Energy Converters – Computer-Aided Design (CAD) and System Dynamics

TECHNISCHE UNIVERSITÄT DARMSTADT

A. Binder

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Lecturer

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Tutorial

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Introduction
Learning outcomes

Understanding of design rules for electrical machines
- scaling laws (typical for power engineering)
  i.e. AC and DC motors and generators

Knowledge of design of AC machinery, here: cage induction machine
- Magnetic circuit and winding topology
- Calculation of resistances and inductances
- Losses and efficiency

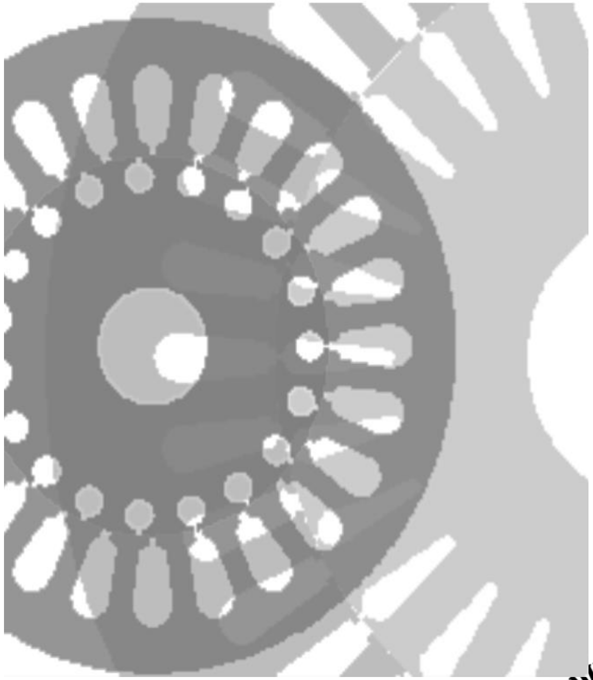
Knowledge of basics in cooling systems and temperature calculation
- applications in electrical motors and generators

Understanding of dynamics of DC and AC machinery
- system dynamics of variable speed drives & space vector theory
- transients in generator systems & power stability

Calculation examples for self-training

Contents

1. Basic design rules for electrical machines
2. Design of Induction Machines
3. Heat transfer and cooling of electrical machines
4. Dynamics of electrical machines
5. Dynamics of DC machines
6. Space vector theory
7. Dynamics of induction machines
8. Dynamics of synchronous machines



Source: SPEED program

Introduction
Organization

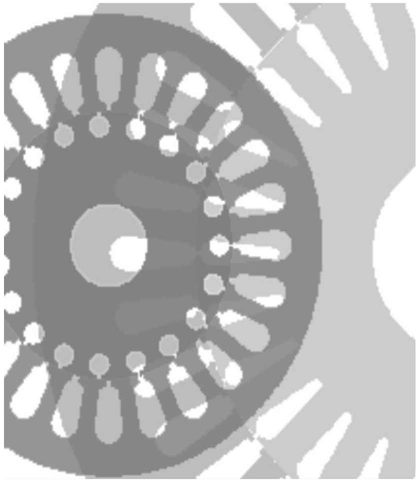
- Moodle:
  Down-load of slides, text book & Collection of exercises

- CityCopies, Holzstraße 5:
  Paper copies of
  slides, text book & Collection of exercises

- Introduction of two computer programs:

  - SPEED program: Induction machine design

  - Matlab/SIMULINK for dynamic calculations



Source:
SPEED program

- Excursion offered

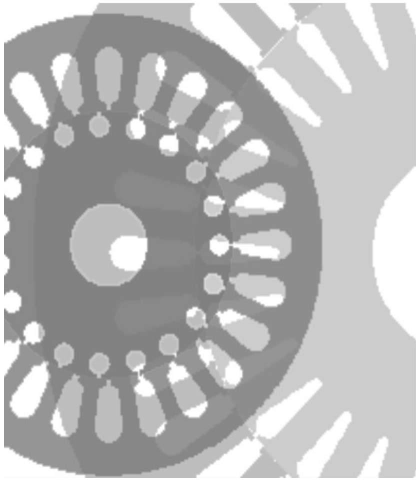
Introduction
Examination

- Examination: In written form

Details see - Moodle
or
- Institute EW Homepage

a) Three calculation examples

b) Theoretical questions



Source:
SPEED program

- Homework:
Dynamics examples – add-on points, if positive examination result

Introduction
Examination

Written examination

Winter term: 2 partial exams

Summer term: 1 exam

Calculation examples & List of theory questions:
see “Collection of Exercises”

- **Used symbols:** **see „Text book“**

- **Greek alphabet:**

$A\ \alpha$	Alpha	$B\ \beta$	Beta	$\Gamma\ \gamma$	Gamma	$\Delta\ \delta$	Delta
$E\ \varepsilon$	Epsilon	$Z\ \zeta$	Zeta	$H\ \eta$	Eta	$\Theta\ \vartheta$	Theta
$I\ \iota$	Jota	$K\ \kappa$	Kappa	$\Lambda\ \lambda$	Lambda	$M\ \mu$	My (mue)
$N\ \nu$	Ny (nue)	$\Xi\ \xi$	Xi	$O\ o$	Omikron	$\Pi\ \pi$	Pi
$P\ \rho$	Rho	$\Sigma\ \sigma$	Sigma	$T\ \tau$	Tau	$Y\ \upsilon$	Ypsilon
$\Phi\ \varphi$	Phi	$X\ \chi$	Chi	$\Psi\ \psi$	Psi	$\Omega\ \omega$	Omega

1. Basic design rules for electrical machines
2. Design of Induction Machines
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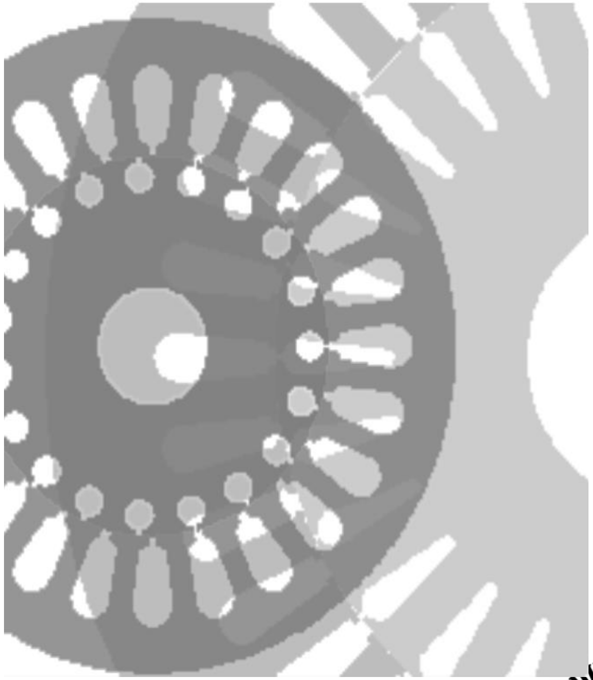
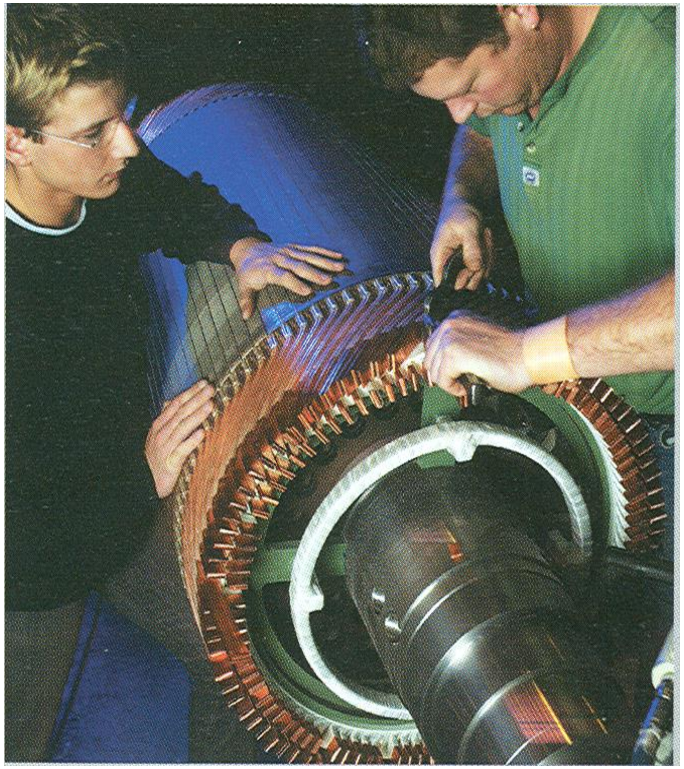


Source: SPEED program

Basic design rules for electrical machines



Source: Winergy, Germany

1. **Basic design rules for rotating machines**

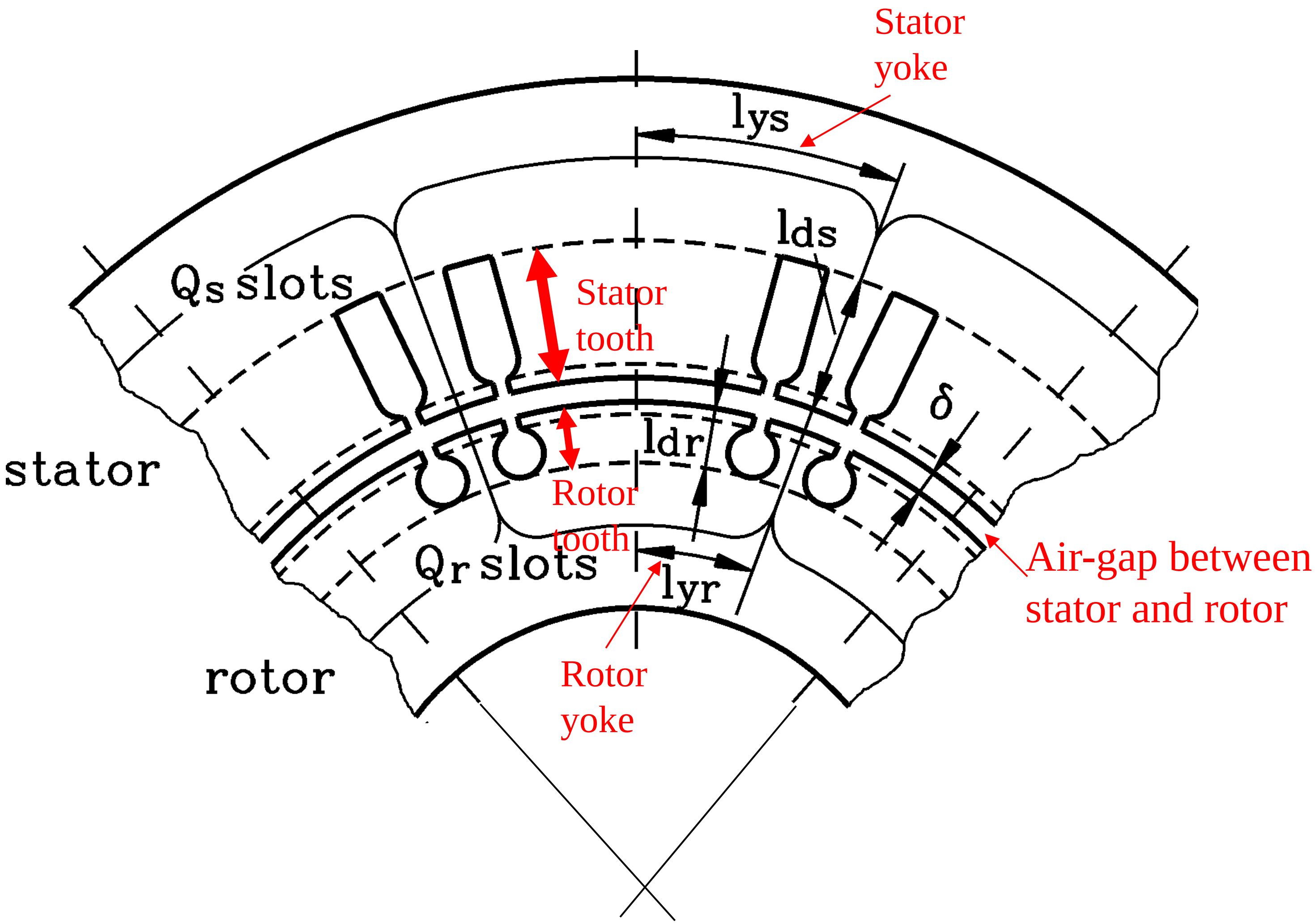
   1.1 Torque generation and internal power

   1.2 Electromagnetic utilization

   1.3 Thermal utilization

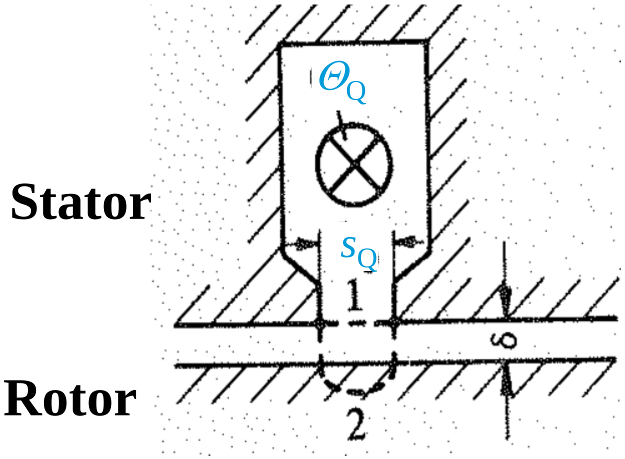
   1.4 Overload capability of AC machines

Cross section of a cage induction machine

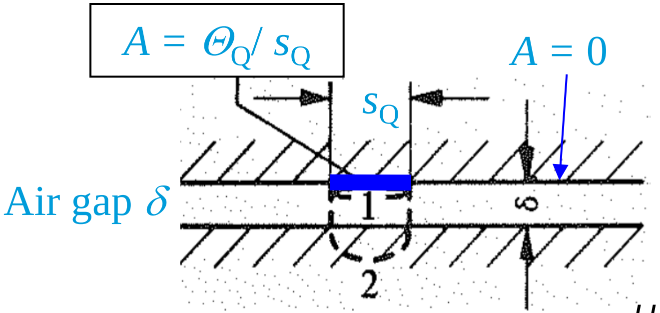


1. Basic design rules for electrical machines

Air gap field, excited by slot conductors



Current loading A introduced to replace the slot geometry



Source:

H. Kleinrath, Studientext 1975, Wiesbaden

Assumption: $\mu_{Fe} \to \infty$: $H_{Fe} = 0$

Ampere´s law: $\Theta_Q = \oint_C \vec{H} \cdot d\vec{s} = \int_{C_{Fe}} \vec{H}_{Fe} \cdot d\vec{s} + \int_{C_\delta} \vec{H}_\delta \cdot d\vec{s} = \int_{C_\delta} \vec{H}_\delta \cdot d\vec{s}$

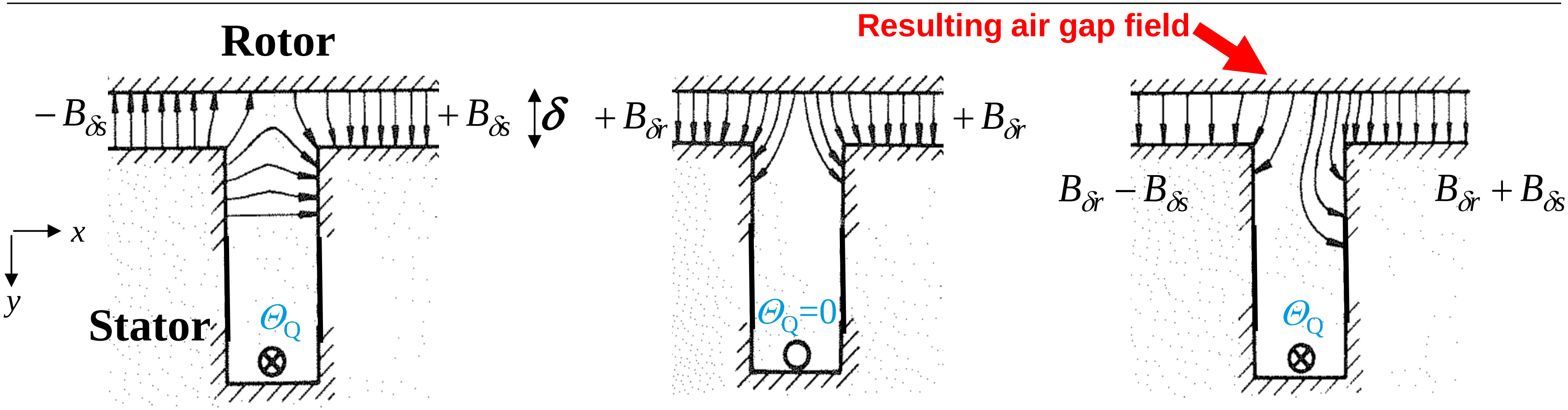
Case 1: Tangential air gap field along path 1: $\Theta_Q = \int_{C_{\delta 1}} \vec{H}_\delta \cdot d\vec{s} = H_{\delta 1} \cdot s_Q \Rightarrow H_{\delta 1} = \Theta_Q / s_Q$

Case 2: Radial air gap field along path 2: $\Theta_Q = \int_{C_{\delta 2}} \vec{H}_\delta \cdot d\vec{s} = H_{\delta 2} \cdot 2\delta \Rightarrow H_{\delta 2} = \Theta_Q / (2\delta)$

The same air gap field is obtained at $\mu_{Fe} \to \infty$
a) for the real slot geometry and
b) for the current loading model.

1. Basic design rules for electrical machines

Resulting air gap field at the slot



- Only stator self field of slot conductor

$$B_{\delta s}=\frac{\mu_0\Theta_Q}{2\delta}$$

- No external rotor field

$$B_{\delta r}=0$$

- No stator self field of slot conductor

$$B_{\delta s}=0$$

- Only external rotor field

$$B_{\delta r}>0$$

Superposition of:

- Stator self field of slot conductor

$$B_{\delta s}=\frac{\mu_0\Theta_Q}{2\delta}$$

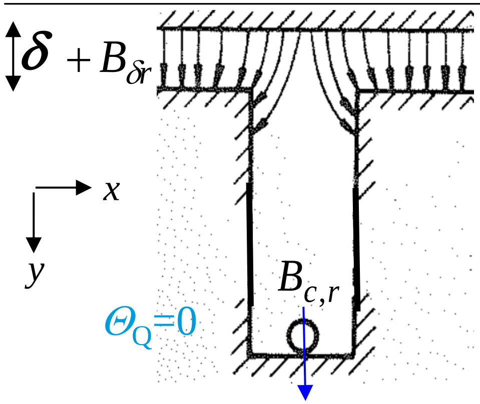
and external rotor field

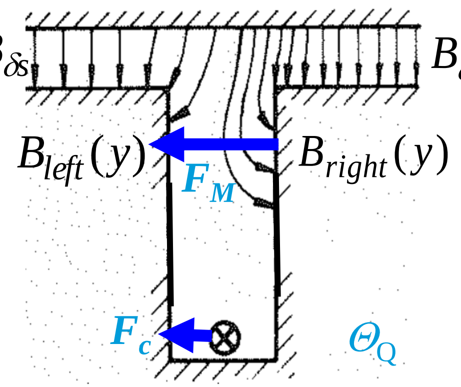
$$B_{\delta r}>0$$

Source: H. Kleinrath, Studientext 1975, Wiesbaden

Note: For simple superposition we have to assume $\mu_{Fe}(x,y)$ = const.

Magnetic force F_e on a slot conductor





Resulting magnetic force F_e:

$$F_e = F_c + F_M$$

ca. 10 % ca. 90 %

$$F_e = l\cdot\Theta_Q\cdot B_{\delta,r}$$

External rotor field is at the slot conductor location very small:

$$B_{c,r} \ll B_{\delta r}$$

LORENTZ-force F_c on the slot conductor is very small:

$$F_c = l\cdot\Theta_Q\cdot B_{c,r}$$

Magnetic pull force F_M due to *MAXWELL* stress f_n on magnetized iron dominates!

Mathematical proof:
Williams-Mamak: IEE Trans. C, 1961, Monograph no. 456U

Source:

H. Kleinrath, Studientext 1975, Wiesbaden

$$F_M = \int_{A_{Fe}} f_n\cdot dA_{Fe} \qquad f_n \approx B_n^2/(2\mu_0)$$

$$F_M = F_{M,right} - F_{M,left}$$

$$F_M = \frac{l}{2\mu_0}\cdot\int_y (B_{right}^2 - B_{left}^2)\cdot dy$$

1. Basic design rules for electrical machines

Simplified model for conductor force F_e

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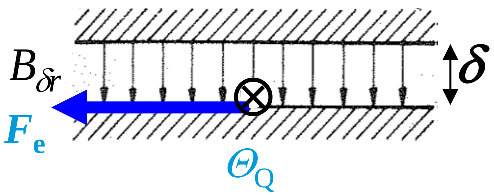
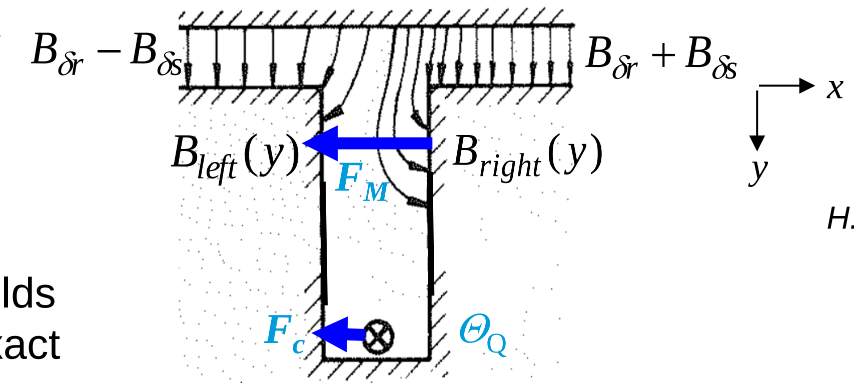




Source:

H. Kleinrath, Studientext 1975, Wiesbaden

LORENTZ-force F_e on a SURFACE conductor yields the same force as the exact model of a slot conductor:

$$F_e = l \cdot \Theta_Q \cdot B_{\delta,r}$$

$$F_e = F_c + F_M$$

$$F_e = l \cdot \Theta_Q \cdot B_{\delta,r}$$

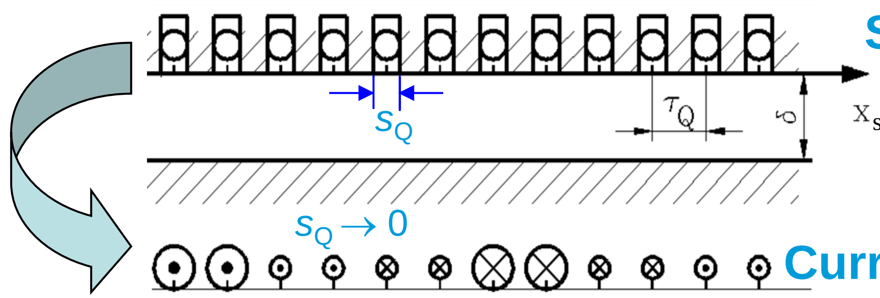
The same force is obtained

a) for the real slot geometry and

b) for the surface conductor model, leading again to a current loading model $A(x_s)$.

Slotting replaced by “discrete” current loading

e.g.: $m = 3$, $q = 2$, $W/\tau_p = 1$



x_s: stator-fixed coordinate

1. Basic design rules for electrical machines

m.m.f. $V(x_s)$ & current loading $A(x_s)$ in AC machines

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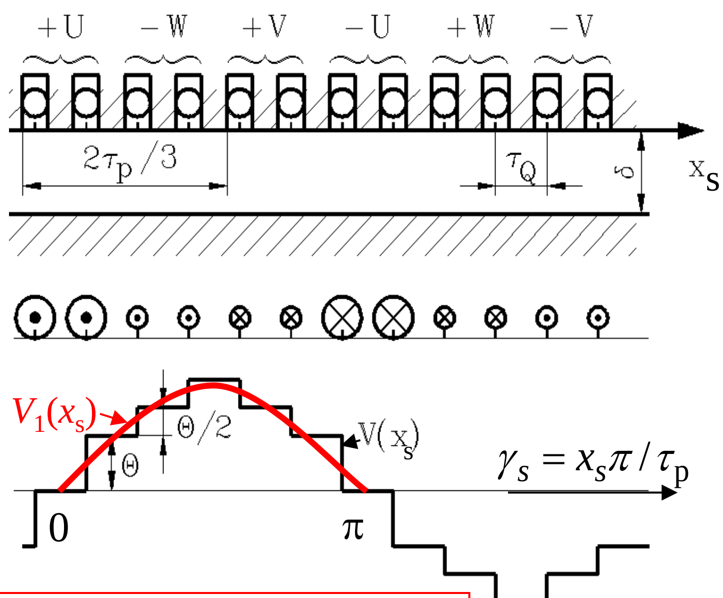
Distributed three-phase winding excites a distributed stator field:



$$B_\delta(\gamma_s) = \mu_0 \cdot V(\gamma_s)/\delta$$

Fundamental of m.m.f. distribution $V(x_s)$ is a sine wave:

$$\hat{V}_1 \cdot \sin(x_s\pi/\tau_p - \omega t) \quad V(x_s) = H_\delta(x_s)\cdot\delta$$

Current loading is derivative of m.m.f.: $V(x_s) = \int A(x_s)\cdot dx_s$

$$A_1(x_s,t) = \frac{d}{dx_s}(\hat{V}_1 \cdot \sin(x_s\pi/\tau_p - \omega t))$$

$$A_1(x_s,t) = \frac{\hat{V}_1\pi}{\tau_p}\cdot\cos(x_s\pi/\tau_p - \omega t)$$

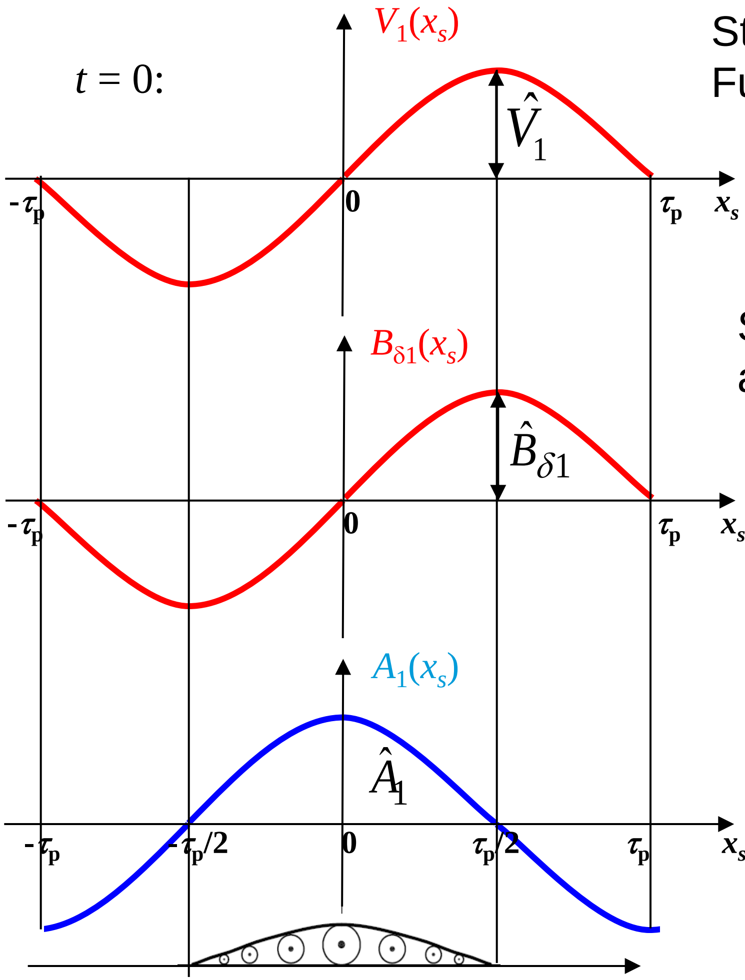
$$\hat{A}_1 = \hat{V}_1\cdot\pi/\tau_p = \left(\frac{\sqrt{2}}{\pi}\cdot\frac{m}{p}\cdot N\cdot k_{w1}\cdot I\right)\cdot\pi/\tau_p$$

Example:
$q = 2$, $m = 3$,
full-pitched coils at time $t = 0$

Fundamental current loading $A_1(x_s)$ is a continuous function, not any longer “discrete” like single conductors!

1. Basic design rules for electrical machines

Fundamental m.m.f. $V_1(x_s)$, air-gap field $B_{\delta 1}(x_s)$ & current loading $A_1(x_s)$



Stator winding:
Fundamental m.m.f. distribution $V(x_s)$ as sine wave:

$$V_1(x_s,t) = \hat{V}_1 \cdot \sin(x_s\pi/\tau_p - \omega t)$$

Stator fundamental air-gap field distribution $B_\delta(x_s)$ as sine wave at $\mu_{Fe} \to \infty$:

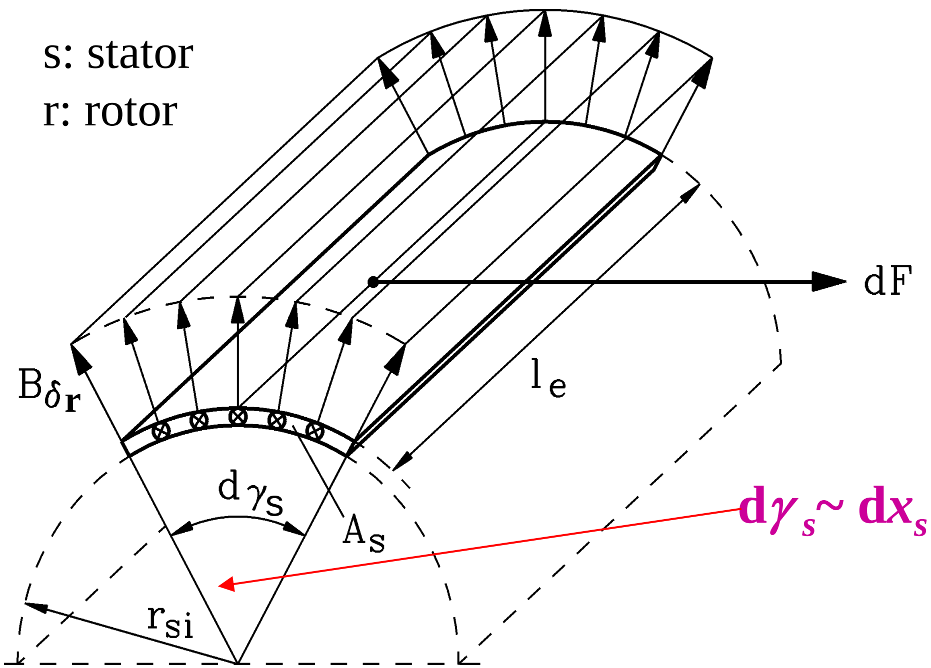
$$B_\delta(x_s,t) = \mu_0 \cdot V(x_s,t)/\delta = \hat{B}_{\delta 1}\sin(x_s\pi/\tau_p - \omega t)$$

Current loading is derivative of m.m.f.:

$$A_1(x_s,t) = dV_1(x_s,t)/dt = \hat{A}_1 \cdot \cos(x_s\pi/\tau_p - \omega t)$$

Fundamental current loading $A_1(x_s)$ is a continuous function,
not any longer “discrete” like single conductors!

1. Basic design rules for electrical machines
LORENTZ force



Stator field and rotor current

OR

rotor field and stator current: e.g.:

$$dF(x_s,t) = dz \cdot i_s(x_s,t) \cdot B_{\delta,r}(x_s,t) \cdot l_e$$

Number of conductors per element dx_s:

$$dz = z \cdot \frac{dx_s}{2p\tau_p}$$

With current loading A_s:

$$A_s(x_s,t) = \frac{z_s \cdot i_s(x_s,t)}{2p\tau_p}$$

***z*: total number of conductors**

Total tangential force:
(acting on stator)

$$F(t) = l_e \cdot \int_0^{2p\tau_p} A_s(x_s,t) \cdot B_{\delta,r}(x_s,t) \cdot dx_s$$

1. Basic design rules for electrical machines

Self-field does <u>not</u> produce resulting tangential force

$$F(t) = l_e \cdot \int_0^{2p\tau_p} A_s(x_s,t) \cdot B_{\delta,s}(x_s,t) \cdot dx_s = 0$$

$$F(t) = l_e \cdot \int_0^{2p\tau_p} A_s(x,t) \cdot B_{\delta,r}(x,t) \cdot dx = l_e \cdot \int_0^{2p\tau_p} A_s(x_s,t) \cdot \left(B_{\delta,s}(x_s,t) + B_{\delta,r}(x_s,t)\right) \cdot dx_s$$

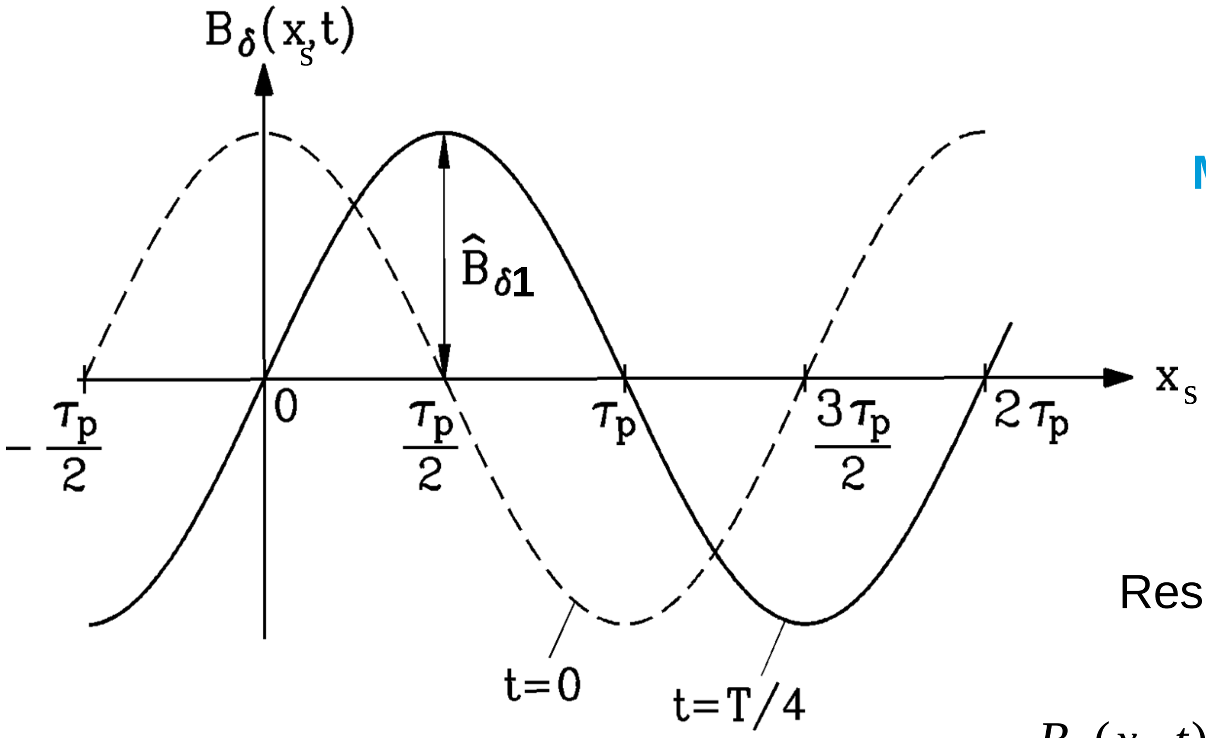
Tangential force on stator: $$F(t) = l_e \cdot \int_0^{2p\tau_p} A_s(x_s,t) \cdot B_\delta(x_s,t) \cdot dx_s$$

Tangential force on rotor: "Actio est reactio" (*Newton´s* 3rd law):

$$F(t) = -l_e \cdot \int_0^{2p\tau_p} A_s(x_s,t) \cdot B_\delta(x_s,t) \cdot dx_s$$

1. Basic design rules for electrical machines

Fundamental sine wave magnetic air gap travelling field in AC machines



Magnetic flux per pole:

$$\Phi_h = \frac{2}{\pi} \cdot \tau_p l_e \cdot \hat{B}_{\delta 1}$$

Resulting air gap field fundamental:

$$B_\delta(x_s,t) = \hat{B}_{\delta,1} \cdot \cos(x_s \pi / \tau_p - \omega t)$$

1. Basic design rules for electrical machines

Torque generation in AC machines by fundamental fields (1)

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$$F(t) = l_e \cdot \int_0^{2p\tau_p} A_s(x_s,t) \cdot B_\delta(x_s,t) \cdot dx_s \qquad \boxed{M_e(t) = F(t) \cdot d_{si}/2} \qquad r_{si} = \frac{d_{si}}{2}$$

Generally the tangential force depends on time due to slotting and phase bands +U, -W, +V, -U, +W, -V. Therefore we have a torque ripple $\Delta M_e(t)$:

$M_e(t) = M_{e,av} + \Delta M_e(t)$

Considering only fundamental fields we get a CONSTANT torque:

$M_e(t) = M_{e,av} \quad \Delta M_e(t) = 0$

$$\Rightarrow \quad F_1 = l_e \cdot \int_0^{2p\tau_p} A_{s,1}(x_s,t) \cdot B_{\delta,1}(x_s,t) \cdot dx_s \qquad \boxed{M_{e,AC} = F_1 \cdot d_{si}/2}$$

1. Basic design rules for electrical machines

Torque generation in AC machines by fundamental fields (2)

- Torque on stator: $M_{e,AC} = \frac{d_{si}}{2} \cdot l_e \cdot \int_0^{2p\tau_p} A_{s,1}(x_s,t) \cdot B_{\delta,1}(x_s,t) \cdot dx_s$ $\quad \gamma_s = x_s \cdot \pi / \tau_p$, $\quad d_{si}\pi = 2p\tau_p$

$$M_{\mathrm{e,AC}} = \int_0^{2\pi p} \hat{A}_{s1} \cos(\gamma_s - \omega t - \varphi_\delta) \cdot \hat{B}_{\delta,1} \cos(\gamma_s - \omega t) \cdot l_e \cdot p \cdot \left(\frac{\tau_p}{\pi}\right)^2 \cdot d\gamma_s$$

$$\gamma_s - \omega t = \xi$$

$$M_{\mathrm{e,AC}} = \int_{-\omega t}^{-\omega t + 2\pi p} \hat{A}_{s1} \cos(\xi - \varphi_\delta) \cdot \hat{B}_{\delta,1} \cos\xi \cdot l_e \cdot p \cdot \left(\frac{\tau_p}{\pi}\right)^2 \cdot d\xi$$

$$M_{\mathrm{e,AC}} = \hat{A}_{s1}\hat{B}_{\delta,1} \cdot l_e \cdot p \cdot \left(\frac{\tau_p}{\pi}\right)^2 \cdot \int_{-\omega t}^{-\omega t + 2\pi p} [\cos\xi \cdot \cos\varphi_\delta + \sin\xi \cdot \sin\varphi_\delta] \cdot \cos\xi \cdot d\xi$$

$$\cos^2\xi = (1 + \cos(2\xi))/2$$
$$\cos\xi \sin\xi = \sin(2\xi)/2$$

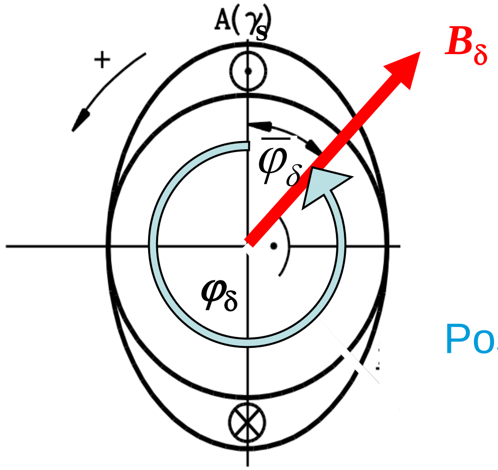
$$\boxed{M_{e,AC} = l_e \cdot (p\tau_p)^2 \cdot \hat{A}_{s1} \cdot \hat{B}_{\delta,1} \cdot \cos\varphi_\delta / \pi}$$

- Torque on rotor: "Actio est reactio" (*Newton*´s 3rd law):

$$M_{e,AC} = -l_e \cdot (p\tau_p)^2 \cdot \hat{A}_{s1} \cdot \hat{B}_{\delta,1} \cdot \cos\varphi_\delta / \pi$$

Example:

Phase shift between stator current loading and air-gap field: φ_δ = 315°



$$M_{e,AC} = l_e \cdot (p\tau_p)^2 \cdot \hat{A}_{s1} \cdot \hat{B}_{\delta,1} \cdot \cos 315° / \pi$$

$$M_{e,AC} = l_e \cdot (p\tau_p)^2 \cdot \hat{A}_{s1} \cdot \hat{B}_{\delta,1} \cdot \frac{1}{\sqrt{2}} \cdot \frac{1}{\pi}$$

Positive torque on the stator = in counter-clockwise direction

$$\overline{\varphi}_\delta = \varphi_\delta - 2\pi$$

Note: Due to $\hat{A}_{s1}\cos(\gamma - \omega t - \varphi_\delta) = \hat{A}_{s1}\cos(\gamma_s - \omega t - (\overbrace{\varphi_\delta - 2\pi})) = \hat{A}_{s1}\cos(\gamma_s - \omega t - \overline{\varphi}_\delta)$

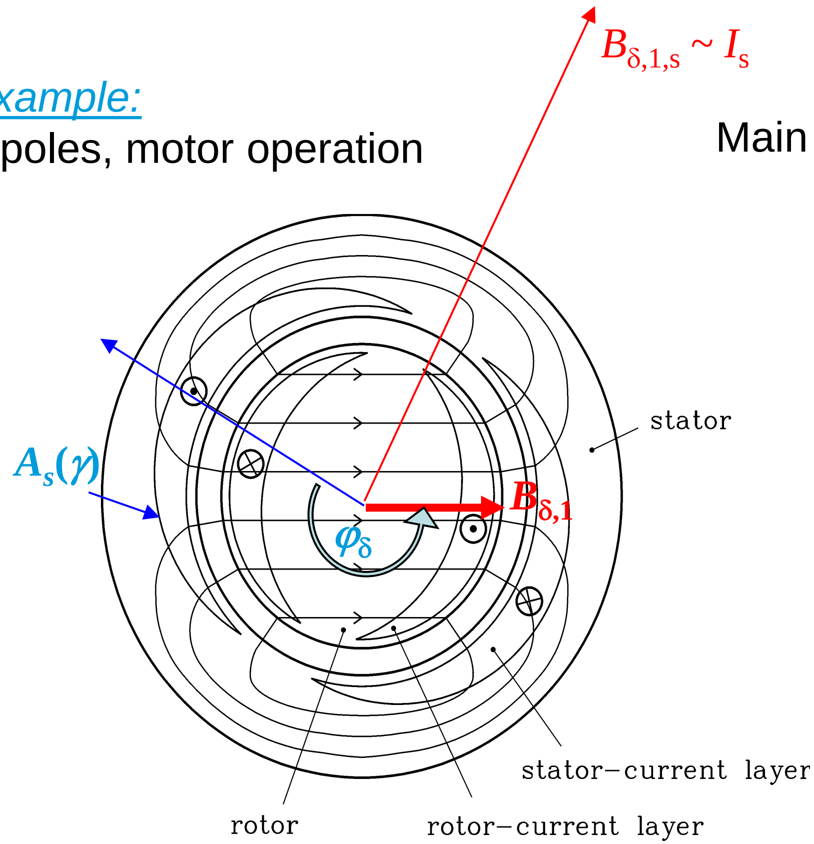
we also get: $\overline{\varphi}_\delta = -45°$ $\quad M_{e,AC} = l_e \cdot (p\tau_p)^2 \cdot \hat{A}_{s1} \cdot \hat{B}_{\delta,1} \cdot \cos(-45°) / \pi$

$$M_{e,AC} = l_e \cdot (p\tau_p)^2 \cdot \hat{A}_{s1} \cdot \hat{B}_{\delta,1} \cdot \frac{1}{\sqrt{2}} \cdot \frac{1}{\pi}$$

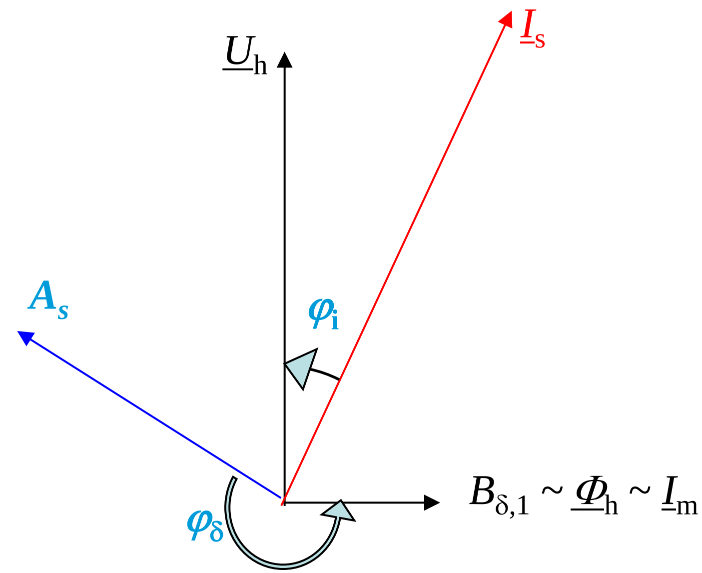
1. Basic design rules for electrical machines

Example: Torque generation in an induction machine

Example:
2 poles, motor operation



Main flux $\Phi_h \sim B_{\delta,1} \sim I_m$ and internal voltage $\underline{U}_h \sim j \cdot \underline{\Phi}_h$:



Internal phase angle between internal voltage and stator current: $\varphi_i = \varphi_\delta - \pi$

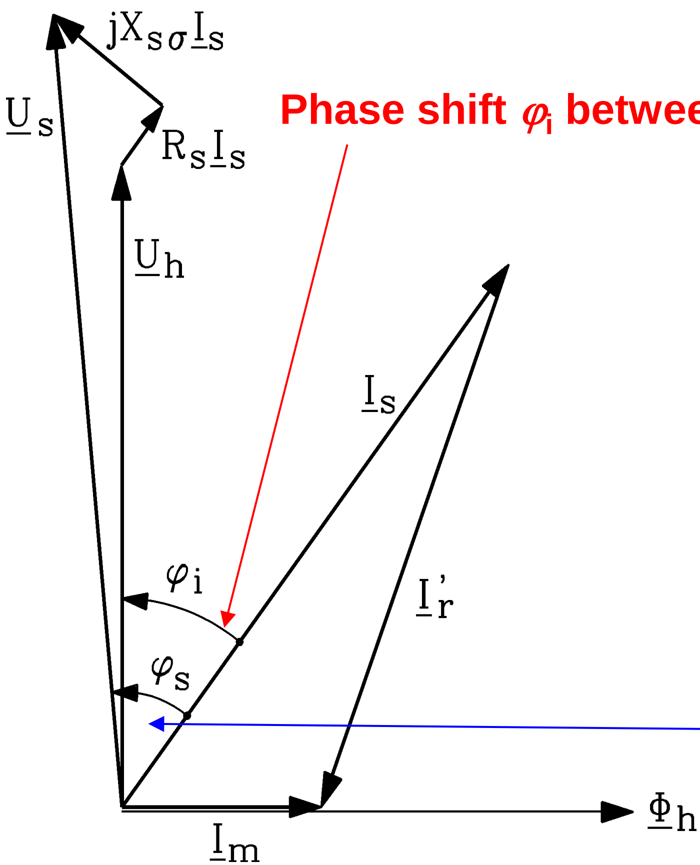
Torque on rotor: $M_{e,AC} = -l_e \cdot (p\tau_p)^2 \cdot \hat{A}_{s1} \cdot \hat{B}_{\delta,1} \cdot \cos\varphi_\delta / \pi = l_e \cdot (p\tau_p)^2 \cdot \hat{A}_{s1} \cdot \hat{B}_{\delta,1} \cdot \cos\varphi_i / \pi$

$$(-\cos\varphi_\delta = \cos\varphi_i)$$

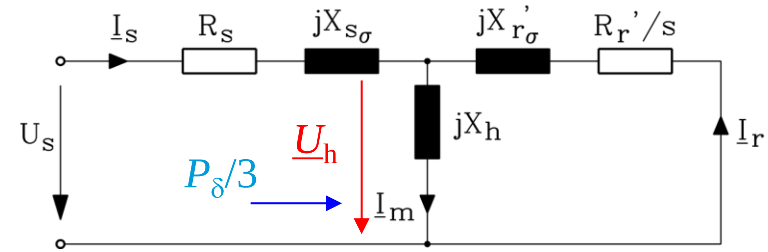
1. Basic design rules for electrical machines

Internal phase shift φ_i in an induction machine

Example: $0 < \varphi_i < \pi/2$: $M_{e,AC} > 0$ on rotor = motor operation



Phase shift φ_i between I_s and U_h!



Internal power:

$$P_\delta = 2\pi n_{syn} M_{e,AC} = 2\pi \frac{f_s}{p} \cdot l_e \cdot (p\tau_p)^2 \cdot \hat{A}_{s1} \cdot \hat{B}_{\delta 1} \cdot \cos\varphi_i / \pi$$

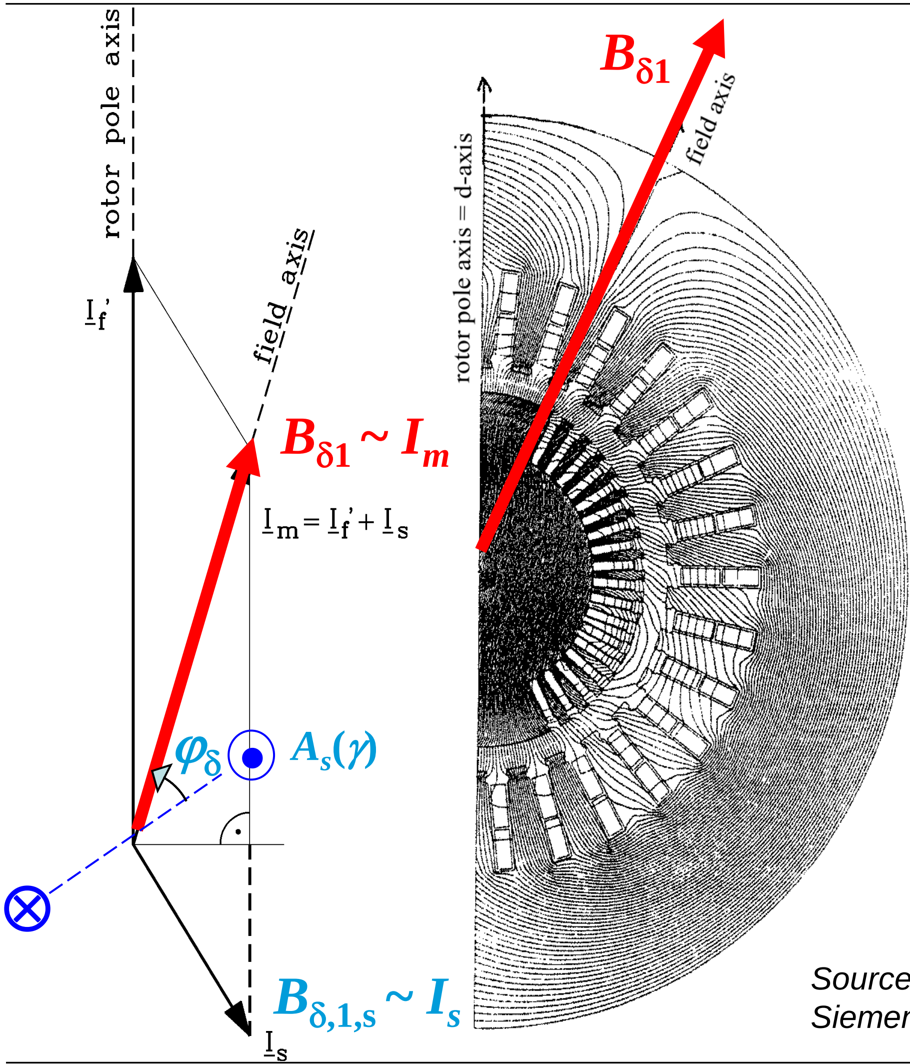
$$U_h = 2\pi f_s \cdot k_{ws1} N_s \Phi_h / \sqrt{2} \qquad \Phi_h = \frac{2}{\pi} \cdot \tau_p l_e \cdot \hat{B}_{\delta 1}$$

$$\hat{A}_{s1} = \frac{\sqrt{2}}{\tau_p} \cdot \frac{m_s}{p} \cdot N_s \cdot k_{ws1} \cdot I_s \qquad \boxed{P_\delta = m_s U_h I_s \cdot \cos\varphi_i}$$

Phase shift φ_s between I_s and U_s!

1. Basic design rules for electrical machines

Synchronous machine: Internal phase shift φ_i



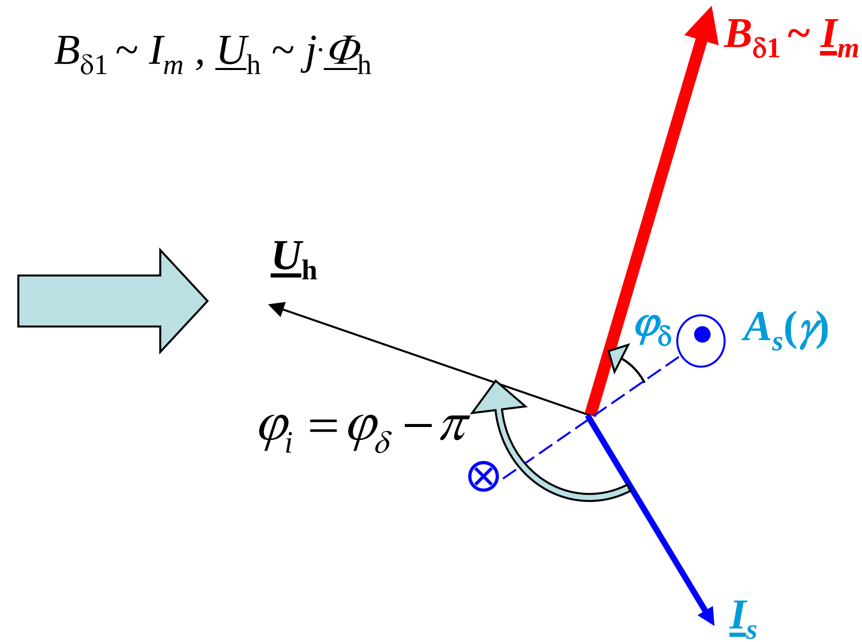
Source: Siemens AG

Example:

Cylindrical rotor, $2p = 2$, generator operation, over-excited, $m_s = 3$, $q_s = 5$

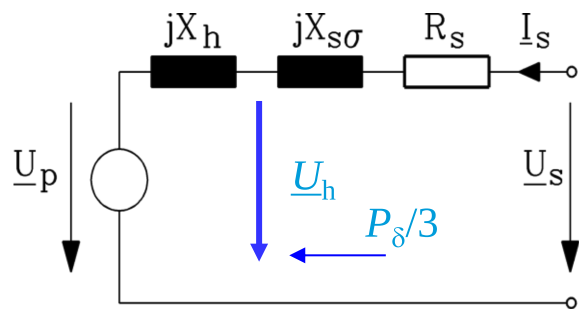
Main flux $\Phi_h \sim B_{\delta 1}$

$B_{\delta 1} \sim I_m$, $\underline{U}_h \sim j \cdot \underline{\Phi}_h$



1. Basic design rules for electrical machines
Phasor diagram of a synchronous machine

Example: $-\pi/2 > \varphi_i > -\pi$: $M_{e,AC} < 0$ on rotor = generator operation

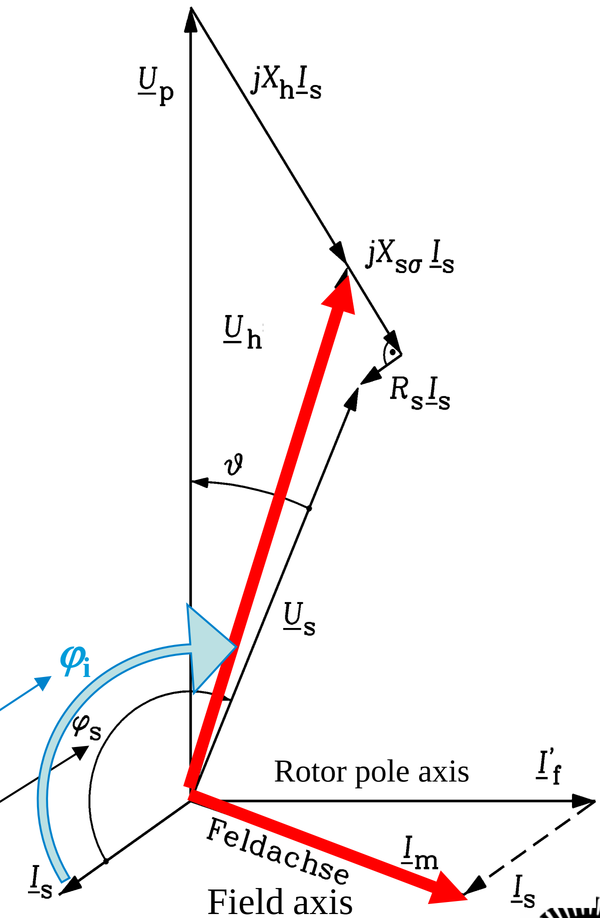


Internal power:

$$P_\delta = 2\pi n_{syn} M_{e,AC} = 2\pi \frac{f_s}{p} \cdot l_e \cdot (p\tau_p)^2 \cdot \hat{A}_{s1} \cdot \hat{B}_{\delta 1} \cdot \cos\varphi_i / \pi$$

$$U_h = 2\pi f_s \cdot k_{ws1} N_s \Phi_h / \sqrt{2} \qquad \Phi_h = \frac{2}{\pi} \cdot \tau_p l_e \cdot \hat{B}_{\delta 1}$$

$$\boxed{P_\delta = m_s U_h I_s \cdot \cos\varphi_i}$$

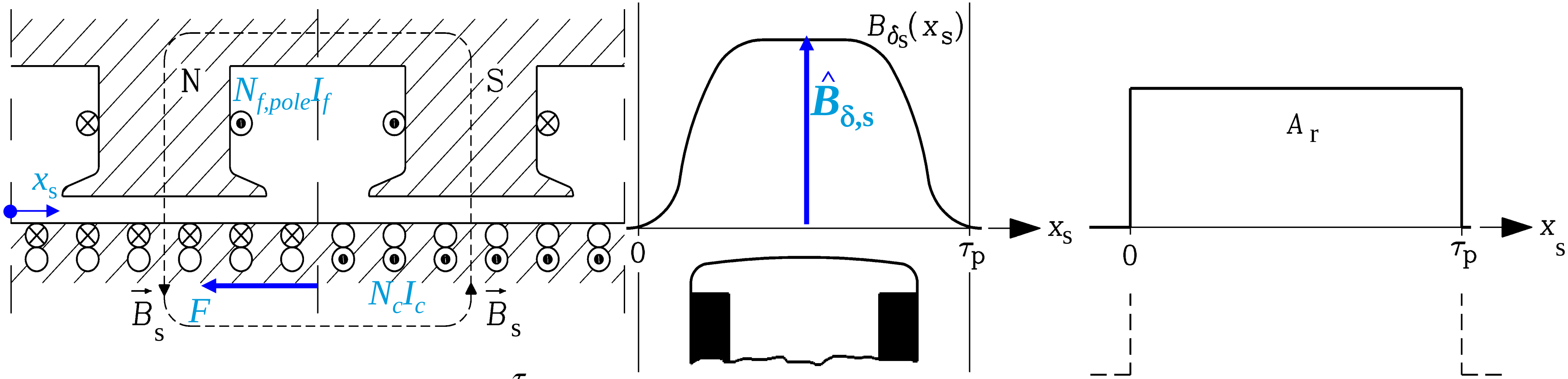


Phase shift φ_i between I_s and U_h

Phase shift φ_s between I_s and U_s

1. Basic design rules for electrical machines

Torque generation in DC machines



$$\Phi = l_e \int_0^{\tau_p} B_{\delta,s}(x_s) \cdot dx_s = \alpha_e \cdot l_e \cdot \tau_p \cdot \hat{B}_{\delta,s} \qquad A_r(x_s) = A_r = \frac{z \cdot I_c}{d_{si} \cdot \pi} = const.$$

Tangential force on rotor:

$$F = l_e \cdot \int_0^{2p\tau_p} A_r(x_s,t) \cdot B_{\delta,s}(x_s,t) \cdot dx_s \qquad M_e = F \cdot d_{si}/2$$

$$M_e = 2p \cdot \frac{d_{si}}{2} \cdot l_e \cdot A_r \cdot \int_0^{\tau_p} B_{\delta,s}(x_s) \cdot dx_s = p \cdot d_{si} \cdot A_r \cdot \Phi$$

$$\boxed{M_{e,DC} = l_e \cdot 2(p\tau_p)^2 \cdot A_r \cdot \alpha_e \hat{B}_{\delta,s}/\pi}$$

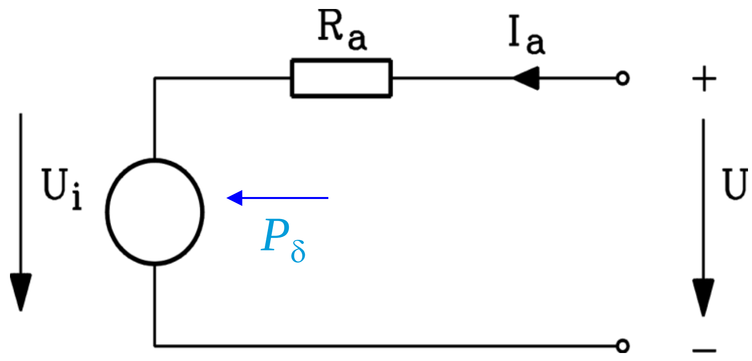
1. Basic design rules for electrical machines

Internal power P_δ in DC machines

$$P_\delta = 2\pi n \cdot M_{e,DC} = 2\pi n \cdot l_e \cdot 2(p\tau_p)^2 \cdot A_r \cdot \alpha_e \hat{B}_{\delta,s} / \pi \qquad d_{si} \cdot \pi = 2p \cdot \tau_p$$

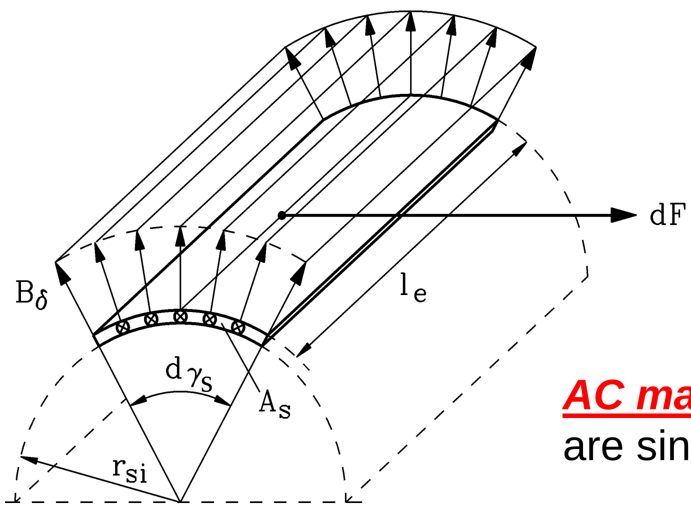
$$U_i = \frac{z \cdot p}{a} \cdot n \cdot \Phi \qquad \Phi = \alpha_e \cdot l_e \cdot \tau_p \cdot \hat{B}_{\delta,s} \qquad A_r = \frac{z \cdot I_c}{d_{si} \cdot \pi} \qquad I_a = 2a \cdot I_c$$

$$\boxed{P_\delta = U_i \cdot I_a}$$



1. Basic design rules for electrical machines

Summary: Torque generation



$$F = l_e \cdot \int_0^{2p\tau_p} A(x_s,t) \cdot B_\delta(x_s,t) \cdot dx_s$$

$$\boxed{M_e = F \cdot d_{si}/2}$$

AC machines: Current loading A_s and air gap flux density B_δ are sinusoidal distributed, phase shift φ_i between I_s and U_h:

Rotor torque: $M_{e,AC} = l_e \cdot (p\tau_p)^2 \cdot \hat{A}_{s1} \cdot \hat{B}_{\delta 1} \cdot \cos\varphi_i / \pi$

DC machines:

Current loading A_r and air gap flux density B_δ constant along $\alpha_e \sim 0.7$ of pole pitch τ_p:

Rotor torque: $M_{e,DC} = l_e \cdot 2(p\tau_p)^2 \cdot A_r \cdot \alpha_e \hat{B}_{\delta,s} / \pi$

1. Basic design rules for electrical machines

Specific air gap thrust τ

Specific air gap thrust: **Force per surface:** $\tau = F/(d_{si}\cdot\pi\cdot l_e)$

Surface: $Area = d_{si}\cdot\pi\cdot l_e = 2p\cdot\tau_p\cdot l_e$

$$F = M_e/(d_{si}/2) = M_e\cdot\pi/(p\tau_p) \qquad \tau = M_e\pi/(2p^2\tau_p^2 l_e)$$

AC machines:

$$\tau_{AC} = \frac{l_e\cdot(p\tau_p)^2}{\pi}\cdot\hat{A}_{s1}\cdot\hat{B}_{\delta 1}\cdot\cos\varphi_i\cdot\pi/(2p^2\tau_p^2 l_e) \qquad \tau_{AC} = \hat{A}_{s1}\cdot\hat{B}_{\delta 1}\cdot\cos\varphi_i/2$$

DC machines:

$$\tau_{DC} = \frac{l_e\cdot 2(p\tau_p)^2}{\pi}\cdot A_r\cdot\alpha_e\hat{B}_{\delta,s}\cdot\pi/(2p^2\tau_p^2 l_e) \qquad \tau_{DC} = A_r\cdot\alpha_e\hat{B}_{\delta,s}$$

1. Basic design rules for electrical machines

Effective current loading

AC machines:

$$\tau_{AC} = \hat{A}_{s1} \cdot \hat{B}_{\delta 1} \cdot \cos\varphi_i / 2$$

$$\tau_{AC} = A_s \cdot k_{w1} \cdot \hat{B}_{\delta 1} \cdot \cos\varphi_i / \sqrt{2}$$

DC machines:

$$\tau_{DC} = A_r \cdot \alpha_e \hat{B}_{\delta,s}$$

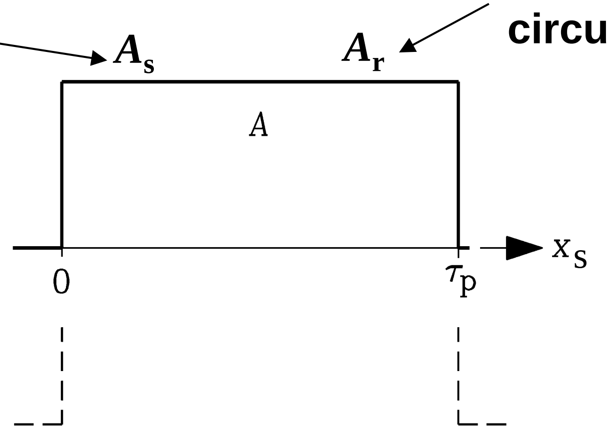
$$A_{s1}(x,t) = \frac{\hat{V}_{s1}\pi}{\tau_p} \cdot \cos(x\pi/\tau_p - \omega t - \varphi_i - \pi)$$

$$A_r(x_s) = A_r = \frac{z \cdot I_c}{d_{si} \cdot \pi} = \frac{z \cdot I_a/(2a)}{2p \cdot \tau_p} = const.$$

$$\hat{A}_{s1} = \hat{V}_{s1} \cdot \pi / \tau_p = \frac{\sqrt{2}}{\pi} \cdot \frac{m_s}{p} \cdot N_s \cdot k_{ws1} \cdot I_s \cdot \pi / \tau_p = \sqrt{2} \cdot k_{w1} \cdot A_s$$

$$A_s = \frac{2 \cdot m_s \cdot N_s \cdot I_s}{2p \cdot \tau_p}$$

DC: Current loading is constant along the air gap circumference!



I_a: Armature current

$I_c = I_a/(2a)$: Coil current

AC: „Fictive" effective current loading is constant along the air gap circumference!

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Electromagnetic torque M_e is determined by air gap flux density B_δ and current loading A (= ampere-turns per unit length) and corresponds with internal power P_δ (air gap power).

- Air gap flux density peak value AC: $\hat{B}_{\delta,1} = 1.0\text{T}$, DC: $\hat{B}_{\delta,s} = 1.0\text{T}$,
- Typical maximum current loading for air cooling with open ventilation: A = 700 A/cm (DC-machines), $\hat{A}_{s1}$ corresponding amplitude for AC-machines

a) DC-machines: $\alpha_e = 0.7$: $\tau_{DC} = A_r \cdot \alpha_e \hat{B}_{\delta,s} = 70000 \cdot 0.7 \cdot 1.0 = 49000\ \text{N/m}^2$ $\cong$ 0.5 bar

b) AC-machines: $k_{ws1} \approx 0.95$, $\hat{A}_{s1} = \sqrt{2} \cdot k_{w1} \cdot A_s = 940\ \text{A/cm}$,

maximum thrust at **$\cos\varphi_i = 1$:** $\tau_{AC} = \hat{A}_{s1} \cdot \hat{B}_{\delta,1} \cdot \cos\varphi_i / 2 = 94000 \cdot 1 \cdot 1/2 = 47000 \cong$ 0.5 bar

In reality: $\cos\varphi_i$ ~ 0.9, so thrust for AC lower than for DC machines.

Summary:
Torque generation and internal power

- Radial and tangential magnetic forces on magnetized iron and on conductors
- Without any rotor eccentricity the sum of radial forces on stator and rotor sum up to zero
- Tangential forces lead to torque
- Equivalent current loading represents slot conductor arrangement
- Fundamental waves for torque in AC machines
- Internal phase angle φ_i between resulting field wave (~ U_h) and current loading (~ I_s)
- DC machines have bigger torque M_e at the same peak current loading A and field B_δ
- Specific air gap thrust τ as tangent force per area in the range of 0.5 … 1 bar

1. **Basic design rules for rotating machines**

   1.1 Torque generation and internal power

   1.2 Electromagnetic utilization

   1.3 Thermal utilization

   1.4 Overload capability of AC machines

1. Basic design rules for electrical machines

Air gap torque M_e, air gap power P_δ, internal apparent power S_δ

Air gap torque: M_e

Air gap power (internal power):

AC machines: $P_\delta = M_e \cdot 2\pi n_{syn} = M_e \cdot 2\pi \cdot (f_s / p)$

DC machines: $P_\delta = M_e \cdot 2\pi \cdot n$

n: Rotor speed !

Internal apparent power:

AC machines: $S_\delta = 3 \cdot U_h \cdot I_s$

DC machines: $P_\delta = U_i \cdot I_a$

DC: $S_\delta = P_\delta$

U_i (= U_h), I_s: Internal stator phase voltage & stator phase current (r.m.s. values!)

U_i, I_a: Induced armature voltage & armature current

1. Basic design rules for electrical machines

Internal apparent power S_δ

Internal apparent power S_δ : Induced voltage x current: $S_\delta = m_s \cdot U_h \cdot I_s$

AC machines: Induced phase voltage r.m.s: $U_h = \sqrt{2}\pi \cdot f_s \cdot N_s \cdot k_{ws1} \cdot \Phi_h$

With $\Phi_h = \frac{2}{\pi} \cdot \tau_p l \cdot \hat{B}_{\delta 1}$, r.m.s. current loading $A_s = \frac{2 \cdot m_s \cdot N_s \cdot I_s}{2 p \tau_p}$ and $d_{si} = 2 p \tau_p / \pi$

we get:

$$S_\delta = m_s U_h I_s = m_s \sqrt{2}\pi \cdot f_s \cdot N_s \cdot k_{ws1} \cdot \Phi_h \cdot I_s = d_{si}^2 l_e \cdot \frac{f_s}{p} \cdot \frac{\pi^2}{\sqrt{2}} k_{ws1} \hat{B}_{\delta 1} A_s$$

DC machines: $S_\delta = P_\delta = U_i \cdot I_a$

Induced armature voltage: $U_i = \frac{z \cdot p}{a} \cdot n \cdot \Phi_h$

With $\Phi_h = \alpha_e \cdot \tau_p l_e \cdot \hat{B}_{\delta,s}$ and current loading $A_r = \frac{z \cdot I_a / (2a)}{2 p \tau_p}$

we get:

$$S_\delta = P_\delta = d_{si}^2 \cdot l_e \cdot n \cdot \pi^2 \cdot \alpha_e \cdot A_r \cdot \hat{B}_{\delta,s}$$

1. Basic design rules for electrical machines

Electromagnetic utilization C as S_δ per volume & speed

Rotor volume: $V_r \approx d_{si}^2 \cdot l_{Fe} \cdot \pi / 4 \approx d_{si}^2 \cdot l_e$

Electromagnetic utilization (*Esson´s* number) *C*:

AC machines:

$$C_{AC} = \frac{S_\delta}{d_{si}^2 \cdot l_e \cdot n_{syn}} = \frac{\pi^2}{\sqrt{2}} \cdot k_{ws1} \cdot A_s \cdot \hat{B}_{\delta 1}$$

$$C_{AC} = \pi^2 \cdot \tau_{AC}$$ **at** $\cos\varphi_i = 1$

DC machines:

$$C_{DC} = \frac{P_\delta}{d_{si}^2 \cdot l_e \cdot n} = \pi^2 \cdot \alpha_e \cdot A_r \cdot \hat{B}_{\delta,s}$$

$$C_{DC} = \pi^2 \cdot \tau_{DC}$$

1. Basic design rules for electrical machines
Electromagnetic utilization (*Esson´s* number) *C*

The internal apparent power S_δ per stator bore volume $d_{si}^2(\pi/4)\cdot l$ (usually neglecting $\pi/4$) and per speed n_{syn} is called
electromagnetic utilization or *Esson´s* number *C* („figure of merit").
It increases with current loading *A* and air gap flux density B_δ.

AC:

$$C_{AC} = \frac{S_\delta}{d_{si}^2 \cdot l_e \cdot n_{syn}} = \frac{\pi^2}{\sqrt{2}} \cdot k_{ws1} \cdot A_s \cdot \hat{B}_{\delta 1}$$

$$A_s = \frac{2 \cdot m_s \cdot N_s \cdot I_s}{2p\tau_p} \quad \text{(r.m.s. current loading)}$$

DC:

$$C_{DC} = \frac{P_\delta}{d_{si}^2 \cdot l_e \cdot n} = \pi^2 \cdot \alpha_e \cdot A_r \cdot \hat{B}_{\delta,s}$$

$$A_r = \frac{z \cdot I_a/(2a)}{2p\tau_p}$$

$$M_N \sim S_\delta / n = C \cdot d_{si}^2 \cdot l_e \sim L^3$$

Machine volume (roughly): $V \approx L^3$

Torque determines size of machine, NOT power !

1. Basic design rules for electrical machines

Mounting of air-air heat exchanger on slip ring induction wind generator

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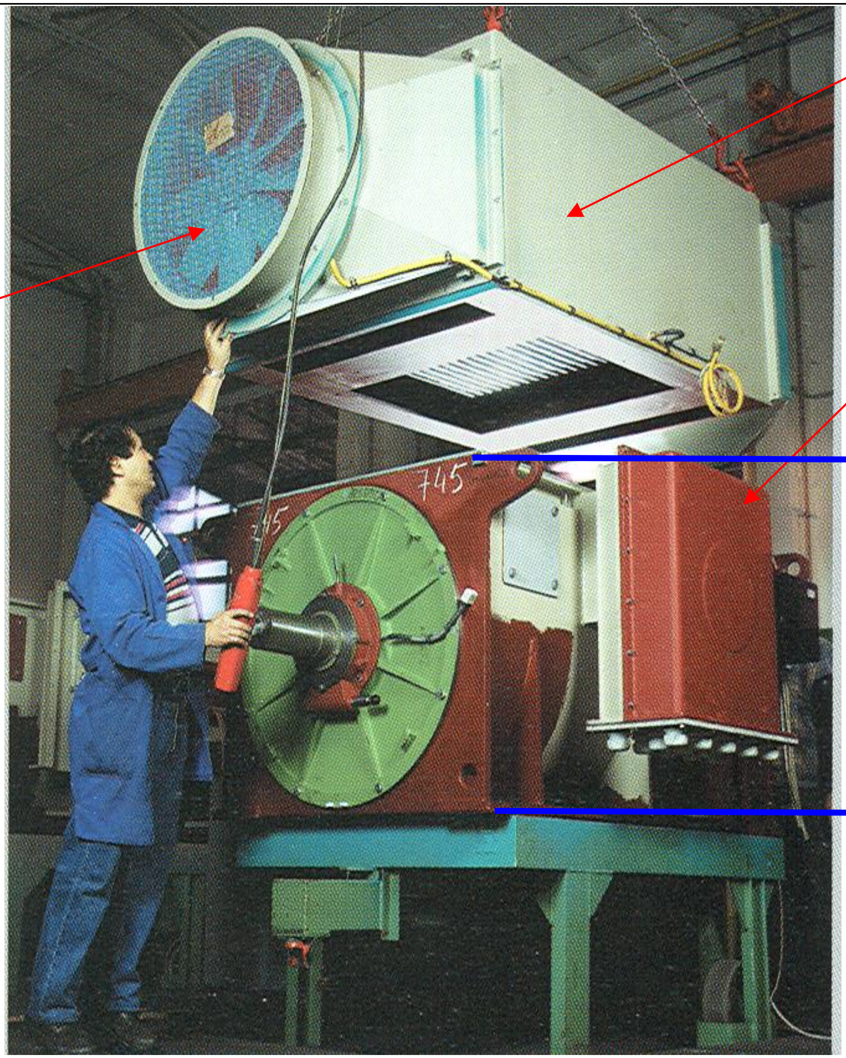
Doubly-fed four-pole induction wind generator

P_N = 1500 kW
n_N = 1800/min

Axial air inlet fan

- Internal “open” ventilation
- Closed air-circuit to reduce air acoustic noise



Air-air heat exchanger

Generator terminal box

L

Source: Winergy Germany

1. Basic design rules for electrical machines

Scaling of power and figure of merit

Example:

AC induction machines: 1500/min
four poles, winding temperature rise 80 K,
air-cooled, open ventilated, for larger rated power

$$C_{AC} = \frac{\pi^2}{\sqrt{2}} \cdot k_{ws1} \cdot A_s \cdot \hat{B}_{\delta 1}$$

rated apparent power	S_N	kVA	100	1000	10000
current loading	A_s	A/cm	300	550	1000
air gap flux density	$\hat{B}_{\delta,1}$	T	1.0	1.05	1.1
Esson´s number	C	kVA·min/m³	3.3	6.4	12.2

Result:

At given rotational speed: Bigger power S_N = bigger torque M_N = bigger machine size L = bigger rotor diameter = higher rotor surface speed = better air cooling!

With a better cooling higher current loadings A are possible,
so electromagnetic utilization C increases with rated power S_N.

Example: AC induction machine:

Four poles, winding temperature rise 80 K, air-cooled, open ventilated

rated apparent power	S_N	10 000 kVA
current loading	A_s	1000 A/cm
air gap flux density	$\hat{B}_{\delta,1}$	1.1 T
Esson´s number	C	12.2 kVA·min/m³

$$C = \frac{\pi^2}{\sqrt{2}} \cdot k_{ws1} \cdot A_s \cdot \hat{B}_{\delta 1} \cong \frac{\pi^2}{\sqrt{2}} \cdot 0.95 \cdot 10^5 \cdot 1.1 \approx 732000 \ \mathrm{VAs/m^3} = 12.2 \ \mathrm{kVA \cdot min/m^3}$$

$$C = 732000 \ \mathrm{VAs/m^3} = 732000 \ \mathrm{N/m^2}$$

$$C = 732000 \ \mathrm{N/m^2} = \pi^2 \cdot \tau_{AC}$$

$$\tau_{AC} = C/\pi^2 = 74176 \ \mathrm{N/m^2} = 0.74 \ \mathrm{bar} \quad \text{at } \cos\varphi_i = 1$$

$$\text{At } \cos\varphi_i = 0.9: \quad \tau_{AC} = 0.74 \cdot 0.9 = 0.67 \ \mathrm{bar}$$

Summary:
Electromagnetic utilization

- Internal apparent power = apparent air gap power S_δ
- Apparent air gap power per rotor volume and speed = electromagnetic utilization
- ESSON´s utilization C = „figure of merit“
- Utilization $C \approx$ 10 x Specific air gap thrust τ
- Utilization C increases with current loading A and air-gap flux density B_δ
- Better cooling = higher current loading A = higher utilization C resp. τ

1. Basic design rules for rotating machines

   1.1 Torque generation and internal power

   1.2 Electromagnetic utilization

   1.3 Thermal utilization

   1.4 Overload capability of AC machines

1. Basic design rules for electrical machines

Armature copper losses P_{Cu}

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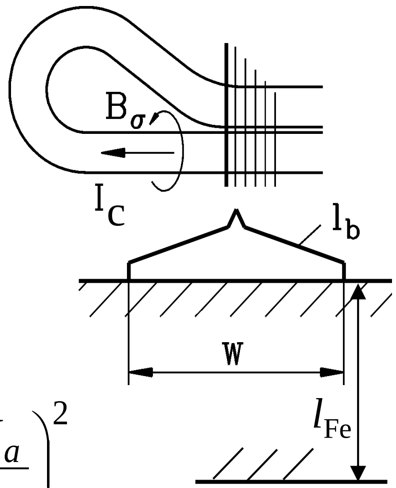
- **AC machines:** m_s phase winding: $P_{Cu} = m_s \cdot R_s I_s^2 = m_s \cdot \frac{N_s \cdot 2(l_{Fe} + l_b)}{\kappa \cdot a_a \cdot A_c} \cdot I_s^2$

$$P_{Cu} = \frac{(l_{Fe} + l_b) \cdot d_{si}\pi}{\kappa} \cdot \frac{I_s / a_a}{A_c} \cdot \frac{2m_s \cdot N_s \cdot I_s}{d_{si}\pi} = \frac{(l_{Fe} + l_b) \cdot d_{si}\pi}{\kappa} \cdot J_s \cdot A_s$$

(c: "conductor", A_c: conductor cross-section)



Current density per conductor: $J = I_c / A_c$ $\quad (I_c = I_s / a_a)$

- **DC machines:** z armature conductors: $P_{Cu} = R_a \cdot I_a^2 = z \cdot R_c \cdot I_c^2 = z \cdot R_c \cdot \left(\frac{I_a}{2a}\right)^2$

$$P_{Cu} = z \cdot \frac{l_{Fe} + l_b}{\kappa \cdot A_c} I_c^2 = \frac{(l_{Fe} + l_b) \cdot d_{si}\pi}{\kappa} \cdot \frac{I_c}{A_c} \cdot \frac{z \cdot I_c}{d_{si}\pi} = \frac{(l_{Fe} + l_b) \cdot d_{si}\pi}{\kappa} \cdot J \cdot A_r$$

- **General result** for AC <u>and</u> DC machines:

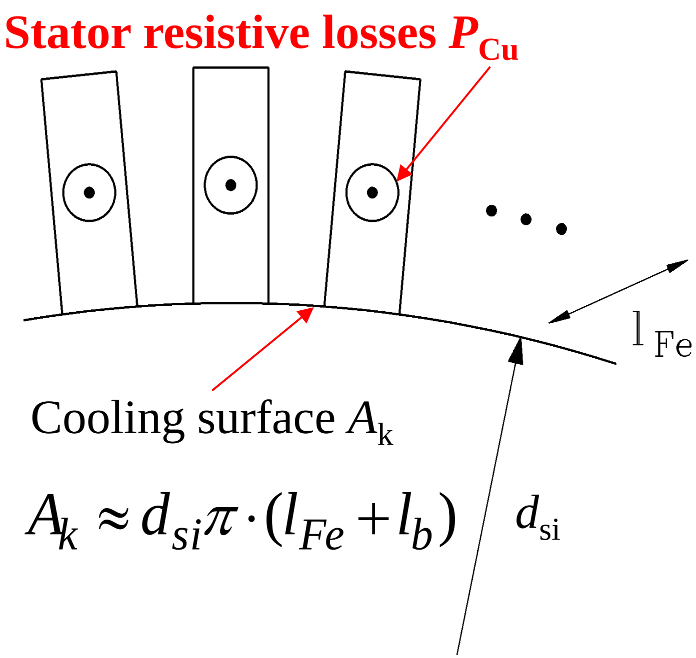
$$P_{Cu} = \frac{(l_{Fe} + l_b) \cdot d_{si}\pi}{\kappa} \cdot J \cdot A$$

1. Basic design rules for electrical machines

Thermal utilization $J_s \cdot A_s$

Heat transfer coefficient α_c, W/(m²K): $P_{Cu} = \alpha_c \cdot A_k \cdot \Delta\vartheta_s$

Steady state temperature rise: $\Delta\vartheta_s = \dfrac{P_{Cu}}{\alpha_c \cdot d_{si}\pi \cdot (l_{Fe} + l_b)} = \dfrac{1}{\alpha_c \cdot \kappa} \cdot J_s \cdot A_s$



$$A_k \approx d_{si}\pi \cdot (l_{Fe} + l_b)$$

$$\Delta\vartheta_s \sim J_s \cdot A_s$$

Result:

1) Temperature rise $\Delta\vartheta$ in armature winding for given machine torque M is determined by product of current density J and current loading A.

2) It may be reduced by

a) superior cooling (increased α_c) or

b) decreased losses (increased κ).

1. Basic design rules for electrical machines

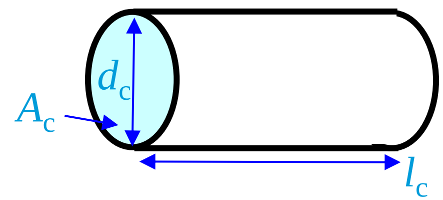
Thermal scaling effect $J(L)$

Increasing motor size: Increasing surface of conductors $d_c\pi \cdot l_c$

Example:

Surface $d_c\pi \cdot l_c$ for round conductor wire (wire diameter d_c & length l_c),

Heat transfer coefficient at conductor surface: α_c



Losses per conductor: (conductor volume: $V_c = A_c l_c \sim L^3$)

$$P_{Cu,c} = R_c I_c^2 = \frac{l_c}{\kappa_c A_c} \cdot I_c^2 = \frac{A_c l_c}{\kappa_c} \cdot J^2 \quad \Rightarrow \quad \frac{P_{Cu,c}}{V_c} = \frac{P_{Cu,c}}{A_c \cdot l_c} = \frac{J^2}{\kappa_c} \Rightarrow P_{Cu,c} \sim L^3 \cdot J^2$$

Temperature rise in conductor:

$$\Delta\vartheta = \frac{P_{Cu,c}}{\alpha_c \cdot d_c\pi \cdot l_c} = \frac{(d_c^2\pi/4)\cdot l_c}{\kappa_c \cdot \alpha_c \cdot d_c\pi \cdot l_c} \cdot J^2 = \frac{d_c}{4 \cdot \alpha_c \cdot \kappa_c} \cdot J^2 \quad \Rightarrow \quad \Delta\vartheta \sim L \cdot J^2 \ (d_c \sim L)$$

Result:

Admissible current density J for SAME temperature rise $\Delta\vartheta$ and cooling α_c lower for bigger machines: $J \sim 1/\sqrt{L}$

1. Basic design rules for electrical machines

Totally enclosed doubly-fed induction wind generator

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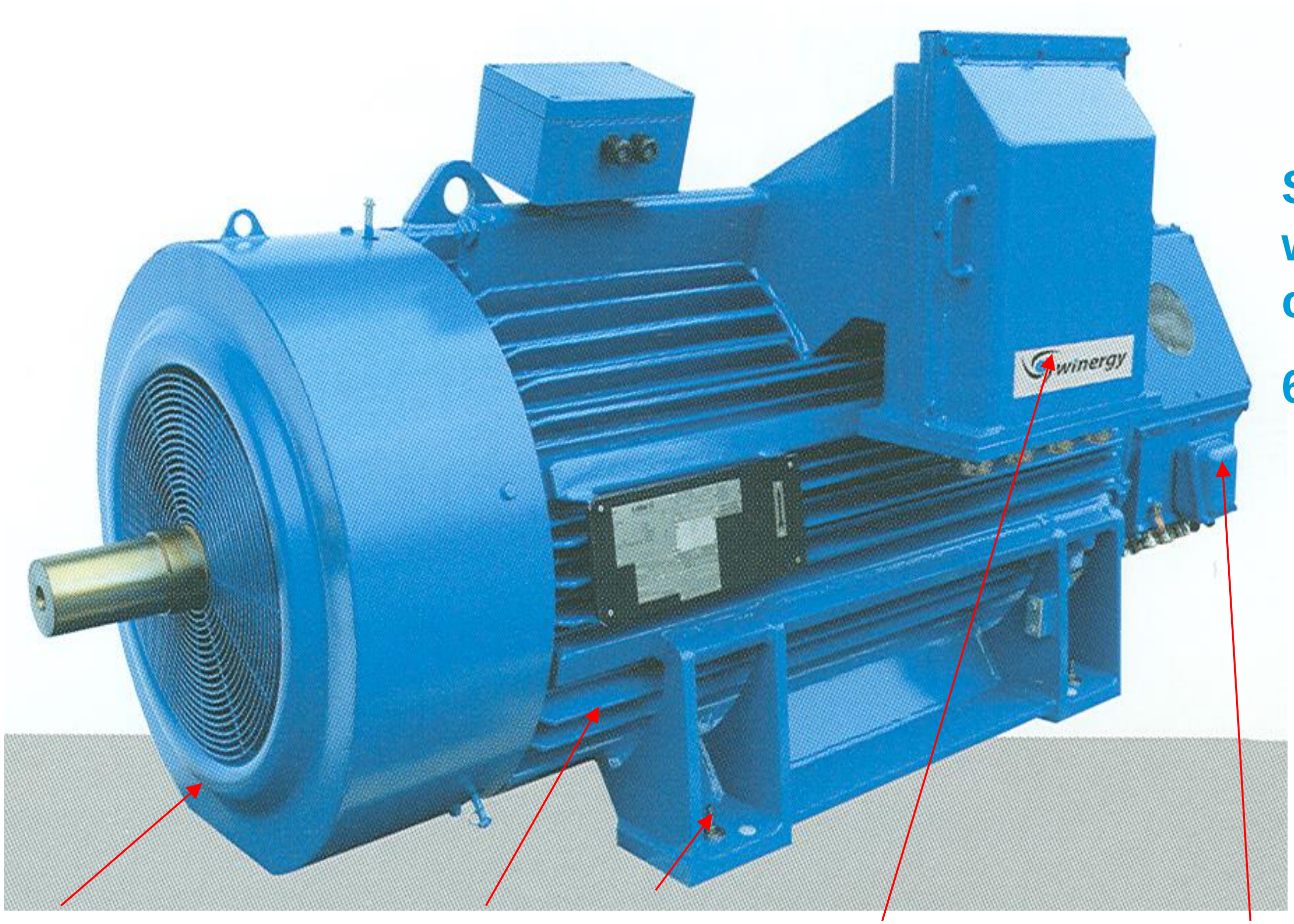

Surface air-cooled with iron-cast cooling fin housing

600 kW at 1155/min

Source:

Winergy

Germany

Fan hood
Shaft mounted fan inside

Cooling fins

Feet

Power terminal box

Slip ring terminal box

1. Basic design rules for electrical machines
Example: AC machines thermal utilization

Totally enclosed, surface cooled, for smaller rated power

a) winding temperature rise $\Delta\vartheta$ = 105 K:

rated power	P_N	kW	5	650
current loading	A_s	A/cm	280	430
current density	J_s	A/mm^2	7.6	5.0
thermal utilization	$A_s \cdot J_s$	A/cm·A/mm^2	2100	2150

b) winding temperature rise $\Delta\vartheta$ = 80 K:

rated power	P_N	kW	4	570
current loading	A_s	A/cm	240	380
current density	J_s	A/mm^2	6.6	4.4
thermal utilization	$A_s \cdot J_s$	A/cm·A/mm^2	1580	1670

Lower admissible winding temperature $\Delta\vartheta$ rise means lower thermal utilization !

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Example:

Thermal utilization at Class F (105K) and Class B (80K), totally enclosed, surface cooled

Rough estimate of power scaling with lower admissible winding temperature:

$$\frac{\Delta\vartheta_B}{\Delta\vartheta_F} = \frac{80}{105} = 0.76 \qquad \frac{(A\cdot J)_B}{(A\cdot J)_F} = \frac{1580}{2100} = 0.76$$

$$A \sim I_s,\ J \sim I_s \Rightarrow \sqrt{A\cdot J} \sim \sqrt{A^2} = A \Rightarrow \frac{A_B}{A_F} = \sqrt{0.76} = 0.87$$

$$\frac{P_{N,B}}{P_{N,F}} = \frac{(B_\delta A)_B}{(B_\delta A)_F} \approx \frac{A_B}{A_F} = \frac{240}{280} = 0.86 \approx 0.8 \quad \Rightarrow \quad \boxed{\frac{P_{N,B}}{P_{N,F}} = \frac{4}{5} = 0.8}$$

Lower admissible winding temperature rise $\Delta\vartheta$ means lower rated power P_N !

1. Basic design rules for electrical machines

Example: AC machines thermal utilization

Air-cooled open ventilated, four-pole induction machines,
temperature rise $\Delta\vartheta$ = 80 K (at ϑ_{amb} = 40 °C), rated speed ≈ 1450/min.

Rated apparent power S_N / kVA	100	1000	10000
$\hat{B}_{\delta,1}$ / T	1.0	1.0	1.0
A_s / A/cm	300	550	1000
C / kVA·min/m³	3.3	6.1	11.1
machine volume ~ L^3 / p.u.	1.0	5.4	29.7
machine size L / p.u.	1.0	1.75	3.1
$1/\sqrt{L}$	1.0	0.75	0.55
$J_s \sim 1/\sqrt{L}$ /A/mm²	6.8	5.1	3.7
$A_s \cdot J_s$ / (A/cm) · (A/mm²)	2040	2850	3700

Result:
Thermal utilization $A \cdot J$ rises with increased rated power S_N.
Increased current density J for big machines needs improved cooling α_c.

Summary:
Thermal utilization

- $I^2 \cdot R$ losses per cooling surface ~ thermal utilization $A \cdot J$
- Reduced current density J at bigger machine size L for same cooling system
- Thermal scaling laws
- Increased thermal utilization $A \cdot J$ leads to increased ESSON´s number $C \sim A \cdot B$
- Thermal Class (B, F, H, …) of insulation system determines thermal utilization

1. Basic design rules for rotating machines

1.1 Torque generation and internal power

1.2 Electromagnetic utilization

1.3 Thermal utilization

1.4 Overload capability of AC machines

1. Basic design rules for electrical machines

Rated stator impedance x_s of AC machines

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$$x_s = \frac{X_s}{Z_N} \quad Z_N = \frac{U_{sN}}{I_{sN}} \quad \text{or} \quad x_d = \frac{X_d}{Z_N} \qquad\qquad x_s = x_h + x_{s\sigma} \approx x_h$$

$$X_h = 2\pi f \cdot \mu_0 \cdot (N_s k_{ws1})^2 \cdot \frac{2m_s}{\pi^2 p} \cdot \frac{\tau_p l_e}{\delta}$$

($\mu_{\mathrm{Fe}} \to \infty$ and no slotting influence)

$$x_h = \frac{X_h I_{sN}}{U_{sN}} \approx \frac{X_h I_{sN}}{U_h} = \frac{2\pi f \cdot \mu_0 (N_s k_{ws1})^2 \cdot \frac{2m_s}{\pi^2 p} \frac{\tau_p l_e}{\delta} \cdot I_{sN}}{\sqrt{2}\pi f \cdot N_s k_{ws1} \cdot \frac{2}{\pi} \tau_p l_e \hat{B}_{\delta 1}} = \frac{\sqrt{2}\mu_0 k_{ws1}}{\pi} \cdot \frac{\frac{2m_s N_s I_{sN}}{2p\tau_p} \cdot \frac{\tau_p}{\delta}}{\hat{B}_{\delta 1}}$$

$$x_h \approx \frac{\sqrt{2}\mu_0 k_{ws1}}{\pi} \cdot \frac{A_s \cdot \frac{\tau_p}{\delta}}{\hat{B}_{\delta 1}} \sim \frac{A_s \cdot \frac{\tau_p}{\delta}}{\hat{B}_{\delta 1}}$$

$$\boxed{\frac{1}{x_s}, \frac{1}{x_d} \approx \frac{1}{x_h} \sim \frac{\hat{B}_{\delta 1}}{A_s} \cdot \frac{\delta}{\tau_p}}$$

1. Basic design rules for electrical machines

Per-unit pull-out power P_{p0}/S_N, P_b/S_N

Synchronous (cylindrical rotor) machines:

$$R_s = 0, X_d = X_q, \vartheta = \pm 90°: \quad \frac{P_{p0}}{S_N} = \pm \frac{3U_p U_{sN} \cdot \sin(90°)/X_d}{3U_{sN} I_{sN}} = \pm \frac{U_p}{U_{sN}} \cdot \frac{U_{sN}/I_{sN}}{X_d} = \pm \frac{u_p}{x_d}$$

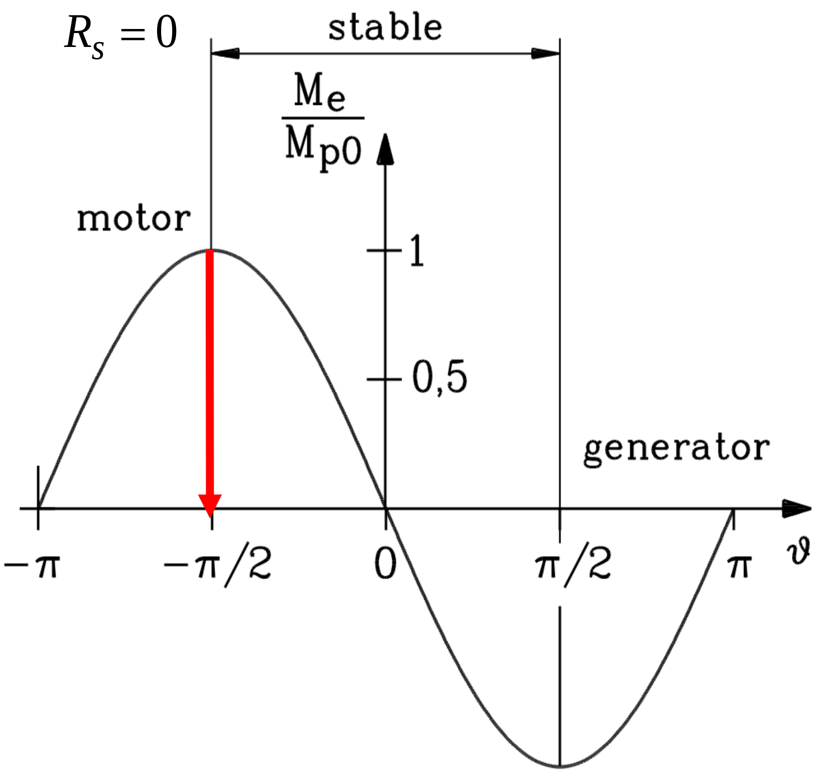
Induction machines:

$$R_s = 0, s = \pm s_b: \quad \frac{P_b}{S_N} \approx \frac{P_{\delta b}}{S_N} = \frac{\frac{\omega_{sN}}{p} \cdot M_b}{3U_{sN} I_{sN}} = \pm \frac{\frac{\omega_{sN}}{p} \cdot 3 \frac{p}{\omega_{sN}} \cdot U_{sN}^2 \frac{1-\sigma}{2\sigma X_s}}{3U_{sN} I_{sN}} = \pm \frac{U_{sN}}{I_{sN}} \cdot \frac{1-\sigma}{2\sigma \cdot X_s} = \pm \frac{1-\sigma}{2\sigma \cdot x_s}$$

1. Basic design rules for electrical machines

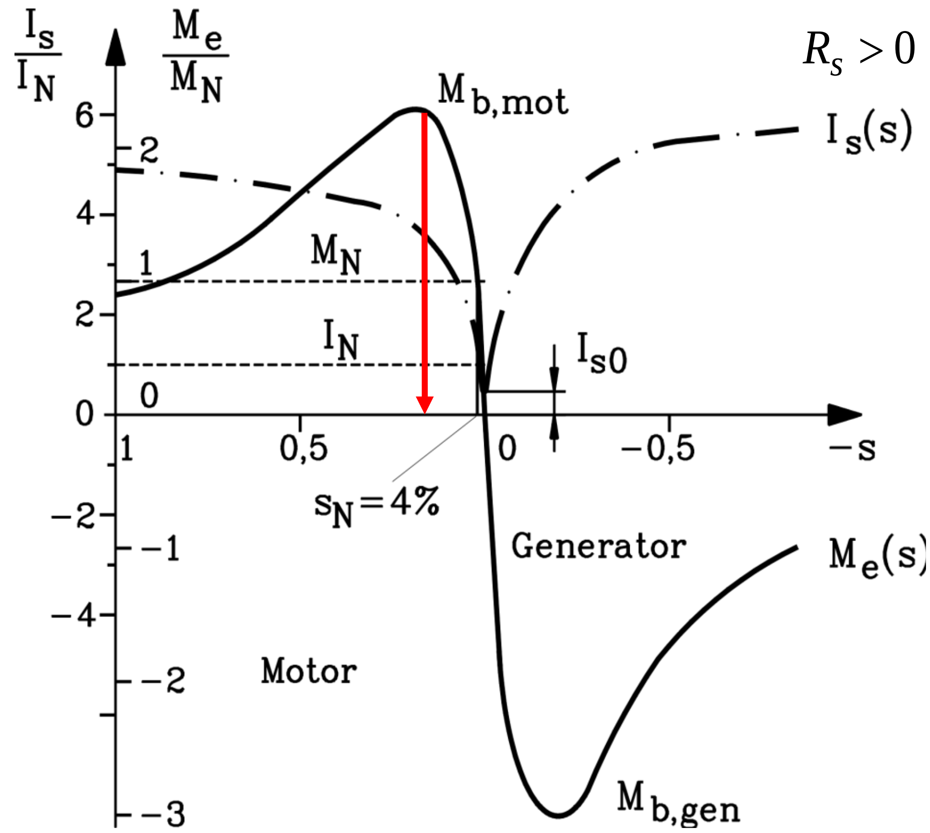
Overload capability of AC machines

Synchronous machines: Pull out torque



$$\frac{P_{p0}}{S_N} = \frac{u_p}{x_d} \approx \frac{1}{x_d} \sim \frac{\hat{B}_{\delta 1}}{A_s} \cdot \frac{\delta}{\tau_p}$$

Induction machines: Breakdown torque



$$\frac{P_b}{S_N} \approx \frac{1-\sigma}{2\sigma \cdot x_s} \sim \frac{1}{\sigma} \cdot \frac{1}{x_s} \sim \frac{1}{\sigma} \cdot \frac{\hat{B}_{\delta 1}}{A_s} \cdot \frac{\delta}{\tau_p}$$

Summary:
Overload capability of AC machines

- Overload capability = Maximum vs. rated torque M_{max}/M_N
- DC machine :
  Maximum torque simply limited by maximum armature current $I_{\mathrm{a,max}} = \mathrm{ca.}\ 2 \cdot I_{\mathrm{N}} \Rightarrow$
  $\Rightarrow M_{\mathrm{max}} = \mathrm{ca.}\ 1.8 \cdot M_{\mathrm{N}}$ (due to increased iron saturation of armature reaction)
- Voltage-fed AC machines:
  Maximum torque given by
  a) Induction machine (IM): Breakdown torque M_{b}
  or
  b) Synchronous machine (SM): Pull-out torque M_{p0}

Note:
  a) IM: Small stray reactance $X_\sigma \approx \sigma \cdot X_{\mathrm{s}}/2$
  b) SM: Small synchronous reactance X_d
  lead to increased maximum torque