tooth flux density B'_d is taken at 1/3 of the tooth height to avoid the tooth m.m.f. integration!





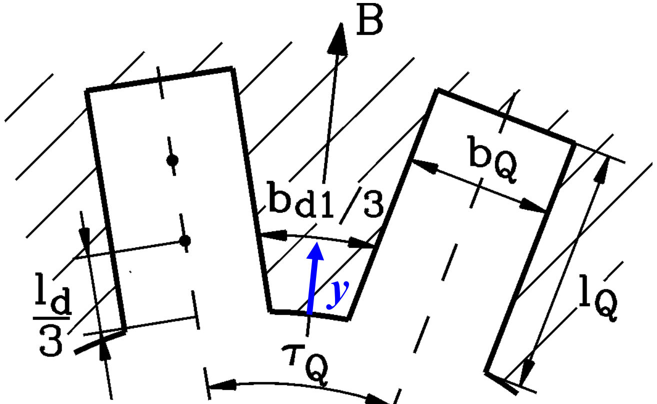
Result: Simplified method:
(1) Neglect slot main flux, (2) Take tooth at 1/3 height

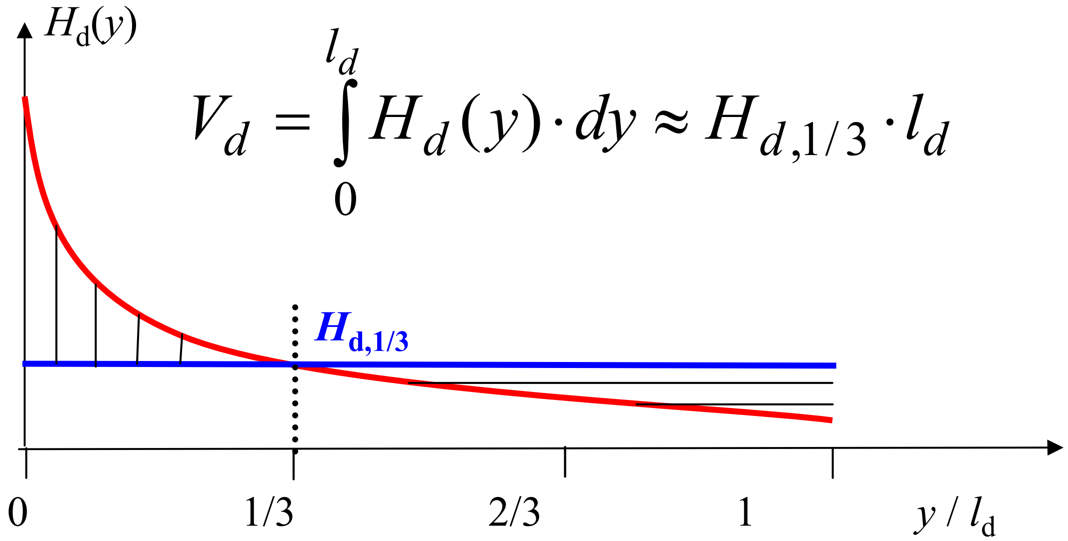
$$B'_{d,1/3} = \frac{B_\delta \cdot \tau_Q \cdot l_e}{k_{Fe} \cdot b_{d,1/3} \cdot l_{Fe}}$$

$$H'_{d,1/3}(B'_{d,1/3}) \approx H_{d,1/3}$$

$$V_d \cong H_{d,1/3} \cdot l_d$$

Tooth flux density at 1/3 of the tooth height for tooth m.m.f.





$$V_d = \int_0^{l_d} H_d(y)\cdot dy \approx H_{d,1/3}\cdot l_d$$

$$B'_{d,1/3} = \frac{B_\delta \cdot \tau_Q \cdot l_e}{k_{Fe}\cdot b_{d,1/3}\cdot l_{Fe}}$$

$$H'_{d,1/3}(B'_{d,1/3}) \approx H_{d,1/3}$$

$$\boxed{V_d \cong H_{d,1/3}\cdot l_d}$$

2. Design of Induction Machines
Example: Teeth and air-gap m.m.f.

500 kW motor: $B_\delta = 0.858$ T, $k_{Fe} = 0.95$, $l_e = 392$ mm, $l_{Fe} = 378$ mm, $d_{si} = 458$ mm, $Q_s/Q_r = 60/50$, $\delta = 1.4$ mm, $\tau_{Qs} = 24.0$ mm, $\tau_{Qr} = 28.8$ mm, iron sheet type III

	h_Q, b_Q (mm)	$l_d = h_Q$ (mm)	$b_{d,1/3}$ (mm)	$B'_{d,1/3}$/T	$H'_{d,1/3}$/A/cm	V_d / A
stator	69.0, 12.5	69.0	13.9	1.615	46.5	321
rotor	43.5, 5.1	43.5	19.9	1.35	8.5	37

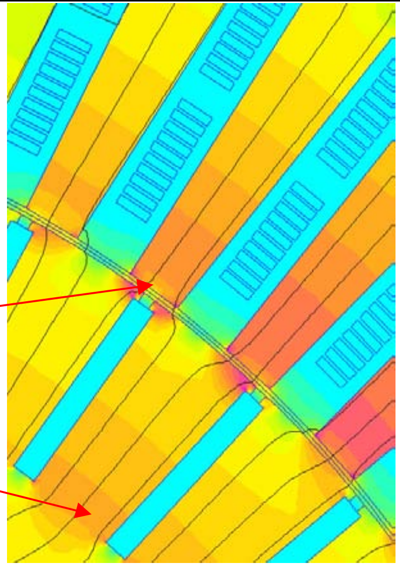
Check of flux density at narrowest tooth width:

	$b_{d,i}$ (mm)	$B'_{d,i}$/T
stator	11.5	$1.95 < 2.4$
rotor	18.0	$1.52 < 2.4$

Limit saturation !

Stator: $b_{ds,i}$

Rotor: $b_{dr,i}$



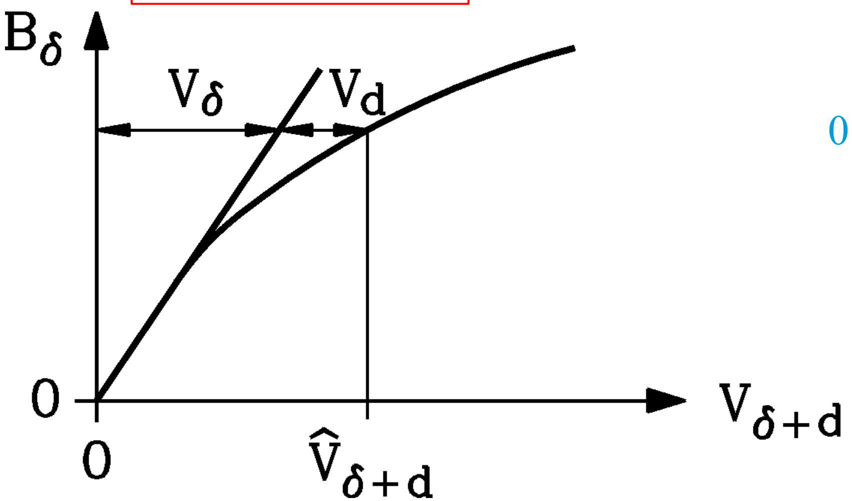
Resulting teeth and air-gap m.m.f.

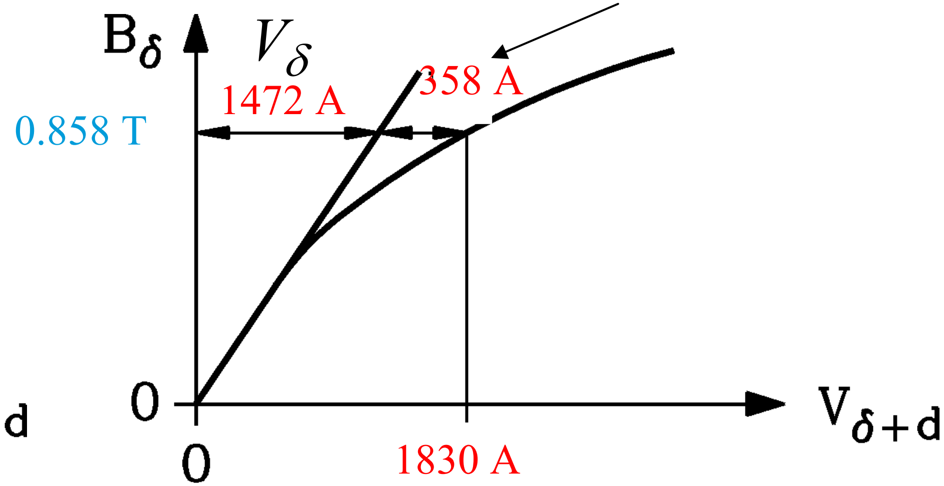
(1) **Neglecting the yoke saturation** ⇒ Start with a certain value of the slot-averaged air-gap flux density B_δ ⇒ Calculate air gap m.m.f. V_δ and the teeth m.m.f. V_{ds} and V_{dr}:

$$\hat{V}_{\delta+d} = V_\delta + V_{ds} + V_{dr} = V_\delta + V_d$$

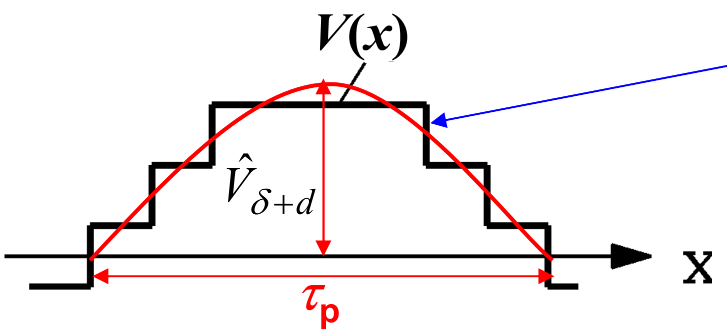
(2) Repeat the calculation for different increasing values B_δ ⇒
⇒ we get the local saturation characteristic: $B_\delta = F(\hat{V}_{\delta+d})$

$$B_\delta = F(\hat{V}_{\delta+d})$$





Distribution of m.m.f. $V(x)$



Distribution of m.m.f. due to distributed stator winding,
a) considering coils placed in slots,
b) but neglecting slot openings

$$\hat{V} = \frac{\sqrt{2}}{\pi} \cdot \frac{m_s}{p} \cdot N_s k_{ws1} I_m$$

I_m : Magnetizing current

(1) Taking only its FOURIER fundamental $\hat{V}_{\delta+d}$ into account
(2) Neglect yoke iron m.m.f.

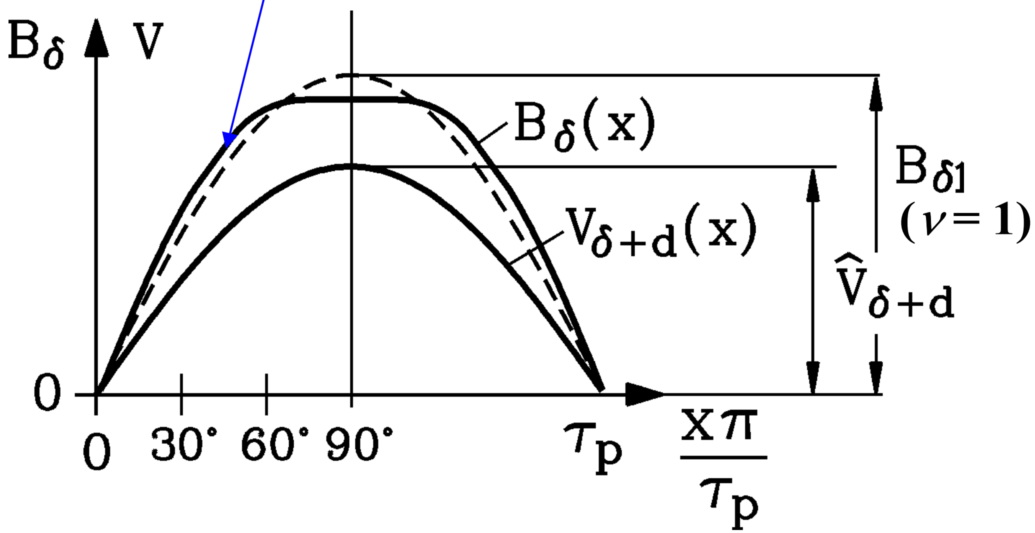
$\Bigg\} \Rightarrow$

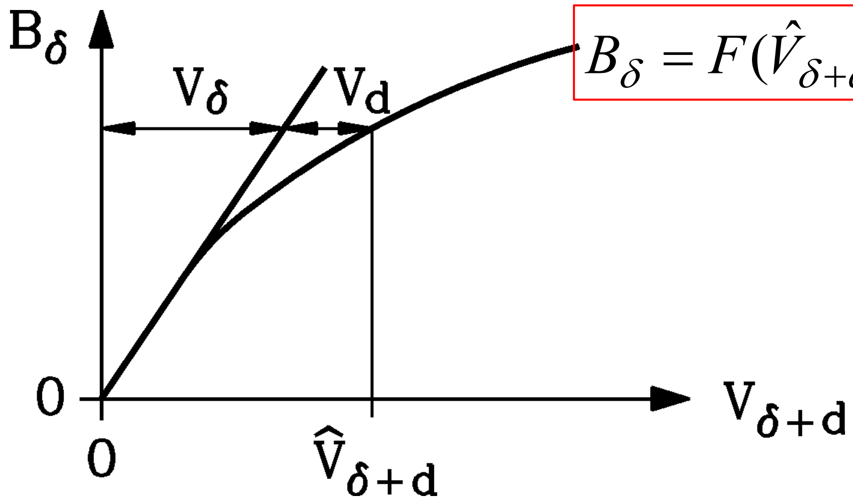
$$\Rightarrow V_{\delta+d}(x) = \hat{V}_{\delta+d} \cdot \sin(x\pi / \tau_p)$$

Result:
Sinusoidal distribution of m.m.f. $V_{\delta+d}(x)$ of air gap and teeth as fundamental of step-like m.m.f. function $V(x)$

Distribution of saturated magnetic air gap flux density

Saturated field distribution, neglecting slotting





$B_\delta = F(\hat{V}_{\delta+d})$

$$V_{\delta+d}(x) = \hat{V}_{\delta+d} \cdot \sin(x\pi/\tau_p)$$

$$B_\delta(x) = F(\hat{V}_{\delta+d} \cdot \sin(x\pi/\tau_p))$$

- **For each amplitude $\hat{V}_{\delta+d}$ and each value x the $B_\delta(x)$-curve is calculated!**

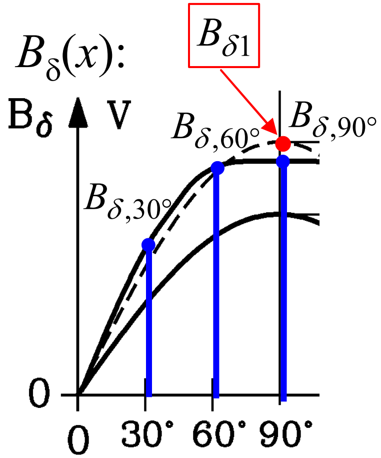
Result:

Tooth saturation leads to flat-topped flux density distribution in air gap

Calculation of $B_{\delta 1}$

- Via FOURIER-series for $\nu = 1$ the air gap amplitude $B_{\delta 1}$ is determined from $B_\delta(x)$:

$$B_{\delta 1} = \frac{1}{\tau_p} \int_0^{2\tau_p} B_\delta(x) \cdot \sin(x\pi/\tau_p) \cdot dx \cong \frac{1}{3} \cdot \left(B_{\delta,30^\circ} + \sqrt{3} \cdot B_{\delta,60^\circ} + B_{\delta,90^\circ}\right)$$



- Result: $B_{\delta 1} = f(\hat{V}_{\delta+d})$

- **Conclusion:** By starting with a certain value of the slot-averaged air-gap flux density B_δ we get the air gap m.m.f. V_δ and the teeth m.m.f. V_{ds} and V_{dr}:

$$\hat{V}_{\delta+d} = V_\delta + V_{ds} + V_{dr}$$

- **Assuming** a sinusoidal distribution of the m.m.f. $V_{\delta+d}(x)$ due to the distributed stator winding we get a non-sinusoidal distribution (usually flat-topped) of $B_\delta(x)$. From that we get via *Fourier*-series the fundamental $B_{\delta 1}$, which induces the sinusoidal internal voltage U_h.

- Repeating the calculation for different increasing values B_δ we get the saturation characteristic!

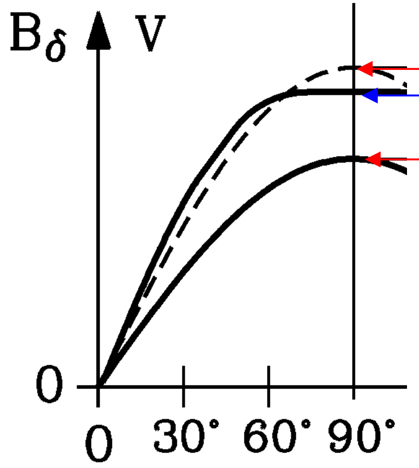
$$U_h \sim B_{\delta 1} = f(\hat{V}_{\delta+d})$$

Simplified calculation of $B_{\delta 1}$

- Detailed calculation via FOURIER-analysis is for hand calculation too long!
- A simplified method was developed by ARNOLD, which allows also a simple consideration of the yoke saturation influence!
- Also this method leads finally to the determination of the internal saturation characteristic:

$$U_h = \sqrt{2}\pi f_s \cdot N_s k_{ws1} \cdot (2/\pi)\tau_p l_e B_{\delta 1}$$

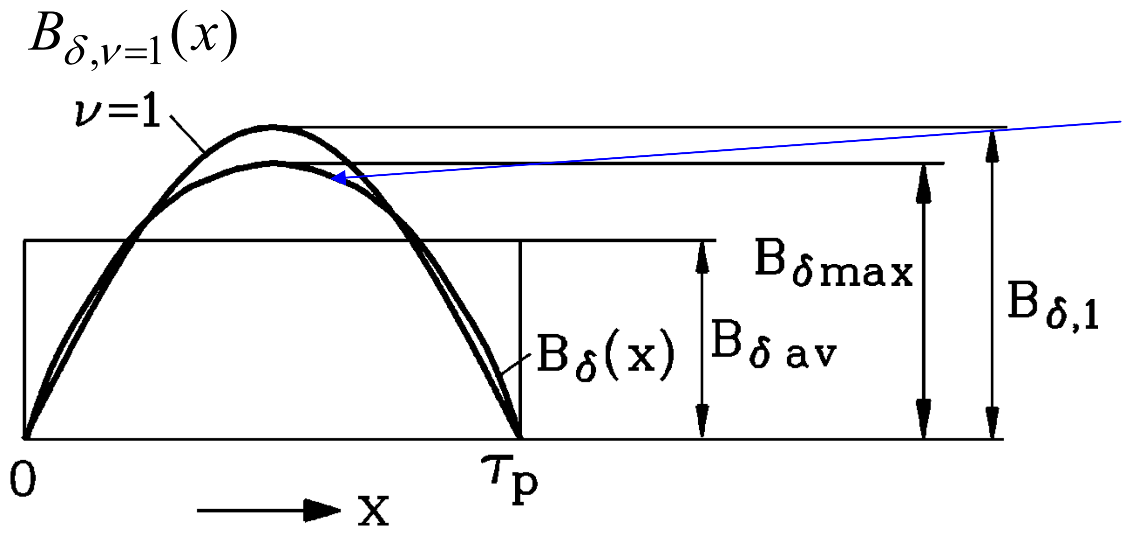
$$U_h \sim B_{\delta 1} = f(\hat{V}_{\delta + d})$$



To be determined:

B_δ = $B_{\delta,max}$: Maximum of flat-topped air-gap flux density distribution

Basic idea of ARNOLD´s method for determination of $B_{\delta 1}$ (1)



Tooth-saturated distribution, neglecting influence of coils in slots, yields flat-topped distribution.

From that, *Fourier* fundamental

$$B_{\delta,\nu=1} = B_{\delta,1}$$

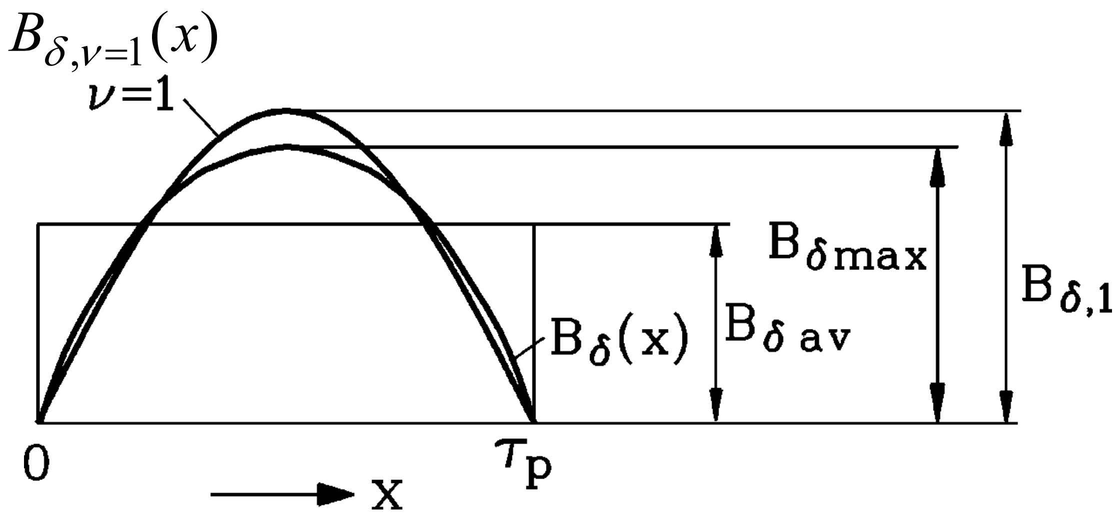
is needed for voltage calculation

$$\Phi_\delta = l_e \int_0^{\tau_p} B_\delta(x) \cdot dx = l_e \cdot \tau_p \cdot B_{\delta,av} \cong l_e \int_0^{\tau_p} B_{\delta,\nu=1}(x) \cdot dx = \frac{2}{\pi} \cdot l_e \cdot \tau_p \cdot B_{\delta,1}$$

The above sketch shows: The air-gap flux per pole Φ_δ of the saturated distribution $B_\delta(x)$ and of fundamental $B_{\delta,\nu=1}(x)$ is nearly the same. So we get:

$$B_{\delta,1} \cong (\pi/2) \cdot B_{\delta,av}$$

Basic idea of ARNOLD´s method for determination of $B_{\delta 1}$ (2)



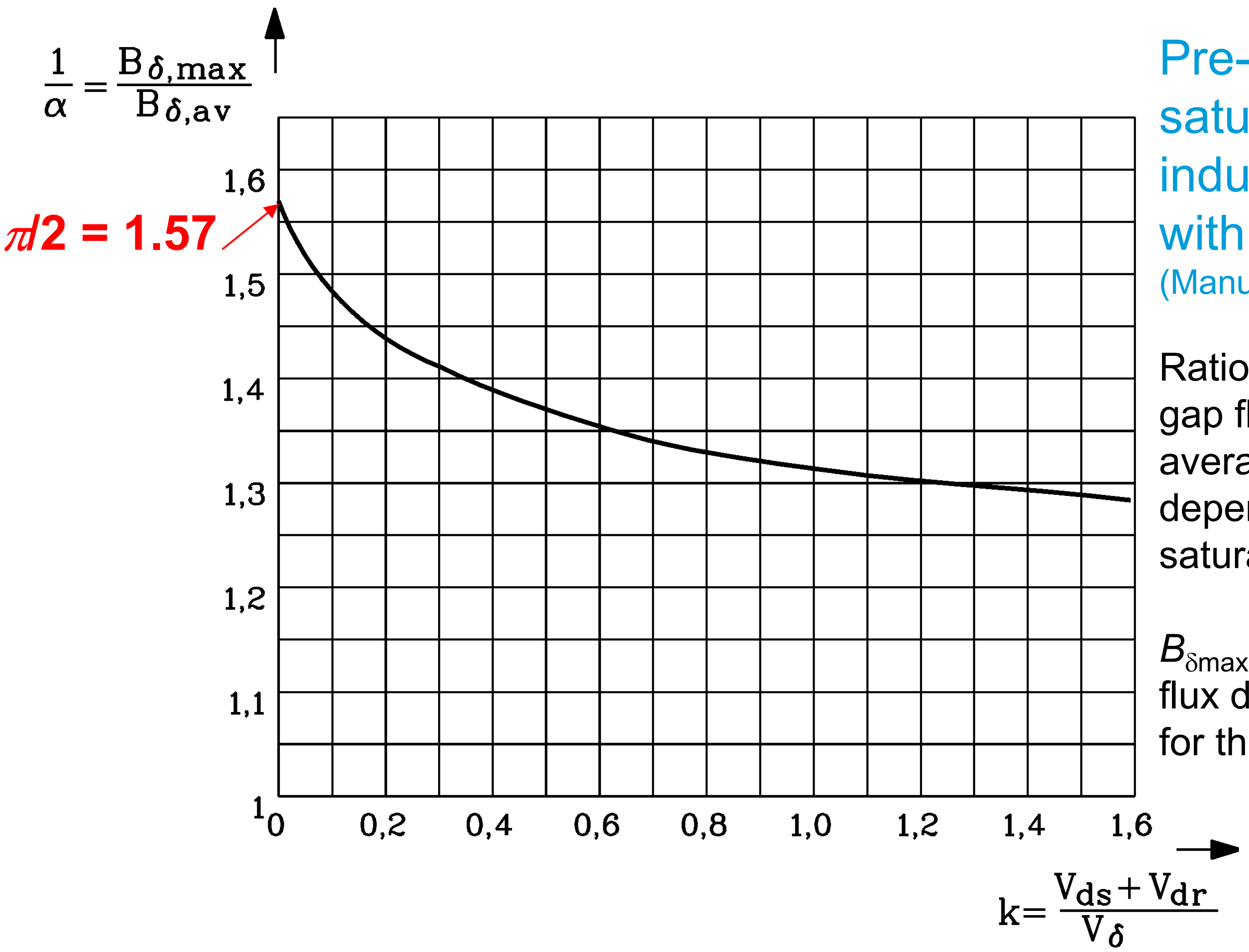
$$B_{\delta,1} \cong (\pi/2) \cdot B_{\delta,av}$$

- Maximum $B_{\delta,max}$ of flat-topped distribution $B_\delta(x)$ must be known for calculating correctly m.m.f. of air gap V_δ.

- $B_{\delta,max}/B_{\delta,av}$ decreases with rising **degree of tooth saturation** V_{ds+dr}/V_δ from (unsaturated) value $\pi/2$ = 1.57 ($B_{\delta,max} = B_{\delta 1}$) down to typically 1.3 (theoretical limit 1 at very high saturation).

2. Design of Induction Machines

ARNOLD´s iterative solution of flat top ratio $B_{\delta,max}/B_{\delta,av}$



Pre-calculated averaged saturation result for many induction machines with iron sheet type III:

(Manufacturer "a")

Ratio of maximum of flat-topped air gap flux density distribution versus average flux density $B_{\delta max}/B_{\delta,av}$, depending on degree of tooth saturation V_{ds+dr}/V_δ

$B_{\delta max} = B_\delta$ is the maximum air-gap flux density value, which is needed for the calculation of V_{ds+dr} and V_δ

Example of ARNOLD´s iteration method

TECHNISCHE UNIVERSITÄT DARMSTADT

550 kW motor, rated voltage 6.6 kV Y, 50 Hz, σ_s = 0.05, N_s = 200, k_{ws} = 0.91

$$U_h = U_N / (\sqrt{3} \cdot (1+\sigma_s)) = 6600 / (\sqrt{3} \cdot 1.05) = 3629 V$$

$$B_{\delta,1} = \frac{U_h}{\sqrt{2}\pi f_s \cdot N_s k_{ws1} \cdot (2/\pi)\tau_p l_e} = \frac{3629}{\sqrt{2}\pi \cdot 50 \cdot 200 \cdot 0.91 \cdot (2/\pi) \cdot 0.36 \cdot 0.392} = 1.0 T$$

$$B_{\delta,av} \cong (2/\pi) \cdot B_{\delta,1} = (2/\pi) \cdot 1.0 = 0.6366 T \quad \Rightarrow \quad B_{\delta,\max} = ?$$

Assumption (1st iteration): $B_{\delta,max}/B_{\delta,av} = B_\delta/B_{\delta,av} = 1.41 \Rightarrow V_{ds+dr}/V_\delta = 0.3$:

$$B_\delta = 1.41 B_{\delta,av} = 1.41 \cdot 0.6366 = 0.897 T$$

$$B'_{ds,1/3} = \frac{B_\delta \cdot \tau_{Qs} \cdot l_e}{k_{Fe} \cdot b_{ds,1/3} \cdot l_{Fe}} = 1.69 T \qquad V_{ds} = 74 \cdot 6.9 = 511 A$$

$$V_{dr} = 13 \cdot 4.35 = 57 A \qquad V_\delta = \frac{B_\delta}{\mu_0} \cdot \delta_e = \frac{0.897}{4\pi \cdot 10^{-7}} \cdot 0.002156 = \underline{\underline{1539 A}}$$

V_{ds+dr}/V_δ = (511+57)/1539 = 0.37

Teeth and air-gap m.m.f.

Assumption (2nd iteration): Average of old and new value:

V_{ds+dr}/V_δ = (0.37 + 0.3)/2 = 0.335

	V_{ds+dr}/V_δ	$B_\delta/B_{\delta,av}$	B_δ	$B'_{ds,1/3}$	$B'_{dr,1/3}$	$H'_{ds,1/3}$	$H'_{dr,1/3}$	V_{ds}	V_{dr}	V_δ	V_{ds+dr}/V_δ
	-	-	T	T	T	A/cm	A/cm	A	A	A	-
1st	0.3	1.41	0.897	1.69	1.42	74	13	511	57	1539	0.37
2nd	0.335	1.40	0.891	1.68	1.41	70	12.1	483	52.6	1529	0.35

After 2nd iteration values V_{ds+dr}/V_δ at beginning and end of iteration differ only by (0.35-0.335)/0.335 = 4.5% < 5% !

We take accuracy limit 5%:

So iteration is ended, taking as final values:

$B_\delta = B_{\delta,max}$ = 0.891 T and V_{ds+dr}/V_δ = 0.35.

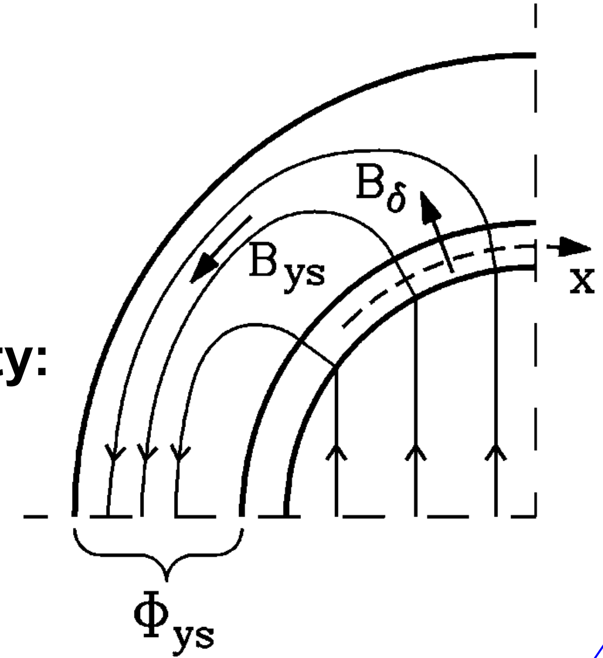
Magnetization of yokes

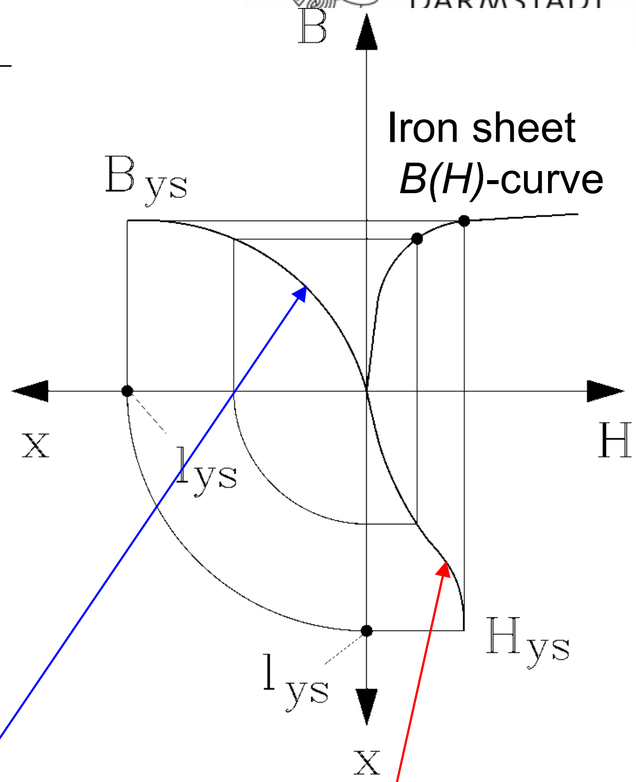
- **Yoke flux:**

$$\Phi_{ys} = (\Phi_{\delta} + \Phi_{\sigma s})/2$$
$$= (1+\sigma_s)\cdot\Phi_{\delta}/2$$

- **Maximum yoke flux density:**

$$B_y = \frac{\Phi_y}{h_y \cdot l_{Fe} \cdot k_{Fe}}$$



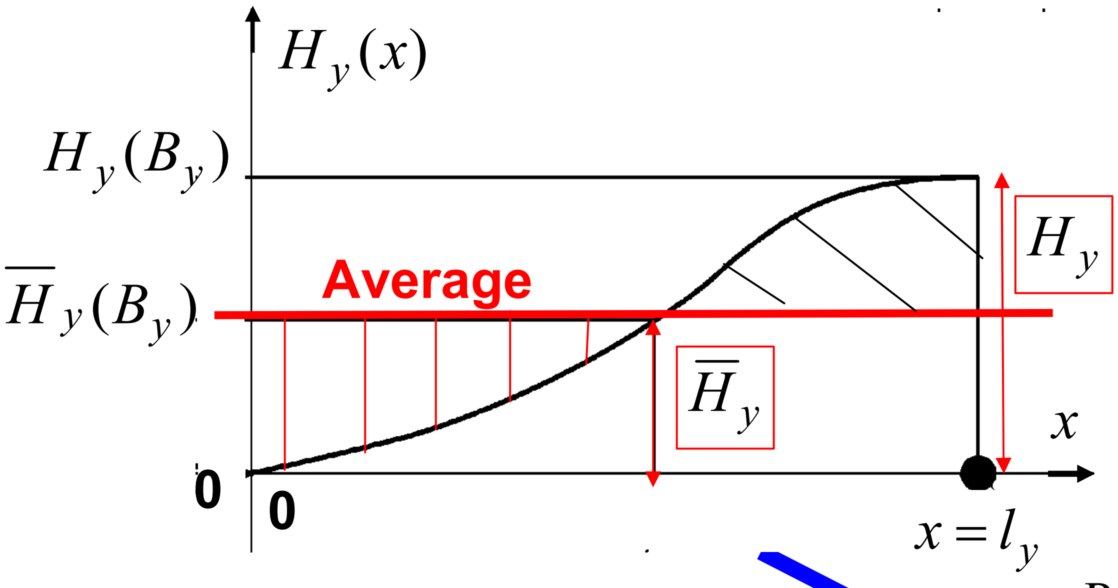


- Yoke flux density $B_y(x)$ is **integral** of air gap flux density distribution $B_\delta(x)$, hence even a flat topped $B_\delta(x)$-distribution leads to a **nearly sinusoidal yoke flux density distribution $B_y(x)$!**

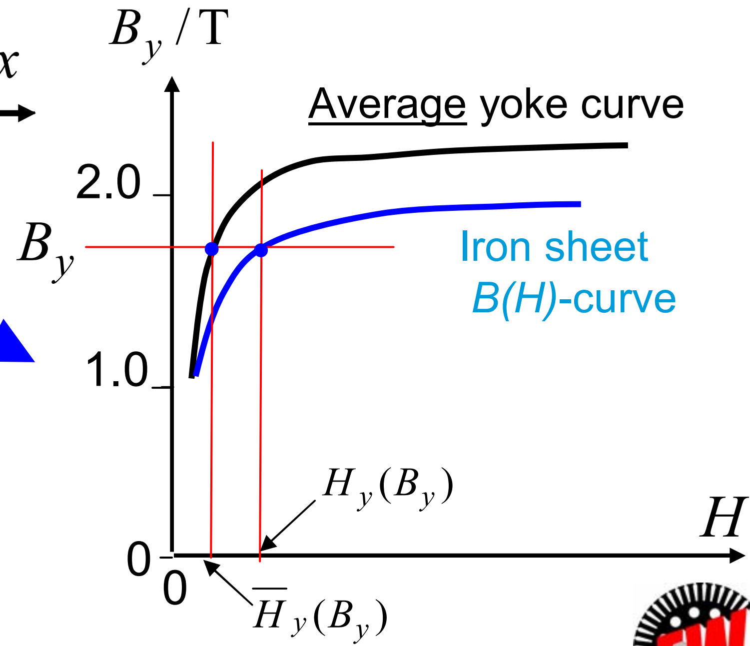
- **But** non-linear *B(H)*-magnetization of iron therefore yields **non-sinusoidal distribution** of magnetic yoke field strength $H_y(x)$!

2. Design of Induction Machines

Magnetic tangential field strength in the yoke



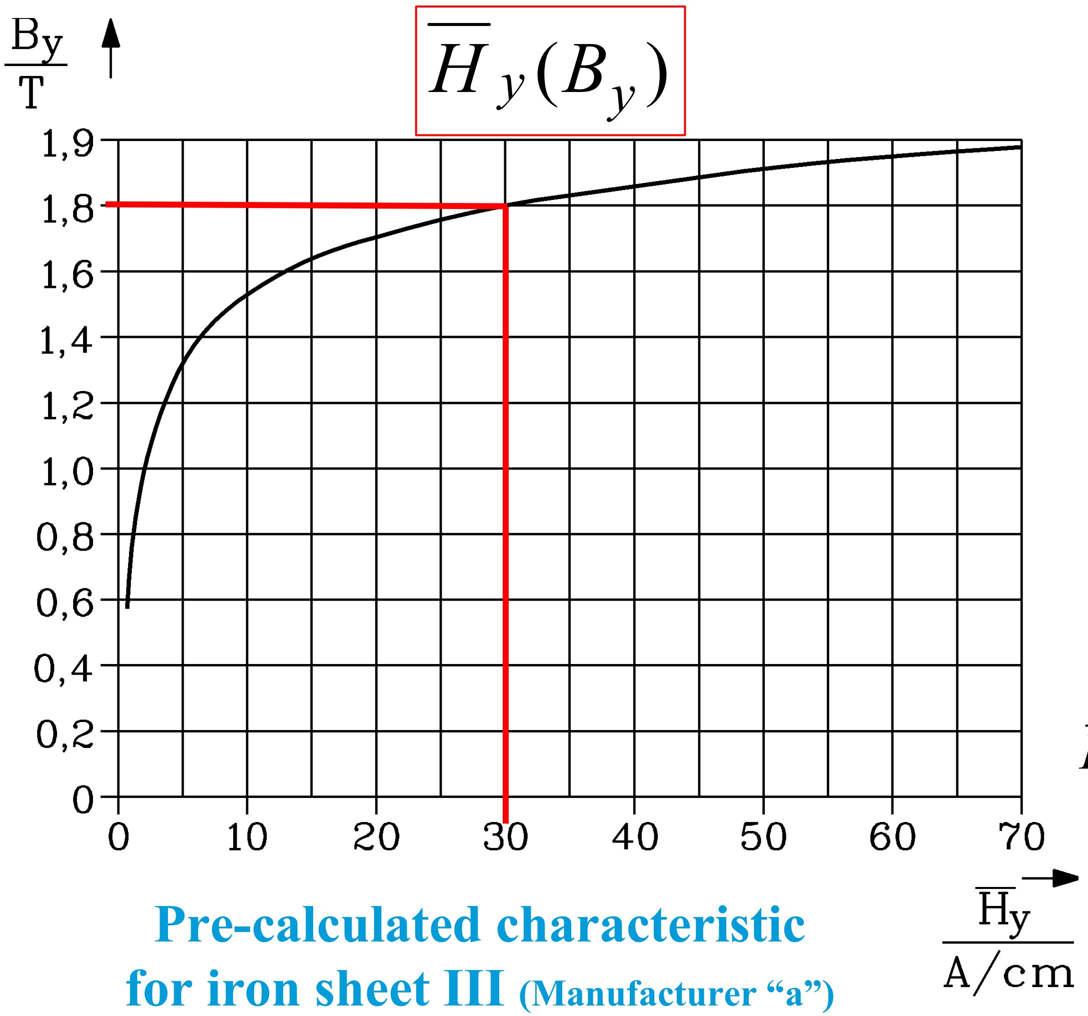
$$H_y(B_y) > \overline{H}_y(B_y)$$



M.m.f. of yoke:

$$V_y = \int_0^{l_y} H_y(x) \cdot dx = l_y \cdot \overline{H}_y$$

Average yoke magnetic field strength: $H_y = \overline{H}_y(B_y)$



Pre-calculated characteristic for iron sheet III (Manufacturer "a")

- **M.m.f. of yoke:**

$$V_y = \int_0^{l_y} H_y(x) \cdot dx = l_y \cdot \overline{H}_y$$

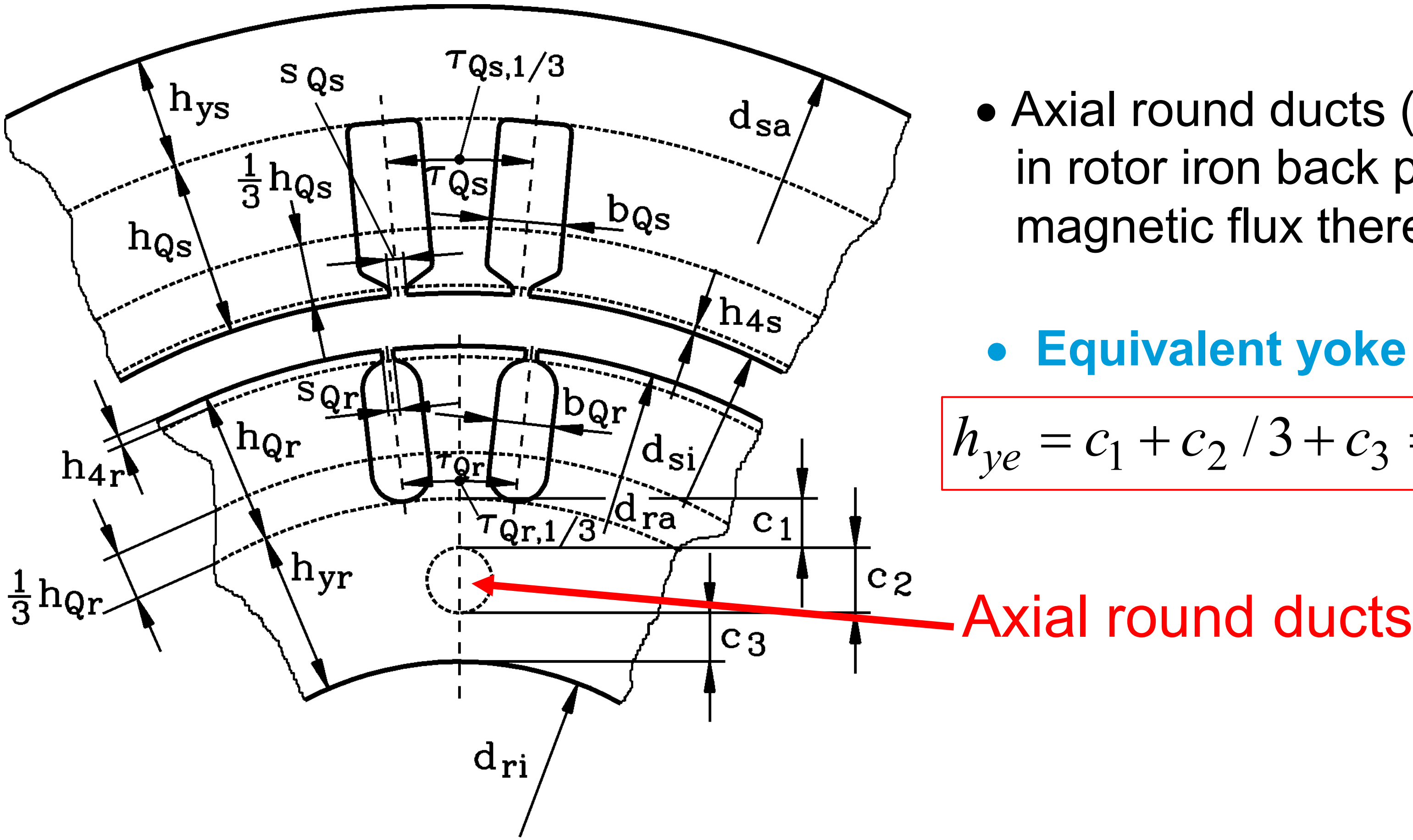
- ***Example:***

- Peak yoke flux density B_y = 1.8 T
- Corresponding field strength (iron sheet type A, III): H_y = 120 A/cm
- Average yoke field strength:

$$\overline{H}_y(1.8\text{T}) = 30\text{A/cm} << H_y(1.8\text{T}) = 120\text{A/cm}$$

2. Design of Induction Machines

Axial cooling ducts: Equivalent yoke height h_{ye}

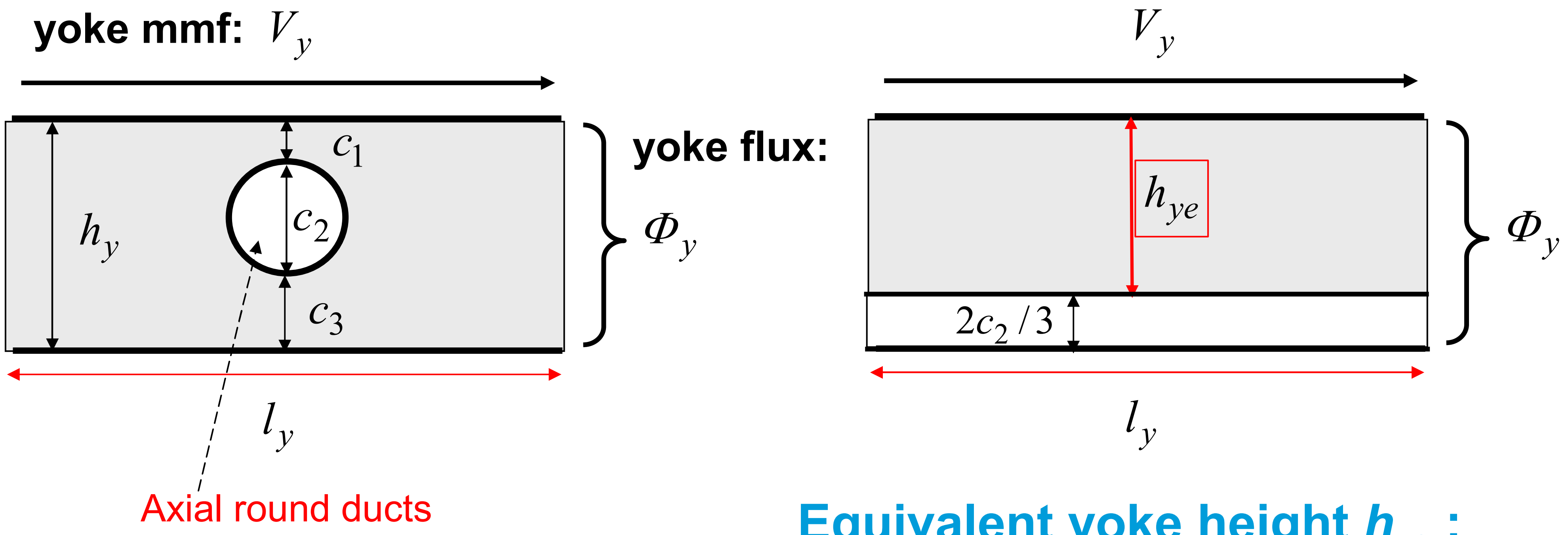


- Axial round ducts (diameter c_2) in rotor iron back prohibit magnetic flux there!

- **Equivalent yoke height h_{ye}:**

$$h_{ye} = c_1 + c_2/3 + c_3 = h_y - (2/3)\cdot c_2$$

Equivalent yoke height h_{ye} due to axial cooling ducts

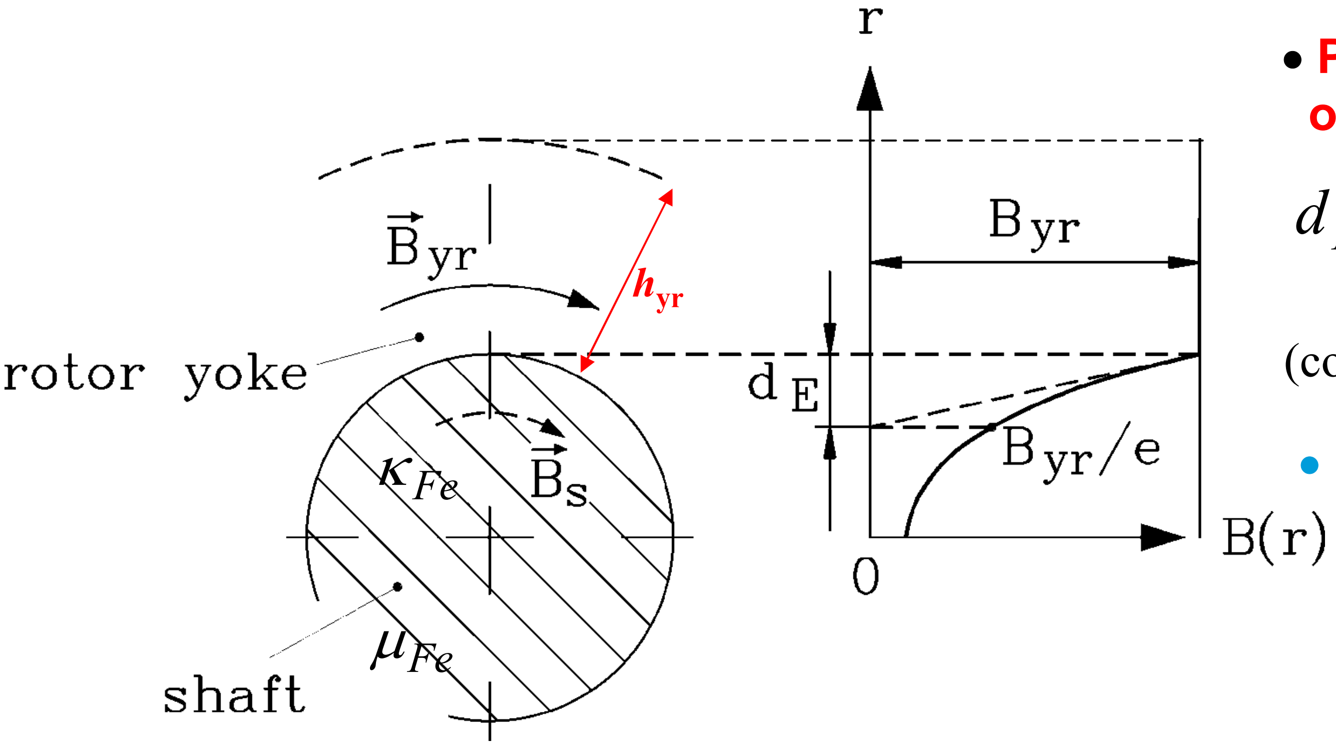


Equivalent yoke height h_{ye} :

$$h_{ye} = c_1 + c_2/3 + c_3 = h_y - (2/3)\cdot c_2$$

Flux penetration in shaft reduces rotor yoke saturation

- Penetration of rotor flux into shaft and reduction of shaft flux by shaft eddy currents = **Skin effect in rotor shaft with rotor frequency** $s \cdot f_s$



- **Penetration depth d_E of yoke flux into iron shaft:**

$$d_E = \frac{1}{\sqrt{\pi \cdot s \cdot f_s \cdot \mu_{Fe} \cdot \kappa_{Fe}}}$$

(constant parameters assumed)

- **Equivalent yoke height $h_{yr,e}$:**

$$h_{yr,e} = h_{yr} + d_E$$

Yoke flux densities

Example: 550 kW 4-pole motor, 6.6 kV Y, 50 Hz, d_{si} = 458 mm, δ = 1.4 mm, l_{ds} = 69 mm, l_{dr} = 43.5 mm, h_{ys} = 77 mm, h_{yr} = 84.1 mm, slip 1.5% shaft diameter d_{ri} = 200 mm, c_2 = 30 mm, B_δ = 0.891 T, $B_{\delta,1}$ = 1.0 T

- Stator maximum yoke flux density: $B_{ys} = \dfrac{\Phi_\delta \cdot (1+\sigma_s)/2}{h_{ys} \cdot l_{Fe} \cdot k_{Fe}}$

$$B_{ys} = \frac{(2/\pi)\cdot B_{\delta,1}\cdot\tau_p\cdot l_e\cdot(1+\sigma_s)/2}{h_{ys}\cdot l_{Fe}\cdot k_{Fe}} = \frac{(2/\pi)\cdot 1.0\cdot 0.36\cdot 0.392\cdot 1.05/2}{0.077\cdot 0.378\cdot 0.95} = 1.70\mathrm{T}$$

- Rotor maximum yoke flux density:

$$d_E = \frac{1}{\sqrt{\pi\cdot s\cdot f_s\cdot \mu_{Fe}\cdot \kappa_{Fe}}} = \frac{1}{\sqrt{\pi\cdot 0.015\cdot 50\cdot 1000\cdot 4\pi\cdot 10^{-7}\cdot 5\cdot 10^6}} = 8.2\mathrm{mm}$$

$$h_{yr,e} = h_{yr} - (2/3)\cdot c_2 + d_E = 84.1 - (2/3)\cdot 30 + 8.2 = 72.3\mathrm{mm}$$

$$B_{yr} = \frac{\Phi_\delta/2}{h_{yr,e}\cdot l_{Fe}\cdot k_{Fe}} = \frac{(2/\pi)\cdot B_{\delta,1}\cdot\tau_p\cdot l_e/2}{h_{yr,e}\cdot l_{Fe}\cdot k_{Fe}} = \frac{(2/\pi)\cdot 1.0\cdot 0.36\cdot 0.392/2}{0.0723\cdot 0.378\cdot 0.95} = 1.73\mathrm{T}$$

2. Design of Induction Machines
Yoke radii, lengths & m.m.f.

$$r_{ys} = (d_{si} + h_{ys})/2 + l_{ds} = (458 + 72)/2 + 69 = 334\text{mm}$$

$$\boxed{l_{ys} = r_{ys} \cdot \pi/(2p) = 262\text{mm}}$$

$$r_{yr} = (d_{si} - h_{yr,e})/2 - l_{dr} - \delta = (458 - 72.3)/2 - 43.5 - 1.4 = 148.25\text{mm}$$

$$\boxed{l_{yr} = r_{yr} \cdot \pi/(2p) = 116.4\text{mm}}$$

- Yoke m.m.f.: According to $\overline{H}_y(B_y)$: $B_{ys} = 1.7T \Rightarrow \overline{H}_{ys} = 20.4\text{A/cm}$
  $B_{yr} = 1.73T \Rightarrow \overline{H}_{yr} = 23.9\text{A/cm}$

$$\boxed{V_{ys} = \overline{H}_{ys} l_{ys} = 20.4 \cdot 26.2 = \underline{\underline{534\text{A}}}}$$

$$\boxed{V_{yr} = \overline{H}_{yr} l_{yr} = 23.9 \cdot 11.64 = \underline{\underline{278\text{A}}}}$$

- Total m.m.f.: $\boxed{V_m = V_\delta + V_{ds} + V_{dr} + V_{ys} + V_{yr}} = \frac{\sqrt{2}}{\pi} \cdot \frac{m_s}{p} \cdot N_s k_{ws1} I_m$

2. Design of Induction Machines

Saturated magnetizing reactance X_h

a) For **infinite iron permeability**: $X_{h,\infty} = 2\pi f_s \cdot \mu_0 \cdot (N_s k_{ws1})^2 \cdot \frac{2 \cdot m_s}{\pi^2 \cdot p} \cdot \frac{l_e \cdot \tau_p}{\delta_e}$

$$X_{h,\infty} = 2\pi \cdot 50 \cdot 4\pi \cdot 10^{-7} \cdot (200 \cdot 0.91)^2 \cdot \frac{2 \cdot 3}{\pi^2 \cdot 2} \cdot \frac{0.392 \cdot 0.36}{0.002156} = 260.2\,\Omega$$

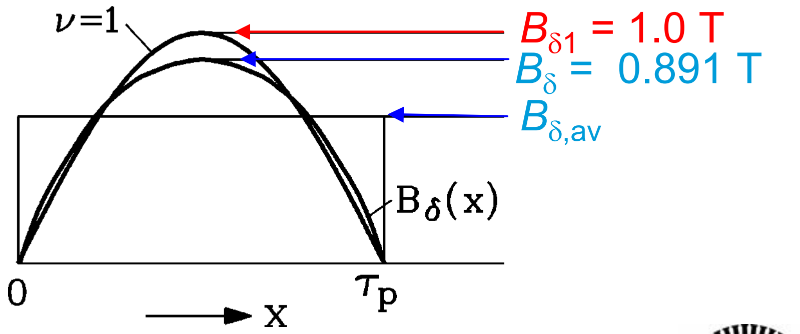
b) With **finite iron permeability:** X_h is reduced by the ratio $V_{\delta,1} / V_m$

$$V_\delta = \frac{B_\delta}{\mu_0} \cdot \delta_e = \frac{0.891}{4\pi \cdot 10^{-7}} \cdot 0.002156 = \underline{\underline{1529\text{A}}}, \quad V_{\delta,1} = \frac{B_{\delta,1}}{\mu_0} \cdot \delta_e = \frac{1.0}{4\pi \cdot 10^{-7}} \cdot 0.002156 = \underline{\underline{1716\text{ A}}}$$

$$V_{\text{ds+dr}} = 483 + 52.6 = 535.6\text{ A}, V_{ys} = 534\text{A}, V_{yr} = 278\text{A}$$

$$V_m = 1529 + 483 + 52.6 + 534 + 278 = 2877\text{A}$$

$$X_h = \frac{V_{\delta,1}}{V_m} \cdot X_{h,\infty} = \frac{1716}{2877} \cdot 260.2 = \underline{\underline{155.2\,\Omega}}$$



Saturated magnetizing current I_m

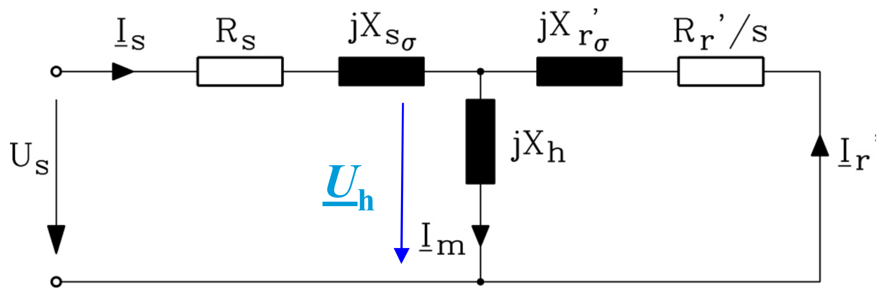
- **Magnetizing current:** $I_m = \dfrac{V_m}{\dfrac{\sqrt{2}}{\pi}\cdot\dfrac{m_s}{p}\cdot N_s k_{ws1}} = \dfrac{2877}{\dfrac{\sqrt{2}}{\pi}\cdot\dfrac{3}{2}\cdot 200\cdot 0.91} = \underline{\underline{23.4\ \text{A}}}$

- Corresponding induced voltage:

$$U_h = \sqrt{2}\pi f_s\cdot N_s k_{ws1}\cdot(2/\pi)\tau_p l_e B_{\delta 1}$$

$$U_h = \sqrt{2}\pi\cdot 50\cdot 200\cdot 0.91\cdot(2/\pi)\cdot 0.36\cdot 0.392\cdot 1.0 = \underline{\underline{3629\ \text{V}}}$$

- Saturated magnetizing reactance: $X_h = U_h / I_m = 3629/23.4 = \underline{\underline{155.1\ \Omega}}$



Summary:
Design of main flux path of magnetic circuit

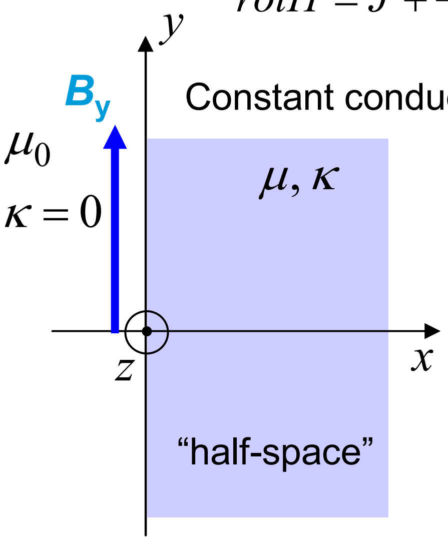
- Air gap, tooth and yoke section for magnetic main flux path
- Different steel sheet grades for low iron losses
- CARTER´s coefficient for influence of slot opening on air-gap MMF
- Equivalent iron length for influence of radial ventilation ducts on air-gap MMF
- At high saturation: Slot parallel flux relieves tooth saturation
- Yoke magnetization is sinusoidal distributed = average yoke MMF curve
- Axial rotor cooling ducts reduce rotor yoke height
- Magnetic shaft relieves rotor yoke saturation
- ARNOLD´s method simplifies magnetic circuit calculation
- Result is the saturated magnetizing (main) inductance

$$B_{\delta 1} = 1\ \mathrm{T} \Rightarrow I_{\mathrm{m}} = 23.4\ \mathrm{A} \leftrightarrow 40\%\ \text{of}\ I_{\mathrm{N}} = 59\ \mathrm{A}$$

Derivation of penetration depth in a massive "half-space" conductor (1)

Repetition

$$rot\vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \approx \vec{J}$$ No wave propagation $$rot\vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad div\vec{B} = 0 \qquad \vec{B} = \mu\vec{H} \qquad \kappa\vec{E} = \vec{J}$$



Constant conductivity κ & permeability μ, two-dimensional field: $\partial\cdot/\partial z = 0,\ B_z = H_z = 0$

$$rot\vec{H} = \begin{pmatrix} \partial H_z/\partial y - \partial H_y/\partial z \\ \partial H_x/\partial z - \partial H_z/\partial x \\ \partial H_y/\partial x - \partial H_x/\partial y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \partial H_y/\partial x - \partial H_x/\partial y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ J_z \end{pmatrix} \qquad \begin{pmatrix} 0 \\ 0 \\ J_z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \kappa E_z \end{pmatrix}$$

$$rot\vec{E} = \begin{pmatrix} \partial E_z/\partial y - \partial E_y/\partial z \\ \partial E_x/\partial z - \partial E_z/\partial x \\ \partial E_y/\partial x - \partial E_x/\partial y \end{pmatrix} = \begin{pmatrix} \partial E_z/\partial y \\ -\partial E_z/\partial x \\ 0 \end{pmatrix} = \begin{pmatrix} -\partial B_x/\partial t \\ -\partial B_y/\partial t \\ 0 \end{pmatrix}$$

$$div\vec{B} = \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} = \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} = 0$$

Impressed AC *B*-field in *y*-direction: $\vec{B}(x,t) = (0, B_0\cdot\cos\omega t, 0),\ x \le 0 \Rightarrow B_x = H_x = 0$

$$\partial B_y/\partial x = \mu J_z = \mu\kappa E_z \qquad \partial E_z/\partial y = 0 \Rightarrow E_z = f(x,t) \qquad \partial E_z/\partial x = \partial B_y/\partial t \qquad \partial B_y/\partial y = 0 \Rightarrow B_y = g(x,t)$$

$$\partial^2 B_y/\partial x^2 = \mu\kappa\partial B_y/\partial t$$ *BULLARD*´s linear partial differential equation of field diffusion

Derivation of penetration depth in a massive "half-space" conductor (2)

Repetition

Stationary (particular) solution of *BULLARD*´s equation: $\partial^2 B_y / \partial x^2 = \mu\kappa \partial B_y / \partial t$

$$B_y(x,t) = \mathrm{Re}\{\underline{B}_y(x)\cdot e^{j\omega t}\} \qquad \partial^2 \underline{B}_y(x)/\partial x^2 - j\omega\mu\kappa \underline{B}_y(x) = 0 \qquad \underline{B}_y(x) = \underline{C}_1 e^{\underline{\lambda}x} + \underline{C}_2 e^{-\underline{\lambda}x} \qquad \underline{\lambda}^2 = j\omega\mu\kappa$$

$$\underline{\lambda} = \sqrt{j\omega\mu\kappa} = \frac{1+j}{\sqrt{2}}\cdot\sqrt{\omega\mu\kappa} = \frac{1+j}{d_E} \qquad d_E = \sqrt{\frac{2}{\omega\mu\kappa}}$$

Penetration depth

Boundary conditions: $\vec{B}(x,t) = (0, B_0\cdot\cos\omega t, 0) = \mathrm{Re}(0, B_0\cdot e^{j\omega t}, 0),\ x \le 0 \quad \Rightarrow \underline{B}_y(0) = B_0 = \underline{C}_1 + \underline{C}_2$

In infinity fields are limited: $\vec{B}(x\to\infty, t) = (0,0,0) \Rightarrow \underline{B}_y(x\to\infty) = 0 \Rightarrow \underline{C}_1 = 0 \Rightarrow \underline{C}_2 = B_0$

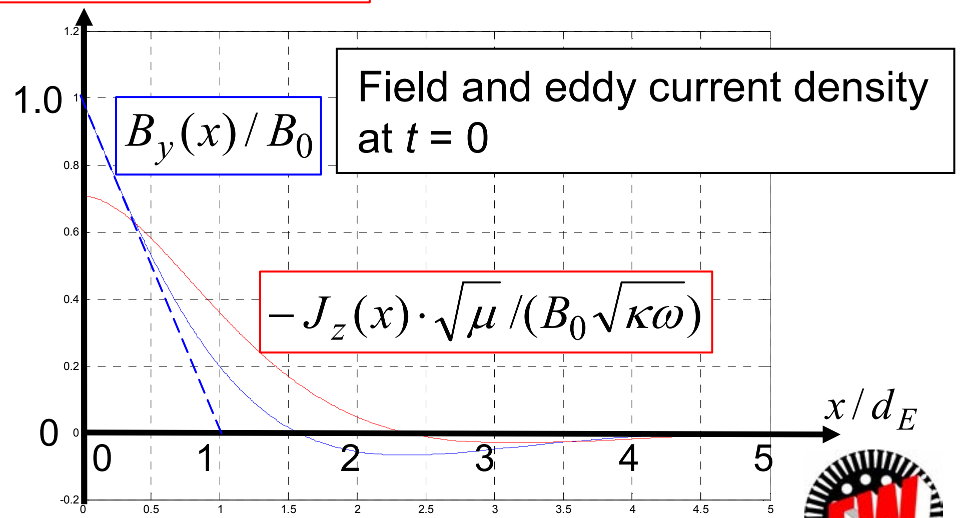
Field solution: $B_y(x,t) = \mathrm{Re}\{B_0 e^{-(1+j)x/d_E}\cdot e^{j\omega t}\} = B_0\cdot e^{-x/d_E}\cdot\cos(\omega t - x/d_E)$

Eddy current density: $J_z(x,t) = \frac{1}{\mu}\frac{\partial B_y}{\partial x}$

$$J_z(x,t) = \frac{1}{\mu}\frac{\partial B_y}{\partial x} = -\mathrm{Re}\left\{\frac{B_0}{\mu}\frac{1+j}{d_E}e^{-(1+j)x/d_E}\cdot e^{j\omega t}\right\}$$

$$J_z(x,t) = \frac{1}{\mu}\frac{\partial B_y}{\partial x} = -\mathrm{Re}\left\{\frac{B_0\sqrt{2}}{d_E\mu}e^{-(1+j)x/d_E}\cdot e^{j(\omega t+\pi/4)}\right\}$$

$$J_z(x,t) = -\frac{B_0\sqrt{\omega\kappa}}{\sqrt{\mu}}e^{-x/d_E}\cdot\cos(\omega t - x/d_E + \pi/4)$$



Derivation of penetration depth in a massive "half-space" conductor (3)

Repetition

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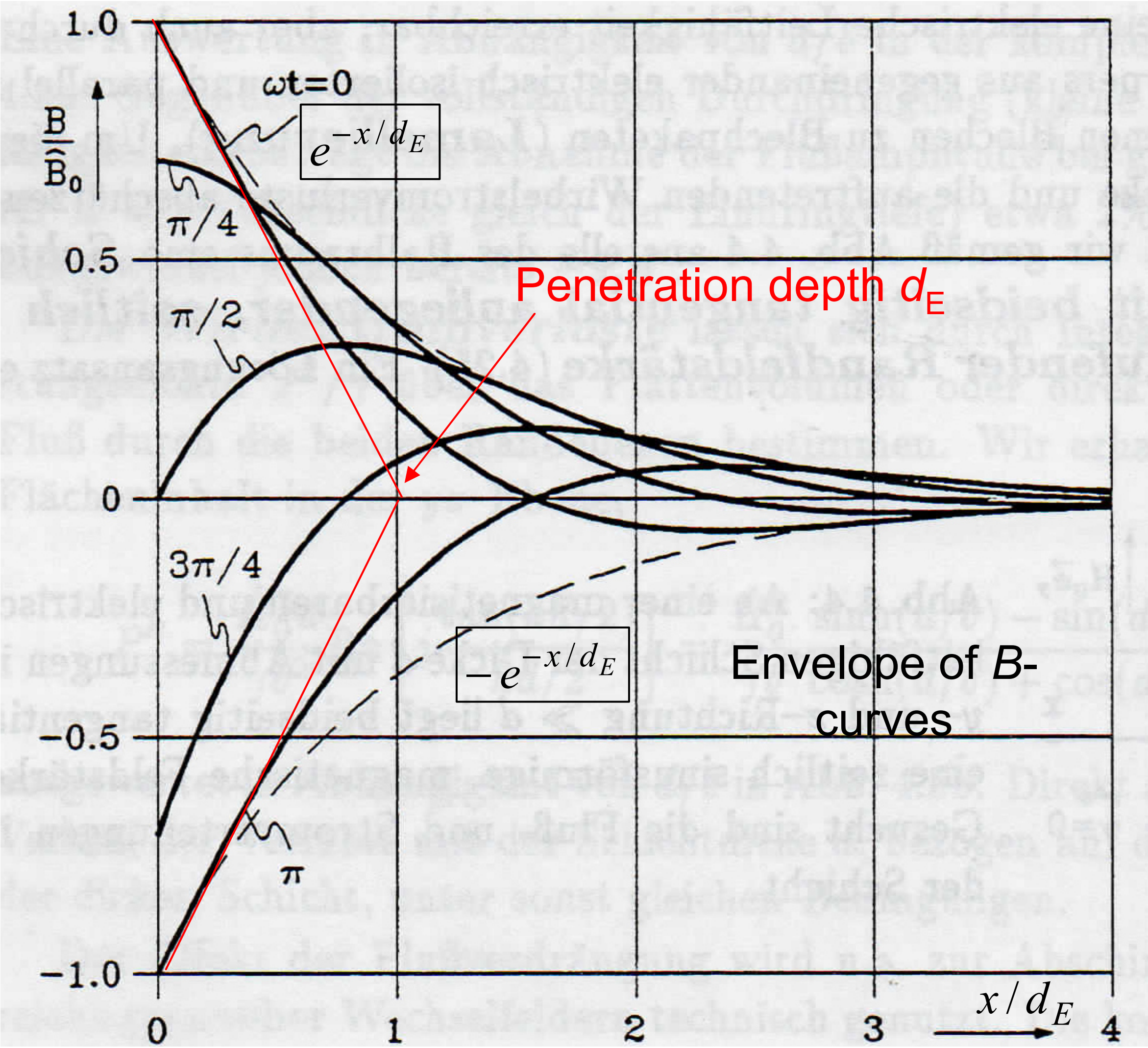


Pulsating damped magnetic flux density in conducting half space

One half oscillation $0 \le \omega t \le \pi$

$$B(x) = B_y(x) = \mu \cdot H_y(x)$$

$$\hat{B}_0 = \mu \cdot \hat{H}_0$$

Source:
A. Prechtl / Theoretische Elektrotechnik, Skript, TU Wien, 1993

TECHNISCHE UNIVERSITÄT DARMSTADT

2. Design of induction machines

2.1 Main dimensions and basic electromagnetic quantities of induction machines

2.2 Scaling effect in electric machines

2.3 Stator winding low and high voltage technology

2.4 Stator winding design

2.5 Rotor cage design

2.6 Wound rotor design

2.7 Design of main flux path of magnetic circuit

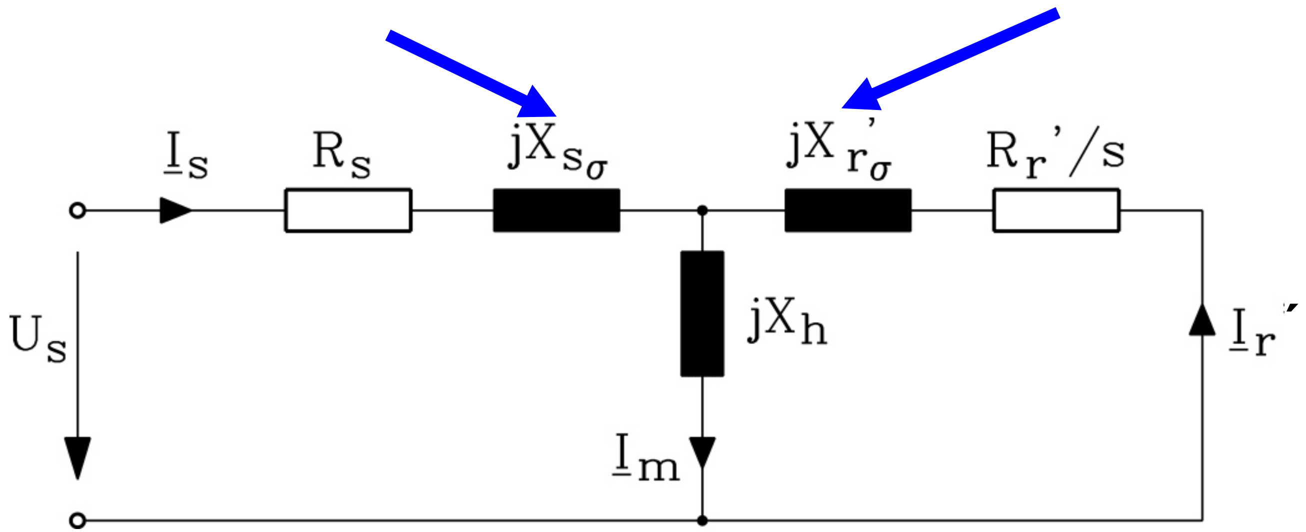
2.8 Stray flux and inductance

2.9 Influence of saturation on inductance

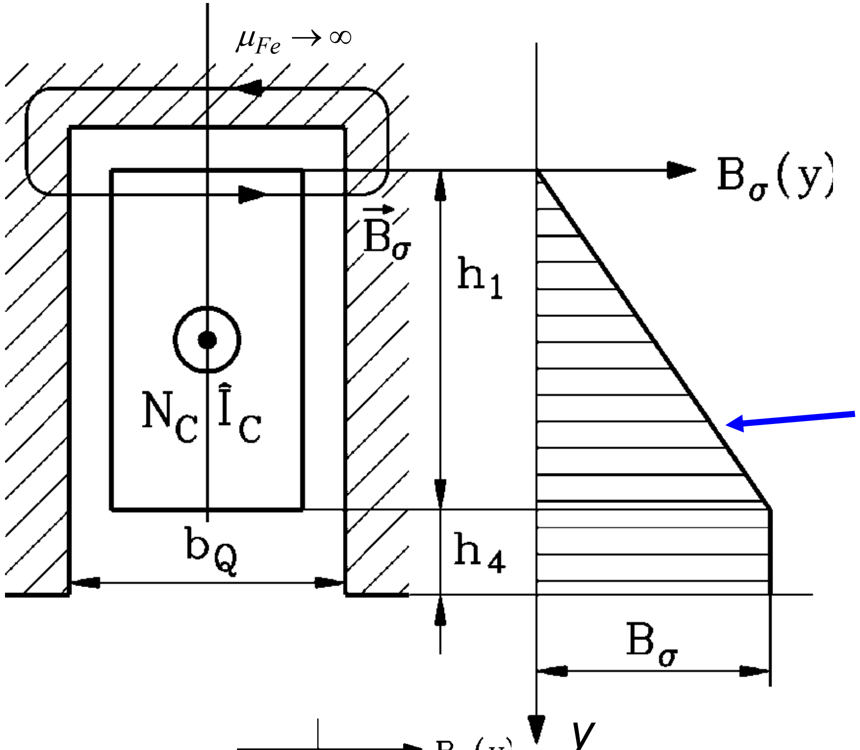
2.10 Masses and losses

Stray fluxes and inductances in the equivalent circuit

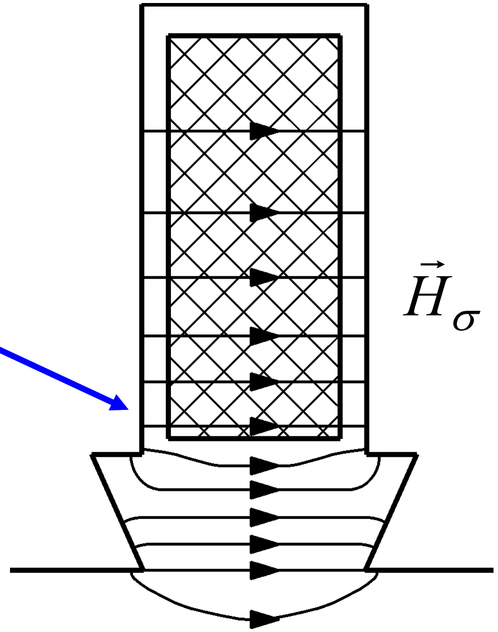
- **Slot stray flux** in stator and rotor slots,
- **Stray flux in winding overhangs** of stator and rotor winding,
- **Flux of harmonics of air gap field**, excited by stator current for stator field harmonics $\nu \neq 1$, and excited by rotor current for rotor field harmonics $\mu \neq 1$,
- **Stray flux due to skew** between stator and rotor slots in cage induction machines. Usually rotor cage is skewed, thus rotor stray inductance is increased.



Slot stray flux density of single layer stator winding

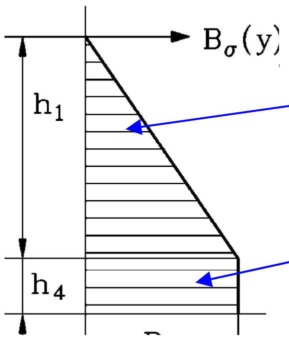


Real flux density B_σ & field strength H_σ in the slot



Idealized flux density in slot for calculation

At $i_c(t) = \hat{I}_c$: Peak slot stray field:



$$B_\sigma(y) = \mu_0 H_\sigma(y) = \mu_0 \frac{N_c \hat{I}_c}{b_Q} \cdot \frac{y}{h_1}, \quad 0 \le y \le h_1$$

$$B_\sigma(y) = \mu_0 \frac{N_c \hat{I}_c}{b_Q}, \quad h_1 \le y \le h_1 + h_4$$

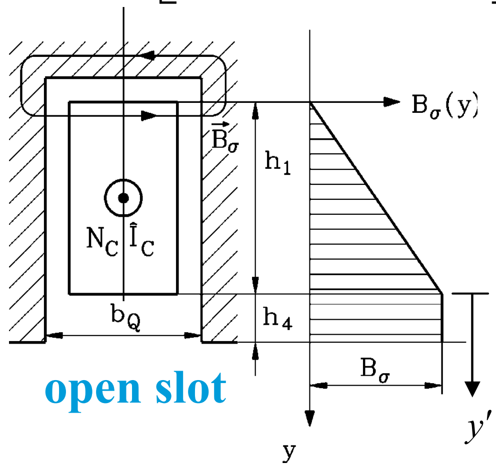
Slot stray inductance calculated from magnetic energy

- **Open slot:** Magnetic energy stored in slot stray field of one coil of a single layer winding (= two coil sides) at peak stray field:

$$W_\sigma = \frac{L_{\sigma c}\hat{I}_c^2}{2} = \int_V \int_0^B (\vec{H}\cdot d\vec{B})dV = \int_V \frac{B_\sigma^2}{2\mu_0}dV = \frac{2 l_e b_Q}{2\mu_0}\cdot \int_0^{h_1+h_4} B_\sigma^2(y)\cdot dy = \frac{\mu_0 l_e \cdot (N_c \hat{I}_c)^2}{b_Q}\cdot \left[\int_0^{h_1}\frac{y^2}{h_1^2}\cdot dy + \int_0^{h_4} dy'\right]$$

$$\frac{1}{b_Q}\int_0^{h_1}\frac{y^2}{h_1^2}\cdot dy = \frac{1}{3}\cdot\frac{h_1}{b_Q} \qquad \frac{1}{b_Q}\cdot\int_0^{h_4} dy' = \frac{h_4}{b_Q}$$

$$L_{\sigma c} = 2W_\sigma / \hat{I}_c^2 = 2\cdot\mu_0\cdot N_c^2\cdot\lambda_Q\cdot l_e \qquad \lambda_Q = \frac{h_1}{3b_Q} + \frac{h_4}{b_Q}$$



open slot

- Series connection of all coils per phase: $L_{\sigma Q} = p\cdot q\cdot L_{\sigma c}$
- In case of *a* parallel branches per phase: $L_{\sigma Q} = p\cdot q\cdot L_{\sigma c}/a^2$

$$N_s = p\cdot q\cdot N_c / a$$

(Single-layer winding)

Stator slot stray inductance: $$L_{s\sigma Q} = \mu_0\cdot N_s^2\cdot\frac{2}{p\cdot q_s}\cdot\lambda_{Qs}\cdot l_e$$

2. Design of Induction Machines

Slot stray inductance for semi-closed slot $L_{s\sigma Q}$

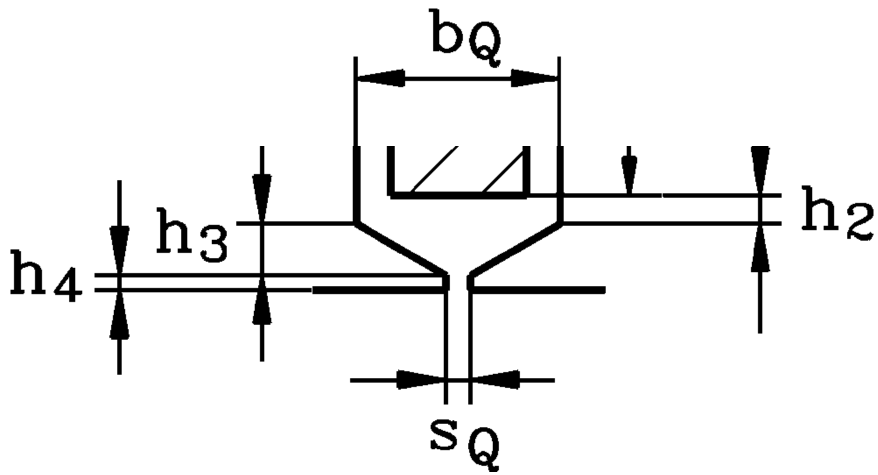
- Single layer winding: $$L_{s\sigma Q} = \mu_0 \cdot N_s^2 \cdot \frac{2}{p \cdot q_s} \cdot \lambda_{Qs} \cdot l_e$$

- Instead of

$$\lambda_Q = \frac{h_1}{3b_Q} + \frac{h_4}{b_Q}$$

we get

$$\lambda_Q = \frac{h_1}{3b_Q} + \frac{h_2}{b_Q} + \frac{h_3}{(b_Q + s_Q)/2} + \frac{h_4}{s_Q}$$



Semi-closed slot

Slot stray flux density of a two-layer winding

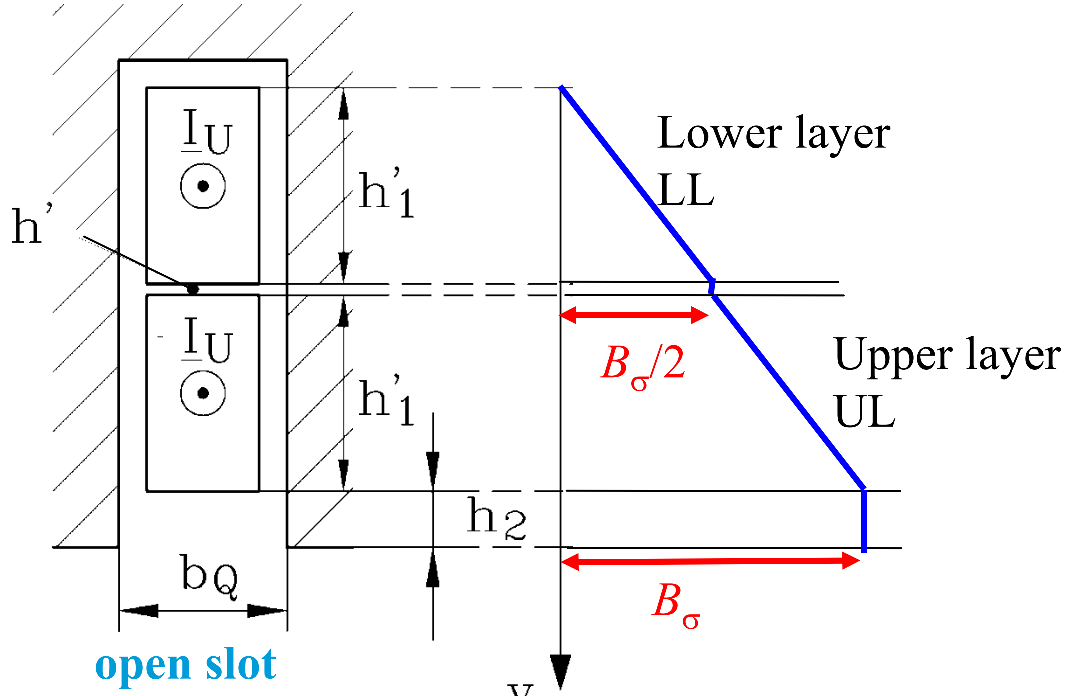
- Open slot, full pitched winding = upper and lower layer have the same coil current:

- Instead of

$$\lambda_Q = \frac{h_1}{3b_Q} + \frac{h_4}{b_Q}$$

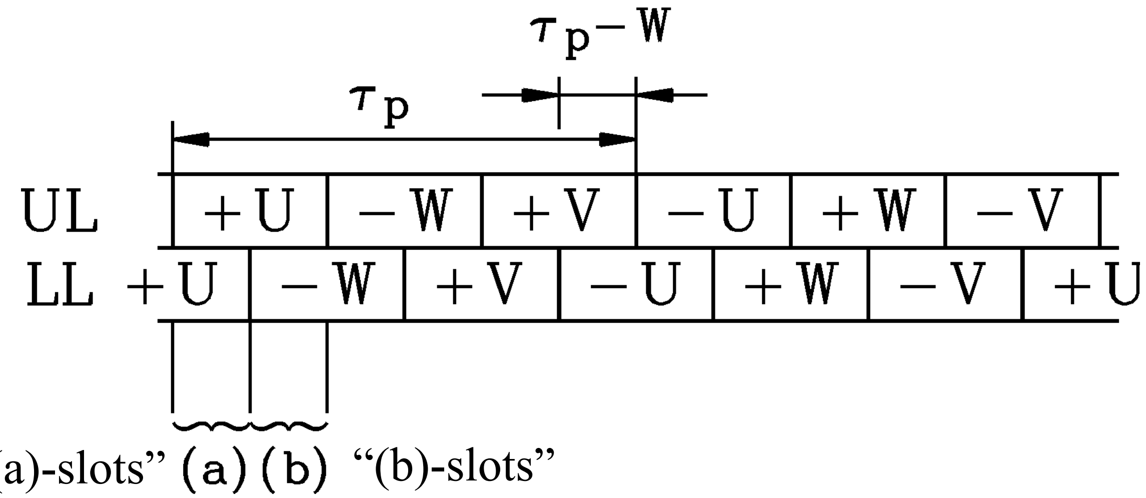
we get with $h_1 \rightarrow 2h_1'$

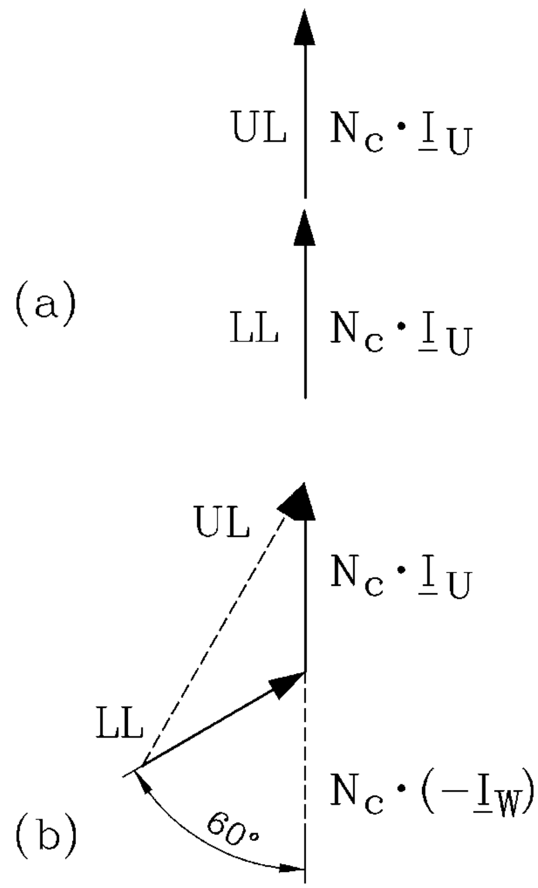
$$\lambda_Q = 2 \cdot \frac{h_1'}{3b_Q} + \frac{h'}{4b_Q} + \frac{h_2}{b_Q}$$



At h': Half flux density $B \sim ½ \Rightarrow w \sim B^2$: $(1/2)^2 = ¼$ in energy density, therefore: $h'/(4b_Q)$= instead of h'/b_Q !

Two layer winding:
Influence of coil pitch $W \neq \tau_p$ on stray inductance



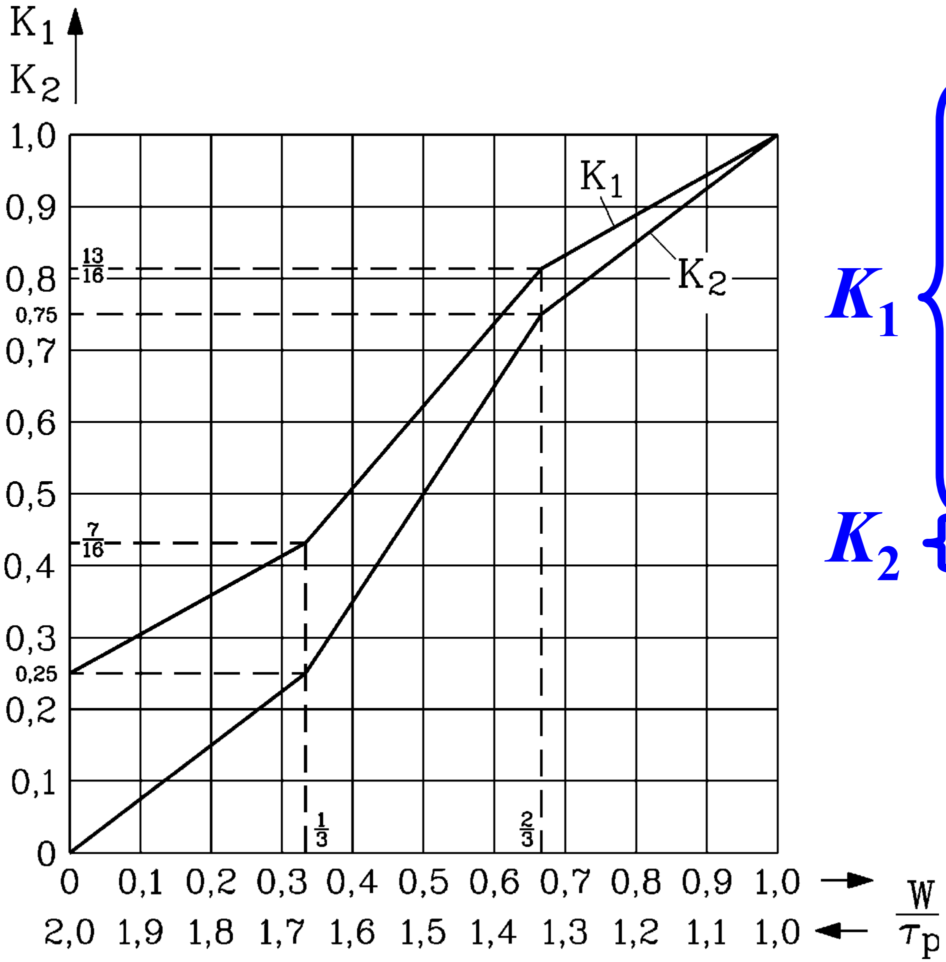


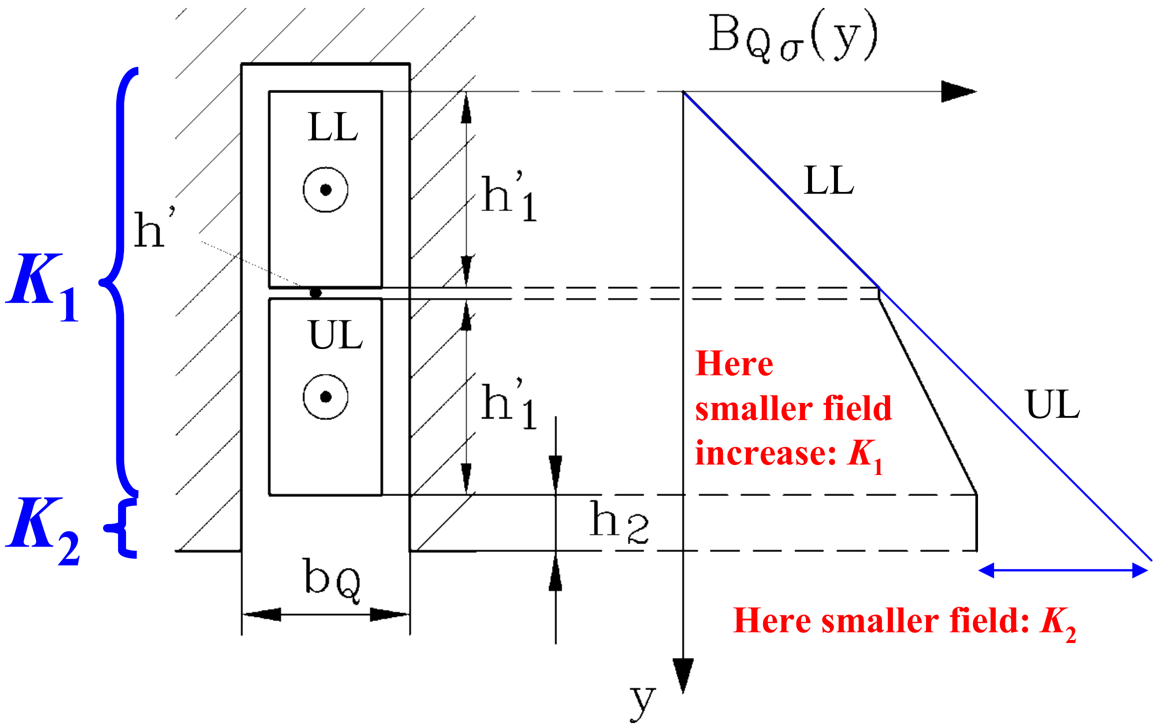
- Phase shift between $\underline{I}_U$ and $-\underline{I}_W$ is 60°, so total ampere turns in (b)-slots are only √3/2 ↔ 86.6% of (a)-slots.
- They excite a smaller stray flux in (b)-slots, thus reducing total slot stray flux - depending on the pitch W/τ_p !
- This is considered by the **slot stray flux reduction functions** K_1, K_2:

$$K_1 = \frac{9}{16} \cdot \frac{W}{\tau_p} + \frac{7}{16} \qquad K_2 = \frac{3}{4} \cdot \frac{W}{\tau_p} + \frac{1}{4} \qquad 2/3 \le W/\tau_p \le 1$$

Functions K_1, K_2: Reduction of slot stray flux due to pitching

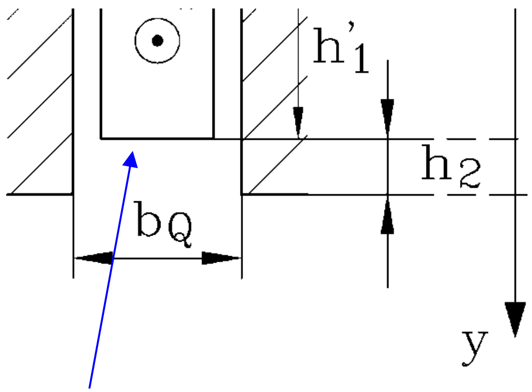
Considering reduction of slot stray flux due to coil pitch W/τ_p in two layer winding





$$\lambda_Q = K_1 \cdot 2 \cdot \frac{h_1'}{3b_Q} + \frac{h'}{4b_Q} + K_2 \cdot \frac{h_2}{b_Q}$$

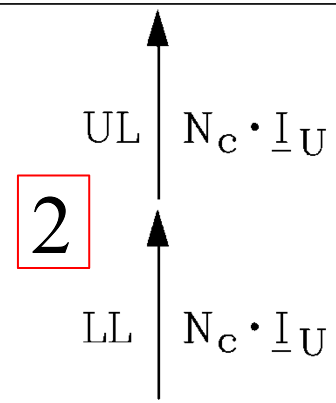
Explanation of factor K_2



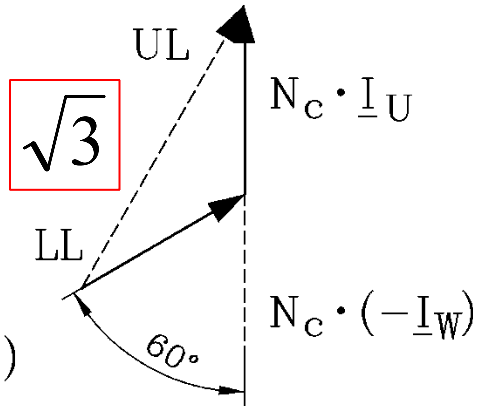
(a) Slot Ampere turns: $\Theta_Q = 2N_c I_U$

(b) Slot Ampere turns: $\Theta_Q = \sqrt{3}\, N_c I_U$

(a)



(b)



- Reduction in magnetic energy in "region" h_2: $W_\sigma = \dfrac{L_{\sigma c}\hat{I}_c^2}{2} \sim \Theta_Q^2$

$$\frac{L_{\sigma c(b)}}{L_{\sigma c(a)}} = \frac{W_{\sigma(b)}}{W_{\sigma(a)}} = \frac{\Theta_{Q(b)}^2}{\Theta_{Q(a)}^2} = \frac{\left(\sqrt{3}\right)^2}{2^2} = \frac{3}{4}$$

- In pitched winding $W/\tau_p = 2/3$ only slots of type (b) occur: Therefore reduction of $L_{\sigma c}$ in the region "h_2" by: $(3/4)\cdot h_2/b_Q$

- Check with K_2: $\dfrac{W}{\tau_p} = \dfrac{2}{3}$: $K_2 \cdot \dfrac{h_2}{b_Q} = \left(\dfrac{3}{4}\cdot\dfrac{W}{\tau_p} + \dfrac{1}{4}\right)\cdot\dfrac{h_2}{b_Q} = \left(\dfrac{3}{4}\cdot\dfrac{2}{3} + \dfrac{1}{4}\right)\cdot\dfrac{h_2}{b_Q} = \dfrac{3}{4}\cdot\dfrac{h_2}{b_Q}$

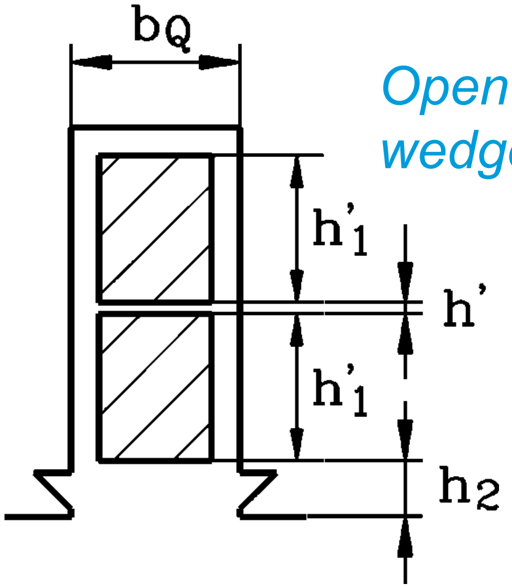
UL: upper layer
LL: lower layer

Influence of stator slot arrangement on slot stray flux

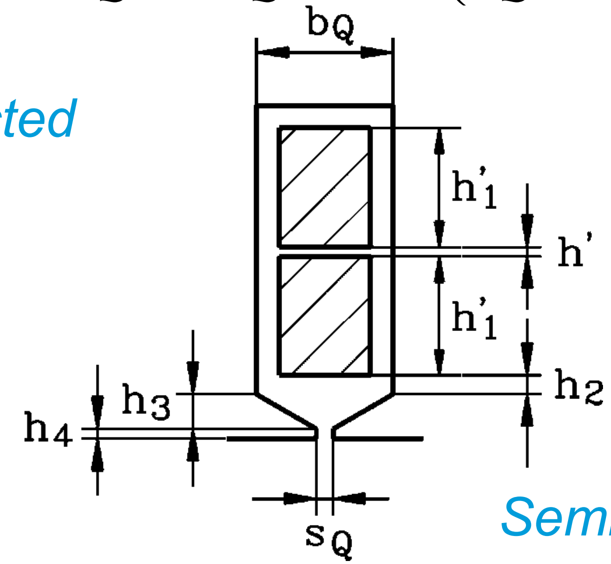
Two-layer winding: λ_Q depends on slot geometry:

1) Open slots: $$\lambda_{Qs} = K_1 \cdot 2 \cdot \frac{h_1'}{3b_Q} + \frac{h'}{4b_Q} + K_2 \cdot \frac{h_2}{b_Q}$$

2) Semi-closed slots: $$\lambda_{Qs} = K_1 \cdot 2 \cdot \frac{h_1'}{3b_Q} + \frac{h'}{4b_Q} + K_2 \cdot \left(\frac{h_2}{b_Q} + \frac{h_3}{(b_Q + s_Q)/2} + \frac{h_4}{s_Q} \right)$$



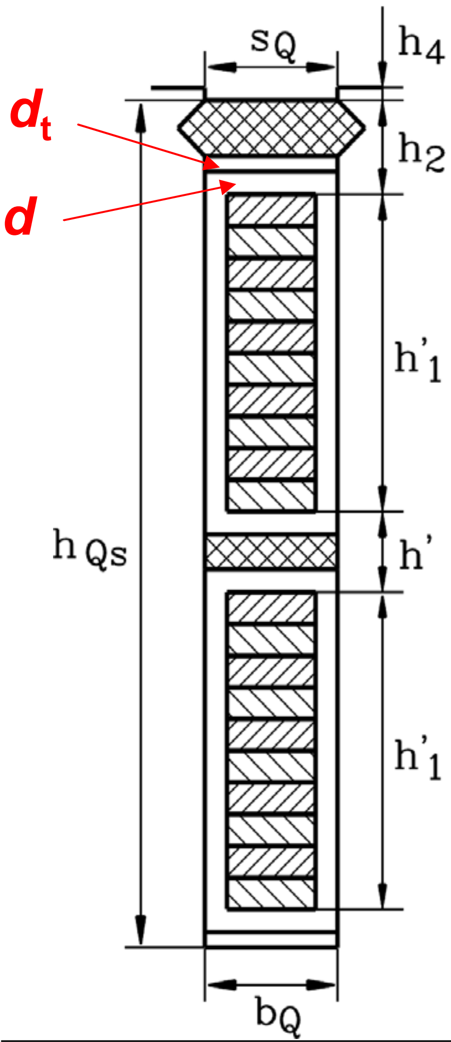
Open slot:
wedge influence neglected



Semi-closed slot

2. Design of Induction Machines

Slot geometry calculation:
Two-layer high voltage stator winding in open slots



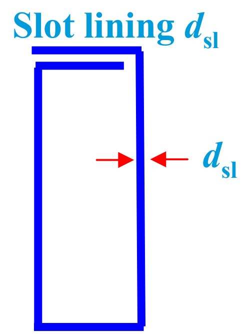
$$h_4 = 0.5mm$$

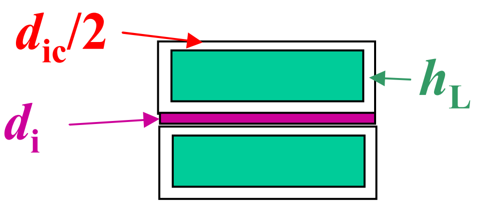
$$h_2 = d_{ic}/2 + d + 2\cdot d_{sl} + d_t + h_{wedge} =$$
$$= 0.4/2 + 2.2 + 2\cdot 0.15 + 0.4 + 4.0 = 7.1mm$$

$$h_1' = N_c \cdot (h_L + d_{ic}) + (N_c - 1)\cdot d_i - d_{ic} = 24.7 - 0.4 = 24.3mm$$

$$h' = Z + 2\cdot d + 2\cdot (d_{ic}/2) = 4 + 2\cdot 2.2 + 2\cdot (0.4/2) = 8.8mm$$

$$h_1' = 24.3mm$$





Conductor arrangement

Stator (open) slot leakage reactance $X_{s\sigma Q}$

Example: $W/\tau_p = 12/15$: $K_1 = \frac{9}{16}\cdot\frac{W}{\tau_p} + \frac{7}{16} = 0.8875$ $\quad K_2 = \frac{3}{4}\cdot\frac{W}{\tau_p} + \frac{1}{4} = 0.85$

$h_1' = 24.3\text{mm}$ $\quad h' = 8.8\text{mm}$ $\quad h_2 + h_4 = 7.6\text{mm}$

$$\lambda_{Qs} = K_1 \cdot 2\frac{h_1'}{3b_Q} + \frac{h'}{4b_Q} + K_2 \cdot \frac{h_2 + h_4}{b_Q} = 0.8875 \cdot 2\frac{24.3}{3\cdot 12.5} + \frac{8.8}{4\cdot 12.5} + 0.85\frac{7.6}{12.5} = \underline{\underline{1.843}}$$

Stator slot leakage reactance (open slot):

$$X_{s\sigma Q} = \omega_s L_{s\sigma Q} = 2\pi f_s \cdot \mu_0 N_s^2 \frac{2}{p\cdot q_s}\lambda_{Qs} l_e =$$

$$= 2\pi 50 \cdot 4\pi \cdot 10^{-7} \cdot 200^2 \cdot \frac{2}{2\cdot 5} \cdot 1.843 \cdot 0.392 = \underline{\underline{2.28}}\,\Omega$$

2. Design of Induction Machines

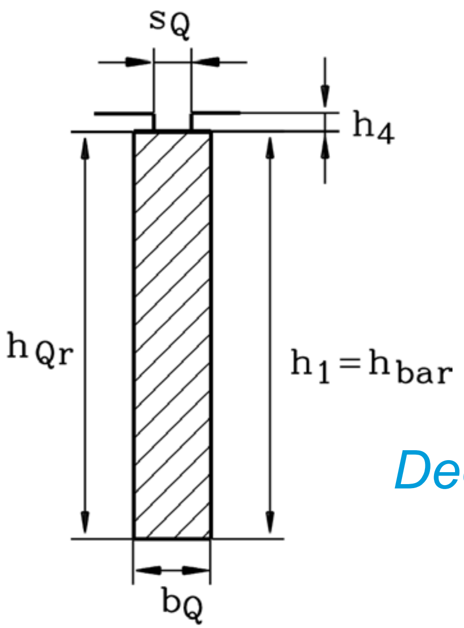
Influence of rotor cage slot arrangement on slot stray flux

- For **cage rotor** per bar: $L_{\sigma,bar} = \mu_0 \cdot \lambda_{Qr} \cdot l_e$

- **Different slot shapes for BIG or LOW current displacement effect:**

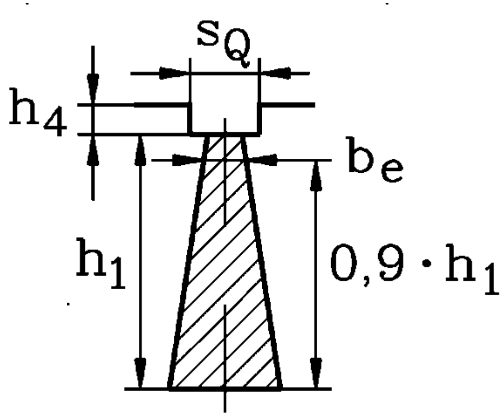
- **BIG displacement:** **Deep bar:** $\lambda_{Qr} = \dfrac{h_1}{3b_Q} \cdot k_L + \dfrac{h_4}{s_Q}$

Wedge bar: $\lambda_{Qr} = \dfrac{h_1}{3b_e} \cdot k_{L,wedge} + \dfrac{h_4}{s_Q}$



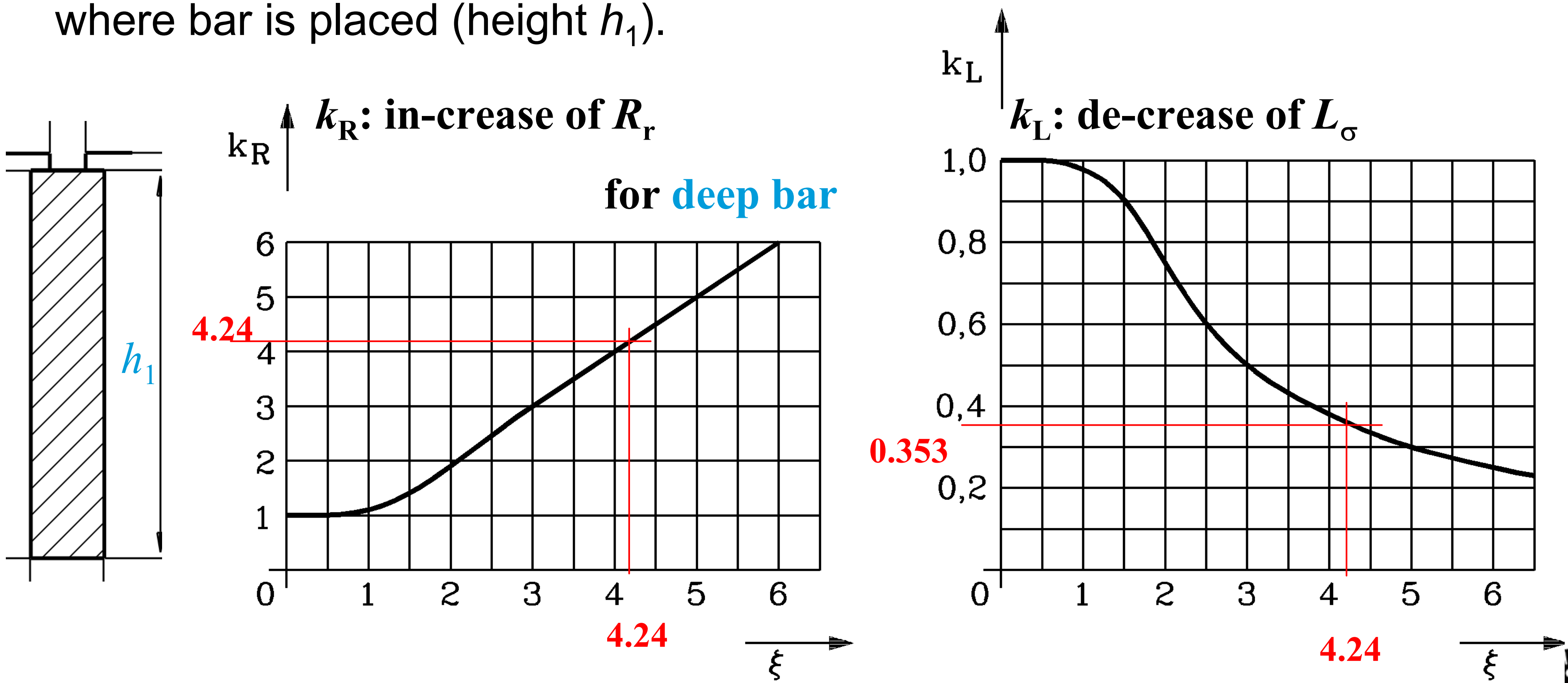
Deep bar

Wedge bar

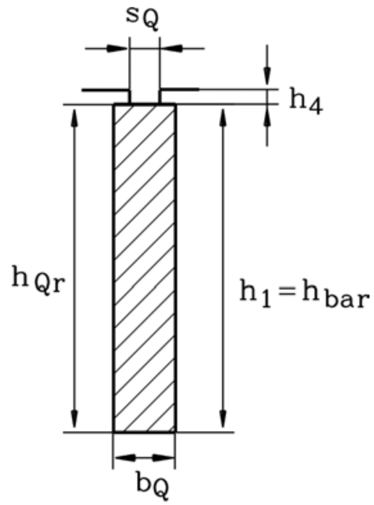


"Current displacement effect" of deep bar k_R, k_L

- It is used for increase of starting torque due to increase of rotor resistance, depending on $\xi = h_1\sqrt{\pi \cdot f_r \cdot \mu_0 \cdot \kappa_{Cu}}$.
- Decrease of slot stray flux must be considered in that section of slot, where bar is placed (height h_1).



Slot stray reactance of deep bar rotor cage $X'_{r\sigma Q}(s)$



Data: $h_1 = 40$ mm, $b_{Qr} = 5.1$ mm, $h_4 = 3.4$ mm, $s_{Qr} = 2.5$ mm

$$\lambda_{Qr} = \frac{h_1}{3b_{Qr}} \cdot k_L + \frac{h_4}{s_{Qr}} = \frac{40}{3 \cdot 5.1} \cdot k_L + \frac{3.4}{2.5} = 2.61 k_L + 1.36$$

a) Start slip *s* = 1 and 20°C: $f_r = 1 \cdot 50 = 50 Hz$

$$\xi = h_1 \sqrt{\pi \cdot f_r \cdot \mu_0 \cdot \kappa_{Cu}} = 0.04 \sqrt{\pi \cdot 50 \cdot 4\pi \cdot 10^{-7} \cdot 57 \cdot 10^6} = 4.24$$

$$k_L = 0.353 \qquad \lambda_{Qr} = \underline{\underline{2.28}}$$

b) Rated slip *s* = 1.5%: $f_r = 0.015 \cdot 50 = 0.75$ Hz: $\xi = 0.519$, $k_L = 0.998$, $\lambda_{Qr} = \underline{\underline{3.96}}$

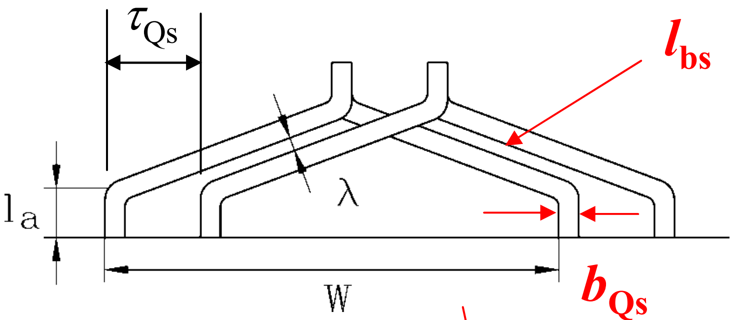
$$\ddot{u}_U = \frac{N_s k_{ws}}{N_r k_{wr}} = \frac{200 \cdot 0.91}{0.5 \cdot 1} = 364, \quad \ddot{u}_I = \ddot{u}_U \cdot \frac{m_s}{Q_r} = 364 \cdot \frac{3}{50} = 21.84$$

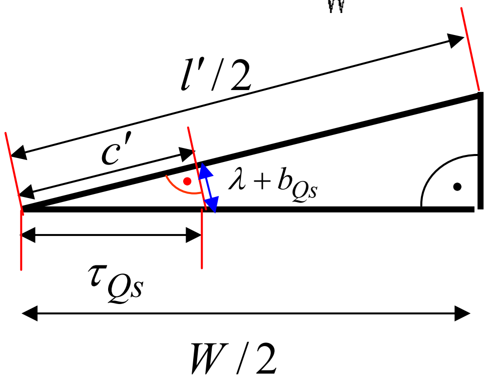
$$X'_{r\sigma Q}(s=1) = \ddot{u}_U \ddot{u}_I \cdot \omega_s L_{r\sigma Q} = \ddot{u}_U \ddot{u}_I \cdot 2\pi f_s \cdot \mu_0 \lambda_{Qr} l_e =$$

$$= 364 \cdot 21.84 \cdot 2\pi 50 \cdot 4\pi \cdot 10^{-7} \cdot 2.28 \cdot 0.392 = \underline{\underline{2.80\ \Omega}}$$

2. Design of Induction Machines

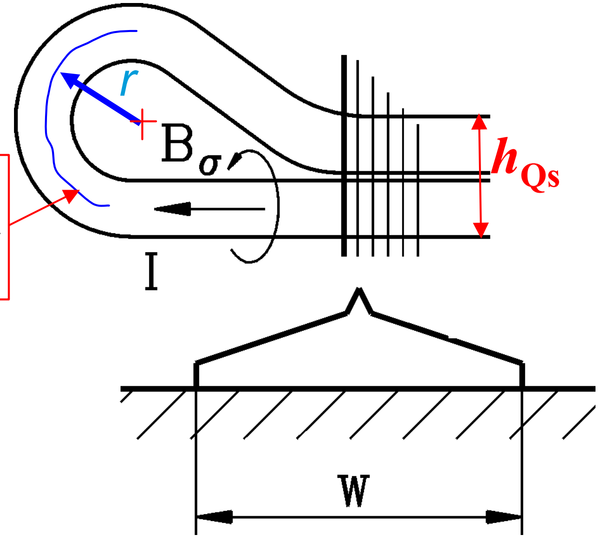
Winding overhang geometry





Minimum theoretical radius $r_{min} = h_{Qs}/4$

$$\pi \cdot r > \pi \cdot \frac{h_{Qs}}{4}$$



$$c' = \sqrt{\tau_{Qs}^2 - (\lambda + b_{Qs})^2}$$

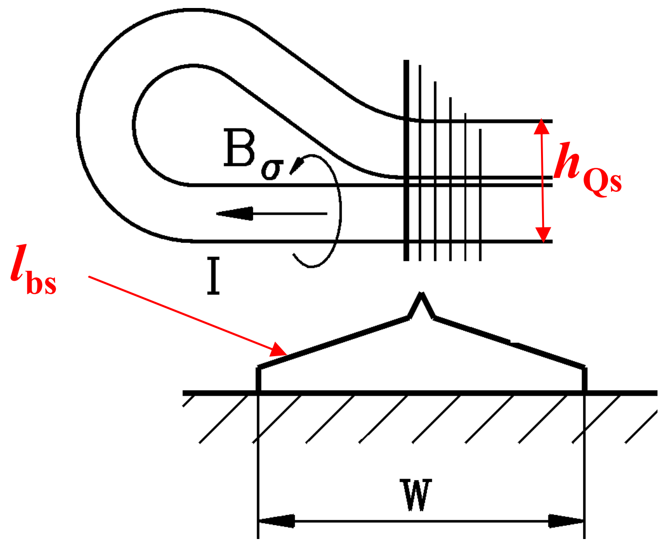
$$\tau_{Qs}/c' = (l'/2)/(W/2) \Rightarrow l' = W \cdot (\tau_{Qs}/c')$$

$$l_{bs} = l' + \pi \cdot \frac{h_{Qs}}{4} + 2 \cdot l_a + \Delta l_b$$

Δl_{bs}: Additional length for coil & series connectors, larger radius $r > r_{min}$ at bend

$$l_{bs} = \frac{W}{\sqrt{1 - \frac{(b_{Qs}+\lambda)^2}{\tau_{Qs}^2}}} + \frac{\pi}{2} \cdot \frac{h_{Qs}}{2} + 2 \cdot l_a + \Delta l_{bs} = \frac{12 \cdot 24}{\sqrt{1 - \frac{(12.5+4)^2}{24^2}}} + \pi \cdot \frac{69}{4} + 2 \cdot 57 + 50 = \underline{\underline{614.8}}\,\mathrm{mm}$$

Stray flux and inductance of winding overhangs $L_{s\sigma b}$



- **Stray flux** is inducing voltage at length l_b of turn per coil

- As all coils excite in winding overhang the stray flux, no influence of slot number or q respectively, appears

- Thus taking l_b instead of l_e and omitting q, we get the expression:

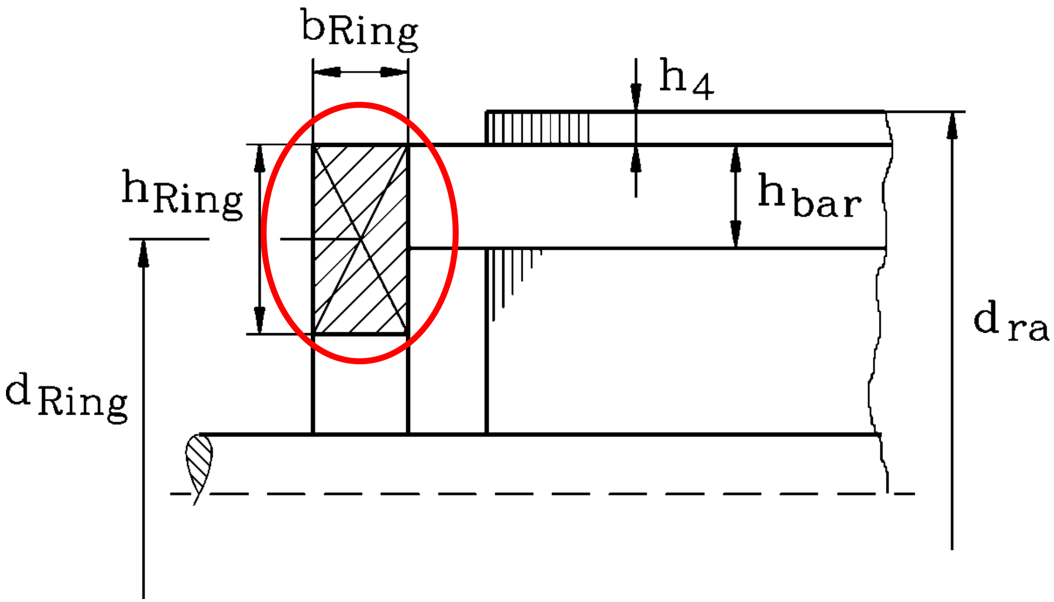
$$L_{s\sigma b} = \mu_0 \cdot N_s^2 \cdot \frac{2}{p} \cdot \lambda_{bs} \cdot l_{bs}$$

- Geometry factor λ_b must be taken from (i) numerical 3D field calculations or from (ii) measurements of single coil model test set-up!

$$l_{bs} = \underline{\underline{614.8\ \mathrm{mm}}},\quad \lambda_{bs} = 0.075 \cdot \left(1 + \frac{l_{bs}}{\tau_p}\right) = 0.075 \cdot \left(1 + \frac{614.8}{360}\right) = \underline{\underline{0.203}}$$

$$X_{s\sigma b} = \omega_s L_{s\sigma b} = 2\pi f_s \cdot \mu_0 N_s^2 \frac{2}{p} \lambda_{bs} l_{bs} = 2\pi 50 \cdot 4\pi \cdot 10^{-7} \cdot 200^2 \cdot \frac{2}{2} \cdot 0.203 \cdot 0.6148 = \underline{\underline{1.97\ \Omega}}$$

Rotor stray flux and inductance of end rings $L'_{r\sigma b}$



From tests $\lambda_{br} = 0.12$
with respect to $l_{br} = \tau_p$

For rotor end ring side adopted (stator-side) formula, including voltage and current transfer ratio:

$$L'_{r\sigma b} = \mu_0 \cdot N_s^2 \cdot \frac{2}{p} \cdot \lambda_{br} \cdot l_{br}$$

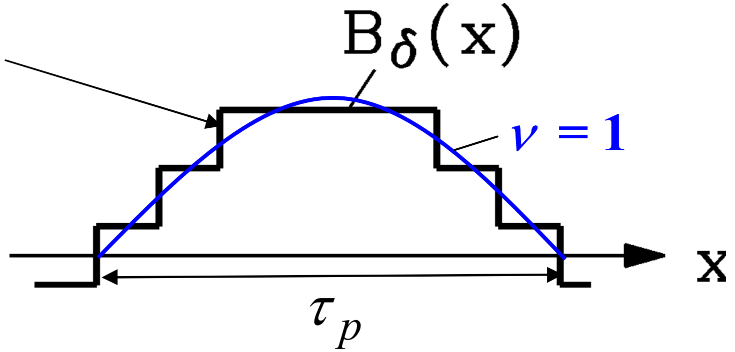
$$X'_{r\sigma b} = \omega_s L'_{r\sigma b} = 2\pi f_s \cdot \mu_0 N_s^2 \frac{2}{p} \lambda_{br} l_{br} = 2\pi 50 \cdot 4\pi \cdot 10^{-7} \cdot 200^2 \cdot \frac{2}{2} \cdot 0.12 \cdot 0.36 = \underline{\underline{0.68\,\Omega}}$$

- A rough estimate for stator winding and cage of induction machine is $\lambda_b \cong 0.3$!

- Here: $\lambda_b = \lambda_{bs} + \lambda_{br} \cdot (l_{br} / l_{bs}) = 0.203 + 0.12 \cdot (0.36 / 0.6148) = 0.2733 \cong 0.3$

Stator harmonic air-gap field waves

- Step-like stator air gap flux density distribution at $\mu_{Fe} \to \infty$: $B_\delta(x,t) = \mu_0 \cdot V_s(x,t)/\delta_e$



- *Fourier* series of stator air-gap field:

$$B_\delta(x,t) = \sum_{\nu=1,-5,7,...}^{\infty} \hat{B}_{\delta\nu} \cdot \cos(\nu\pi x/\tau_p - \omega_s t)$$

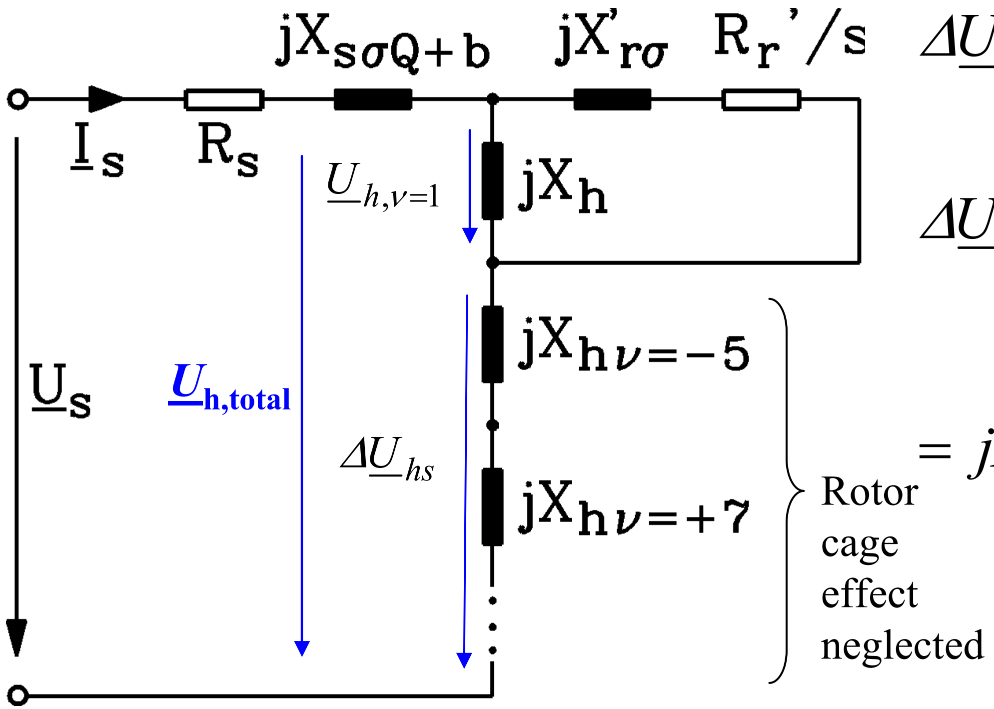
$$\frac{\hat{B}_{\delta\nu}}{\hat{B}_{\delta 1}} = \frac{\hat{V}_{s\nu}}{\hat{V}_{s1}} = \frac{k_{ws,\nu}}{k_{ws,1} \cdot \nu}$$

- Each harmonic self-induces stator winding with stator frequency f_s, thus adding up to the total self-induced stator voltage $\underline{U}_{h,total}$

Harmonic stator leakage inductance $L_{s\sigma O}$

$$B_\delta(x,t) = \sum_{\nu=1,-5,7,\ldots}^{\infty} \hat{B}_{\delta\nu} \cdot \cos(\nu\pi x/\tau_p - \omega_s t) \qquad \frac{\hat{B}_{\delta\nu}}{\hat{B}_{\delta 1}} = \frac{k_{ws,\nu}}{k_{ws,1}\cdot\nu}$$

- Each harmonic self-induces stator winding with stator frequency f_s, thus adding up self-induced stator voltage $\underline{U}_{h,total}$



$$\Delta\underline{U}_{hs} = \sum_{|\nu|>1} \underline{U}_{h\nu} = \sum_{|\nu|>1} jX_{h\nu}\cdot\underline{I}_s$$

$$\Delta\underline{U}_{hs} = \sum_{|\nu|>1} jX_{h\nu}\cdot\underline{I}_s =$$

$$= jX_{h,\nu=1}\cdot\underline{I}_s\cdot\underbrace{\sum_{|\nu|>1}\left(\frac{k_{ws\nu}}{\nu\cdot k_{ws1}}\right)^2}_{\sigma_{Os}} = jX_{s\sigma O}\cdot\underline{I}_s$$

„Harmonic stator stray coefficient" σ_{Os}

$$s = 0 : U_{h,total} = \omega_s \cdot L_{h,total} \cdot I_s \qquad L_{h\nu} = \mu_0 \cdot (N_s k_{ws\nu})^2 \cdot \frac{2 \cdot m_s}{\pi^2 \cdot \nu \cdot p} \cdot \frac{l_e \cdot \tau_p / \nu}{\delta_e} \sim \frac{k_{ws\nu}^2}{\nu^2}$$

$$U_{h,total} = \sum_{\nu=1}^{\infty} U_{hs,\nu} = \sum_{\nu=1}^{\infty} \omega_s L_{h\nu} I_s = \omega_s L_{h,total} I_s$$

$$L_{h,total} = \sum_{\nu=1}^{\infty} L_{h\nu} = L_{h,\nu=1} \cdot \sum_{\nu=1}^{\infty} \frac{L_{h\nu}}{L_{h,\nu=1}} = L_{h,\nu=1} \cdot \sum_{\nu=1}^{\infty} \frac{(k_{ws,\nu} / \nu)^2}{(k_{ws,1} / 1)^2}$$

Definition of „harmonic stray coefficient":

$$\sigma_{Os} = \sum_{\nu=1}^{\infty} \left(\frac{k_{ws,\nu}}{\nu \cdot k_{ws,1}} \right)^2 - 1$$

$$L_{h,total} = (1 + \sigma_{Os}) \cdot L_{h,\nu=1} = L_{h,\nu=1} + \sigma_{Os} \cdot L_{h,\nu=1} = L_h + L_{s\sigma\sigma}$$

$$m_s = 3: L_{h,total} = \sum_{\nu=1,-5,7,\ldots}^{\infty} L_{h\nu} = (1 + \sigma_{Os}) \cdot L_{h,\nu=1}$$

Harmonic stator leakage (stray) inductance $L_{s\sigma O}$

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- It is calculated from fundamental magnetizing inductance (where main flux saturation may be included): $L_{s,\sigma,O} = \sigma_{Os} \cdot L_h$
- **Harmonic stray coefficient σ_{Os} !**

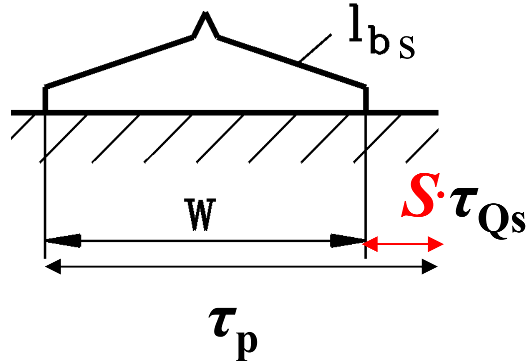
$$\sigma_{Os} = \sum_{|\nu|>1} \left(\frac{k_{ws\nu}}{\nu \cdot k_{ws1}} \right)^2$$

A) For full-pitched winding, $m_s = 3$: $\sigma_{Os} = \left(\frac{\pi}{3k_{ws,1}} \right)^2 \cdot \frac{5q_s^2 + 1}{6q_s^2} - 1$

B) For pitched winding, $m_s = 3$: $W = (m_s \cdot q_s - S) \cdot \tau_{Qs}$

Values $100\sigma_{Os}$ from Table

$100\sigma_{Os}$	$q = 1$	2	3	4	5
$S = 0$	9.662	2.844	1.406	0.890	0.648
1	9.662	2.354	1.149	0.738	0.549
2	9.662	2.844	1.109	0.624	0.437
3		2.844	1.406	0.688	0.411
4		2.844	1.429	0.890	0.500



Harmonic stray coefficient $100\sigma_{Os}$

2. Design of Induction Machines

Minimum harmonic stator leakage inductance at $W/\tau_p = 0.8$

For pitched stator winding with $W/\tau_p = 4/5 = 0.8$

the harmonic leakage inductance is minimum,
because no 5th harmonic of stator field occurs,
which in a 3-phase winding is the biggest contribution to harmonic leakage inductance.

$$k_{p,\nu=-5} = \sin\left(\nu \cdot \frac{W}{\tau_p} \cdot \frac{\pi}{2}\right) = -\sin\left(5 \cdot \frac{4}{5} \cdot \frac{\pi}{2}\right) = -\sin(2\pi) = 0$$

$100\sigma_{Os}$	$q = 1$	2	3	4	5	$q \to \infty$
$S = 0$	9.662	2.844	1.406	0.890	0.648	0.22
1	9.662	2.354	1.149	0.738	0.549	
2	9.662	2.844	1.109	0.624	0.437	
3		2.844	1.406	0.688	0.411	
4		2.844	1.429	0.890	0.500	
W/τ_p		5/6=**0.83**	7/9=**0.78**	10/12=**0.83**	12/15=**0.8**	

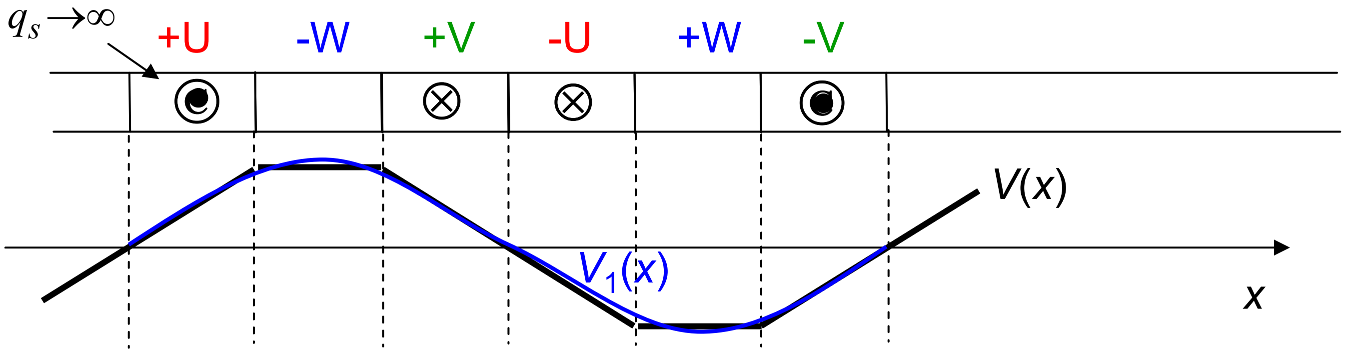
Harmonic stator leakage at infinite slot count $q \to \infty, m = 3$

E.g.: Full-pitched winding, m_s = 3: $L_{s,\sigma,O} = \sigma_{Os} \cdot L_h$

$$W = \tau_p: \quad \sigma_{Os}\Big|_{q_s\to\infty} = \lim_{q_s\to\infty}\left(\frac{\pi}{3k_{ws,1}}\right)^2 \cdot \frac{5q_s^2+1}{6q_s^2} - 1 = \left(\frac{\pi}{3\cdot(3/\pi)}\right)^2 \cdot \frac{5}{6} - 1 = 0.0022 > 0$$

$$W = \tau_p: \quad k_{ws,1} = k_{ds,1}\cdot k_{ps,1} = k_{ds,1}$$

$$\lim_{q_s\to\infty} k_{ds,1} = \lim_{q_s\to\infty} \frac{\sin(\pi/6)}{q_s\cdot\sin\left(\frac{\pi}{6q_s}\right)} \underset{\xi=\frac{\pi}{6q_s}}{=} \lim_{\xi\to 0}\frac{0.5}{\frac{\sin\xi}{\xi}}\cdot\frac{6}{\pi} = \frac{0.5\cdot 6}{\pi} = \frac{3}{\pi}$$

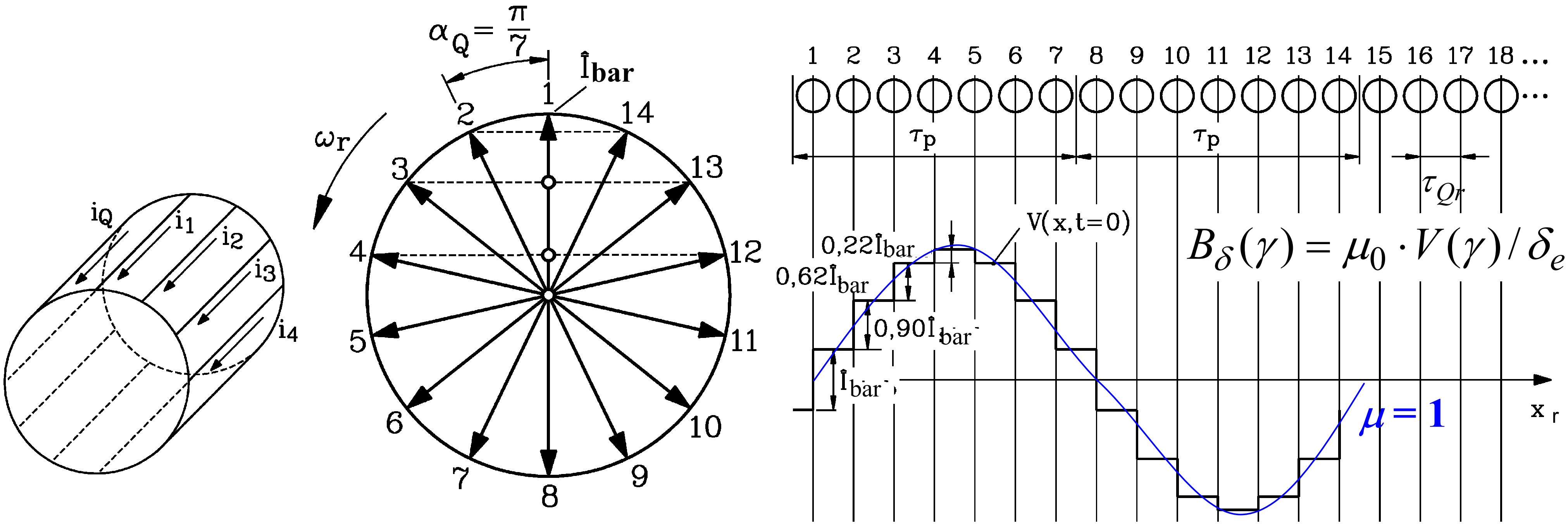


Result:

Even at infinitely big slot count the m.m.f. V(x) is not sinusoidal due to the 6 zones per pole-pair of the phase belt.

$$\sigma_{Os}\Big|_{q_s\to\infty} = 0.0022 > 0$$

M.m.f. of cage current distribution $V_r(x_r, t)$



- **Symmetrical m_r-polyphase bar current system:**
- ***Example:*** Q_r = 28 bars, $2p$ = 4, Q_r / p = 14, phase shift $\alpha_Q = 2\pi p / Q_r = \pi / 7$

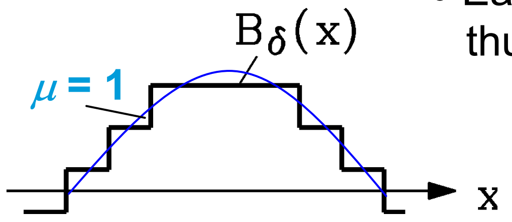
$$V_r(x_r,t) = \sum_{\mu=1,\ldots}^{\infty} \frac{\sqrt{2}}{\pi} \cdot \frac{Q_r}{p} \cdot \frac{1}{2} \cdot \frac{1}{\mu} \cdot I_{bar} \cdot \cos\left(\frac{\mu \cdot \pi \cdot x_r}{\tau_p} - 2\pi \cdot f_r \cdot t\right)$$

$$\mu = 1 + \frac{Q_r}{p} g_r \qquad g_r = 0, \pm 1, \pm 2, \ldots$$

$\mu = 1, -13, 15, -27, 29, \ldots$

Rotor harmonic inductance $L'_{r\sigma O}$

$$B_{\delta r}(x_r,t) = \sum_{\mu=1+g_r\cdot Q_r/p}^{\infty} \hat{B}_{\delta\mu}(I_r)\cdot\cos(\mu\cdot\pi\cdot x_r/\tau_p - \underbrace{\omega_r}_{s\cdot\omega_s}\cdot t) \qquad \frac{\hat{B}_{\delta\mu}}{\hat{B}_{\delta,\mu=1}} = \frac{1}{\mu}$$



- Each harmonic self-induces rotor cage with rotor frequency f_r, thus adding up self-induced rotor harmonic voltage $j\omega_r\cdot\sigma_{Or}\cdot L_{h,\nu=1}\cdot\underline{I}_r$

$$L'_{r\sigma O} = \sum_{|\mu|>1}^{\infty} L'_{h\mu} = \sum_{|\mu|>1}^{\infty} \ddot{u}_{U\mu}\cdot\ddot{u}_{I\mu}\cdot L_{h\mu} = \sigma_{Or}\cdot L_{h,\nu=1}$$

$$L'_{h\mu} = \mu_0\cdot(N_s k_{ws\mu})^2\cdot\frac{2\cdot m_s}{\pi^2\cdot\mu\cdot p}\cdot\frac{l_e\cdot\tau_p/\mu}{\delta_e} \sim \frac{k_{ws\mu}^2}{\mu^2}$$

$$\mu = 1+\frac{Q_r}{p}g_r \qquad g_r = 0,\pm1,\pm2,\ldots \qquad \sigma_{Or} = \sum_{|\mu|>1}^{\infty}\frac{k_{ws\mu}^2}{\mu^2\cdot k_{ws1}^2}$$

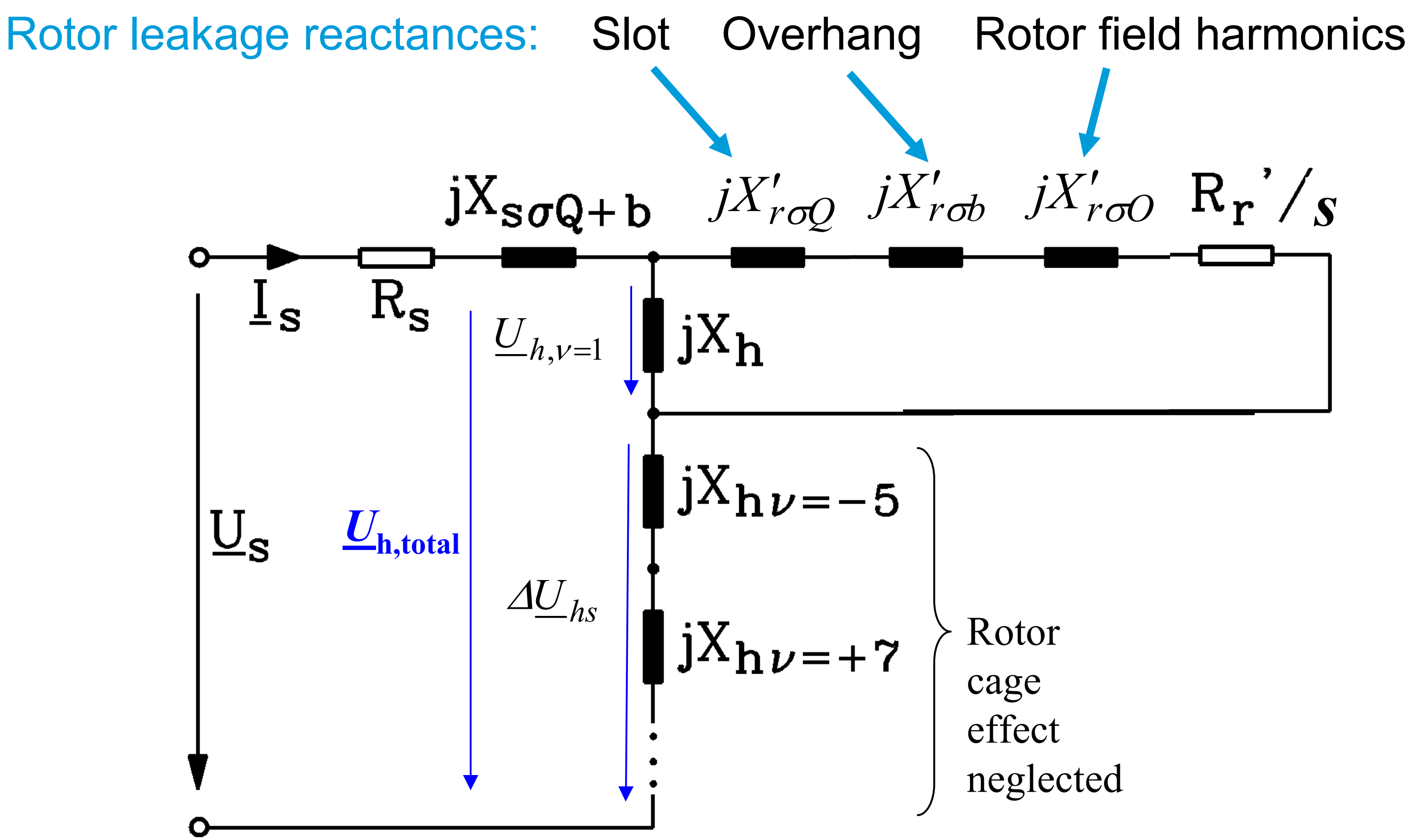
- Summation of series gives exact formula for rotor harmonic leakage coefficient e.g. via magnetic energy method:

$$\sigma_{O,r} = \frac{1}{\left(\frac{\sin(p\cdot\pi/Q_r)}{p\cdot\pi/Q_r}\right)^2} - 1$$

2. Design of Induction Machines

Rotor leakage reactances

$$X'_{r\sigma O} = j\omega_s \cdot \sigma_{Or} \cdot L_{h,\nu=1}$$

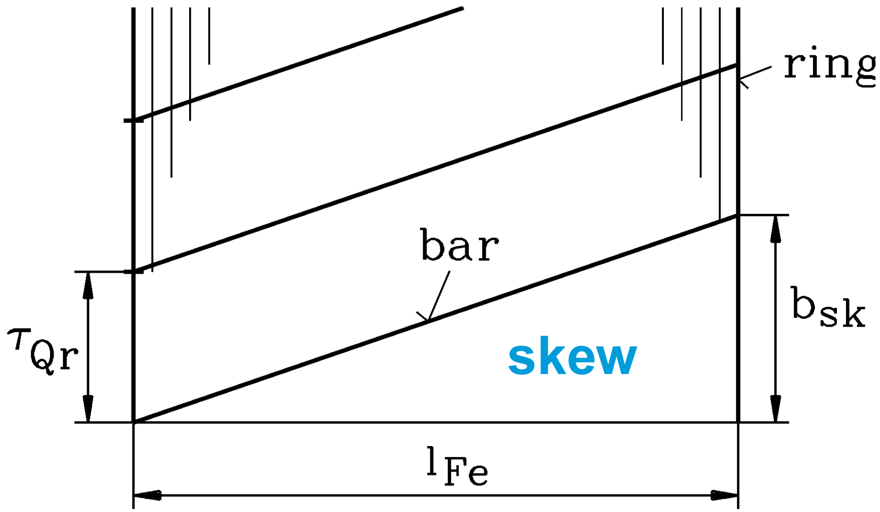


2. Design of Induction Machines

Rotor harmonic and skew leakage reactance

- Each stator harmonic wave is decoupled partially from the rotor cage by the skew between stator and rotor winding
- With die-cast rotor cages usually the rotor cage is skewed
- With brazed rotor copper cages often the stator slots are skewed.
- For high voltage windings with parallel-sided open slots a special sheet punching method acc. to HUBER is used: „HUBER-Stanzen“

Skewed rotor bars of cage rotor:



- **Exact formula:**

$$\sigma_{O+sk,r} = \frac{1}{\left(\frac{\sin(p\cdot\pi/Q_r)}{p\cdot\pi/Q_r}\right)^2 \cdot \left(\frac{\sin[(p\cdot\pi/Q_r)\cdot(b_{sk}/\tau_{Qr})]}{(p\cdot\pi/Q_r)\cdot(b_{sk}/\tau_{Qr})}\right)^2} - 1$$

Separation of rotor harmonic & skew leakage reactance

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- **Skew leakage:** $\sigma_{\mathrm{sk,r}} = \sigma_{\mathrm{O+sk,r}} - \sigma_{\mathrm{O,r}}$
- **Good approximation, as** $p\pi / Q_{\mathrm{r}} << 1$ e.g.: $\frac{p\pi}{Q_r} = \frac{2\pi}{50} = 0.13 << 1$

Harmonic reactance **skew leakage**

$$\sigma_{O+sk,r} \cong \frac{1}{3} \cdot \left(\frac{p \cdot \pi}{Q_r}\right)^2 + \frac{1}{3} \cdot \left(\frac{p \cdot \pi}{Q_r}\right)^2 \cdot \left(\frac{b_{sk}}{\tau_{Qr}}\right)^2$$

$$\sigma_{\mathrm{O,r}} = \frac{1}{\left(\frac{\sin a}{a}\right)^2} - 1 \Rightarrow \quad a = \frac{p\pi}{Q_r} << 1 : \frac{\sin a}{a} \approx \frac{1}{a} \cdot \left(a - \frac{a^3}{6}\right),$$

$$\sigma_{\mathrm{O,r}} = \frac{1}{\left(1 - \frac{a^2}{6}\right)^2} - 1 \approx \frac{1}{1 - \frac{a^2}{3}} - 1 \approx 1 + \frac{a^2}{3} - 1 = \frac{a^2}{3}$$

Stator and rotor harmonic leakage inductance

Example: 500 kW-cage induction motor:

Harmonic stator reactance: $\sigma_{Os}(q_s = 5, S = 3) = 0.411/100$

$$X_{s\sigma O} = \omega_s L_{s\sigma O} = \sigma_{Os} \cdot X_h = (0.411/100) \cdot 155.1 = \underline{\underline{0.64\ \Omega}}$$

No-load saturation

Stray inductance of skewed rotor: Rotor skew $b_{sk} = \tau_{Qs} = 24$ mm

$$b_{sk} / \tau_{Qr} = \tau_{Qs} / \tau_{Qr} = Q_r / Q_s = 50/60$$

$$\sigma_{O+sk,r} = \frac{1}{\left(\frac{\sin(2\pi/50)}{2\pi/50}\right)^2 \cdot \left(\frac{\sin[(2\pi/50)\cdot(50/60)]}{(2\pi/50)\cdot(50/60)}\right)^2} - 1 = 0.896/100$$

$$X'_{r\sigma O} = \omega_s L'_{r\sigma O} = \sigma_{Or} \cdot X_h = (0.896/100) \cdot 155.1 = \underline{\underline{1.39\ \Omega}}$$

No-load saturation

Rotor harmonic stray inductance: $\underline{0.82\ \Omega}$
Skew leakage stray reactance: $\underline{0.57\Omega}$

$$0.82 + 0.57 = \underline{\underline{1.39\ \Omega}}$$

2. Design of Induction Machines
Resulting stator and rotor leakage inductances

Example: Data: 550 kW 4-pole cage induction motor, 6.6 kV Y, 50 Hz

Saturated magnetizing reactance: $X_h = 155.1\,\Omega$

Resulting stator leakage reactance:

$$X_{s\sigma} = X_{s\sigma Q} + X_{s\sigma b} + X_{s\sigma O} = 2.28 + 1.97 + 0.64 = 4.89\,\Omega$$

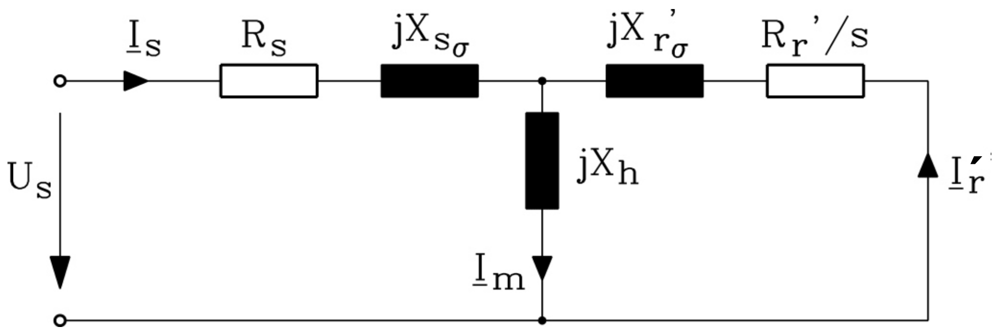
Resulting rotor leakage reactance (UNskewed): (End ring leakage: 0.68 Ω!)

a) Stand still (locked rotor, *s* = 1):

$$X'_{r\sigma} = X'_{r\sigma Q} + X'_{r\sigma b} + X'_{r\sigma O} = 2.80 + 0.68 + 0.82 = 4.30\,\Omega$$

b) Rated slip 1.5%: $X'_{r\sigma} = 6.37\,\Omega$

c) Arbitrary slip: $X'_{r\sigma} = 3.20\,\Omega \cdot k_L(s) + 3.18\,\Omega$



Relative stator and rotor leakage inductances σ_s, σ_r

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Example: Data: 550 kW 4-pole cage induction motor, 6.6 kV Y, 50 Hz

Saturated magnetizing reactance: $X_h = 155.1\,\Omega$

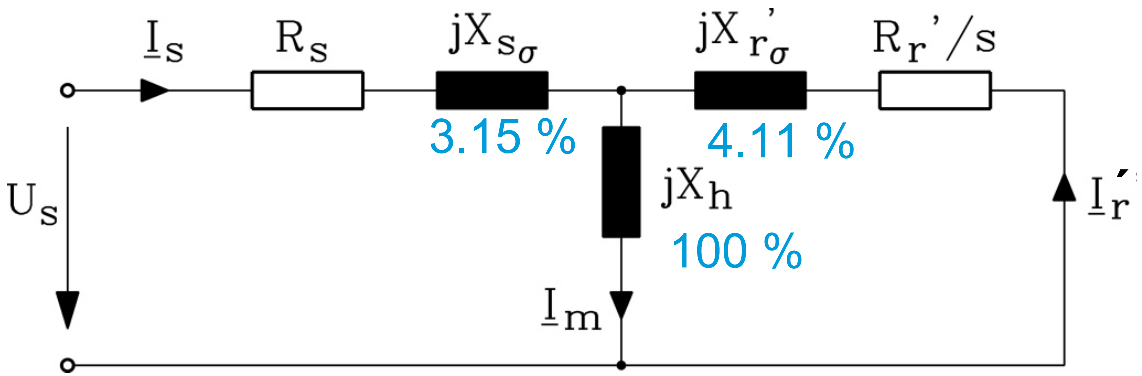
Resulting stator leakage reactance: $X_{s\sigma} = 4.89\,\Omega$

Resulting rotor leakage reactance (UNskewed): Rated slip: $X'_{r\sigma} = 6.37\,\Omega$

Relative leakage:

$$\sigma_s = X_{s\sigma} / X_h = 4.89 / 155.1 = 0.0315$$ estimated: 0.05

$$\sigma_r = X'_{r\sigma} / X_h = 6.37 / 155.1 = 0.0411$$



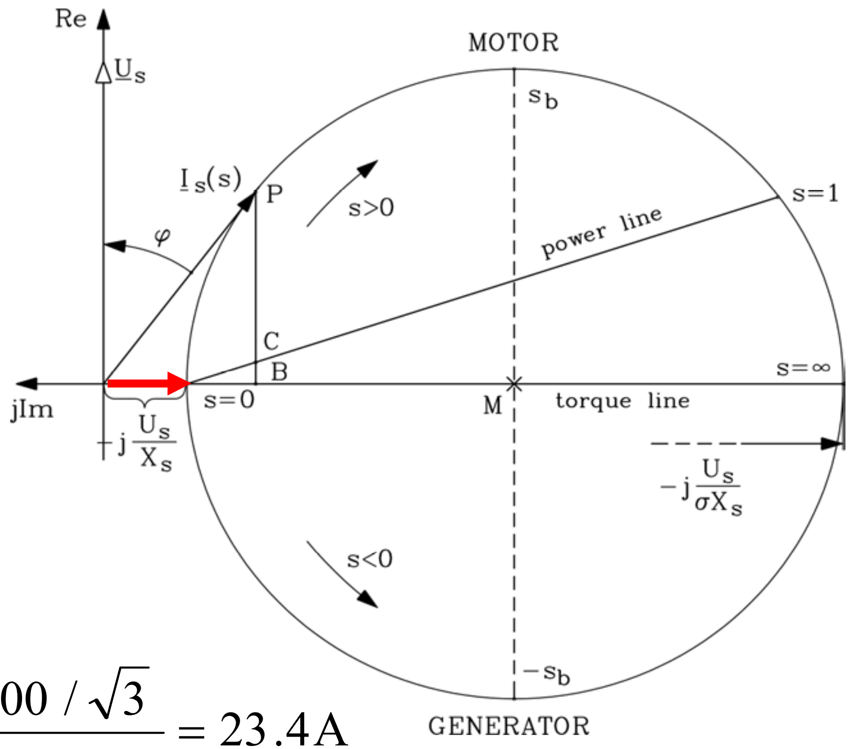
Result:
Rotor slot count $Q_r = 50 < Q_s = 60$:
Higher harmonic leakage $\Rightarrow$ higher resulting leakage

2. Design of Induction Machines

Root locus of stator current phasor at fixed stator voltage

- Amplitude and phase angle of stator current phasor $\underline{I}_s$ depend on load (= slip *s*).
- If machine parameters are constant ⇒ locus diagram of stator current phasor is a circle.
- Simplification: If stator resistance is neglected (**R_s = 0**) ⇒ circle centre M located on abscissa (***HEYLAND*'s circle**).

No-load: $\underline{I}_s(s=0) = -j\dfrac{U_s}{X_s}$



Example: *500 kW induction motor no-load current*

$$\underline{I}_s(s=0) \approx -j\frac{U_s}{X_s} = -j\frac{6600/\sqrt{3}}{160.0} = -j\cdot 23.8\text{A}$$

$$X_s = X_{s\sigma} + X_h = 4.89 + 155.1 = 160.0\ \Omega$$

$$Z_s = \sqrt{R_s^2 + X_s^2} = \sqrt{0.74^2 + 160.0^2} = 160.0\ \Omega$$

estimated: σ_s = 0.05: $I_s(s=0) \approx \dfrac{U_s}{X_h\cdot(1+\sigma_s)} = \dfrac{6600/\sqrt{3}}{155.1\cdot 1.05} = 23.4\text{A}$

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Summary:
Stray flux and inductance

- Slot, overhang and harmonic stray flux
- Skewing increases stray flux
- Coil pitching ~ 0.8 reduces slot stray flux and harmonic stray flux
- Stator and rotor stray flux are of the same order of magnitude
- Results are the stray inductances for the equivalent circuit
- Note: $L_s, L_r, M \Rightarrow \sigma = 1 - M^2/(L_s \cdot L_r)$ can be measured directly, but not $L_{s\sigma}, L_{r\sigma}$

$$L_{s\sigma} = L_s - \ddot{u} \cdot M \quad L_{r\sigma} = L_r - M / \ddot{u} \quad \ddot{u}: \text{arbitrary}$$

$$\sigma \cdot L_s \approx L_{s\sigma} + \ddot{u}^2 \cdot L_{r\sigma} \quad \text{but not exactly}$$

$$X_h = 155.1\,\Omega \quad X_{s\sigma} = 4.89\,\Omega \quad X'_{r\sigma} = 6.37\,\Omega$$

$$\sigma = 1 - \frac{X_h^2}{(X_h + X_{s\sigma}) \cdot (X_h + X'_{r\sigma})} = 1 - \frac{155.1^2}{(155.1 + 4.89) \cdot (155.1 + 6.37)} = 0.069$$

2. Design of induction machines

2.1 Main dimensions and basic electromagnetic quantities of induction machines

2.2 Scaling effect in electric machines

2.3 Stator winding low and high voltage technology

2.4 Stator winding design

2.5 Rotor cage design

2.6 Wound rotor design

2.7 Design of main flux path of magnetic circuit

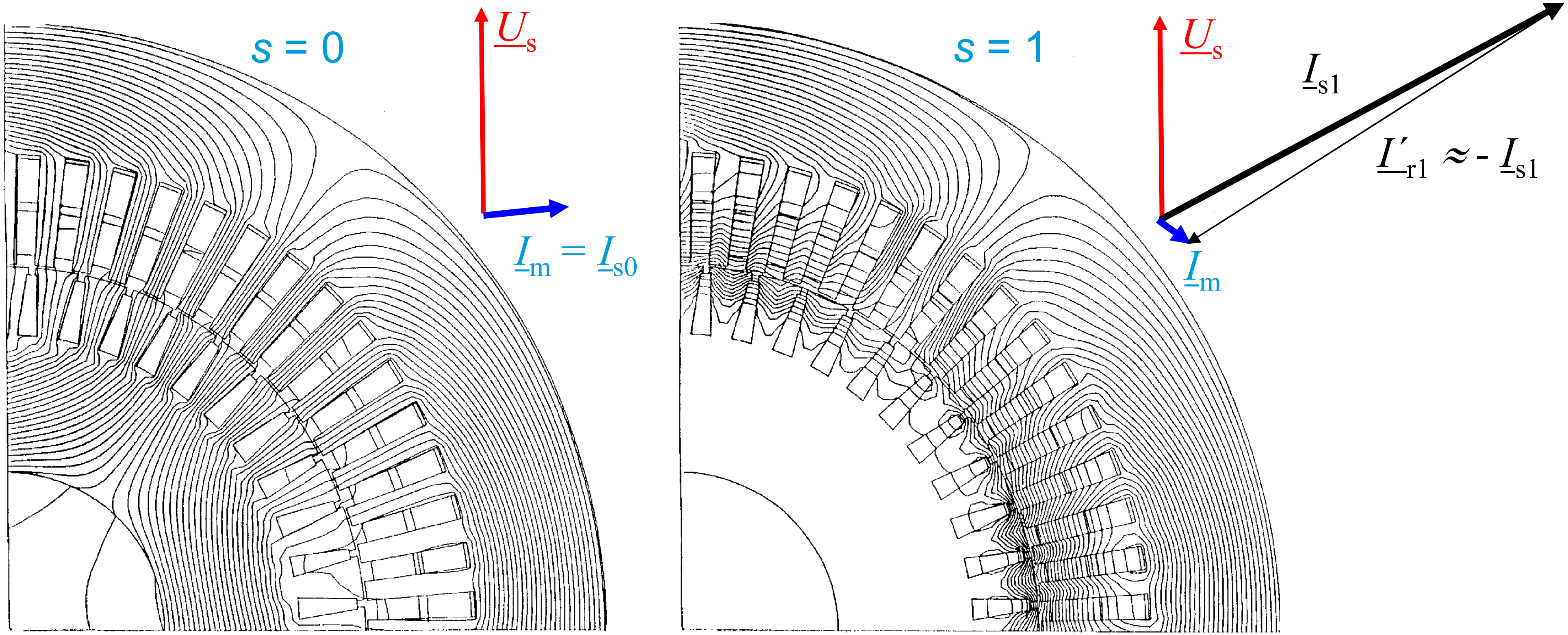
2.8 Stray flux and inductance

2.9 Influence of saturation on inductance

2.10 Masses and losses

2. Design of Induction Machines

Numerically calculated influence of saturation on inductances

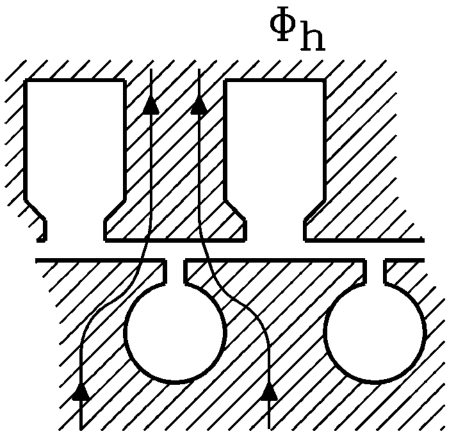


a) At no-load (s = 0, rotor current zero) b) At stand still (locked rotor) s = 1

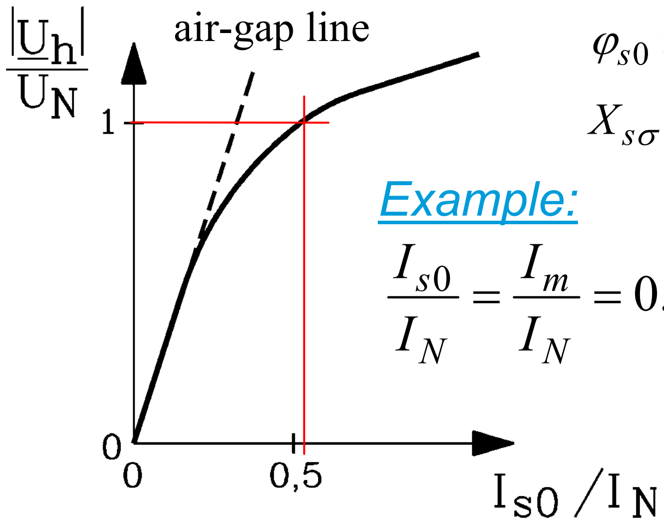
Numerically calculated two-dimensional magnetic flux density B of a three-phase, 4-pole high voltage cage induction machine with wedge rotor slots (Q_s / Q_r = 60/44) at rated voltage

2. Design of Induction Machines

Saturation of teeth and yokes by main flux (s = 0)



a) main flux path



b) measured "no-load"-characteristic

$$\underline{U}_h = \underline{U}_s - (R_s + j \cdot X_{s\sigma}) \cdot \underline{I}_{s0}$$

$$\varphi_{s0} : \angle(\underline{U}_s, \underline{I}_{s0})$$

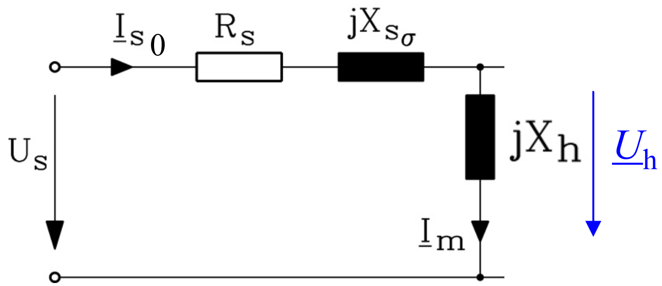
$$X_{s\sigma} \approx X_\sigma / 2$$

X_σ from test at $s = 1$

Example:

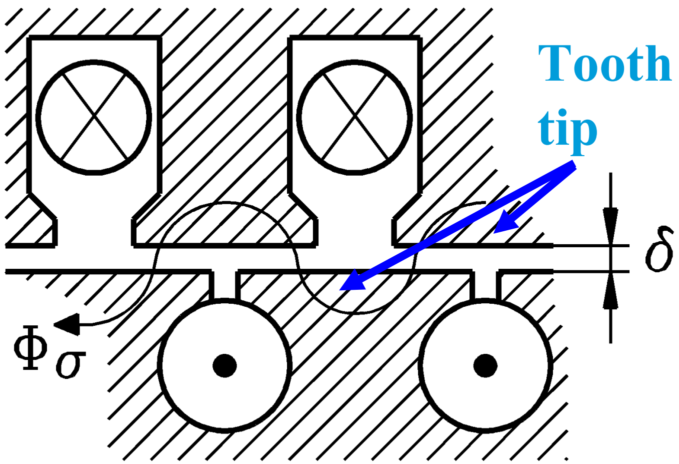
$$\frac{I_{s0}}{I_N} = \frac{I_m}{I_N} = 0.55$$

Saturation of $X_h = U_h/I_0$ is measured!

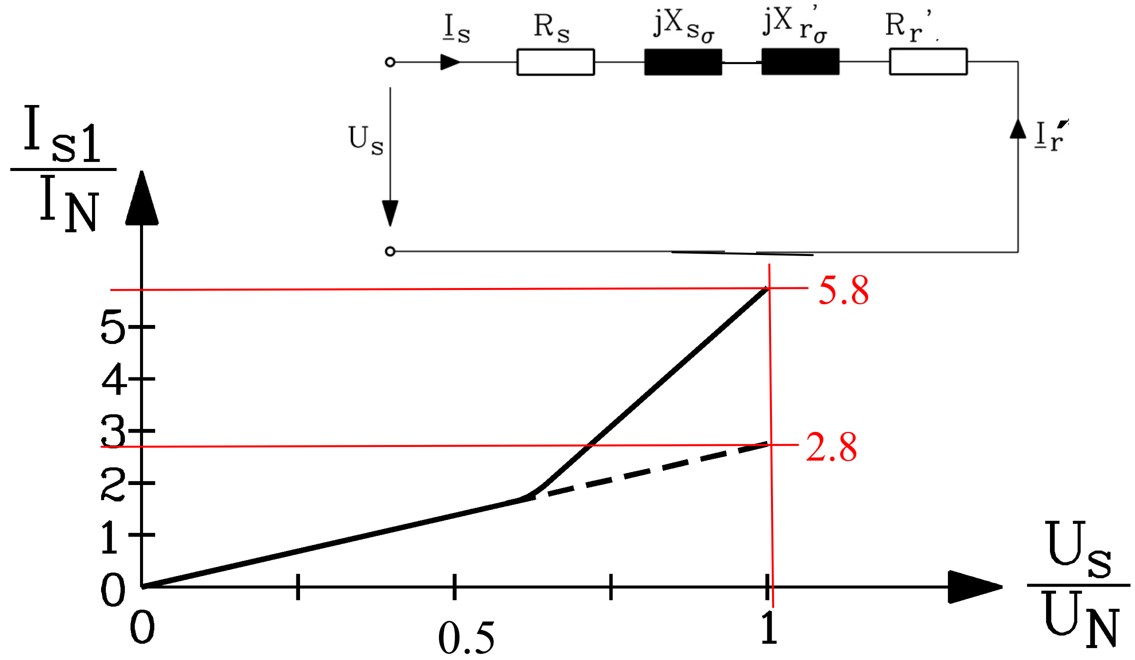


2. Design of Induction Machines

Saturation of tooth tips by zig-zag stray flux (s = 1)



a) Zig-zag stray flux path



b) Measured “locked rotor”-characteristic

- Saturated $X_\sigma = X_{s\sigma} + X'_{r\sigma}$ is measured at s = 1 via reactive power Q_s by $X_\sigma \cong Q_s/(3(I_{s1})^2)$!
- Saturation of leakage flux causes increase of locked rotor current e.g. from 2.8 to 5.8, which should be kept as low as possible !   For details see the text-book!

Summary:
Influence of saturation on inductance

- Main flux saturation reduces main inductance L_h
- No-load current I_{s0} is increased by main flux saturation (teeth & yokes)
- Stray fluxes especially at open slots pass large parts of air, so they are usually not influenced by saturation
- Closed slots & harmonic stray flux are influenced by tooth tip saturation
- Stray flux saturation reduces stray inductances $L_{s\sigma}$, $L_{r\sigma}$
- Short-circuit current I_{s1} (= starting current) is increased by stray flux saturation

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2. Design of induction machines

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2.10 Masses and losses

- Determination of **efficiency** η for demanded output power P_{out} requires knowledge of losses P_d

$$\eta = \frac{P_{out}}{P_{out} + P_d}$$

- **Loss components** are
- Stator and rotor ohmic losses,
- Friction and windage losses,
- Iron losses (mainly in stator iron for $0 \leq s \leq 2s_N$),
- Brush losses in case of slip ring induction machines,
- Additional no-load losses such as tooth pulsation and surface losses,
- Additional load losses such as stator and rotor eddy current losses in conductors.

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550 kW cage induction motor, 6.6 kV Y, 50 Hz, unskewed rotor

Electrical input power $P_{e,in}$	**574 921 W**
Stator winding losses $P_{Cu,s}$	7 739 W
Total iron losses P_{Fe}	6 779 W
Stray load losses $P_{ad,1}$ (= 0.5% of 574 921 W)	2 875 W
Air gap power P_{δ}	557 528 W
Rotor cage losses $P_{Cu,r}$ (slip: 0.814%)	4 538 W
Friction and windage losses P_{fr+w}	2 670 W
Mechanical output power $P_{m,out}$	**550 320 W (≅ 550 kW)**
Efficiency η	95.72 %

2. Design of Induction Machines
Stator winding resistance per phase & copper mass

$$\rho(\vartheta) = \rho(20°C)\cdot(1+\alpha_{\vartheta}\cdot\Delta\vartheta) \qquad \alpha_{\vartheta} = 1/(255\ \mathrm{K}) \qquad \Delta\vartheta = \vartheta - 20°C$$

$$R_s = \frac{1}{\kappa}\cdot\frac{N_s\cdot 2\cdot(L+l_b)}{a\cdot a_i\cdot A_{TL}}$$

ρ: resistivity

κ: conductivity $\kappa = 1/\rho$

Example: *Hot resistance value !*

$$\rho_{Cu}(75°C)/\rho(20°C) = (1+(75-20)/255) = \underline{\underline{1.22}}$$

$$R_s = \frac{1}{\kappa}\cdot\frac{N_s\cdot 2\cdot(L+l_b)}{a\cdot a_i\cdot A_{TL}} = \frac{1}{57\cdot 10^6/1.22}\cdot\frac{200\cdot 2\cdot(0.458+0.6148)}{1\cdot 1\cdot 12.42\cdot 10^{-6}} = \underline{\underline{0.74}}\,\Omega$$

$$m_{Cu,s} = \gamma_{Cu} m_s N_s \cdot 2(L+l_b)\cdot a\cdot a_i A_{TL} =$$

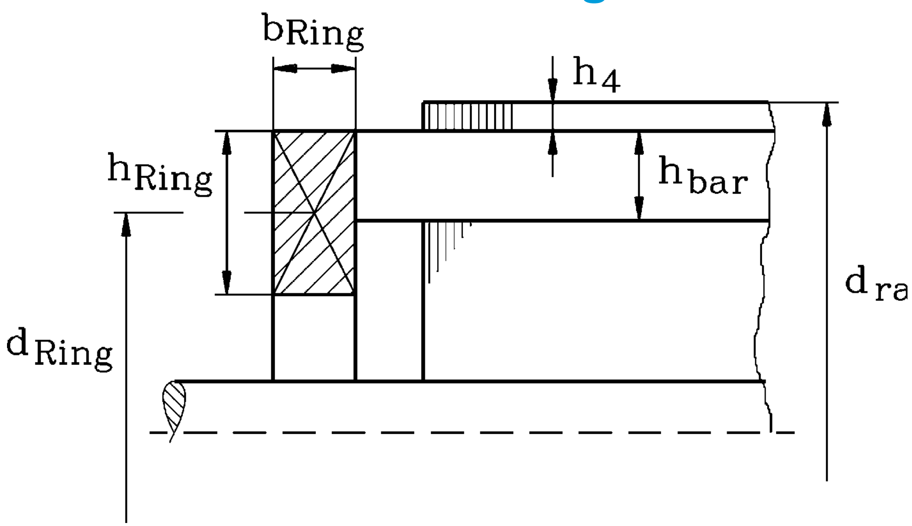
$$= 8900\cdot 3\cdot 200\cdot 2\cdot(0.458+0.6148)\cdot 1\cdot 1\cdot 12.42\cdot 10^{-6} = \underline{\underline{142.3}}\,\mathrm{kg}$$

2. Design of Induction Machines

Rotor ring segment resistance

$$R_r = R_{bar} + \Delta R^*_{Ring} = \frac{1}{\kappa} \cdot \frac{l_e \cdot k_R + L - l_e}{A_{Cur}} + \Delta R_{Ring} \cdot \frac{1}{2\sin^2(\pi p / Q_r)}$$

Geometrical data of rings:



$$d_{Ring} = d_{si} - 2\delta - 2h_4 - h_{Ring} =$$

$$= 458 - 2 \cdot 1.4 - 2 \cdot 3.4 - 40 = 408.4 \text{ mm}$$

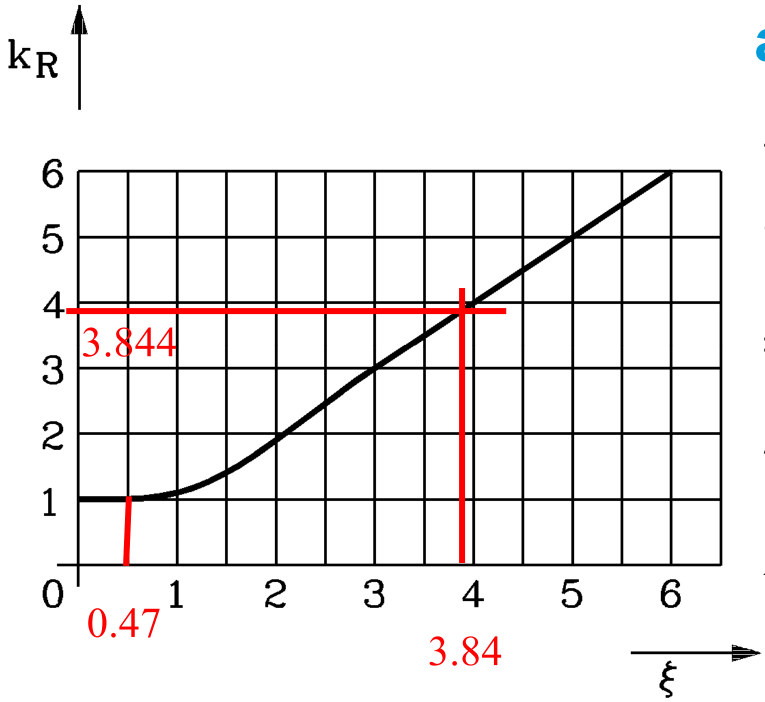
$$d_{Ring} \cdot \pi / Q_r = 408.4 \cdot \pi / 50 = 25.66 \text{ mm}$$

$$\Delta R_{Ring} = d_{Ring} \pi / (\kappa \cdot Q_r \cdot A_{Ring}) = 0.4084 \cdot \pi / ((57/1.22) \cdot 10^6 \cdot 50 \cdot 800 \cdot 10^{-6}) = 0.69\ \mu\Omega$$

$$\Delta R^*_{Ring} = \Delta R_{Ring} \cdot \frac{1}{2\sin^2(\pi p / Q_r)} = 0.69 \cdot \frac{1}{2\sin^2(\pi \cdot 2/50)} = 22.0\ \mu\Omega$$

Rotor bar resistance at 75°C

k_R: Increase of R_r for deep bar *(k_R depends on slip due to current displacement)*



a) At stand still (locked rotor) *s* = 1.0:

$f_r = 1 \cdot 50 = 50$ Hz:

$$\xi = h_1 \sqrt{\pi \cdot f_r \cdot \mu_0 \cdot \kappa_{Cu}} =$$

$$= 0.04 \sqrt{\pi \cdot 50 \cdot 4\pi \cdot 10^{-7} \cdot (57/1.22) \cdot 10^6} = 3.84$$

$$k_R = 3.844$$

$$R_{bar} = \frac{(0.392 \cdot 3.844 + (0.458 - 0.392))}{57/1.22 \cdot 10^6 \cdot 200 \cdot 10^{-6}} = 168.32\ \mu\Omega$$

b) Rated slip *s* = 0.015:

$f_r = 0.015 \cdot 50 = 0.75$ Hz: $\quad \xi = 0.47$,

$k_R = 1.0043$, $R_{bar} = 49.2\ \mu\Omega$

Rotor bar and ring segment resistance R'_r

$$R'_r = \ddot{u}_U \ddot{u}_I R_r$$

Rated slip s = 0.015: $f_r = 0.015 \cdot 50 = 0.75$ Hz:

$$R_r = R_{bar} + \Delta R^*_{Ring} = 49.2 + 22.0 = \underline{\underline{71.2\ \mu\Omega}}$$

$$R'_r = \ddot{u}_U \ddot{u}_I R_r = 364 \cdot 21.84 \cdot 0.0000712 = \underline{\underline{0.566\ \Omega}}$$

Cage losses:

$$P_{Cu,r} = Q_r R_r I_r^2 = m_s R'_r I'^2_r$$

$$\begin{cases} P_{Cu,r} = Q_r R_r I_r^2 = Q_r \cdot \ddot{u}_I^2 R_r \cdot (I_r / \ddot{u}_I)^2 = \\ = Q_r \cdot \dfrac{m_s}{Q_r} \ddot{u}_U \ddot{u}_I R_r \cdot I'^2_r = m_s \cdot R'_r \cdot I'^2_r \end{cases}$$

Rotor current I'_r to be calculated from T-equivalent circuit for chosen slip s !

Cage mass: Q_r bars 2 rings

$$m_{Cu,r} = 8900 \cdot \left[50 \cdot 200 \cdot 10^{-6} \cdot 0.458 + 2 \cdot 800 \cdot 10^{-6} \cdot 0.4084 \cdot \pi\right] = \underline{\underline{59.0}}\ \text{kg}$$

2. Design of Induction Machines

Friction and windage losses *(air-cooled machines)*

- Friction losses in ball or sleeve bearings $P \sim n^2$
- Windage losses: Power consumption of shaft mounted fan $P \sim n^3$
- At high speed or big power machinery: Air friction losses of rotor $P \sim n^3$

Details: see Lecture „Large generators and high power drives“ (2+1)

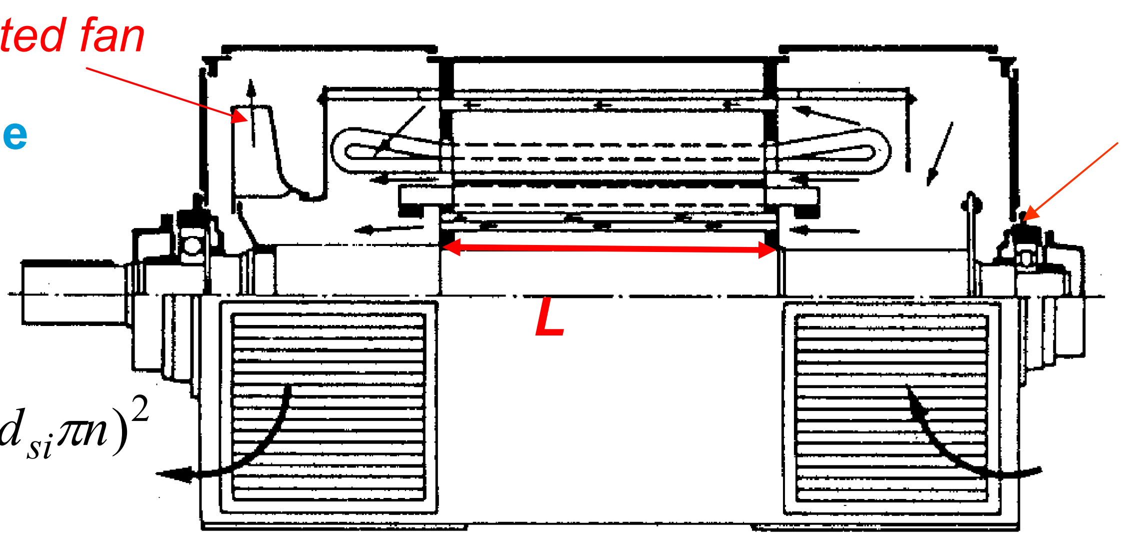
Semi-empirical equations to calculate losses !

Open-ventilated cage induction machine

O: Rotor surface

$$P_{fr+w} \sim O \cdot v^2$$

$$P_{fr+w} \sim (\pi \cdot d_{si} \cdot L) \cdot (d_{si} \pi n)^2$$



Source: ABB, Switzerland

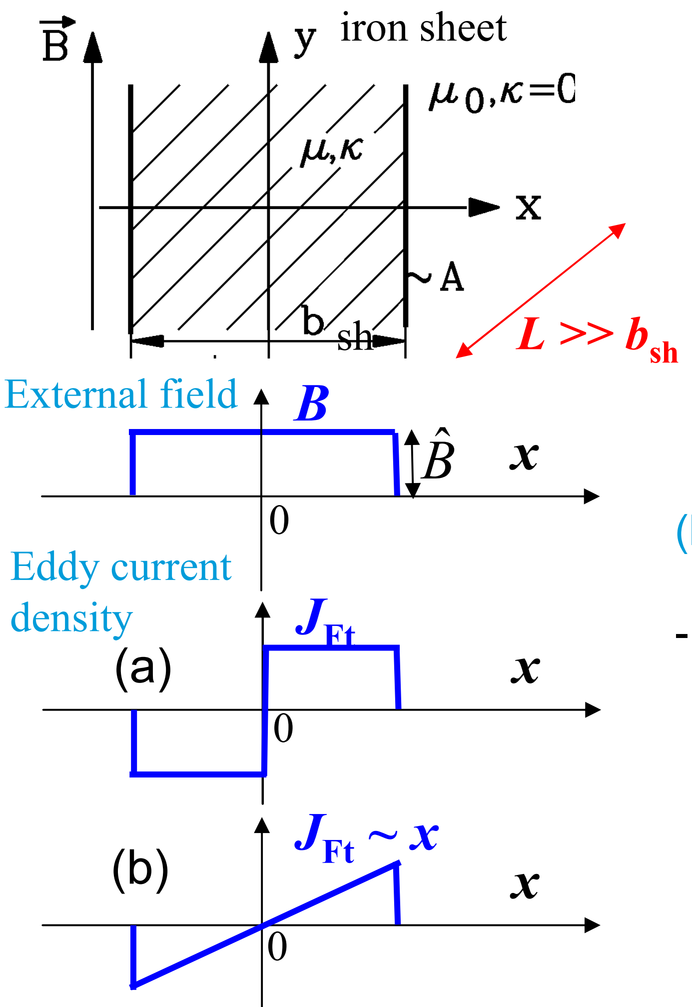
Example:

500 kW, 4-pole induction motor: Synchronous speed n = 1500/min:

$$P_{fr+w} \approx 10 \cdot d_{si}^3 \cdot L \cdot \pi^2 \cdot n^2 = 10 \cdot 0.458^3 \cdot 0.458 \cdot \pi^2 \cdot (1500/60)^2 = \underline{\underline{2714\ \text{W}}}$$

2. Design of Induction Machines

Eddy currents in conductive sheets without their self field



(a) Approximate calculation with sinusoidal varying flux:

- Induced voltage: $U_i = \omega\Phi/\sqrt{2} = \sqrt{2}\pi f \cdot (b_{sh} L \cdot \hat{B})$
- Electrical field strength (loop $2L+2b_{sh} \approx 2L$): $E = U_i/(2L)$
- Eddy current density: $J_{Ft} = \kappa \cdot E = \kappa \cdot \pi \cdot f \cdot b_{sh} \cdot \hat{B}/\sqrt{2}$
- Eddy current losses per volume:

$$P_{Ft}/V = J_{Ft}^2/\kappa = \kappa \cdot (\pi \cdot f \cdot b_{sh} \cdot \hat{B})^2/2$$

(b) Considering, that an inner "loop" has a smaller flux $\Phi \sim x$ than an "outer" loop, yields less eddy currents J_{Ft} within the sheet.

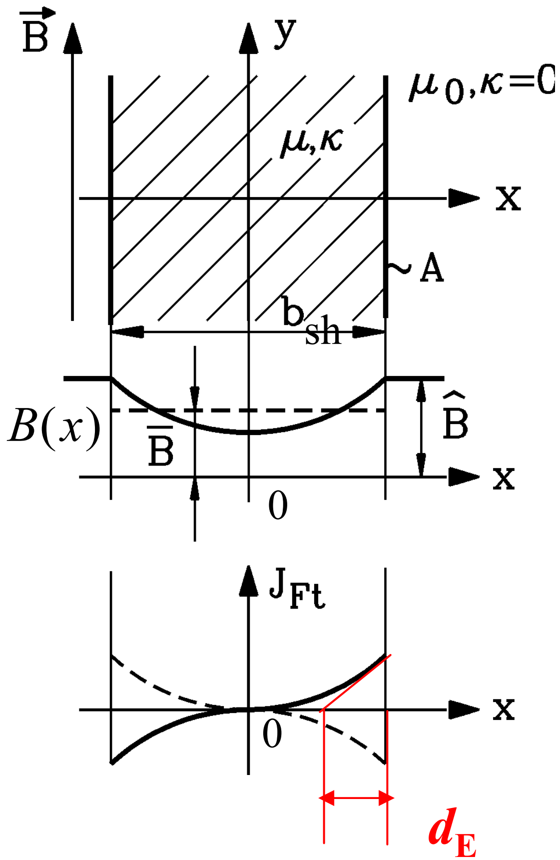
- Neglecting the eddy current self field reaction, we get a "triangular" eddy current density $J_{Ft} \sim x$ with a 1/3 loss density.

$$P_{Ft}/V = \kappa \cdot (\pi \cdot f \cdot b_{sh} \cdot \hat{B})^2/(2 \cdot 3)$$

$$\int_0^1 x^2 \cdot dx = \frac{1}{3}$$

$$P_{Ft}/V = \frac{4 \cdot \kappa}{3} \cdot \left(\frac{\pi}{2\sqrt{2}} \cdot f \cdot b_{sh} \cdot \hat{B} \right)^2$$

Eddy current losses P_{Ft} in conductive sheets with their self field



$$P_{Ft} / V = \frac{4 \cdot \kappa}{3} \cdot \left(\frac{\pi}{2\sqrt{2}} \cdot f \cdot b_{sh} \cdot \overline{B} \right)^2 \cdot k_W$$

$$d_E = 1 / \sqrt{\pi \cdot f \cdot \mu \cdot \kappa}$$

d_E: Penetration depth of $B(x)$ in iron sheet

$$\zeta = b_{sh} / d_E$$

Attenuation factor $k_W \leq 1$ due to eddy currents self field

$$k_W = \frac{3}{\zeta} \cdot \frac{\sinh\zeta - \sin\zeta}{\cosh\zeta - \cos\zeta}$$

J_{Ft}: Eddy current density in iron sheet ($\Rightarrow$ *Foucault* losses)

$$k_W \approx 1, \quad 0 \leq \zeta << 1$$

$$k_W \approx 3/\zeta, \quad \zeta >> 1$$

b_{sh}: Sheet thickness

$B(x)$: Attenuated flux density in iron sheet due to the opposing self field of the eddy currents

$$\text{e.g.}: d_E = b_{sh}/2 \Rightarrow \zeta = b_{sh}/d_E = 2$$

2. Design of Induction Machines

"Thin" and "thick" conductive sheets

$$P_{Ft}/V = \frac{4\cdot\kappa}{3}\cdot\left(\frac{\pi}{2\sqrt{2}}\cdot f\cdot b_{sh}\cdot\overline{B}\right)^2\cdot k_W$$

A) "Thin sheet": Small eddy currents J_{Ft}: Their self-field is negligibly small !

$\zeta \le 2$ $\quad$ $\zeta < 2 \;\Rightarrow\; k_W \cong 1 \;\Rightarrow\; \overline{B} \cong \hat{B}$ is evenly distributed in the sheet

$d_E \ge b_{sh}/2$

$$P_{Ft}/V = \frac{4\cdot\kappa}{3}\cdot\left(\frac{\pi}{2\sqrt{2}}\cdot f\cdot b_{sh}\cdot\overline{B}\right)^2 \sim f^2\cdot b_{sh}^2\cdot\hat{B}^2$$

B) "Thick sheet": Big eddy currents J_{Ft}: Their self field attenuates the *B*-field!

$\zeta > 2$ $\quad$ $\zeta > 2\ldots3 \;\Rightarrow\; k_W \cong 3/\zeta \;\Rightarrow\; \overline{B} << \hat{B}$: *B* repulsed to sheet sides

$$P_{Ft}/V = \frac{b_{sh}\cdot\overline{B}^2}{2}\cdot\sqrt{\frac{\pi^3\cdot f^3\cdot\kappa}{\mu}}$$

Sheet volume: $V = A\cdot b_{sh}$

2. Design of Induction Machines

Eddy current losses in “thin” vs. “thick” sheets

Example:

f = 50 Hz, $\overline{B}$ = 1T, $\kappa_{Fe} = 10^7$ S/m (pure iron), $\mu_{Fe} = 1000\mu_0$, $\gamma_{Fe} = 7850$ kg/m^3

	thin sheet b_{sh} = 0.5 mm	thick sheet b_{sh} = 50 mm
ζ	0.7	70
k_W	1	0.043 (= 3/70)
P_{Ft}/V / W/dm^3	10.3	4390.5
$p_{Ft} = P_{Ft}/(V \cdot \gamma_{Fe})$ / W/kg	1.3	559

$$d_E = 1/\sqrt{\pi \cdot f \cdot \mu \cdot \kappa} = 1/\sqrt{\pi \cdot 50 \cdot 1000 \cdot 4\pi \cdot 10^{-7} \cdot 10^7} = 0.71\,\text{mm}$$

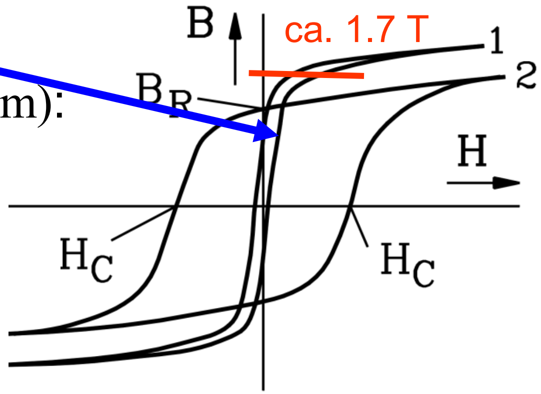
- Eddy current losses p_{Ft} per kg at $\overline{B} = 1$ T, 50 Hz, are much bigger for thick sheets, so laminated iron stack with thin insulated sheets must be used to interrupt I_{Ft}.

- Adding Si to Fe increases sheet resistance (κ_{Fe} drops), and reduces p_{Ft}.

2. Design of Induction Machines

Determination of iron losses

- **Eddy current** (*Foucault*: Ft) and **hysteresis losses** (Hy) in iron stack, measured in *EPSTEIN* frame



- Losses in W/kg: at 1 T, per kg mass of iron stack (f in Hz, b_{sh} in mm):

$$p_{10}(f) = p_{Hy} \cdot (f/50) + p_{Ft} \cdot (f/50)^2 \cdot (b_{sh}/0.5)^2$$

At 1 T, 50 Hz: $v_{10} = p_{Hy} + p_{Ft} \cdot (b_{sh}/0.5)^2$

At 1.5 T, 50 Hz: $v_{15} \approx (1.5\mathrm{T}/1.0\mathrm{T})^2 \cdot v_{10} = 2.25 \cdot v_{10}$

b_{sh} = 0.5 mm	Si	v_{10}	hysteresis losses p_{Hy}	eddy-current losses p_{Ft}
	%	W/kg	W/kg	W/kg
steel sheet (C < 0.08%)	0	≈ 4.2	≈ 2.9 *)	≈ 1.3 *)
sheet type III	1	2.3	1.6	0.7
sheet type IV	3.5	1.7	1.3	0.4

*) not specified, but typically 70% hysteresis losses, 30% *Foucault* losses

Adding of % of Si decreases *B* in *B*(*H*)-curves

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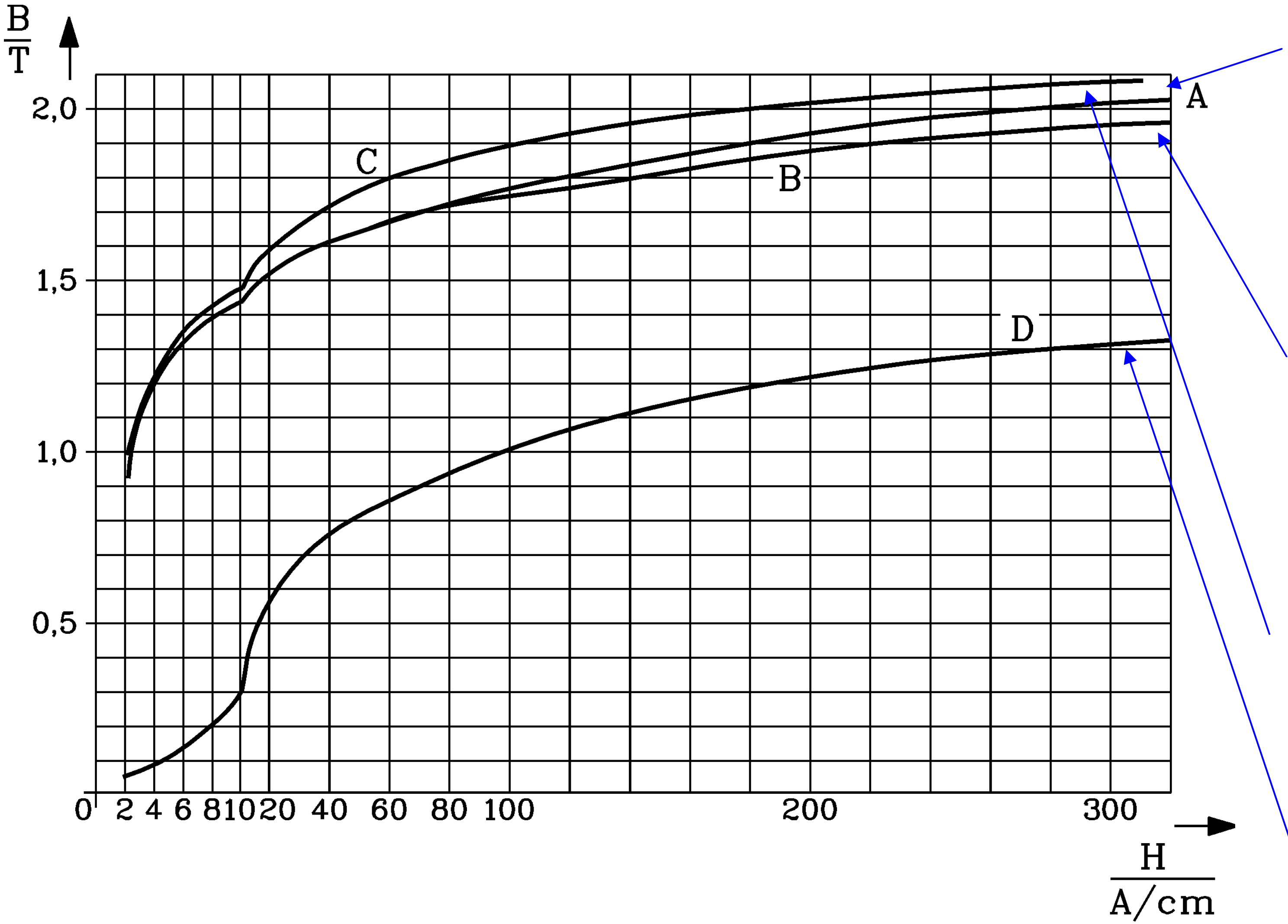


A: sheet type III
1% Si
v_{10} = 2.3 W/kg
Hysteresis: 1.6 W/kg
Foucault: 0.7 W/kg

B: sheet type IV
3.5% Si
v_{10} = 1.7 W/kg
Hysteresis: 1.3 W/kg
Foucault: 0.4 W/kg

C: steel sheet
(C < 0.08%)
0 % Si
v_{10} > 4.2 W/kg

D: cast iron

2. Design of Induction Machines

Calculation of stator iron losses (1)

- **Teeth iron losses:** $\hat{B} = \overline{B}$

$$P_{Fe,d} = k_{Vd} \cdot \left(\frac{B_{d,1/3}}{1.0} \right)^2 \cdot v_{10} \cdot k_f \cdot m_d$$

- **Yoke iron losses:**

$$P_{Fe,y} = k_{Vy} \cdot \left(\frac{B_{ys}}{1.0} \right)^2 \cdot v_{10} \cdot k_f \cdot m_y$$

- **Loss increase** due to manufacturing (e.g. punching):

Teeth: $k_{Vd} = 1.8 \ldots 2.0$

Yoke: $k_{Vy} = 1.3 \ldots 1.5$

- Influence of frequency: $k_f = \dfrac{p_{Hy} \cdot \left(\frac{f}{50} \right) + p_{Ft} \cdot \left(\frac{f}{50} \right)^2}{v_{10}}$ $\quad b_{sh} = 0.5\ \mathrm{mm}$

- Masses:

Stator teeth: $m_{ds} = \gamma_{Fe} \cdot \left\{ \left[(d_{si} + 2l_{ds})^2 - d_{si}^2 \right] \cdot (\pi / 4) - Q_s \cdot A_{Qs} \right\} \cdot l_{Fe} \cdot k_{Fe}$

Stator yoke: $m_{ys} = \gamma_{Fe} \cdot \left[d_{sa}^2 - (d_{sa} - 2h_{ys})^2 \right] \cdot (\pi / 4) \cdot l_{Fe} \cdot k_{Fe}$

Calculation of stator iron losses (2)

- *Example:*

550 kW motor, **50 Hz, iron sheet type IV**, v_{10} = 1.7 W/kg, d_{si} = 458 mm, δ = 1.4 mm, l_{ds} = 69 mm, l_{dr} = 43.5 mm, h_{ys} = 77 mm, shaft d_{ri} = 200 mm, h_{yr} = 84.1 mm

$$B_{ys} = 1.70\,\mathrm{T}, \qquad B'_{ds,1/3} = 1.68\,\mathrm{T},$$

$$k_{\mathrm{Vy}} = 1.5, \qquad k_{\mathrm{Vd}} = 1.8,$$

$$m_{ys} = 456\,\mathrm{kg} \qquad m_{ds} = 175\,\mathrm{kg}$$

At rated speed rotor frequency is very small (< 1 Hz),
so **rotor iron losses can be neglected !**

- Stator iron losses: f = 50 Hz: $k_f = 1$:

$$P_{Fe,ds} = 1.8 \cdot \left(\frac{1.68}{1.0}\right)^2 1.7 \cdot 1 \cdot 175 = \underline{\underline{1511}}\,\mathrm{W}$$

$$P_{Fe,ys} = 1.5 \cdot \left(\frac{1.70}{1.0}\right)^2 1.7 \cdot 1 \cdot 456 = \underline{\underline{3360}}\,\mathrm{W}$$

- *Resulting (stator) iron losses:* 4871 W

Additional no-load losses *(depending on main flux!)*

- Rotor surface losses (eddy currents):

Non-sinusoidal air gap field (due to stator slots) induces with slot frequency in the conductive rotor surface,
where the rotor sheet insulation is partially bridged due to milling process !

- Tooth pulsation losses = Slot-frequent iron losses:

Non-sinusoidal air gap field (due to stator slots) causes flux pulsation with slot frequency in rotor teeth, hence increasing iron losses.

Stator tooth flux pulsation usually much smaller, as rotor slot openings are semi-closed or closed.

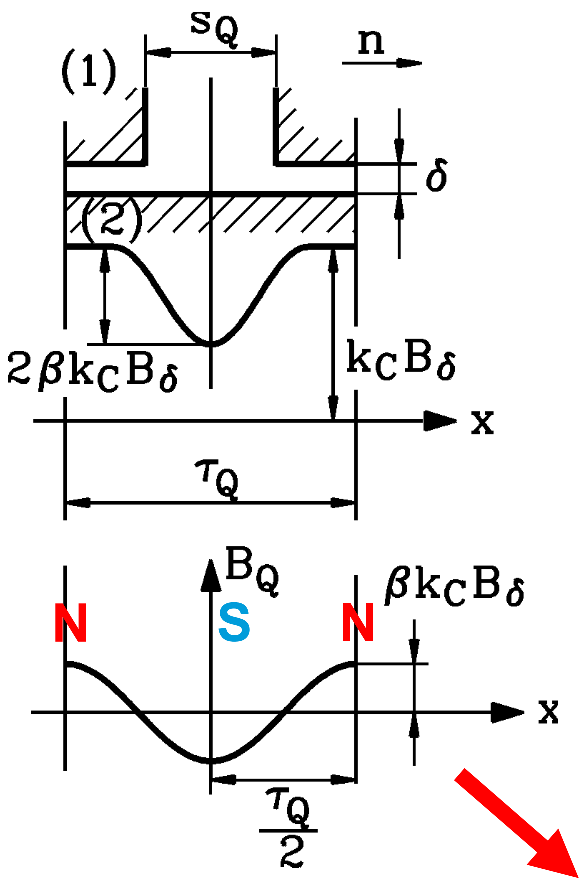
- Rotor cage harmonic currents (bar currents):

Stator slot-ripple air gap field also induces with slot frequency in the rotor cage !

Stator slot frequency: $f_{Qs} = n \cdot Q_s$

Slot ripple coefficient β

Experts only



Air gap field distortion under open stator slot

CARTER: Air gap flux density ripple at rotor surface !

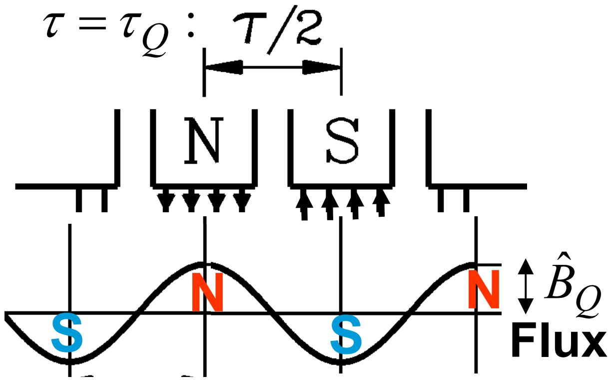
Note: For infinite s_Q/δ the value of $\beta = 0.5$.

$$\beta = \frac{1}{2}\cdot\left(1 - \frac{2/h}{\sqrt{1+(2/h)^2}}\right)$$

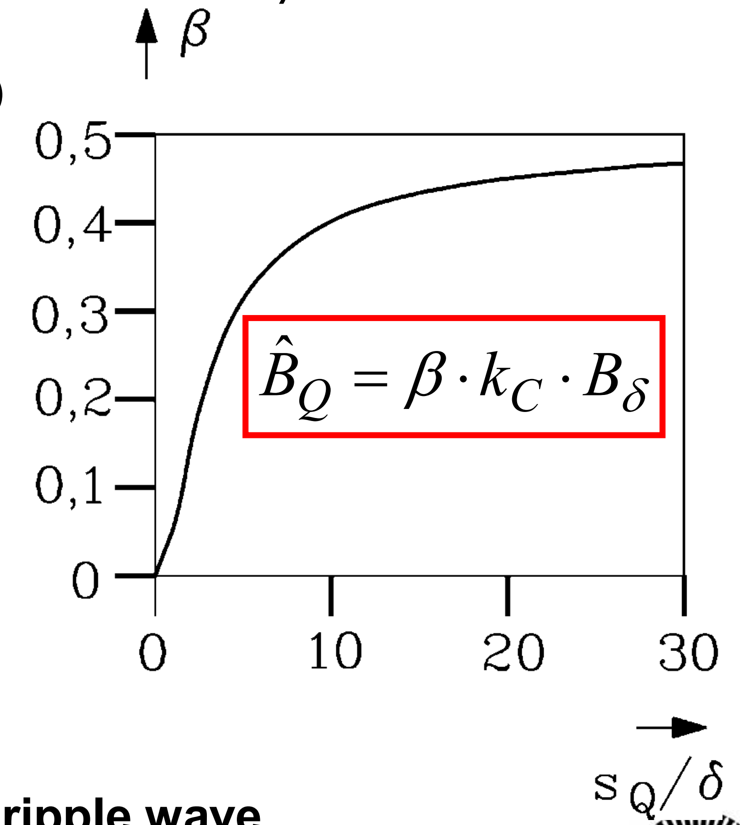
$$h = s_Q/\delta$$

"Equivalent" pole sequence:

$\tau = \tau_Q$:



Flux ripple wave



2. Design of Induction Machines

Rotor surface losses P_{Or}

Experts only

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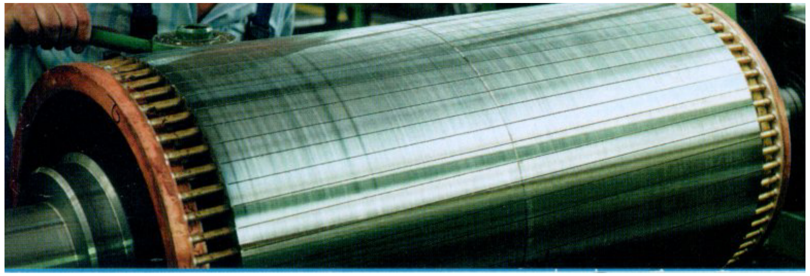
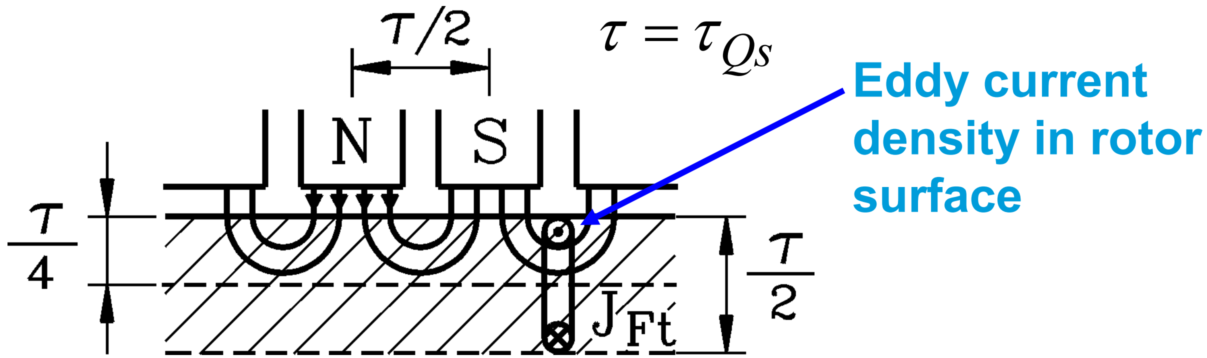


Source: Breuer, Germany



- In case of **conductive rotor iron** slot-frequent flux ripple B_Q in rotor causes induction of eddy current density J_{Ft} with penetration of about $\tau_{Qs}/4$ (Use thick sheet model: „b_{sh}“ = $\tau/2$).
- Eddy current density distribution resembles one half of calculation model **of thick sheet.**
- **BUT: Rotor iron is laminated**, but lamination is partially bridged due to tooling. Experiments show, that **measured surface losses** of iron stack with 0.5 mm sheet type IV are only $k_{exp} = 8 \ldots 13\ \%$ of "thick sheet" equation.
- Surface layer volume:

$$V_{Or} = Q_r \cdot (\tau_{Qr} - s_{Qr}) \cdot l_{Fe} \cdot \tau_{Qs} / 4$$

Fundamental sine wave: $B_\delta(x) = B_\delta \cdot \sin(x\pi/\tau_p) = B_\delta \cdot \sin(\gamma)$

$$\frac{1}{2\pi}\int_0^{2\pi} \sin^2\gamma \cdot d\gamma = \frac{1}{2} = \left(\frac{1}{\sqrt{2}}\right)^2$$

$$P_{Or} = Q_r \cdot (\tau_{Qr} - s_{Qr}) \cdot l_{Fe} \cdot (\tau_{Qs}/4)^2 \cdot \left(\frac{2}{\pi} \cdot \beta \cdot k_{Cs} \cdot \frac{B_\delta}{\sqrt{2}}\right)^2 \cdot (Q_s \cdot n)^{1.5} \cdot \sqrt{\frac{\pi^3 \cdot \kappa}{\mu}} \cdot k_{exp}$$

2. Design of Induction Machines

Calculation of rotor surface losses

- ***Example:*** 60 stator slots, 1500/min

Slot frequency $f_{Q_s} = Q_s \cdot n = 60 \cdot 1500 / 60 = \underline{\underline{1500\,\mathrm{Hz}}}$

$$h = s_{Qs} / \delta = 12.5 / 1.4 = 8.93 \qquad \beta = \frac{1}{2} \cdot \left(1 - \frac{2/8.93}{\sqrt{1 + (2/8.93)^2}}\right) = 0.391$$

Flux density amplitude: $\hat{B}_{Qs} = \beta \cdot k_{Cs} \cdot B_\delta = 0.391 \cdot 1.5 \cdot 0.891 = \underline{\underline{0.52\ \mathrm{T}}}$

Rotor surface: $A_r = Q_r \cdot (\tau_{Qr} - s_{Qr}) \cdot l_{Fe} = 50 \cdot (0.0288 - 0.0025) \cdot 0.378 = 0.497\ \mathrm{m}^2$

Experimental factor: $k_{\exp} = 0.08$, $\mu_{\mathrm{Fe}} = 1400\mu_0$ (estimated), $\kappa_{\mathrm{Fe(Steel)}} = \text{ca. } 4\ \mathrm{MS/m}$

- Surface losses:

$$P_{Or} = 0.497 \cdot (0.024/4)^2 \cdot \left(\frac{2}{\pi} \cdot \frac{0.52}{\sqrt{2}}\right)^2 \cdot 1500^{1.5} \cdot \sqrt{\frac{\pi^3 \cdot 4 \cdot 10^6}{1400 \cdot 4\pi \cdot 10^{-7}}} \cdot 0.08 = \underline{\underline{1220\,\mathrm{W}}}$$

Rotor tooth flux Φ_{dr}: Pulsation $\Delta\Phi_{dr}$

Experts only

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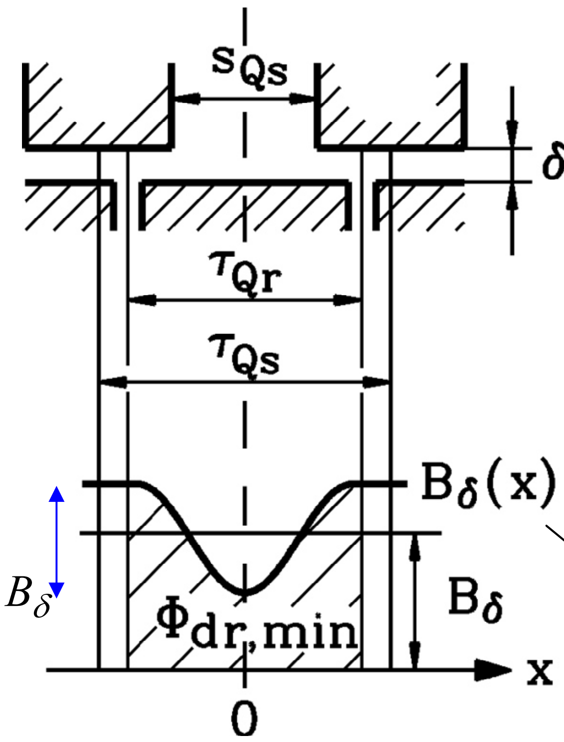
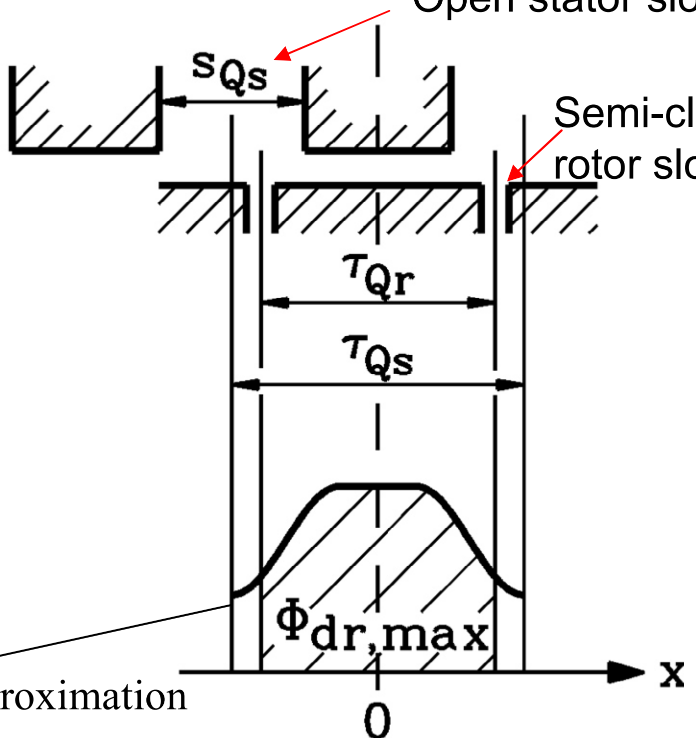
$2\hat{B}_{Q_s} =$
$= 2\beta \cdot k_{Cs} \cdot B_\delta$



Minimum rotor tooth flux

Example:
$Q_s < Q_r$

Sine-wave approximation of $B_\delta(x)$-ripple



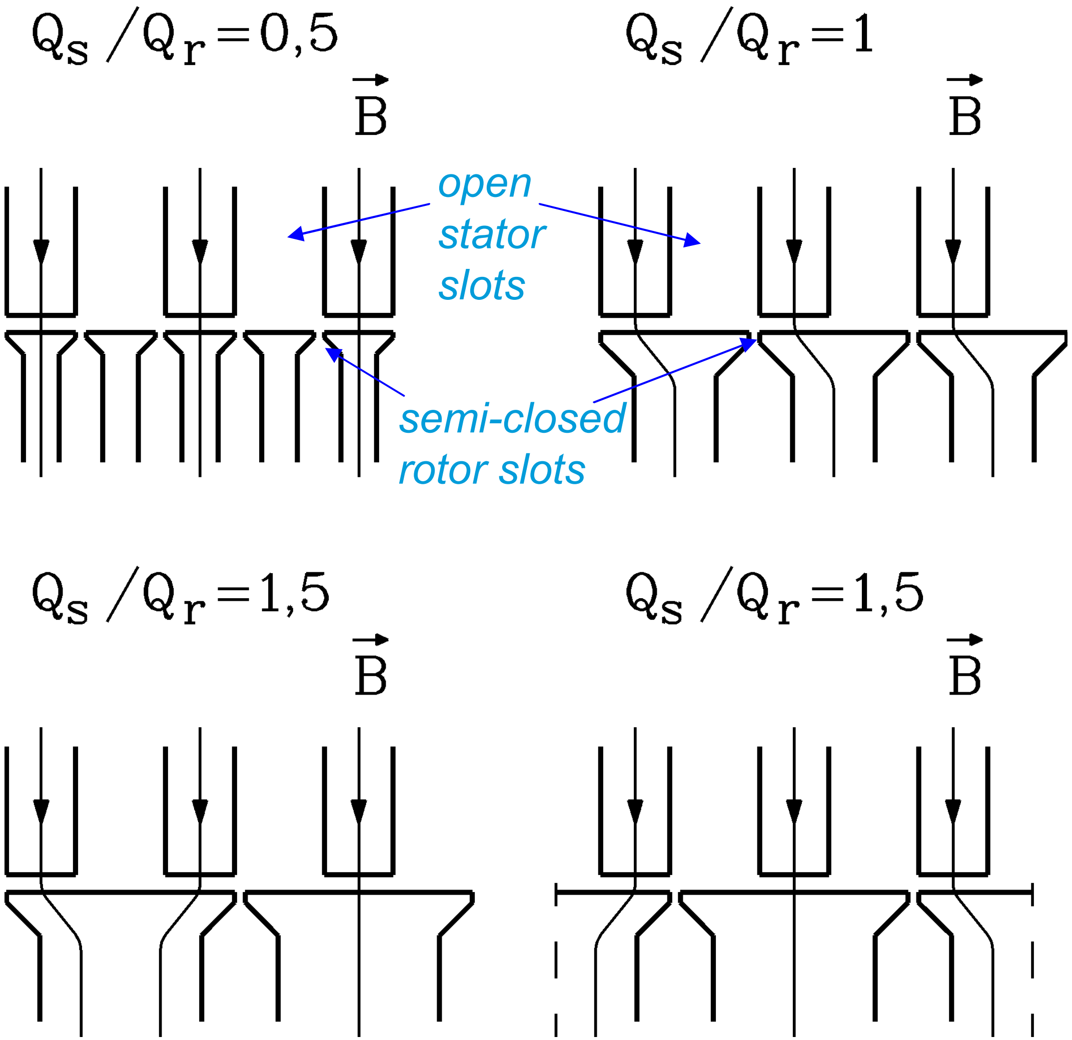
$$\frac{\tau_{Qr}}{\tau_{Qs}} = \frac{Q_s}{Q_r}$$

Maximum rotor tooth flux

Pulsation of rotor tooth flux density:

$$\frac{\Delta B_{dr}}{B_{dr}} = \frac{\Phi_{dr,\max} - \Phi_{dr,\min}}{2\Phi_{dr,av}} = \beta \cdot k_{Cs} \cdot \frac{\sin(\pi \cdot Q_s / Q_r)}{\pi \cdot Q_s / Q_r}$$

Rotor tooth flux pulsation depends on ratio Q_s/Q_r



Q_s/Q_r = 0.5:

Flux pulsation between 0 and 200% (Average: 100%)

Q_s/Q_r = 1 (forbidden !):

No flux pulsation: 0%

$$\frac{\Delta B_{dr}}{B_{dr}} \sim \frac{\sin(\pi \cdot Q_s / Q_r)}{\pi \cdot Q_s / Q_r}$$

Q_s/Q_r = 1.5:

Flux pulsation between 66% and 133%

2. Design of Induction Machines

Slot ratio Q_s/Q_r influence on tooth flux pulsation

Open stator slots, semi-closed or closed rotor slots

Q_s/Q_r	0.5	1.0	1.5
Flux pulsation between:	0% … 200%	0%	66% ... 133%
Pulsation amplitude:	100%	0%	33%
$\left\|\frac{\Delta B_{dr}}{B_{dr}}\right\| \sim \left\|\frac{\sin(\pi \cdot Q_s / Q_r)}{\pi \cdot Q_s / Q_r}\right\|$	0.64	0	0.21
	0.64 = 100%	0%	0.21 = 33%

2. Design of Induction Machines

Calculation of rotor tooth pulsation losses $P_{puls,r}$

- High frequency f_{Qs}: Eddy current losses $P_{Ft,Qs} \sim f_{Qs}^2$ in teeth with stator slot frequency f_{Qs} dominate over hysteresis losses $P_{Hy,Qs} \sim f_{Qs}$, which are neglected
- Damping influence of self-field of the induced harmonic rotor cage currents $I_{r,\mu}$ neglected!

$$P_{puls,r} \approx P_{Ft,Q_s} = k_{Vd} \cdot \left(\frac{\Delta B_{d,1/3}}{1.0}\right)^2 \cdot p_{Ft} \cdot \left(\frac{f_{Q_s}}{50}\right)^2 \cdot m_{dr}$$

- ***Example:*** 60 stator slots, 1500/min, slot frequency: f_{Qs} = 1500 Hz, m_{dr} = 127.8 kg
  Fundamental sine wave: $B_{dr,1/3}(x) \sim B_{dr,1/3} \cdot \sin(x\pi/\tau_p) \Rightarrow$ Losses proportional $(B_{dr,1/3}/\sqrt{2})^2$

  Tooth flux density pulsation amplitude: $B'_{dr,1/3} = 1.41\,\mathrm{T} \approx B_{dr,1/3}$

$$\Delta B_{dr,1/3} = \frac{B_{dr,1/3} \cdot \beta \cdot k_{Cs}}{\sqrt{2}} \cdot \frac{\sin(\pi \cdot Q_s / Q_r)}{\pi \cdot Q_s / Q_r} = \frac{1.41 \cdot 0.391 \cdot 1.5}{\sqrt{2}} \cdot \frac{\sin(\pi \cdot 60/50)}{\pi \cdot (60/50)} = -0.091\mathrm{T}$$

- **Losses:** $P_{puls,r} = 1.8 \cdot \left(\frac{0.091}{1.0}\right)^2 \cdot 0.4 \cdot \left(\frac{1500}{50}\right)^2 \cdot 127.8 = \underline{\underline{688\,\mathrm{W}}}$

  (Sheet type IV: p_{Ft} = 0.4 W/kg)

Additional no-load losses

- **Additional no-load losses are occurring already at no-load !**
- **Tooth pulsation, rotor surface & harmonic bar current losses at no-load**
- **They are measured during no-load test !**
- **They are included in measured iron losses at no-load !**

Example:

550 kW-cage induction motor: "Measured" iron losses $P_{Fe,meas}$:

Eddy current losses }
Hysteresis losses } **4871 W**
Additional no-load losses 1220 W + 688 W

$P_{Fe,meas}$ **= 4871 W + 1220 W + 688 W = 6779 W**

Note: Harmonic bar current losses neglected here for simplicity.

2. Design of Induction Machines

Additional load losses $P_{ad,1}$ (“stray load losses”)

- Stator winding eddy current losses due to slot stray flux (*Field´s* losses)
- Increased rotor surface and tooth flux pulsation losses due to load-increased slot-harmonic air gap field
- Rotor cage harmonic currents induced by increased stator field harmonics
- Inter-bar harmonic currents in skewed rotor cages between adjacent bars

} $\boldsymbol{P_{ad,1}}$

- **Standard IEC EN 60034-2:**

  Large machines: Estimate $P_{ad,1}$ as 0.5% of electric power P_e of machine!

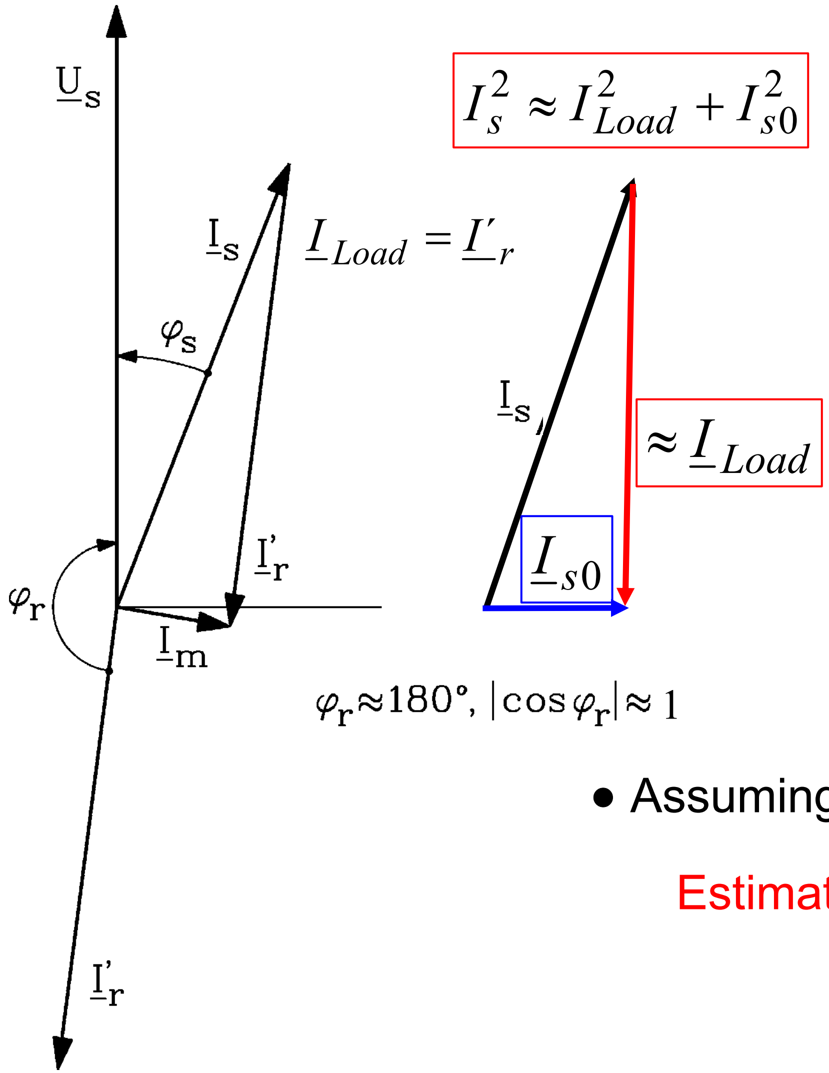
$$P_{\mathrm{ad,1,N}} = 0.005 \cdot P_{\mathrm{eN}}$$

Smaller machines < 100 kW: Measurements show typically ~ 1.5% ... 2% … 3%.

$$P_{\mathrm{ad,1,N}} = (0.015...0.03) \cdot P_{\mathrm{eN}}$$

2. Design of Induction Machines

Estimate for the load current I_{Load}



- Load current = "Primed" rotor current!

$$\underline{I}_{Load} = \underline{I}'_r = \underline{I}_m - \underline{I}_s$$

- No-load current: $\underline{I}_s(s=0) = \underline{I}_{s0}$

- Estimate for load current:

$$\underline{I}_{Load} \approx \underline{I}_{s0} - \underline{I}_s$$

- Assuming purely inductive no-load current = phase shift: 90°

Estimate for load current: $I_{Load} \approx \sqrt{I_s^2 - I_{s0}^2}$

2. Design of Induction Machines

Load-dependent stray load losses $P_{ad,1}$

- Load current: $\underline{I}_{Load} = \underline{I}'_r \approx \underline{I}_{s0} - \underline{I}_s$

- Stray load losses: $P_{ad,1} \sim u_i^2 / R \sim B^2 / R \sim I_{Load}^2 / R$

$$P_{\mathrm{ad,1,N}} = 0.005 \cdot P_{\mathrm{eN}} \sim I_{Load,N}^2 = I_{sN}^2 - I_{s0}^2$$

- Stray load losses at arbitrary load:

$$\boxed{P_{ad,1} = 0.005 \cdot P_{eN} \cdot (I_s^2 - I_{s0}^2)/(I_{sN}^2 - I_{s0}^2)}$$

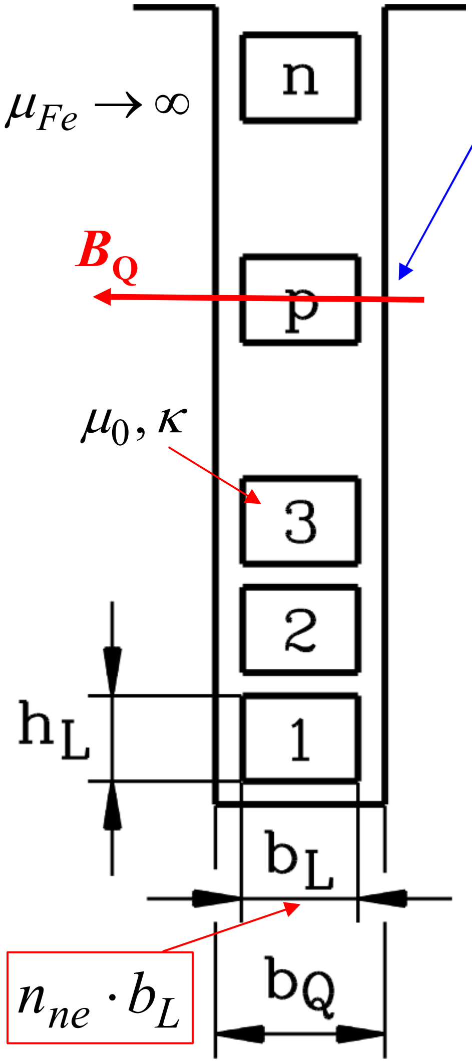
- *Example:*

550 kW motor, rated efficiency 94.4 %, $P_{in,N} = P_{eN} = P_{out,N} / \eta_N = 550/0.944 = 582.6\,\mathrm{kW}$

Stray load losses at rated current: $P_{ad,1,N} = 0.005 \cdot P_{eN} = 0.005 \cdot 582.6 = \underline{\underline{2.91\,\mathrm{kW}}}$

- *Example:* Half-load $M = M_N / 2 \rightarrow I_{Load} \approx I_{Load,N} / 2 \rightarrow P_{ad,1} \approx P_{ad,1,N} \cdot 0.5^2 = P_{ad,1,N} / 4$

Eddy current losses in stator winding (*Field´s losses*)



Stator slot stray flux density B_Q: at e.g.: $n = 2N_c = 2 \cdot 10 = 20$

It induces eddy currents in slot conductors, causing there losses, which may be described by increased "AC resistance"

Average rise of AC resistance per phase:

$$k_{R,av} = R_{AC} / R_{DC} = \frac{k_n \cdot l_e + L - l_e + l_b}{L + l_b} \geq 1$$

Field & Emde

$$k_n = \varphi(\xi) + \frac{n^2 - 1}{3} \cdot \psi(\xi) \qquad \xi = h_L / d_E = h_L \cdot \sqrt{\pi \cdot \mu_0 \cdot f_s \cdot \kappa \cdot (n_{ne} \cdot b_L / b_Q)}$$

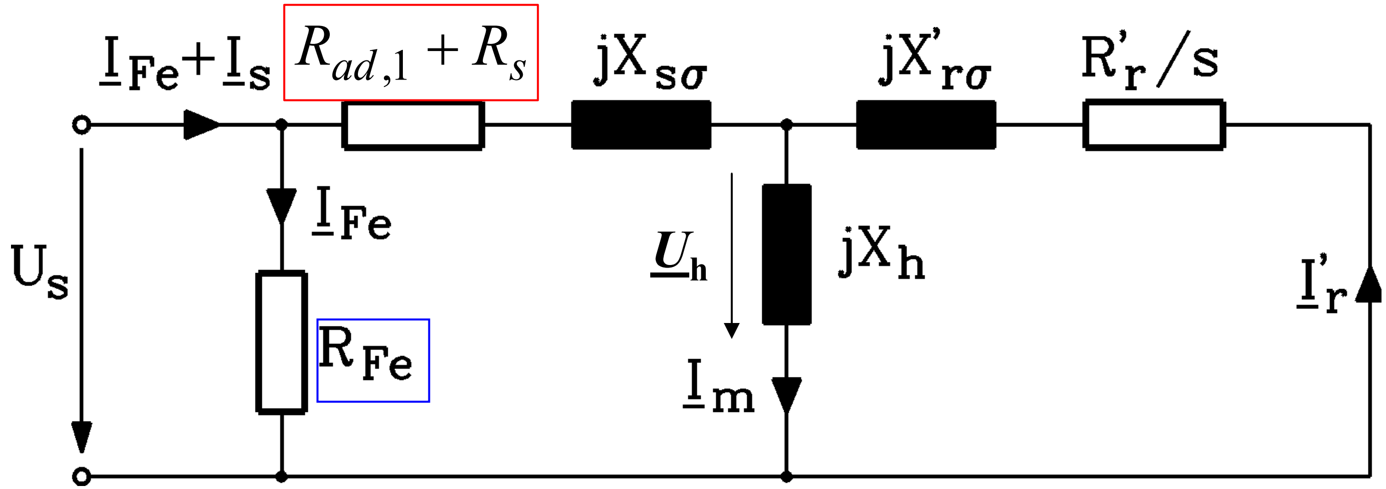
$\varphi(\xi) \geq 1, \psi(\xi) \geq 0$: See text book! $(W / \tau_p = 1)$

Example: $\xi = 0.0018 \cdot \sqrt{\pi \cdot 4\pi \cdot 10^{-7} \cdot 50 \cdot 57 \cdot 10^6 \cdot (1 \cdot 7.1/12.5)} = 0.144$

$\varphi(0.144) = 1.0000382$, $\psi(0.144) = 0.0001433$ at 20°C

$$k_n = 1.0000382 + \frac{20^2 - 1}{3} \cdot 0.0001433 = \underline{\underline{1.019}}$$ negligibly small !

T-equivalent circuit per phase



"Equivalent iron resistance" R_{Fe}: $P_{Fe} = m_s \cdot U_h^2 / R_{Fe}$

Simplification: U_s **instead of** U_h: $R_{Fe} = m_s \cdot U_s^2 / P_{Fe} = 3 \cdot (6600/\sqrt{3})^2 / 6779 = \underline{\underline{6425}}\ \Omega$

Often: Further simplification:

$$R_{Fe+fr+w} = m_s \cdot U_s^2 /(P_{Fe} + P_{fr+w}) = 3 \cdot (6600/\sqrt{3})^2 /(6779 + 2714) = \underline{\underline{4588}}\ \Omega$$

Simplification: "Equivalent series resistance" $R_{ad,1}$ on stator side, although most part of additional losses $P_{ad,1}$ occur on rotor side!

$$P_{ad,1} \cong m_s \cdot R_{ad,1} \cdot I_s^2 \qquad R_{ad,1} = P_{ad,1} /(m_s \cdot I_N^2) = 2910/(3 \cdot 59^2) = \underline{\underline{0.278}}\ \Omega$$

2. Design of Induction Machines

Example: Loss balance

550 kW motor, 6.6 kV, 50 Hz, unskewed rotor: $I_{sN} = 59.04\ \text{A}$

$\underline{U}_s = U_s = 6600/\sqrt{3}\ \text{V},\ f_s = 50\ \text{Hz}$, at 75°C: $R_s = 0.74\ \Omega$, $R_{ad,1} = 0.278\ \Omega$, $R_{Fe} = 6425\ \Omega$,

$X_{s\sigma} = 4.89\ \Omega$, $X_h = 155.1\ \Omega$ (saturated value), $X'_{r\sigma} = (3.2 \cdot k_L(s) + 3.18)\,\Omega$,

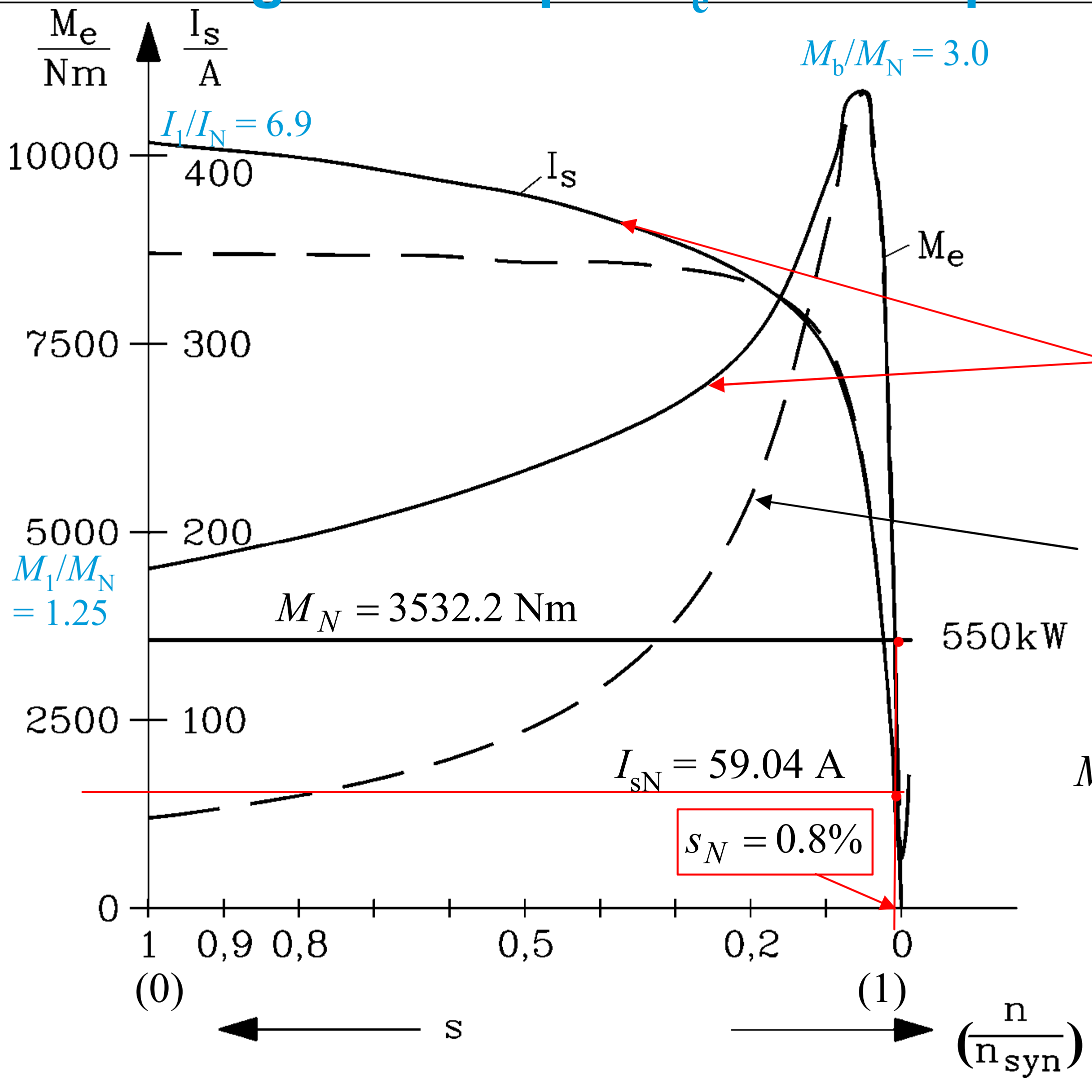
$R'_r = (0.3335 \cdot k_R(s) + 0.2334)\,\Omega$

$$\cos\varphi_s = \frac{574921}{\sqrt{3} \cdot 6600 \cdot 59.04} = 0.852$$

Electrical input power $P_{e,in}$	**574 921 W**
Stator winding losses $P_{Cu,s}$	7 739 W
Total iron losses P_{Fe}	6 779 W
Stray load losses $P_{ad,1}$ (= 0.5% of 574 921 W)	2 875 W
Air gap power P_δ	557 528 W
Rotor cage losses $P_{Cu,r}$ (slip: 0.814%)	4 538 W
Friction and windage losses P_{fr+w}	2 670 W
Mechanical output power $P_{m,out}$	**550 320 W (≅ 550 kW)**
Efficiency η	95.72 %

2. Design of Induction Machines

Calculated stator current I_s & electromagnetic torque M_e versus speed n



550 kW, 4 pole, three phase cage induction motor, 6.6 kV. 50 Hz

Solid lines:
***with** current displacement* (k_L, k_R) in rotor bars considered,

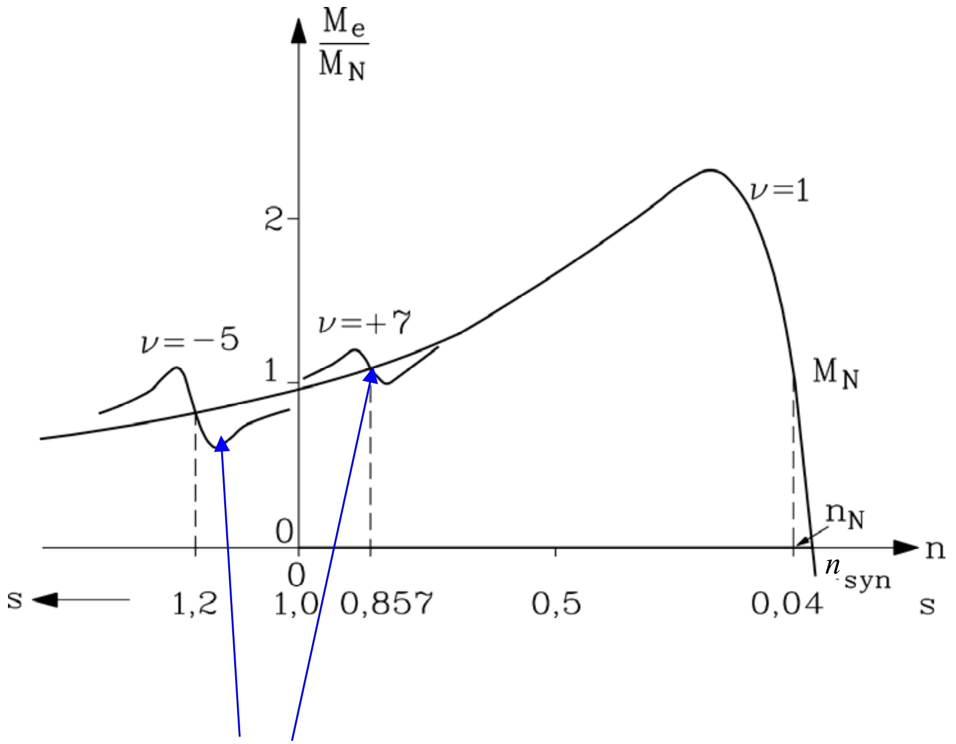
Dashed lines:
without considering current displacement:
($k_L = 1$, $k_R = 1$) $\Rightarrow$ **too low torque is calculated !**

$$M_N = \frac{P_{mN}}{2\pi \cdot n_N} = \frac{550320}{2\pi \cdot (1487.79/60)} = 3532.2 \text{ Nm}$$

$$s_N = \frac{1500 - 1487.79}{1500} = 0.814\%$$

$$I_s(s=1)/I_{sN} = 410/59.04 = 6.9$$

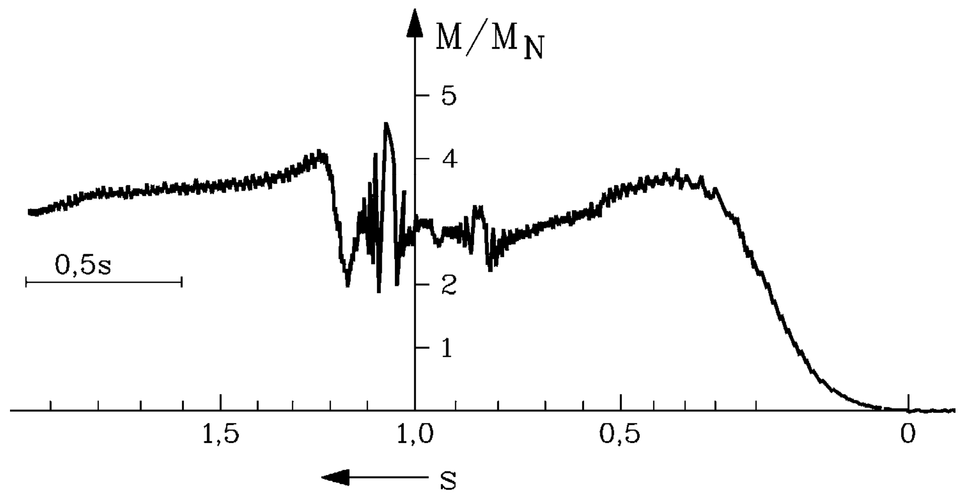
Influence of stator harmonic field waves on torque characteristic *M*(*n*)



Additional asynchronous torque component due to 5th and 7th harmonic stator field wave

Skewing helps due reduce these asynchronous harmonic torque components!

Example: Measured *M*(*n*) resp. *M*(*s*)



Measured at run-up from $s = 2$ to $s = 0$:
Linear time scale = Non-linear slip scale

Details: See Lecture:
Motor development for electrical drive systems (2 + 1)

2. Design of Induction Machines

Total „active“ mass of machine

Stator winding (without winding insulation)	142.3 kg
Rotor cage	59.0 kg
Winding mass:	**201.3 kg**
Stator teeth	175.0 kg
Stator yoke	456.0 kg
Stator iron mass:	**631.0 kg**
Rotor teeth	127.8 kg
Rotor yoke	210.2 kg
Rotor iron mass:	**338.0 kg**
Total active mass:	**1170.3 kg**

550 kW, 4 poles, cage induction machine, open ventilated, Th. Cl. B, 50 Hz

Thermal Class B: 80 K temperature rise: 550 kW/ 1170.3 kg = 0.47 kW/kg.

Thermal Class F: 105 K temperature rise: $\sqrt{105/80}\cdot 0.47 = 0.53$ kW/kg.

A typical ratio of **"power per active mass"** is **0.5 kW /kg** !

Typical modern high-voltage (HV) squirrel-cage induction motor

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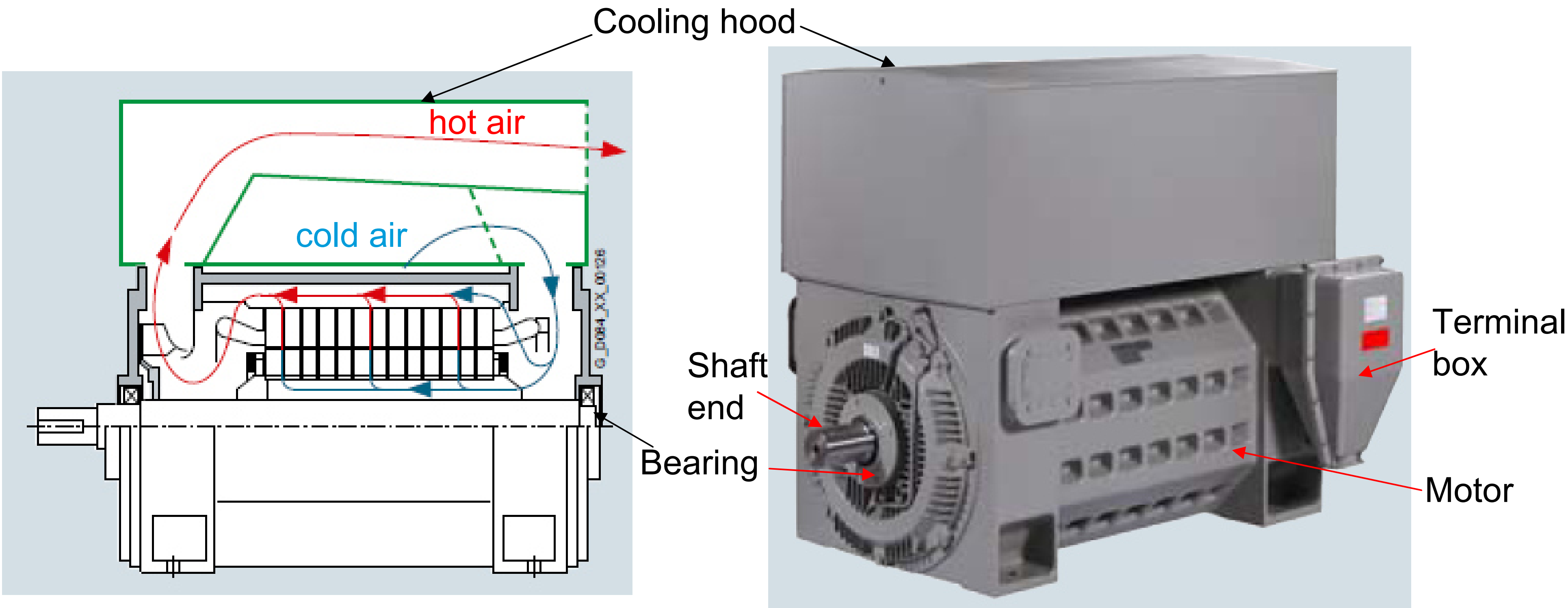


Open-circuit air ventilation (IC01, IP23)

(International cooling IC)
(International protection IP)

Source: Siemens AG, Germany, Catalogue D84.9, 2015

Foot-mounted motor (IMB3)

(International mounting IM)

Rated power 500 kW S1
(S1: continuous duty)

Data of a typical modern HV squirrel-cage induction motor

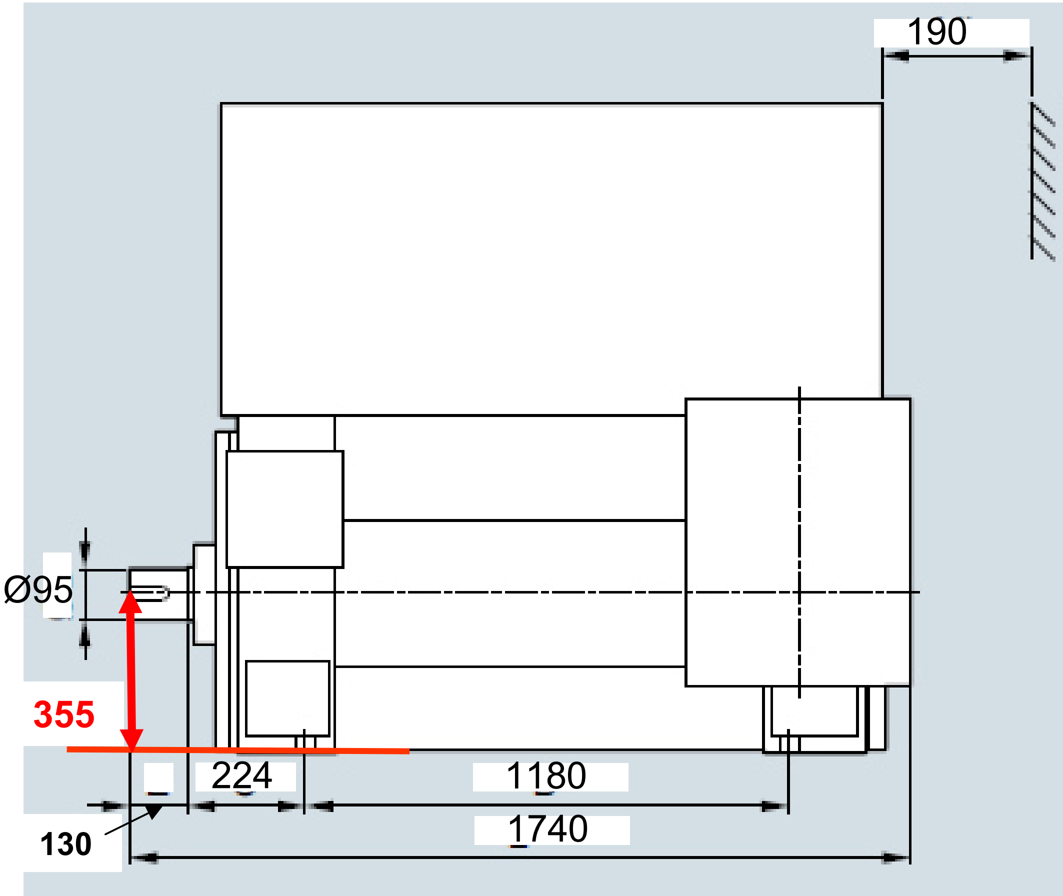
500 kW, 6 kV±5%, Y 50 Hz, 59 A,
$\eta = 94.5\%$, $\cos\varphi = 0.87$, $2p = 4$,
$n_N = 1479/\text{min}$, $M_N = 3229$ Nm
$M_b/M_N = 2.4$, $M_1/M_N = 0.9$, $I_1/I_N = 5.2$

IMB3, IC01: 1 m^3/s air flow

Above 1000 m sea-level:
Power de-rating necessary!

Insulation system:
Thermal Class F (155°C), but utilized only:
Th. Cl. B (130°C) to insulation increase life span

Motor total mass: 2200 kg
Rotor mass: 422 kg
Rotor polar moment of inertia: $J = 6.8$ kgm²



Source: Siemens AG, Germany, Catalogue D84.9, 2015

Frame size (shaft height) = 355 mm (standardized value)

International Protection IP („Schutzart“)

- The IP Code (International Protection Marking, IEC 60529, EN 60529), classifies and rates the degree of protection provided against intrusion of
a) parts: body parts such as hands and fingers, dust, accidental contact, and of
b) water
by mechanical casings and electrical enclosures

- Standards for electrical machines: IEC 60034-5

- IPxy: International Protection
x: 1. Number: describes protection against intrusion of parts
y: 2. Number: describes protection against intrusion of water (no protection against oil!

International Protection IP: Examples

- ***Examples:***

IP00:	Motor housing is open	Motor housing is open
IP23:	Parts with diameter >12 mm cannot intrude	Spray water with intrusion angle up to 60° from vertical line cannot intrude
IP44:	Parts with diameter >1 mm cannot intrude	Splashing of water from all directions cannot intrude
IP67:	Dust may not intrude	Powerful water jets under defined pressure and time duration may not intrude

International Mounting IM („Bauart“)

- The kind of fixing the electric machine to a base, e.g. by bolts, for transferring the machine torque into the supporting structure, is defined by the nomenclature of IM

- Standards IEC 60034-7

- IM xy: International Mounting
  x: 1st letter: B: horizontal shaft, V: vertical shaft
  y: 2nd number: e.g.: Mounted on feet or via a flange

- ***Examples:***
  IM B3: Mounted on feet; with horizontal shaft
  IM B5: Mounted via housing flange; with horizontal shaft

- "B": Bearings are built within end-shields of E-machine
  "D": Separate bearings mounted directly on supporting frame

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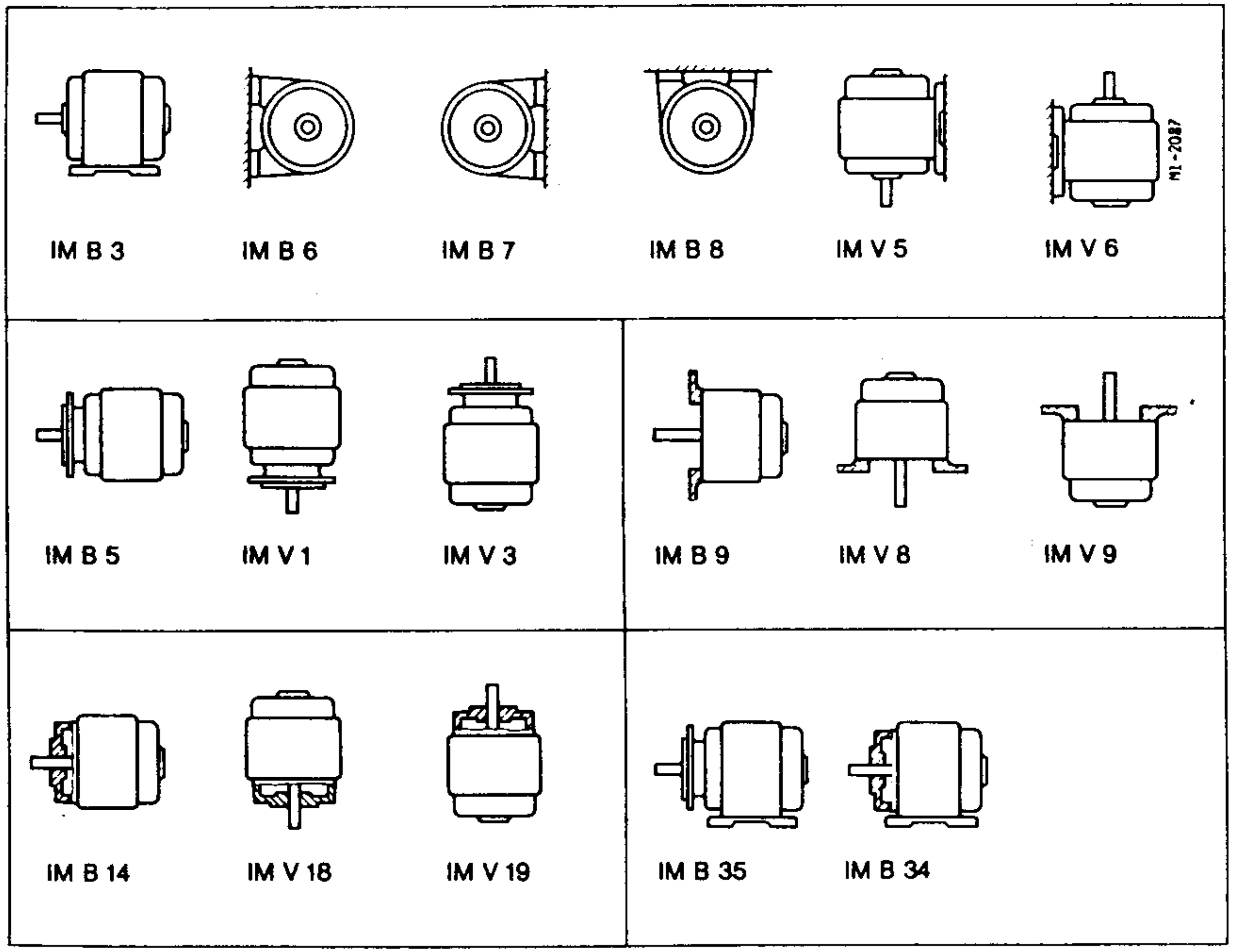
International Mounting IM



Source:
IEC standards, IEC60034-7

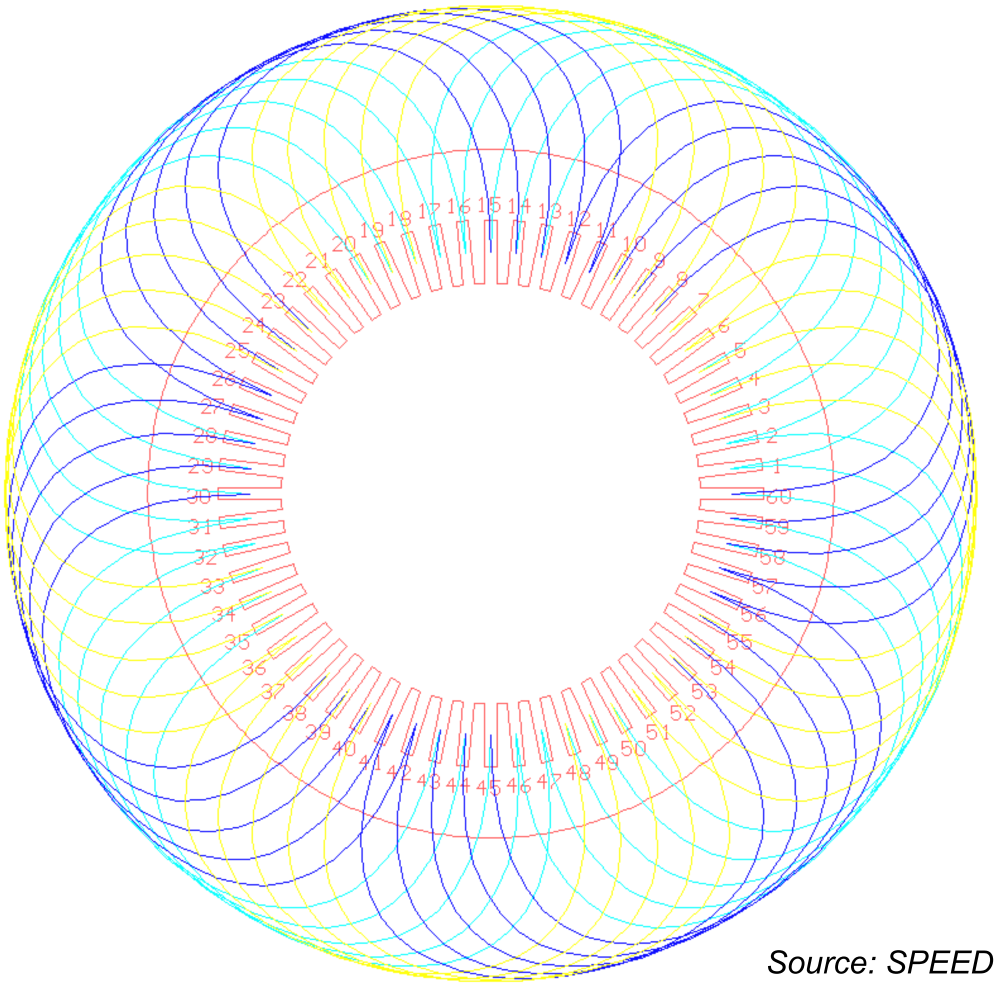
Summary:
Masses and losses

- No-load and load-dependent loss components
- Iron and friction losses already at no-load
- No-load additional losses due to tooth flux pulsations and surface eddy currents
- $I^2 \cdot R$-losses at load and additional load losses
- Loss balance for efficiency determination
- Active masses = Iron masses for flux and copper masses for current flow (e.g.: 0.55 kW/kg)
- Inactive masses = constructive parts (bearings, shaft, housing, cooling system) ($\rightarrow$ Power vs. total mass: e.g.: 0.23 kW/kg) *)

*) In cars higher: > 1 kW/kg (see: Lecture „Motor development (2+1)“)
upper limit today: ca. 5 kW/kg (by: a) elevated speed, b) special materials)

Appendix 1: Induction Motor Computation with the SPEED program

Induction Motor Computation with SPEED: Stator winding



5 slots per pole and phase

3 phases

Two-layer winding

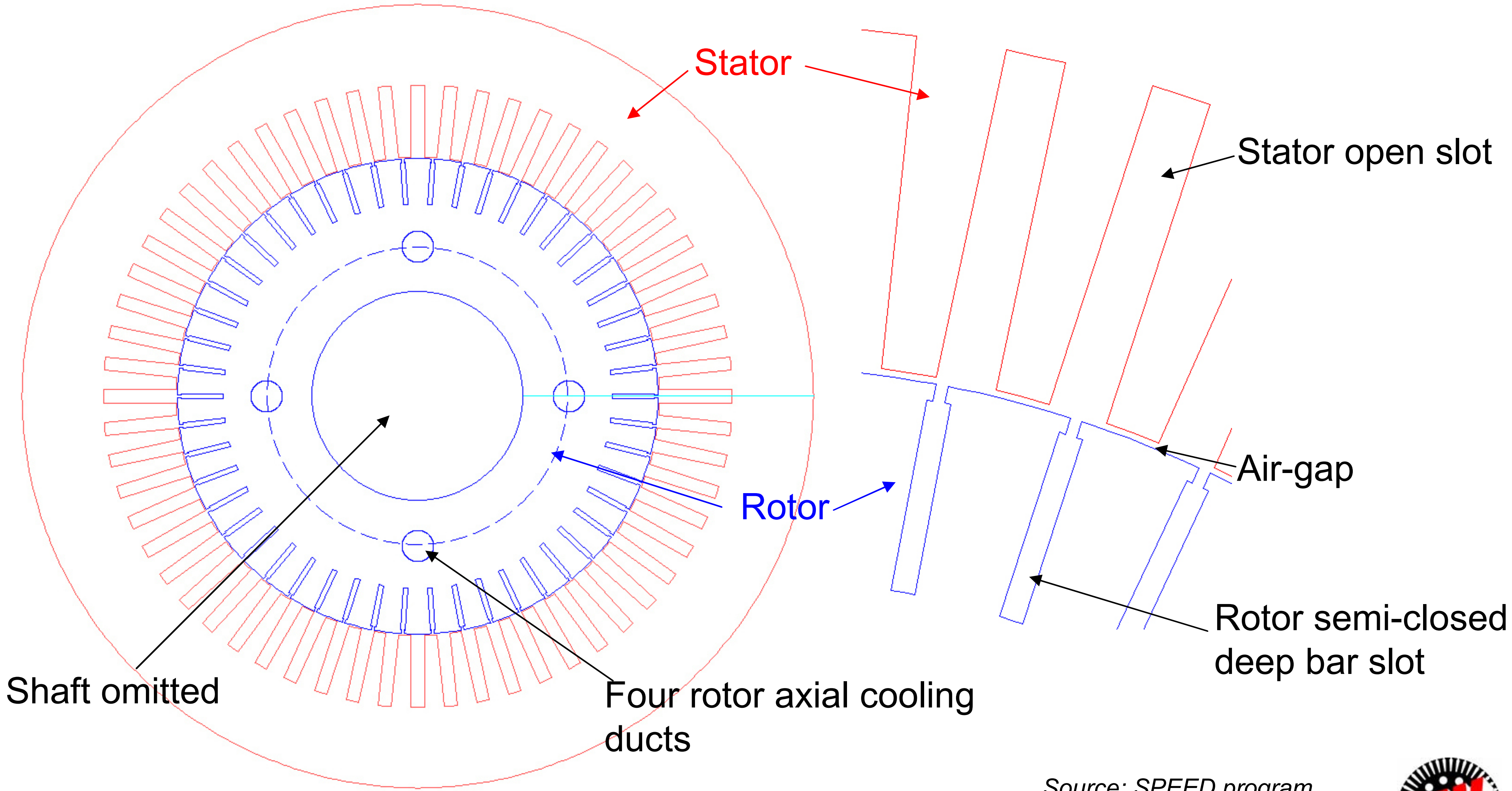
Pitching of coils 12/15

Slot step: 1 $\rightarrow$ 13

U **V** **W**

Source: SPEED program

Energy Converters – CAD and System Dynamics
Induction Motor Computation with SPEED: Geometry



Source: SPEED program

Induction Motor Computation with SPEED: Results 50 Hz

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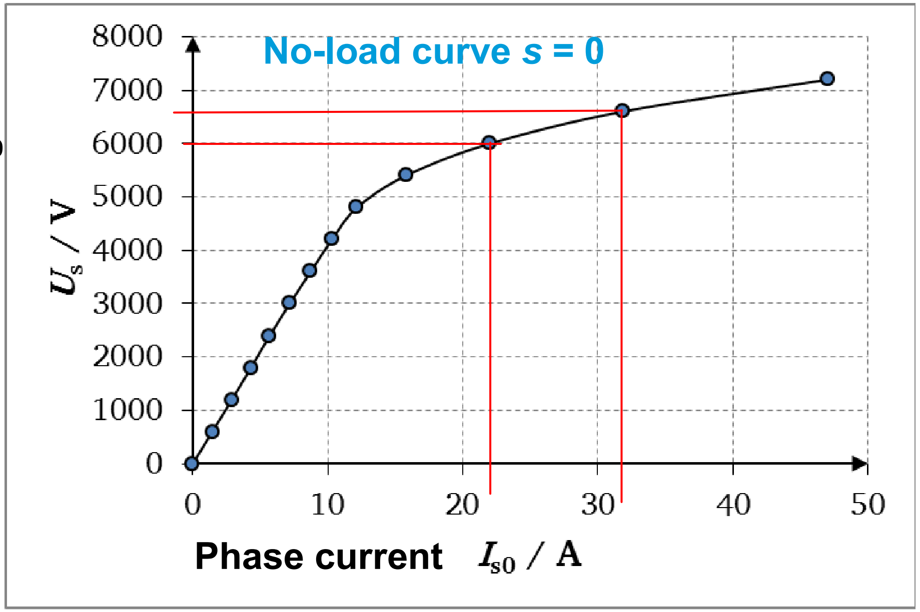
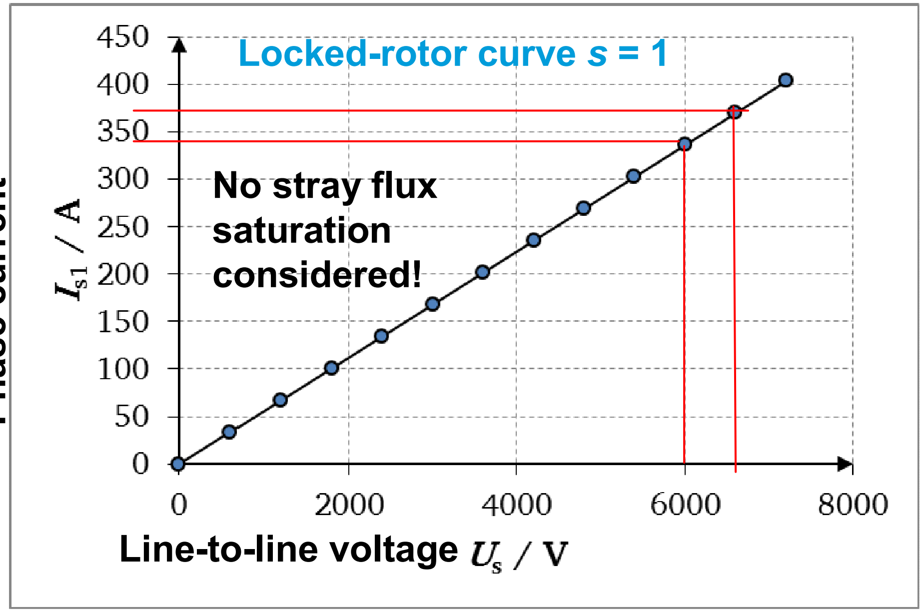




No-load current at 6 kV: **22.06 A** $(= 0.4 \cdot I_N)$ $I_N = 59$ A

at 6.6 kV: 31.92 A (23.8 A analytical)

SPEED: a) no shaft = higher rotor yoke saturation
b) not exactly the same *B*(*H*)-curve of iron

Locked-rotor current at 6 kV: **336 A** (396 A, FEM) $(= 6.7 \cdot I_N)$

at 6.6 kV: 370 A (410 A, analytical)

Source: SPEED program

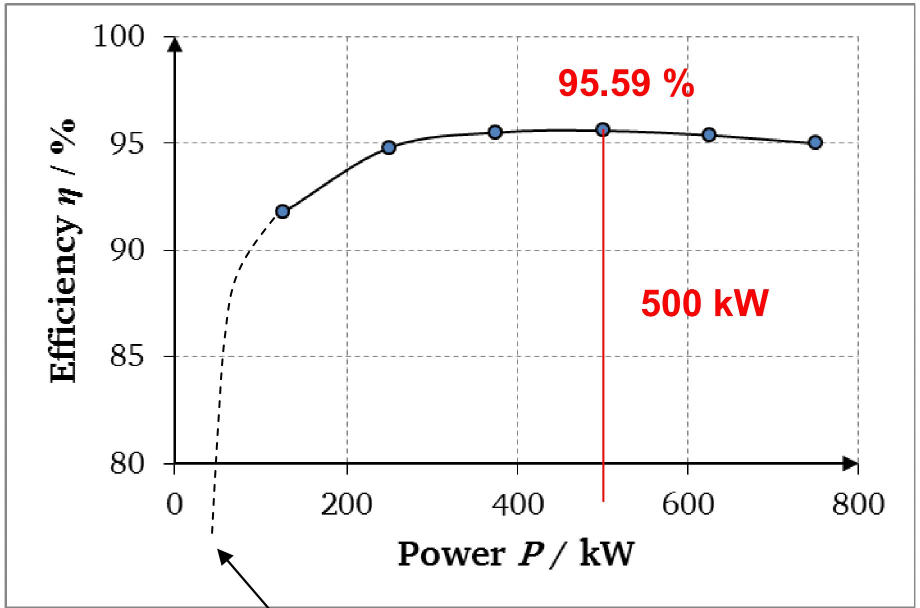
Induction Motor Computation with SPEED: Loss balance 50 Hz

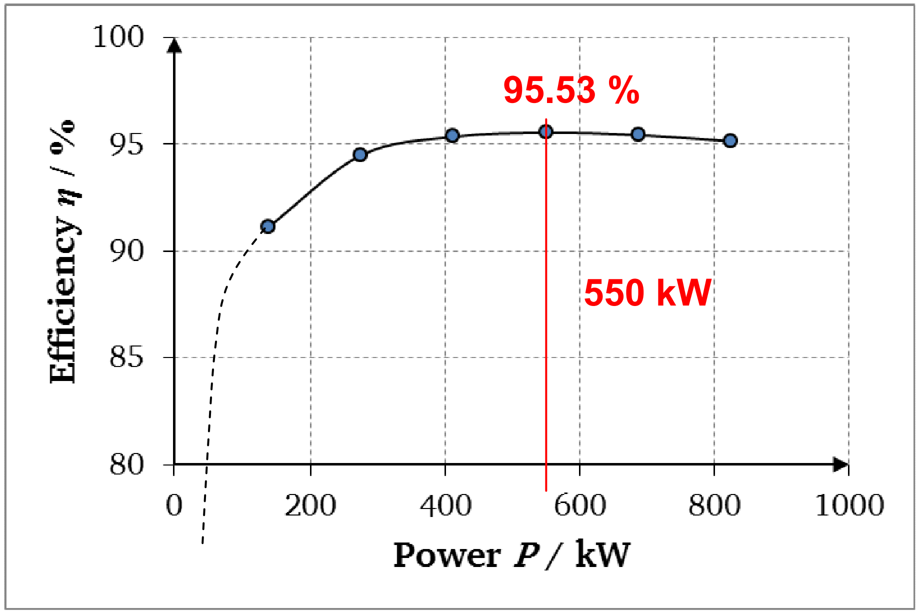
Source: SPEED program

	P_N = 500 kW, U_{sN} = 6000 V Y I_{sN} = 59 A, s = 0.89 %	P_m = 550 kW, $U_{s,LL}$ = 6600 V Y I_s = 64 A, s = 0.82 % (I_s = 59.04 A, s = 0.814 %)
Electrical input power $P_{e,in}$	523 070 W	575 680 W (574 921 W analytical)
Stator winding losses $P_{Cu,s}$	7 445 W	8 743 W (7 739 W)
Total iron losses P_{Fe}	5 959 W	6 932 W (6 779 W)
Stray load losses $P_{ad,1}$ (= 0.5% of 500 kW)	2 513 W	2 764 W (2 875 W)
Air gap power P_δ	507 153 W	557 241 W (557 528 W)
Rotor cage losses $P_{Cu,r}$	4517 W	4 594 W (4 538 W)
Friction and windage losses P_{fr+w}	2643 W	2 648 W (2 670 W)
Mechanical output power $P_{m,out}$	499 993 W (≅ 500 kW)	549 999 W (550 320 W) (≅ 550 kW)
Efficiency η	95.59 %	95.53 % (95.72 %)

Induction Motor Computation with SPEED: Efficiency vs. load

U_{sN} = 6000 V Y   50 Hz $U_{s,LL}$ = 6600 V Y





Zero efficiency at zero power! ,
$\eta = 0$ at $P_{m,out} = 0$!

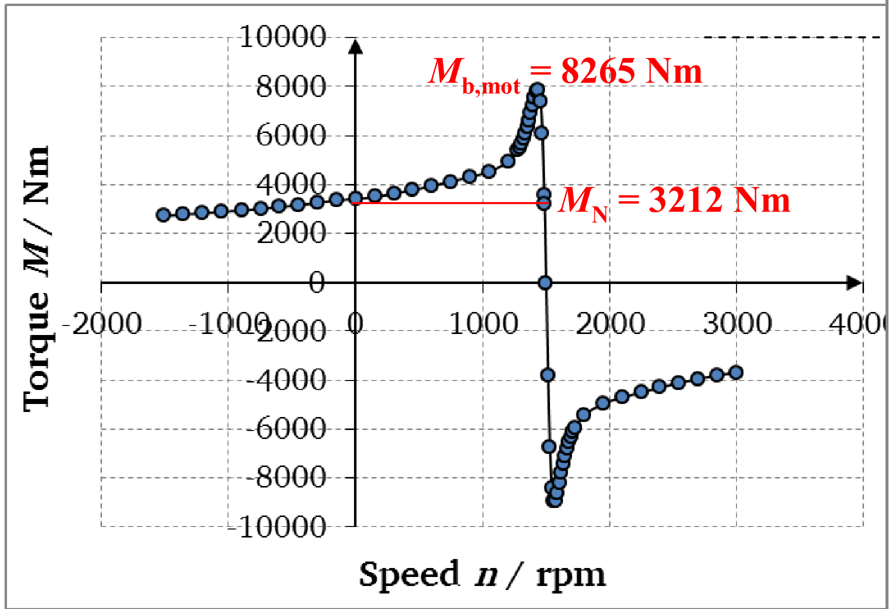
Source: SPEED program

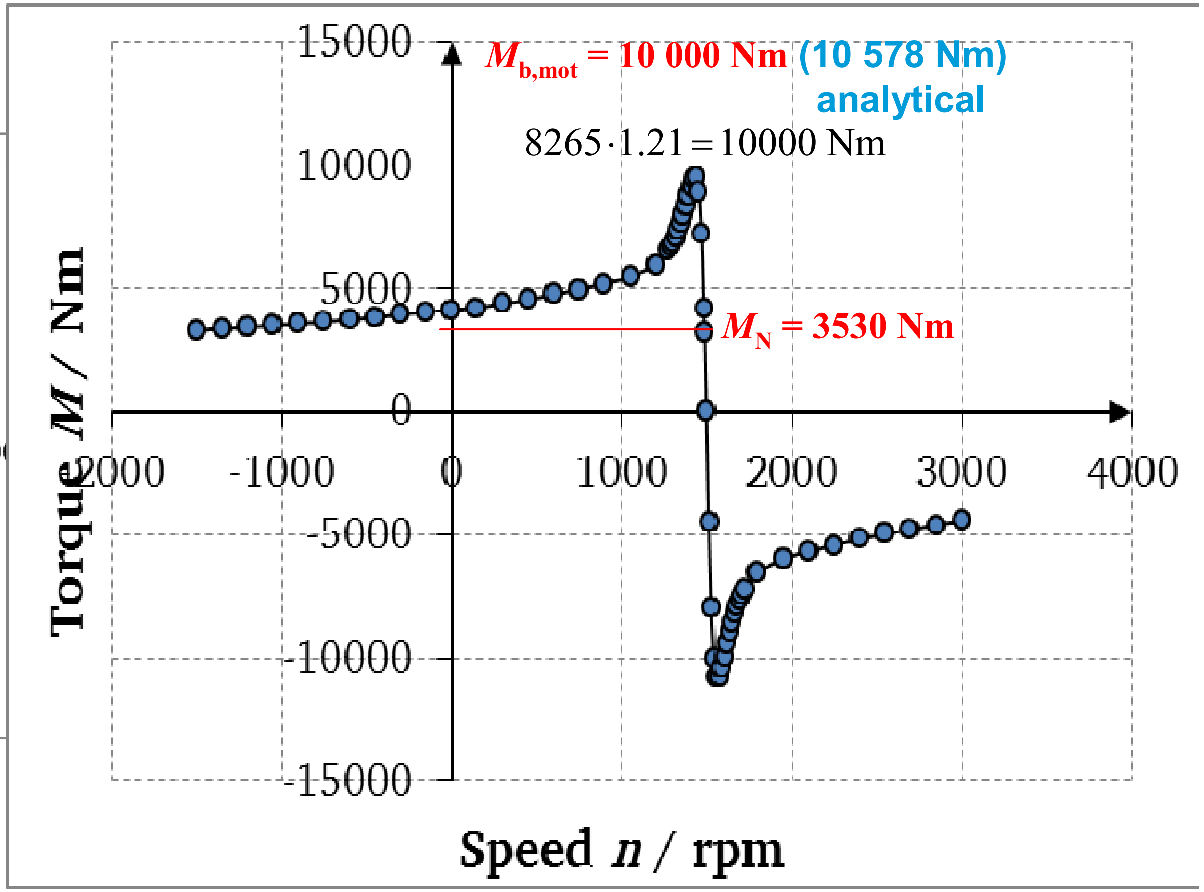
Induction Motor Computation with SPEED: Torque curve (50 Hz)

Source: SPEED program

$U_{s,LL} = 6600$ V Y

$U_{sN} = 6000$ V Y



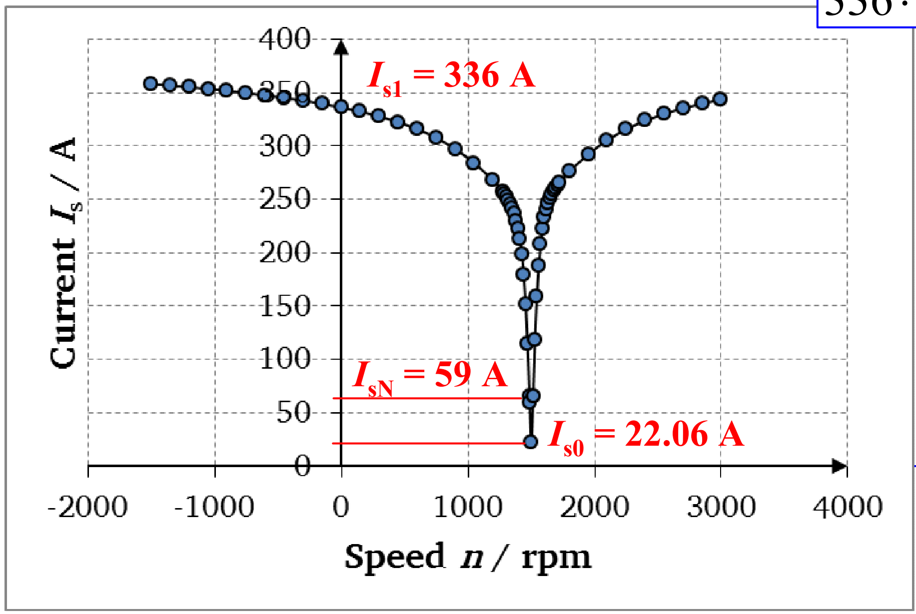


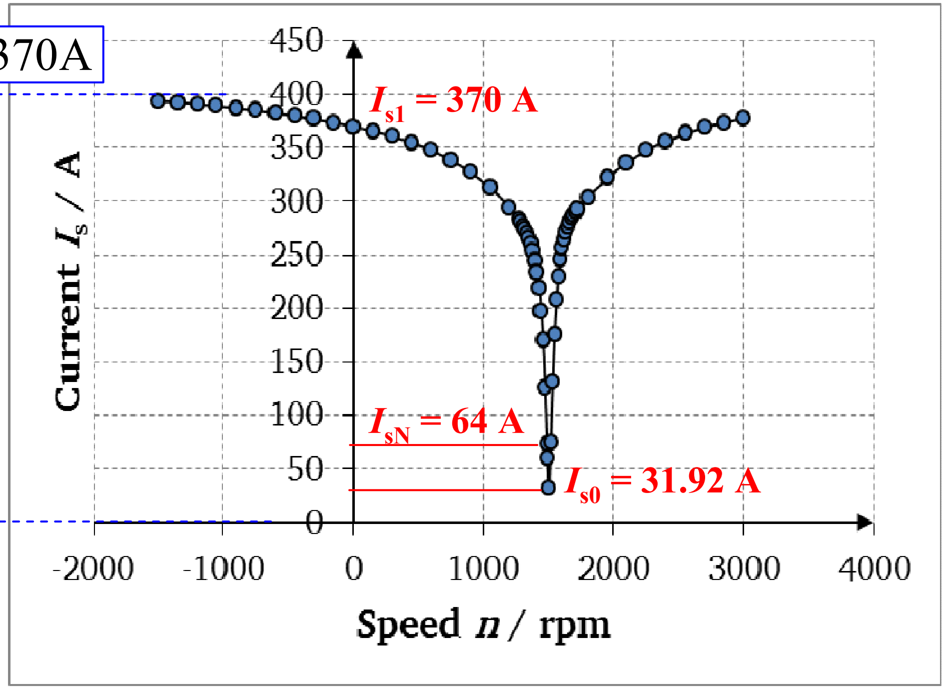
$$\frac{M_{6.6kV}}{M_{6kV}} = \frac{6.6^2}{6^2} = 1.21$$

Induction Motor Computation with SPEED: Stator current curve

$U_{sN} = 6000$ V Y   50 Hz $U_{s,LL} = 6600$ V Y

$336 \cdot 1.1 = 370$A



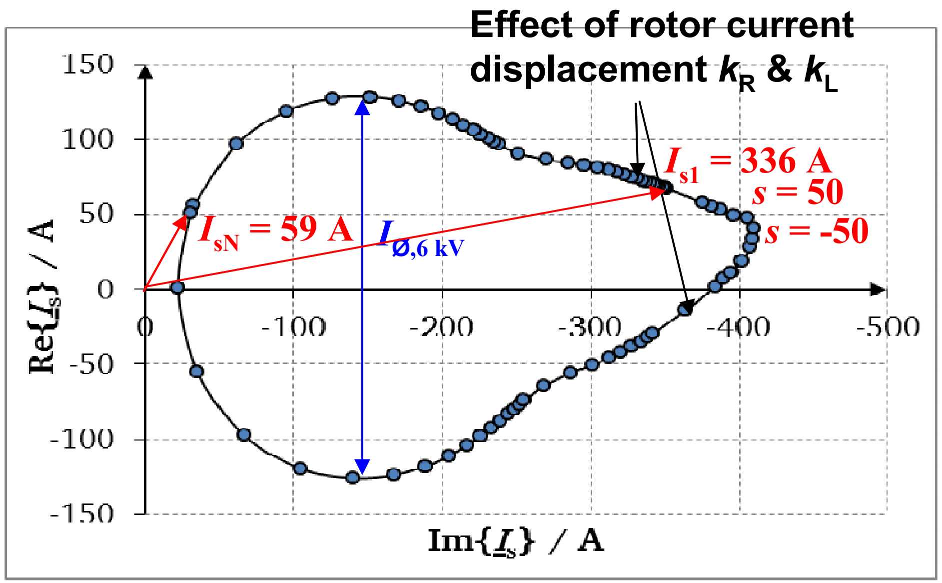


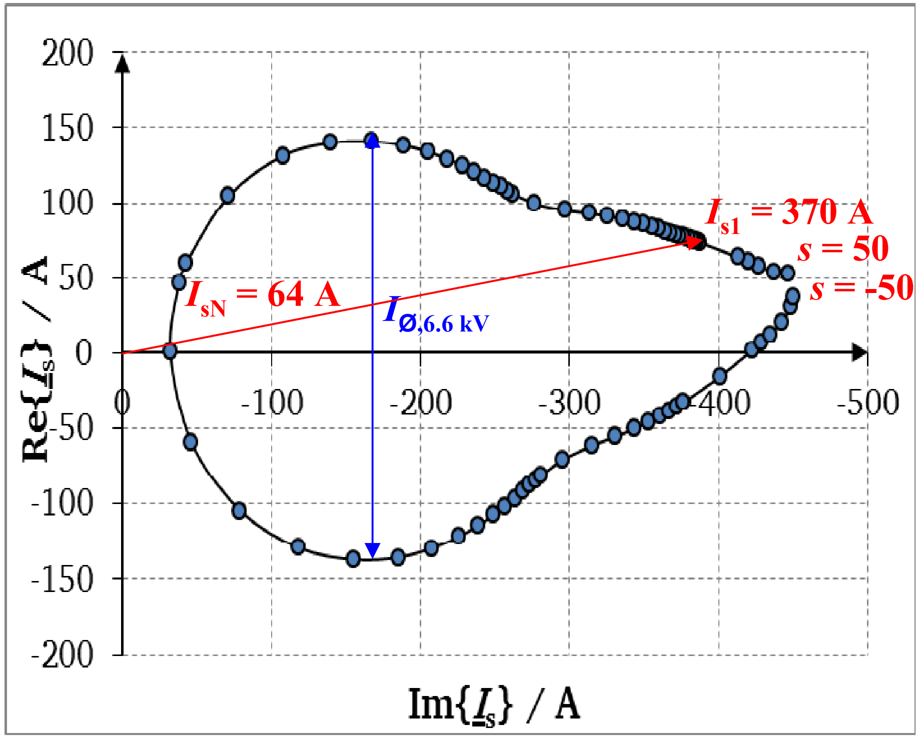
$$\frac{I_{s1,6.6kV}}{I_{s1,6kV}} = \frac{370\ \text{A}}{336\ \text{A}} = 1.1 = \frac{U_{s,LL}}{U_{s,N}} = \frac{6.6\ \text{kV}}{6\ \text{kV}} = 1.1$$

Source: SPEED program

Induction Motor Computation with SPEED:
Stator current root locus

U_{sN} = 6000 V Y   50 Hz $U_{s,LL}$ = 6600 V Y





$$\frac{I_{\phi,6.6\mathrm{kV}}}{I_{\phi,6\mathrm{kV}}} = \frac{6.6}{6} = 1.1$$

Source: SPEED program

Appendix 2: Induction Motor Computation with the Finite-Element-Method (FEM)

2. Design of Induction Machines

2D numerical solution of quasi-stationary MAXWELL equations

2D Flux density vector: $\vec{B}(x,y,t) = (B_x, B_y, 0)$ infinitely long in *z*-direction

2D Vector potential: $\vec{A} = (0,0,A_z(x,y,t))$ current density: $\vec{J} = (0,0,J_z(x,y,t))$

$$\vec{B} := \mathrm{rot}\vec{A} \quad \mathrm{div}\vec{A} := -\varepsilon\cdot\mu\cdot\partial\varphi/\partial t \approx 0$$

$$\mathrm{rot}\vec{E} = -\partial\vec{B}/\partial t = -\partial(\mathrm{rot}\vec{A})/\partial t \Rightarrow \vec{E} = -\partial\vec{A}/\partial t - \mathrm{grad}\,\varphi$$

$$\mathrm{rot}(\vec{B}/\mu) = \mathrm{rot}\vec{H} = \vec{J} + \underbrace{\partial\vec{D}/\partial t}_{\approx 0} \approx \vec{J} = \mathrm{rot}(\mathrm{rot}\vec{A}/\mu)$$

$$\mu = \mathrm{const}:$$

$$\mathrm{rot}(\mathrm{rot}\vec{A}) = \mathrm{grad}\,(\overbrace{\mathrm{div}\vec{A}}^{:=0}) - \nabla^2\vec{A} = \mu\cdot\vec{J} = -\nabla^2\vec{A}$$

$$-\nabla^2\vec{A} = -\Delta\vec{A} = \mu\cdot\vec{J} = \mu\cdot\kappa\cdot(-\partial\vec{A}/\partial t - \mathrm{grad}\,\varphi)$$

$$\Delta\vec{A} = -\mu\cdot\kappa\cdot(-\mathrm{grad}\,\varphi) + \mu\cdot\kappa\cdot\partial\vec{A}/\partial t$$

$$\Delta\vec{A} = -\mu\cdot\vec{J}_{\mathrm{imp}} + \mu\cdot\kappa\cdot\partial\vec{A}/\partial t$$

$$\frac{\partial^2 A_z}{\partial x^2} + \frac{\partial^2 A_z}{\partial y^2} = -\mu\cdot J_{z,\mathrm{imp}} + \mu\cdot\kappa\cdot\frac{\partial A_z}{\partial t}$$

A_z: *z*-component magnetic vector potential (unknown)

μ : Magnetic permeability (given)

κ : Electrical conductivity (given)

$J_{z,\mathrm{imp}}$: Impressed *z*-component of current density (given)

Or: gradφ: Impressed electric potential gradient (given)

Time-stepping solution needed

2. Design of Induction Machines

FE-Solutions for the analysis of el.-magn. field problems

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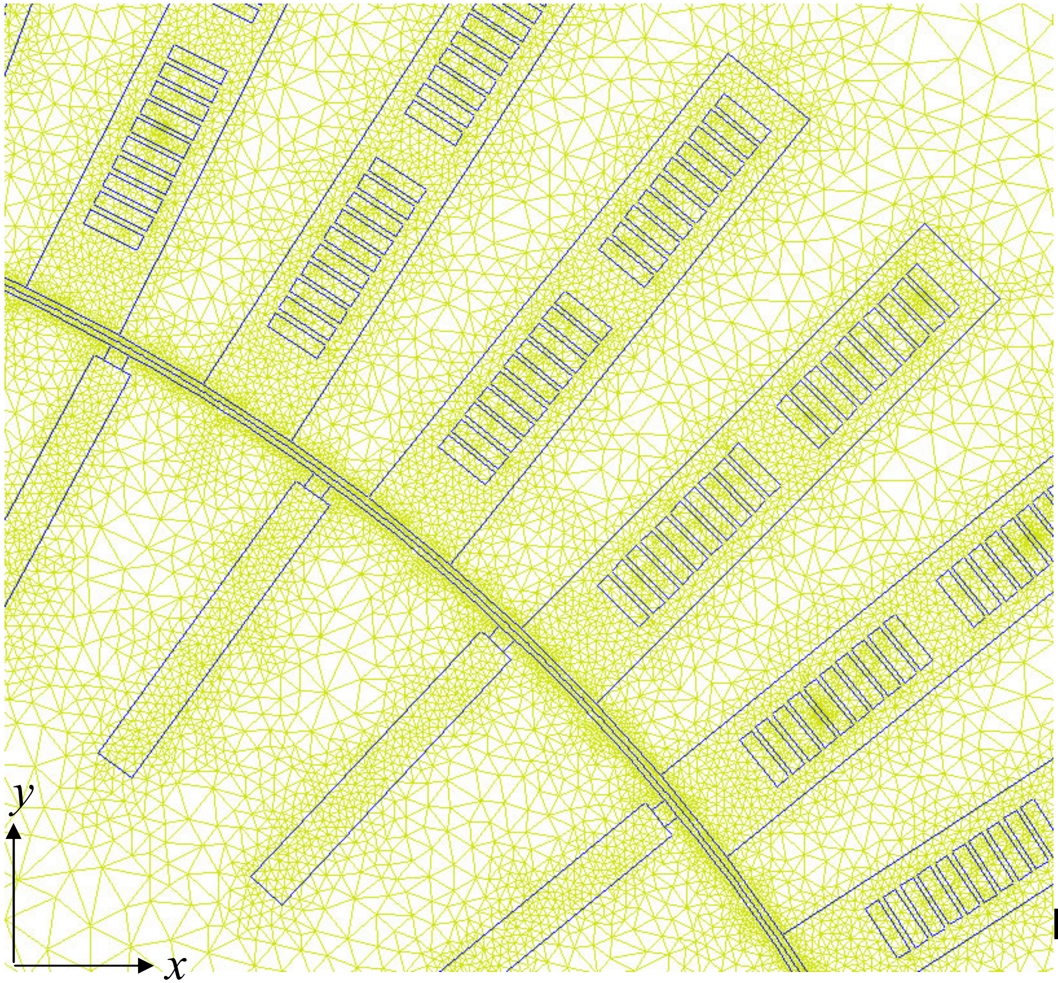
1) ***Steady state FE-analysis: ("DC")*** $\partial\,.\,/\partial t = 0$
   - + Calculation of fluxes and inductance
   - + Determination of forces and torques
   - + Saturation effects, De-magnetization of magnets

2) ***Frequency domain FE-analysis: ("AC")*** $\partial\,.\,/\partial t \Rightarrow j\omega$ ...
   - + Time-harmonic eddy currents
   - + Time-harmonic induced voltages
   - + Fixed-saturation problems

3) ***Time stepping FE-analysis: ("transient")***
   - + Arbitrary time variation (eddy-currents, moving rotors, …)
   - + Transient phenomena: (e.g. sudden short-circuits, …)

2. Design of Induction Machines

2D triangle finite element (FE) mesh



$$\frac{\partial^2 A_z}{\partial x^2} + \frac{\partial^2 A_z}{\partial y^2} = -\mu \cdot J_z + \mu \cdot \kappa \cdot \frac{\partial A_z}{\partial t}$$

μ : Permeability is constant per finite element

κ: Electrical conductivity is constant per FE

Special case ("AC"):

Harmonic time functions:

ω : angular frequency

$$A_z(x,y,t) = \mathrm{Re}\left\{\underline{A}_z(x,y) \cdot e^{j\omega t}\right\}$$

$$J_z(x,y,t) = \mathrm{Re}\left\{\underline{J}_z(x,y) \cdot e^{j\omega t}\right\}$$

$$\frac{\partial^2 \underline{A}_z}{\partial x^2} + \frac{\partial^2 \underline{A}_z}{\partial y^2} = -\mu \cdot \underline{J}_z + j \cdot \omega \cdot \mu \cdot \kappa \cdot \underline{A}_z$$

No time-stepping needed, but *B*(*H*) must be linear!

Open source software FEMM free at http://femm.foster-miller.net/wiki/HomePage

Induction Motor *B*-field (flux density) plot at no-load *s* = 0, 6 kV

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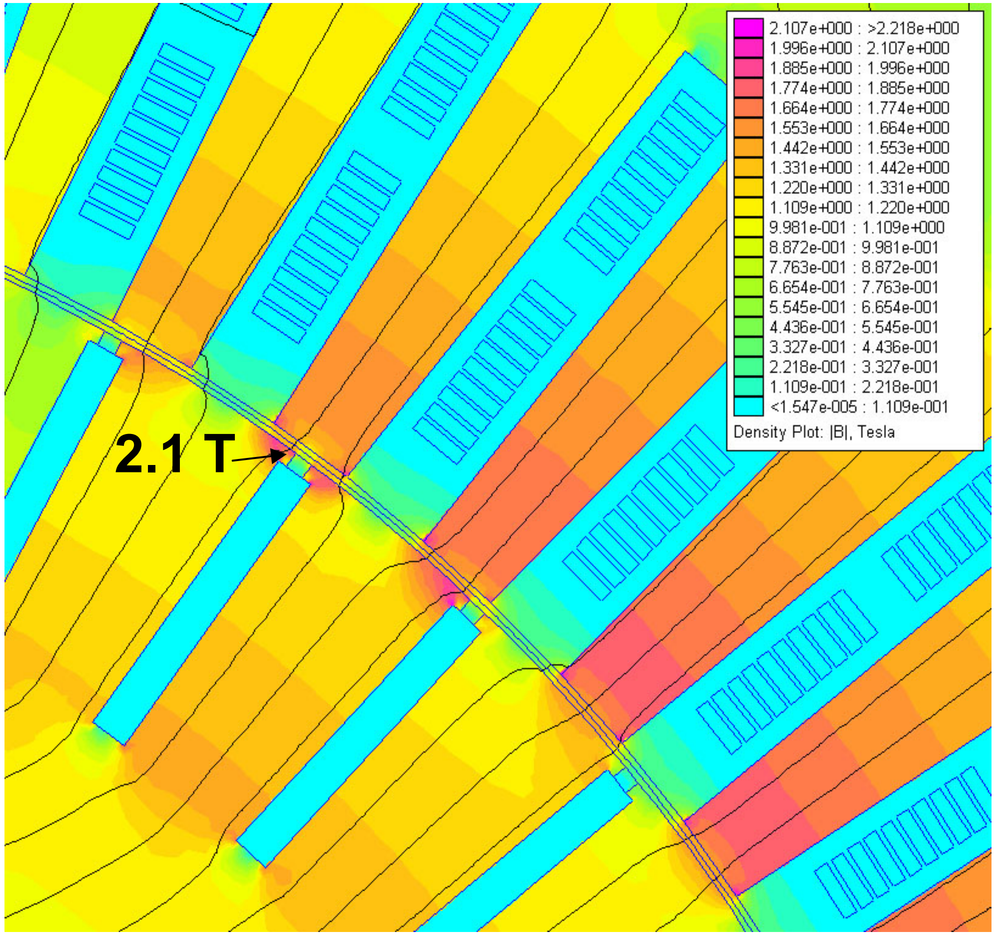
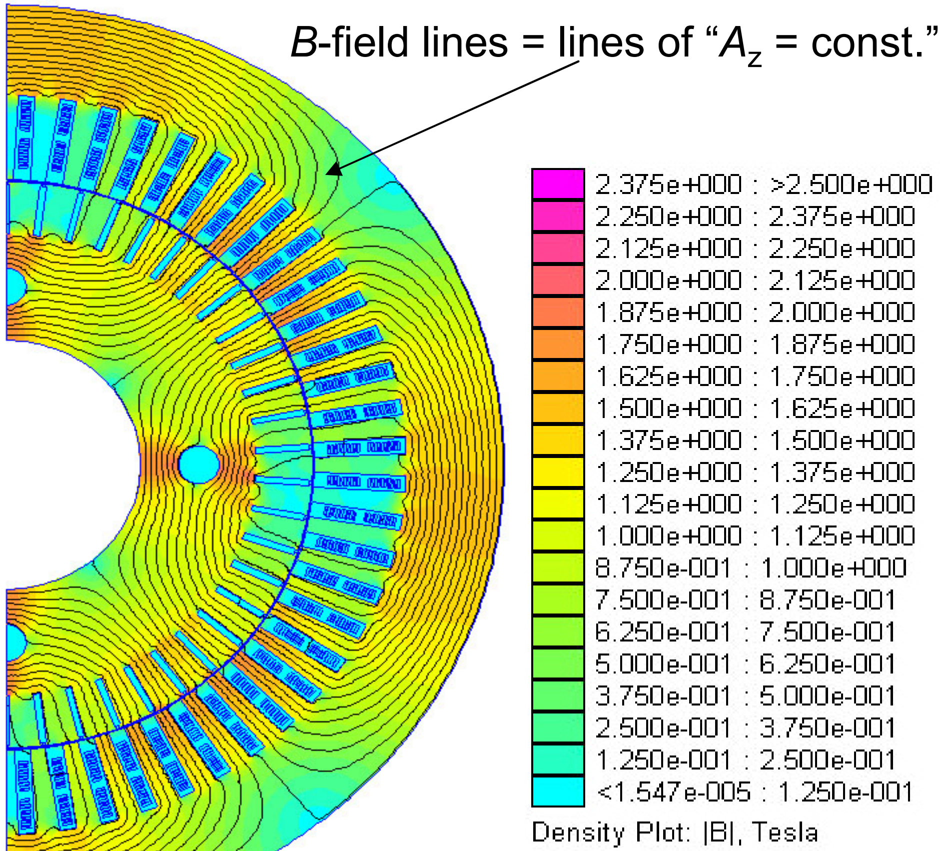




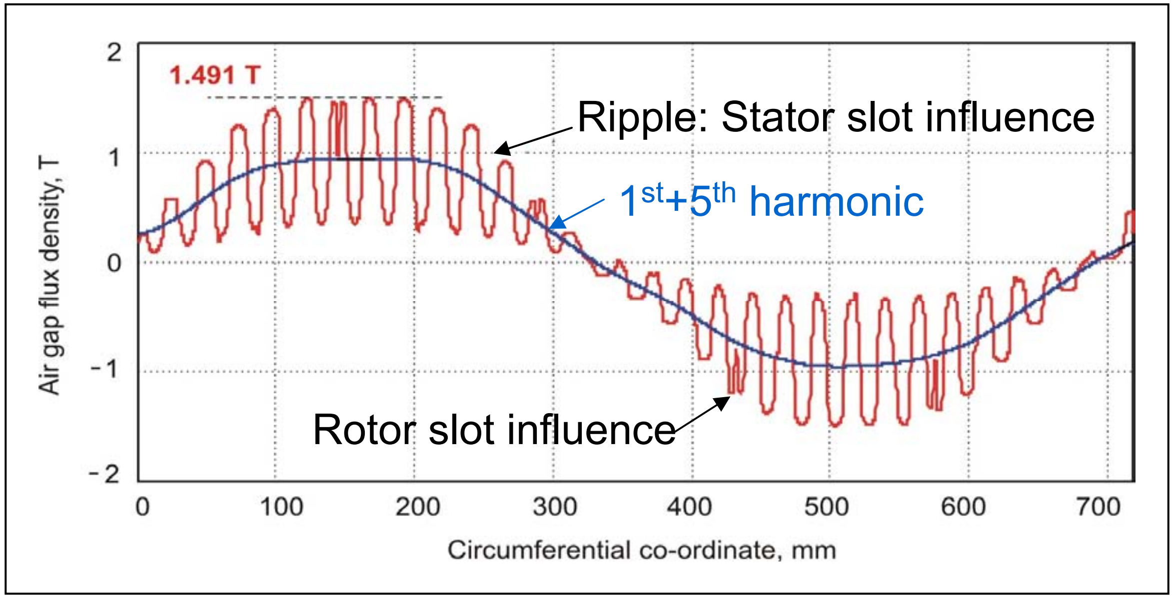
***f* = 50 Hz**

- Cross section of a three-phase induction motor (P_N = 500 kW), no-load: $P_{m,out} = 0$:
- No-load flux density *B*: Rated voltage U_N = 6 kV, *s* = 0, rotor current zero (FEMM program)

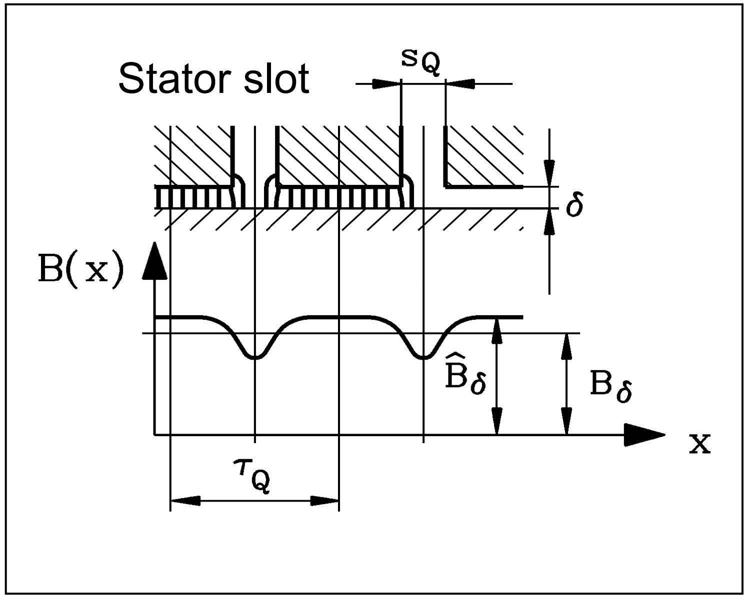
Induction Motor radial *B*-field at no-load 6 kV in the air gap

Open stator slots of high voltage machines yield considerable flux density ripple !

Radial flux density component in middle air-gap $\delta/2$



Idealized geometry for calculation



Flux fringing in air gap due to slotting

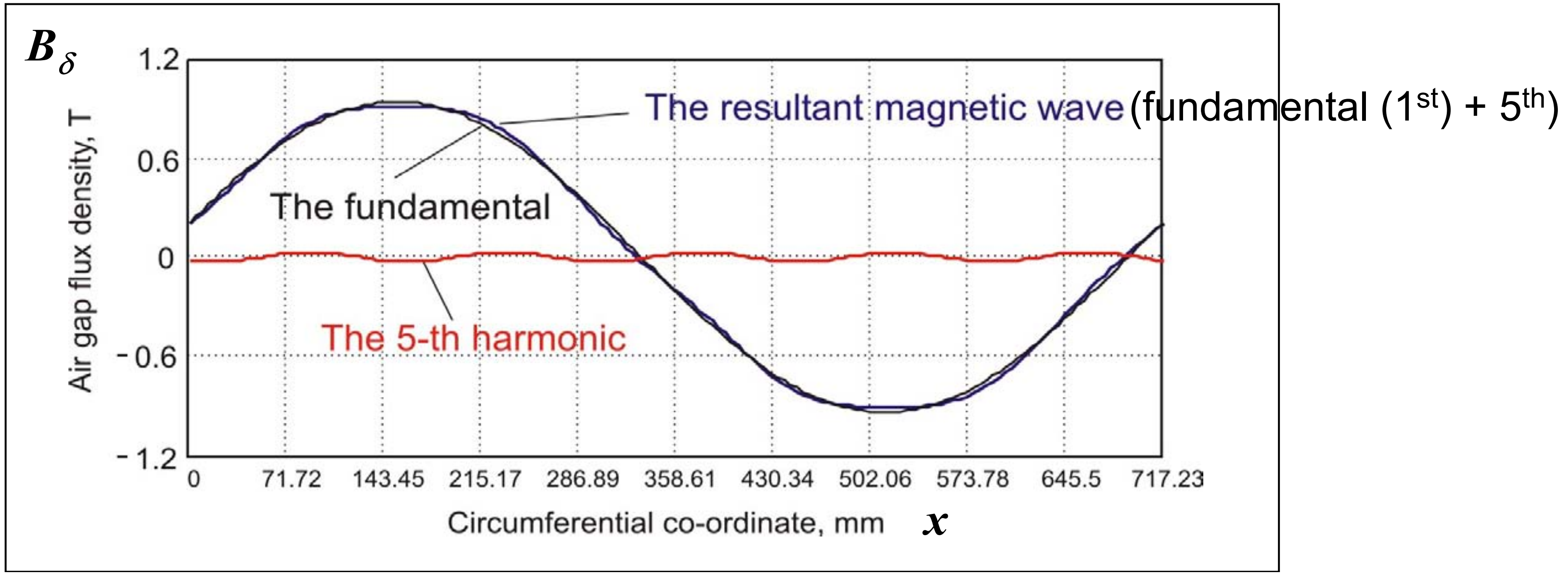
Source: FEMM program

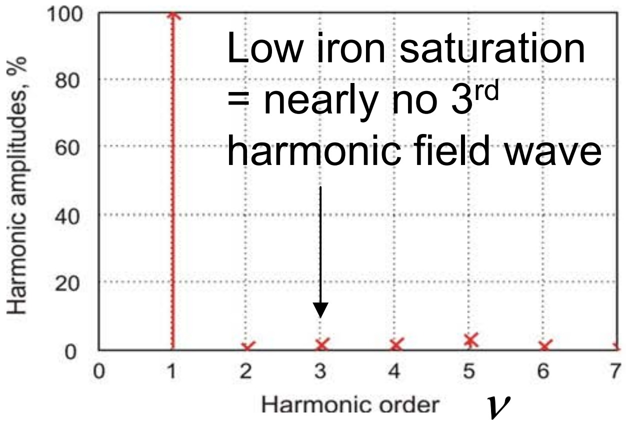
CARTER´s coefficient:

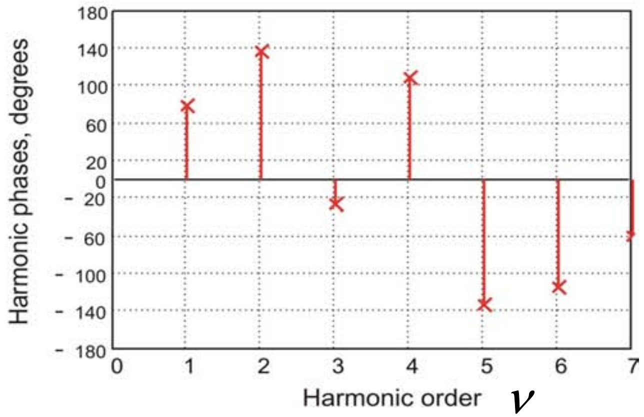
$$k_C = \hat{B}_\delta / B_\delta = \frac{\tau_Q}{\tau_Q - \zeta(h)\cdot\delta}$$

2. Design of Induction Machines

FOURIER-series of radial *B*-field at no-load 6 kV in the air gap



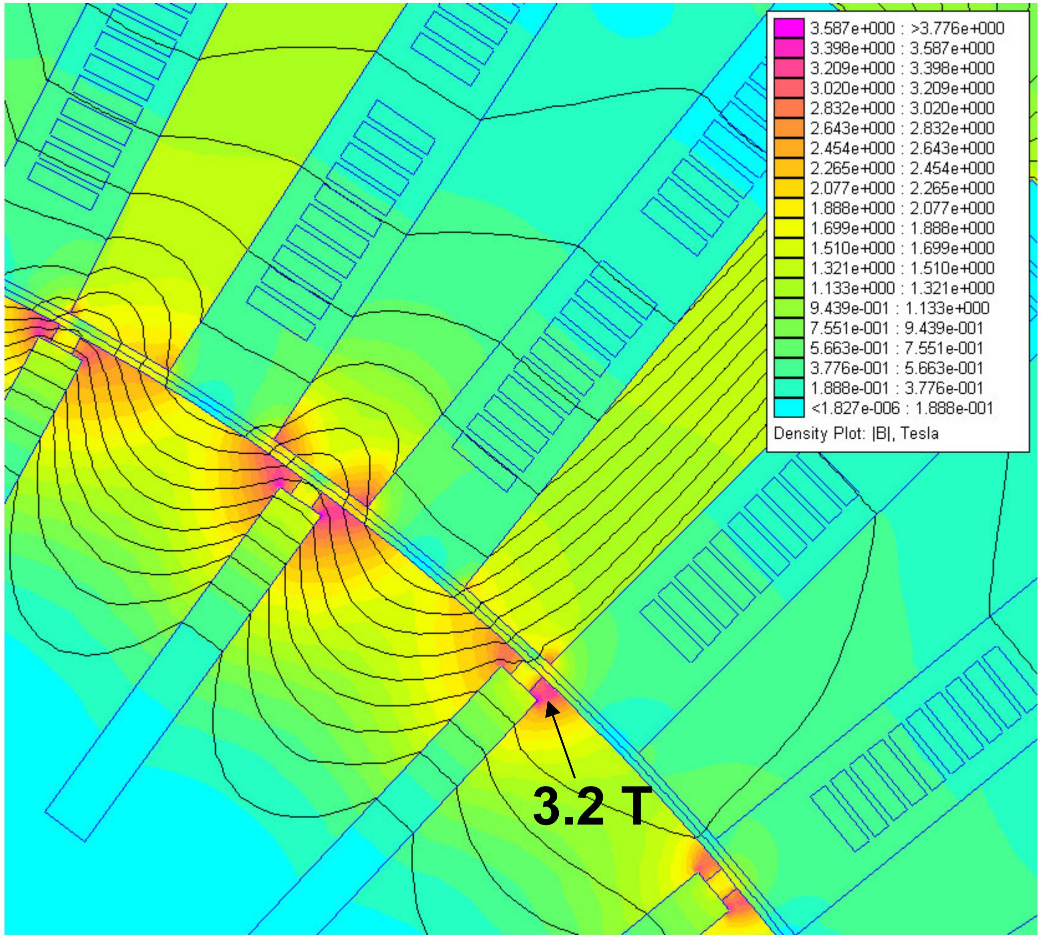


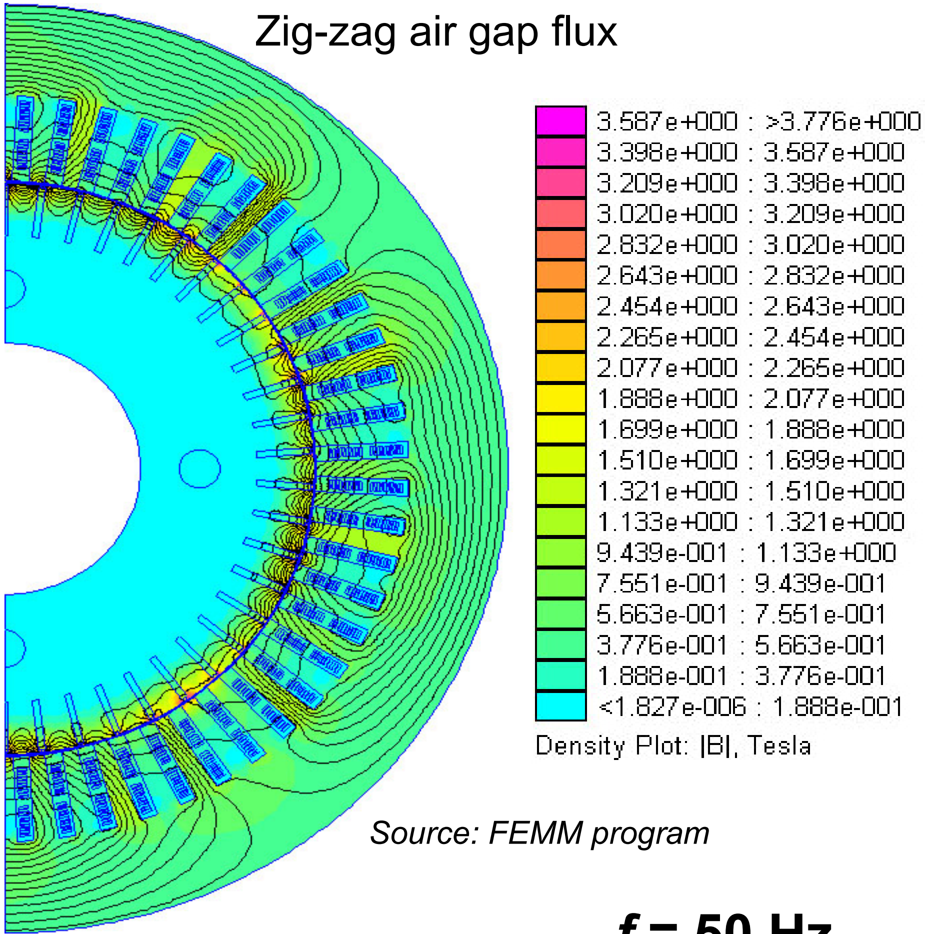


Source: FEMM program

2. Design of Induction Machines

Induction Motor *B*-field (flux density) plot at locked rotor (s = 1)





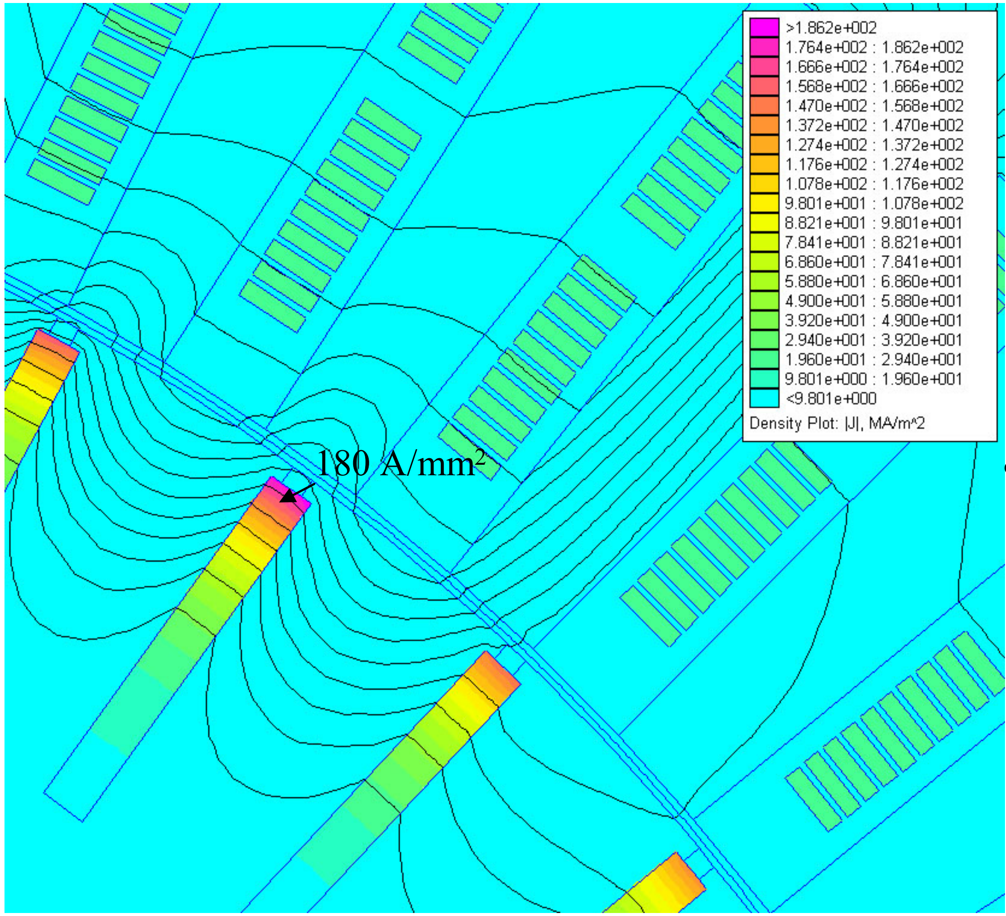
Source: FEMM program

***f* = 50 Hz**

- Cross section of a three-phase induction motor P_N = 500 kW: Stand still s =1.
- Harmonic calculation ("AC"): Flux density plot at U_N = 6 kV, ωt = 0, constant μ per element

Induction Motor *B*-field and rotor bar current density plot at locked rotor (*s* = 1, 6 kV)

Source: FEMM program



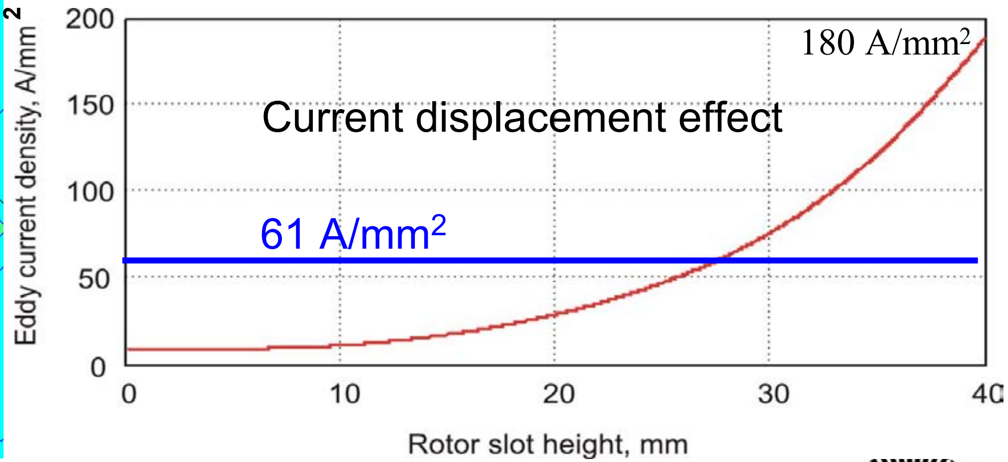
$$I_s(s=1) \approx \frac{U_s}{\sqrt{(R_s + R'_r)^2 + (X_{s\sigma} + X'_{r\sigma})^2}}$$

$$I_s(s=1) \approx 6.6 \cdot I_{sN} = 396\,A \approx I'_r(s=1)$$

$$I_{bar} = \ddot{u}_I \cdot I'_r(s=1) = 21.84 \cdot 6.6 \cdot 60 = 8650\ \mathrm{A}$$

Rotor bar current density:

$$\hat{J}_{bar} = \sqrt{2} \cdot 8650\ \mathrm{A}/(40 \cdot 5) = 61\,\mathrm{A/mm}^2$$



Summary:
Induction Motor Computation with the Finite-Element-Method (FEM)

- 2D and 3D no-load and load calculations, solving MAXWELL`s equations
- Different commercial solvers available
- Analytical programs much faster, but no detailed saturation calculation
- Numerical calculation for prototype development
- Analytical programs for daily use (changes of voltage, power, etc.)

- Numerical methods for field calculation: see Proffs. deGersem & Schöps
- Seminar on numerical E-machines calculation offered (Dr.-Ing. B. Funieru)

Seminary: Design of Electrical Machines and Actuators with Numerical Field Calculation (SS 0+3) (Dr.-Ing. B. Funieru, CST company, Darmstadt)

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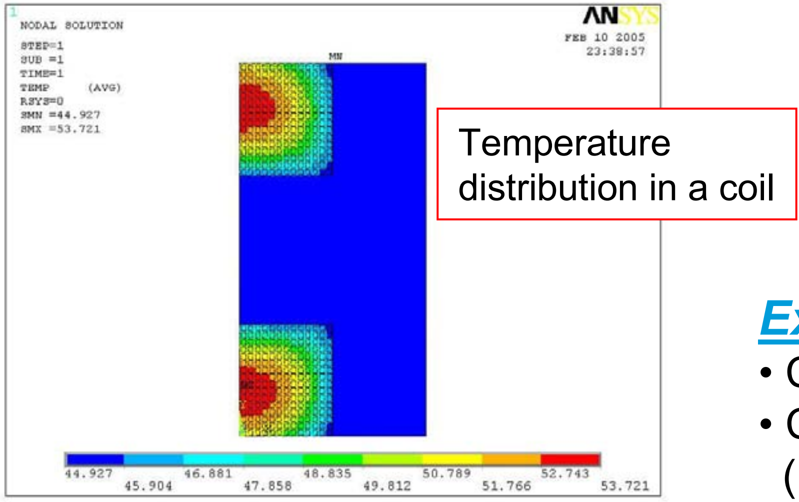
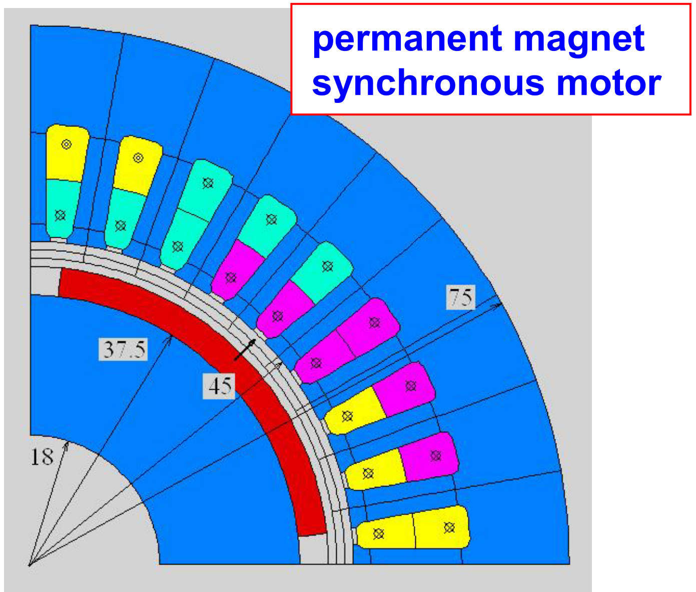
Contents

- The Finite Element (FE) Method Fundamentals
- Presentation of FEMAG, ANSYS and SPEED
- Basic methods to optimise a FE model
- Specific use of the programs for magnetic and thermal cases





Examples:

- Calculation of an electromagnetic actuator (FEMAG)
- Calculation of a permanent magnet synchronous motor (FEMAG & SPEED)
- Thermal calculation of an actuator (ANSYS)

Examination:

- Activity during seminary (33%)
- Project report (33%)
- Presentation of the project results and oral examination of the theoretical part (33%)

Seminary data: see TUCaN

Example: PM synchronous motor – Time stepping solution

- 2D FE *steady state* calculation with *stepwise rotor movement* (air gap re-meshing)

- Stator winding fed by AC current source *(impressed current system)* , depending on the rotor position angle α and the angular speed $d\alpha/dt \sim n$

- **Determination of:**

↳ Flux per phase, induced voltage, forces, torque, losses

↳ Machine parameters (inductances, flux linkages, …): L_d, L_q, ψ_m

↳ Machine operation quantities: $M, I, U, P = f(n)$

PM synchronous motor – Time stepping solution

FE-Calculation with 2D solver FEMAG (ETH Zürich, Prof. K. Reichert)

Analytical EM-Design => CAD

Parameter input

Optimization

GP-FEM-Analysis:
- DC
- AC
- Transient

El.-magnetic Fields

EM-FEM-Analysis:
- Synchron.-Motor
- BLDC-Motor
- COM-Motor
- SR-Motor
- Induction-Motor
- Linear-Motor

Magnetization Demagnetization

EM-Models

Performance characteristics

PE-Simulation

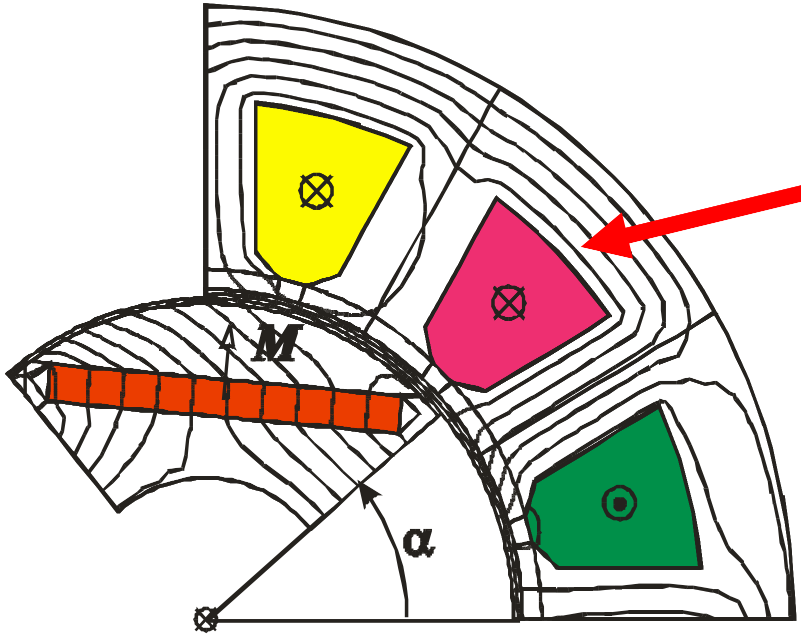
(2008)

Source: FEMAG, Reichert, PROFEMAG company

Step 1: Generation of FE mesh

Example:

4-pole PM synchronous motor, $m = 3, q = 1$, single-layer winding,1 pole section:

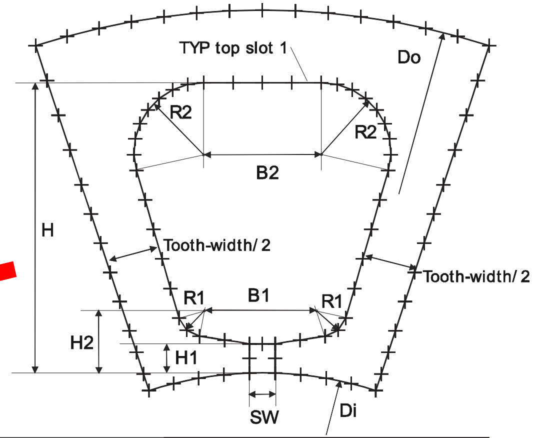


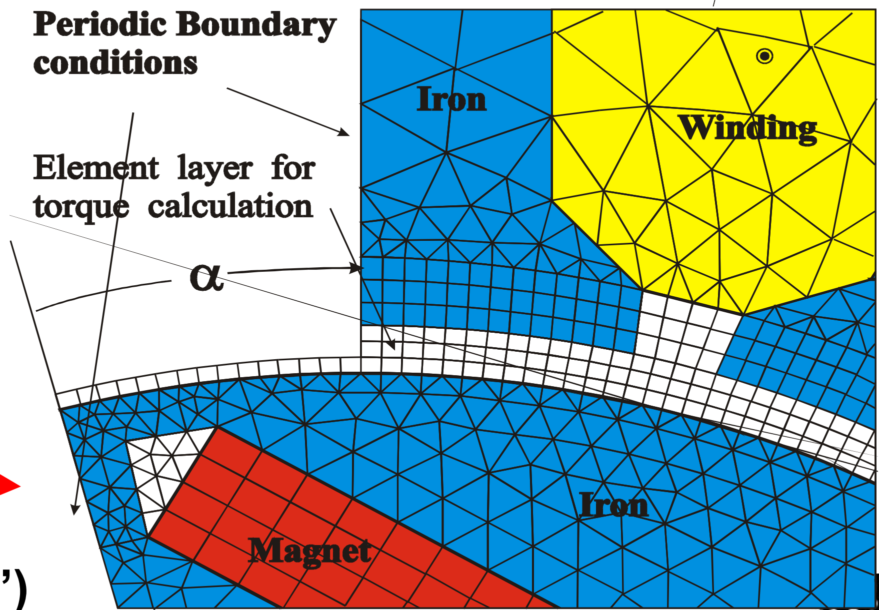
Buried rotor magnet

Source: FEMAG, Reichert, PROFEMAG company

Semi-closed stator slot:

Parameter-defined slot geometry





Automatic mesh generation ("Mesher")

<u>Step 2:</u> Moving rotor no-load (currents = 0)

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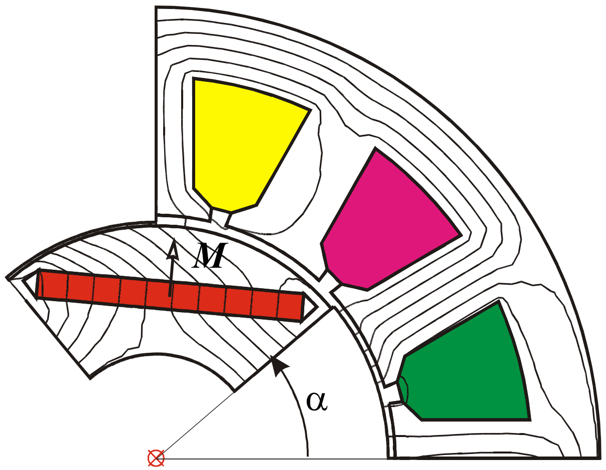
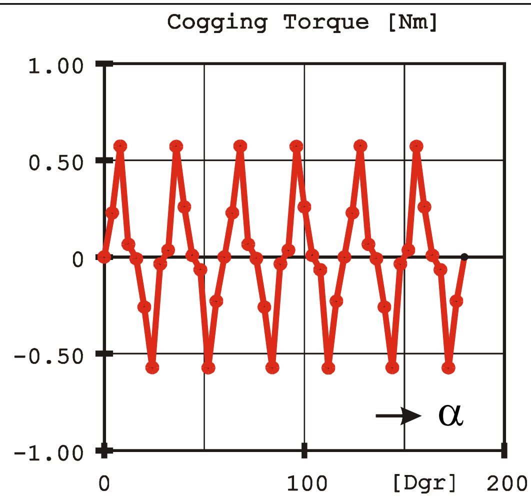
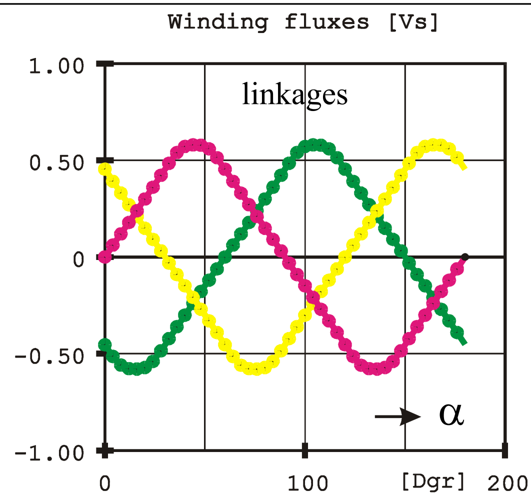
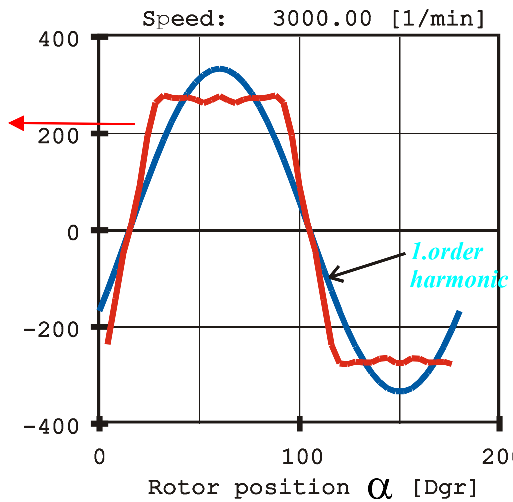


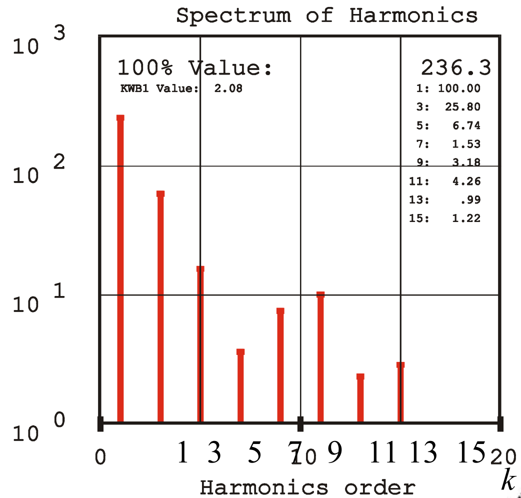
Skew angle = 0

k	(%)
1:	100.00
3:	25.80
5:	6.74
7:	1.53
9:	3.18
11:	4.26
13:	.99
15:	1.22







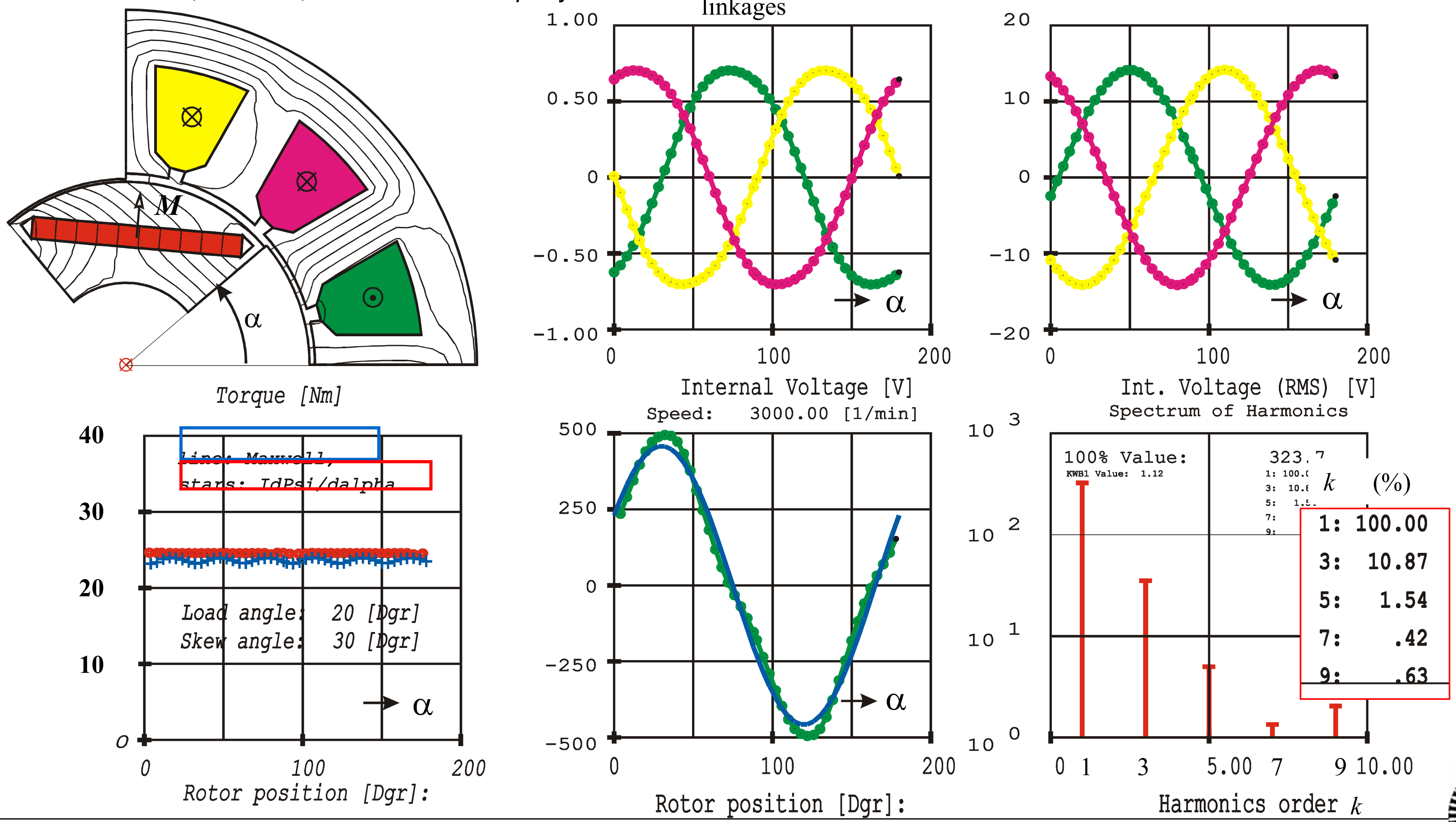


Steady State FE-Analysis

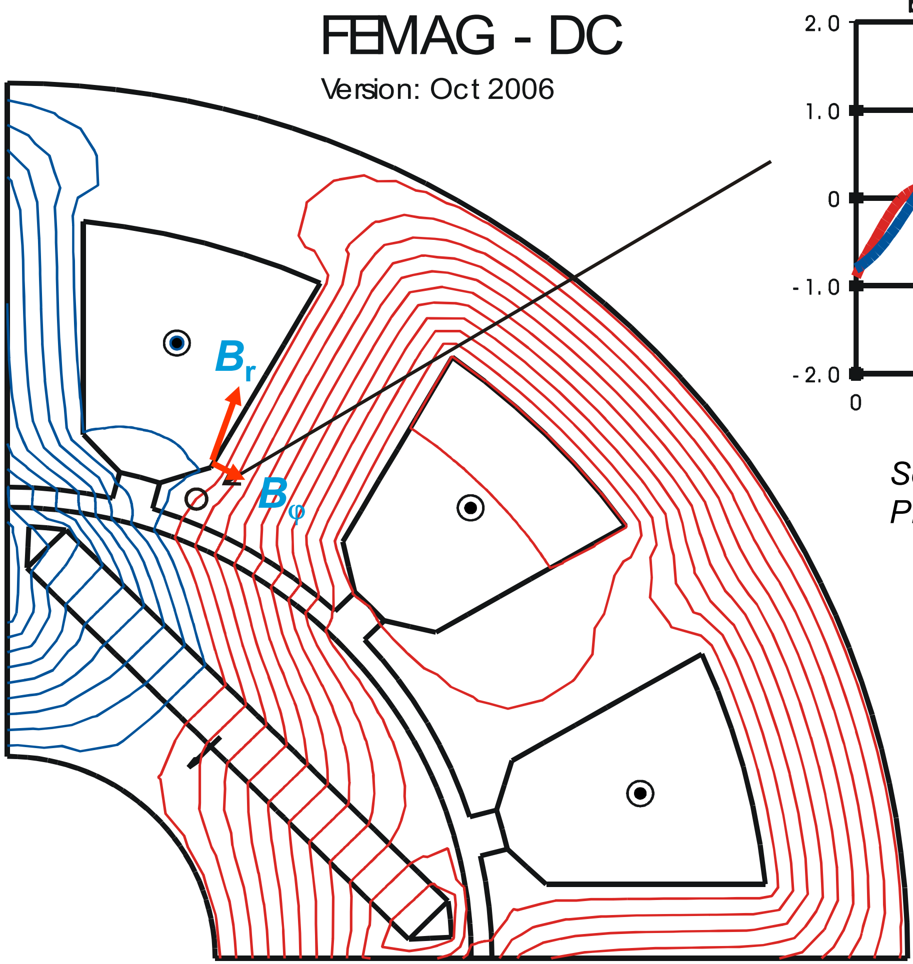
Source: FEMAG, Reichert, PROFEMAG company

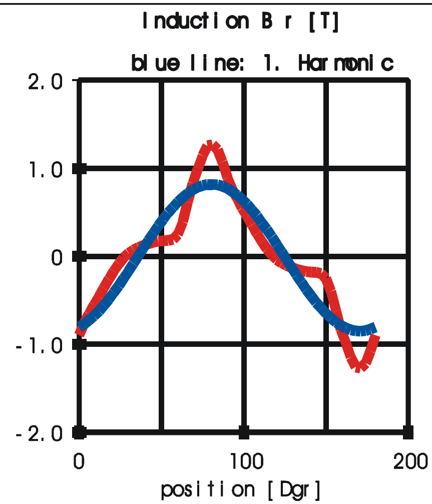
<u>Step 3:</u> Load simulation with impressed current system

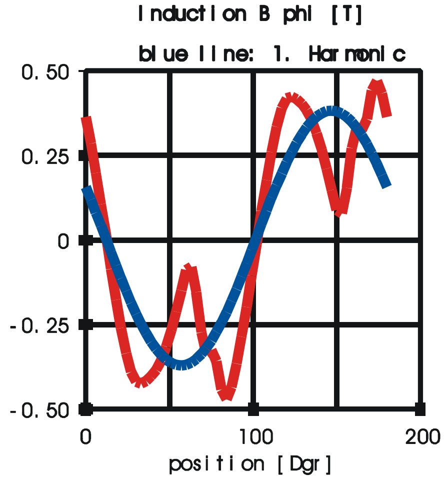
Source: FEMAG, Reichert, PROFEMAG company



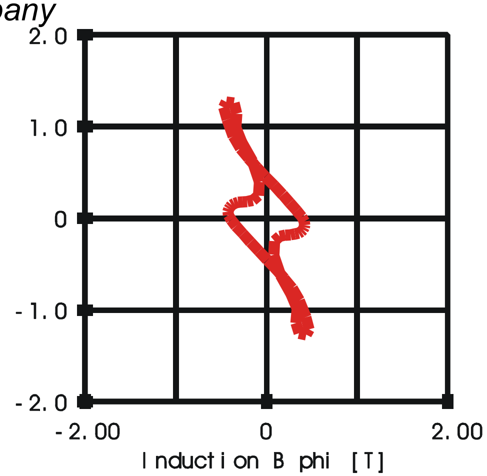
<u>Step 4:</u> Post processing: Flux density *B* at load, depending on rotor position angle α and on stator currents







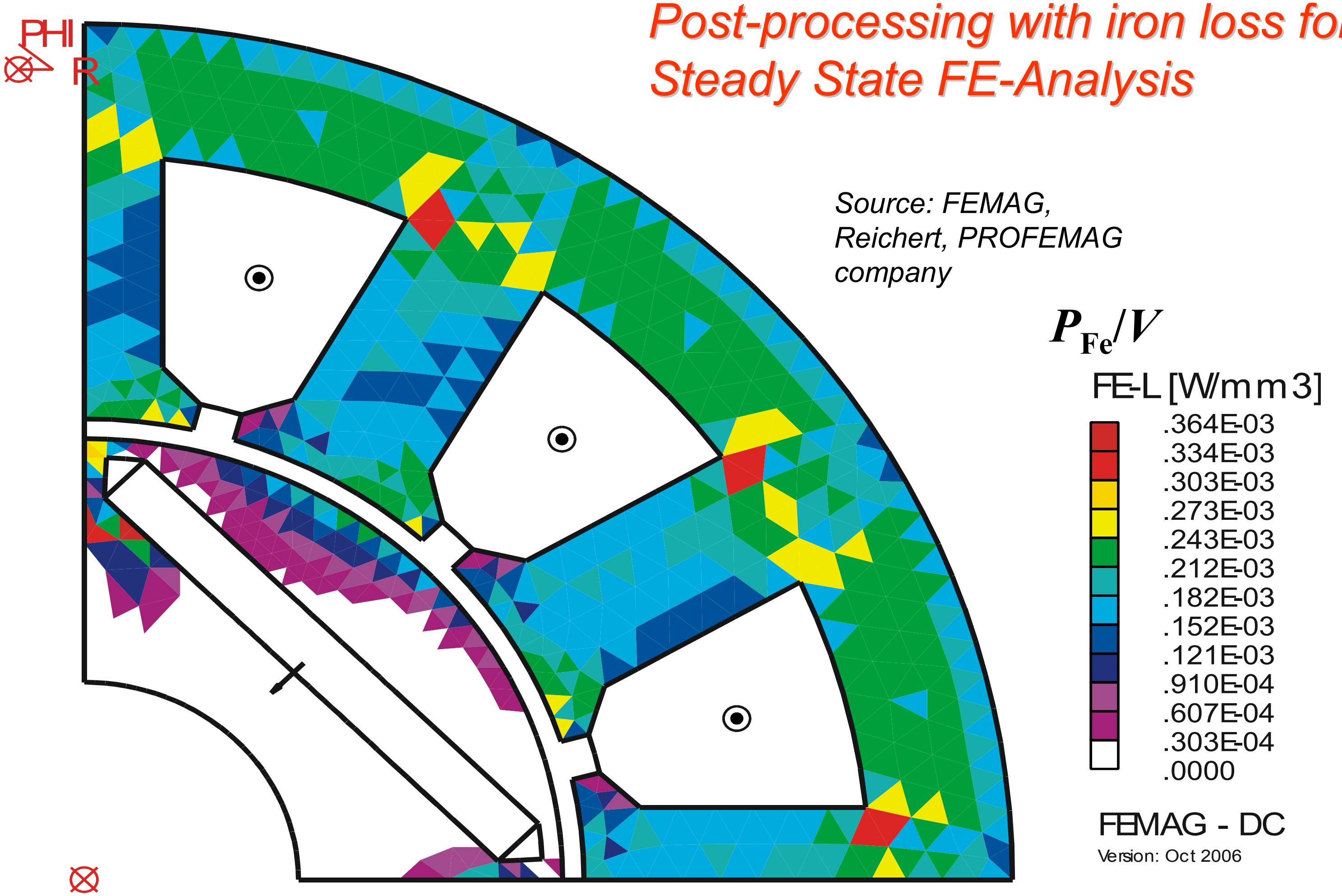
Source: FEMAG, Reichert, PROFEMAG company



Steady State FE-Analysis

Step 5: Local iron loss distribution at load

Post-processing with iron loss formula:
Steady State FE-Analysis



FE-Simulation with movement of rotor (program FEMAG)

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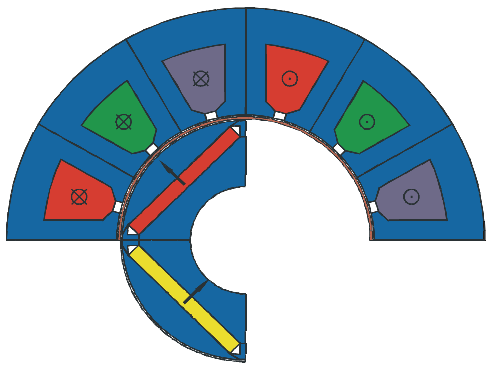
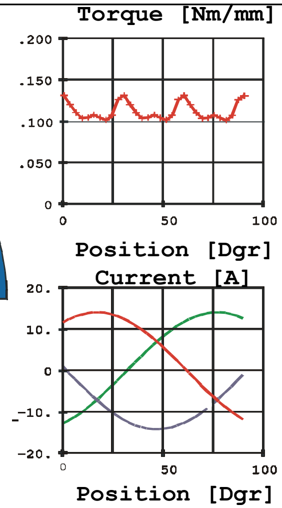




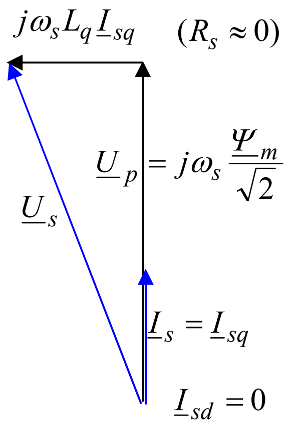
BLDC- Motor Fieldweakening

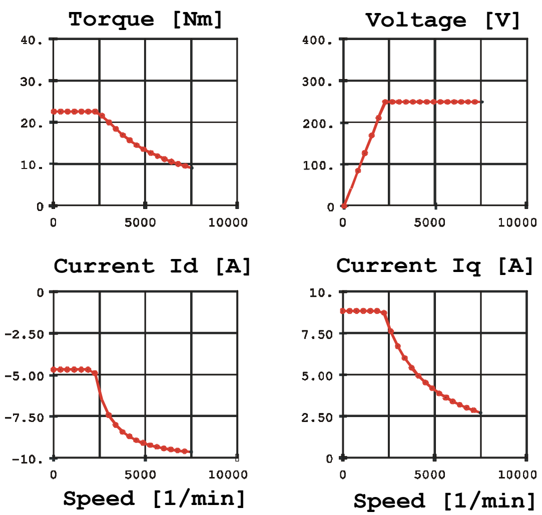
Machine data from FE-Simulation:
No-load and load calculation

*** PM-Synchron-Motor (Rot.) ***			
Wdg.Voltage (operat. limit) (RMS)	[V]	=	250.0
Wdg.Current (operat. limit) (RMS)	[A]	=	10.00
Curr.angle (=0: optim,>0:const)	[Deg]	=	0.0
Winding resistance stator	[Ohm]	=	0.0
Inductance Ld	[H/mm]	=	.3289E-03
Inductance Lq (lin)	[H/mm]	=	.5846E-03
Inductance Lq (1.5*In)	[H/mm]	=	.5846E-03
Inductance Lq (2.5*In)	[H/mm]	=	.5846E-03
Stator end-winding inductance	[H]	=	0.0
Wdg.No-load PM flux (RMS)	[Vs/mm]	=	.3056E-02
Effect. armature length	[mm]	=	100.0
Number of Phases m	(>= 2)	=	3.000
Number of Pole pairs p	(>= 1)	=	2.000
Max. current (RMS): = psi_m/Ld	[A]	=	9.291
Rel. number winding turns (wdg.1)	[pu]	=	1.000
Rotor Speed (1/min): from	0	to	7500
Number of steps:			20.00

$0 \le \omega_s \le \omega_N$:

$0 \le U_s \le U_{sN}$





Performance characteristics from

L_d, L_q, Ψ_m machine model

Source: FEMAG, Reichert, PROFEMAG company

Summary:
Design of Induction Machines

- Detailed analytical calculation method for induction machines
- These methods are usually programmed as „analytical“ machine programs (e.g. SPEED)
- 2D and 3D numerical finite element or finite difference programs as alternative
- Numerical programs for detailed prototype investigations
- Further information on E-machines:
  - Motor development for electrical drive systems (Lecture, 2+1)
  - Large generators and high power drives (Lecture, 2+1)
  - Design of Electrical Machines & Actuators with Numerical Field Calculation (Seminary, 0+3)